ABOUT USING THE MINIMUM ENERGY DISSIPATION TO FIND THE STEADY-STATE FLOW DISTRIBUTION IN NETWORKS

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ABSTRACT

The incorrect analysis of the flow distribution through HVAC duct-networks has an economical and environmental impact. The existence of *negative* head loss coefficients at branched junctions poses a difficulty. However the dissipated energy is inherently positive and simplifies the solution. The paper explores the use of a variational method based on the minimization of the dissipated mechanical energy to find the actual steady-state flow distribution through a network. To our knowledge, Robert Niven was the first to propose/explore this idea but unfortunately discarded the method. The paper begins with a short explanation, afterwards extends previous outcomes [1] and ends with an example.

INTRODUCTION

The Minimum Entropy Production principle (MinEP) is an approximate variational characterization of steady states for thermodynamically open systems maintained out of equilibrium. Originally formulated within the framework of linear irreversible thermodynamics [2]. Along the years afterwards, many authors have tried to apply it to different fields of science. In concrete, much more recently, Robert Niven focused on its application to flow networks. He arrived close to our proposal in [3], but unfortunately discarded it as general method.

Our approach is very practical and stems from computation difficulties with traditional methods, mainly in HVAC return networks. The cause of the difficulties is the possible appearance of *negative* head loss coefficients in branched junctions (see Schmandt and Herwig [4] or [5]). This phenomena also occurs in other flow systems [6].

The operating point of a duct-network usually is found by intersecting the fan curve with the equivalent resistance curve of the network system. During iteration, while trying to find the latter, it may happen that the resistance at some section becomes negative [5] and therefore a special search algorithm must be devised. The method proposed was devoted, among other purposes, to simplify this.

NOMENCLATURE

$\dot{m}$	$[kg/s]$	Mass flow rate
w	[J/kg]	Work transfer per unit of mass
e_m	[J/kg]	Mechanical energy per unit of mass
P_T	[W]	Power
$\dot{S}$	[W/K]	Entropy rate
p	$[Pa]$	Pressure
T	[K]	Temperature
u	$[J/kg]$	Internal energy por unit of mass
$\bar{v}$	[m/s]	Mean velocity
z	[m]	Height
g	$[m/s^2]$	Gravitational acceleration
L	[m]	Length
D	[m]	Diameter
$nsect$	[-]	Number of network sections
sign()	[-]	Sign function

Special characters

α	[-]	Correction factor for the kinetic energy (laminar $= 2$, turbulent≈ 1)
φ	$[J/kg]$	Mechanical energy dissipation per unit of mass
$\widehat{\varphi}$	$[J/m^3]$	Mechanical energy dissipation per unit of volume
$\dot{\Phi}$	$[W]$	Dissipation rate
ψ_j	[-]	Volume flow ratio $\dot{V}_j/\dot{V}_T$ at section j
l_i	[-]	Volume flow ratio $\dot{V}_{l_i}/\dot{V}_T$ at loop i
ρ	$[kg/m^3]$	Density
ε	[-]	Roughness
ϕ	[-]	Roughness scale
$\circ$	[-]	Function composition
$\mathcal{D}$	[-]	Total derivative

Subscripts

sh	Shaft
gen	Generated
h	Hydraulic
fit	Fit value

THE ENERGY DISSIPATION

It is well known that the energy balance from point 1 to 2 in a conduit can be expressed per unit of mass, for an incompressible fluid, as:

$$(u_2-u_1)+\left(\frac{p_2}{\rho}-\frac{p_1}{\rho}\right)+\left(\frac{\alpha_2\cdot\bar{v}_2^2}{2}-\frac{\alpha_1\cdot\bar{v}_1^2}{2}\right)+(g\cdot z_2-g\cdot z_1)= \\ =-w_{sh,12}+q_{12} \quad (1)$$

where $w_{sh,12}$ and q_{12} are the shaft work and the heat transferred between both points, respectively. In this case the internal energy u changes due to the heat transfer and/or the irreversible conversion of mechanical energy into internal energy, which we call energy dissipation. Following Herwig et al. [7] equation (1) can be rewritten in a split form as:

$$\left(\frac{p_2}{\rho}-\frac{p_1}{\rho}\right)+\left(\frac{\alpha_2\cdot\bar{v}_2^2}{2}-\frac{\alpha_1\cdot\bar{v}_1^2}{2}\right)+(g\cdot z_2-g\cdot z_1)=\Delta e_{m,21}= \\ =-w_{sh,12}-\varphi_{12} \\ (u_2-u_1)=q_{12}+\varphi_{12} \tag{2}$$

where $e_m=p/\rho+\alpha\bar{v}^2/2+g\cdot z$. Equation (2), makes more explicit the previous statement by using the φ symbol for the *dissipated energy*. It is the extended Bernoulli's equation. Notice that $\varphi>0$. The rate of energy dissipation is given by:

$$\dot{\Phi}=\varphi\cdot\dot{m} \tag{3}$$

Without lossing generality, we assume that the fluid and the surroundings remain at the same temperature. Therefore φ and $\dot{\Phi}$ are related to the entropy generation as:

$$\begin{aligned}\varphi&=T\cdot\dot{S}_{gen}/\dot{m}\\ \dot{\Phi}&=T\cdot\dot{S}_{gen}=\dot{m}\cdot\varphi=\dot{V}\cdot\rho\cdot\varphi=\dot{V}\cdot\widehat{\varphi}\end{aligned} \tag{4}$$

Herwig et al. in [8] proposed to relate the dissipation inside straight conduits to the Darcy-Weisbach friction factor f_D as:

$$\varphi_{12}=\underbrace{f_D\cdot\frac{L_{12}}{D_h}}_{C}\cdot\frac{\bar{v}^2}{2}\quad,\quad\widehat{\varphi}_{12}=f_D\cdot\frac{L_{12}}{D_h}\cdot\frac{\rho\bar{v}^2}{2} \tag{5}$$

Finally, it can be shown (see [8] or [1]), that the pressure drop is related to the dissipation as:

$$\Delta p_{12}=p_1-p_2=\widehat{\varphi}_{12}=\varphi_{12}\cdot\rho \tag{6}$$

which corresponds, although written differently, to equation (3.12) in [9].

THE ENERGY DISSIPATION MINIMISATION

Tree-shaped networks

The methods employed in finding the steady-state of a network, make explicit use of the *energy* and *mass* conservation. On one hand, the energy must be conserved along any path joining two points. It is usually stated: "the *head* change (energy per unit of fluid weight) in a *closed loop* must be zero". On the other, at each node the ingoing mass flows must be equal to the outgoing ones.

Assuming that all the components of the network are passive (i.e. $w_{sh}=0$ see equation (2)) $\Delta e_{m,21}=-\varphi_{12}$. For the loop shown in figure (1), the energy conservation is written just as:

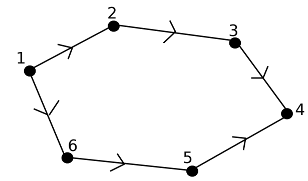

Figure 1. Example of a network loop.

$$0=\varphi_{12}+\varphi_{23}+\varphi_{34}-\{\varphi_{54}+\varphi_{65}+\varphi_{16}\} \tag{7}$$

The following question arises: how could eq.(7) be also obtained for each independent loop of a flow network but using the energy dissipation minimisation? The detailed answer for tree shaped-networks can be found in [1]. Here we summarise the main ideas.

The network energy dissipation rate can be computed as:

$$\dot{\Phi}=(\varphi_T\cdot\rho)\cdot\dot{V}_T=\widehat{\varphi}_j\cdot\dot{V}_T=\sum_{j=1}^{nsect}(\varphi_j\cdot\rho)\cdot\dot{V}_j=\sum_{j=1}^{nsect}\widehat{\varphi}_j\cdot\dot{V}_j \tag{8}$$

where $\dot{V}_T$ is a constant reference volume flow rate. Dividing equation (8) by $\dot{V}_T$ we get (note: assuming all $\dot{V}$ are positive):

$$F(\psi_1,\psi_2,\ldots,\psi_{nsect})=\widehat{\varphi}_T=\sum_{j=1}^{nsect}\widehat{\varphi}_j\cdot\psi_j \tag{9}$$

However, the ψ_j are not independent in equation (9). In what follows we use x_i for the independent variables. Usually one or more flow rates are imposed or fixed as a constraint. The problem is finding the steady-state distribution of this forced flow through the network. For instance, tree-shaped duct networks like the one in figure (2) were analysed in [1]. In this case $\dot{V}_T$ represents the design supply or return total flow rate. The independent variables are x_1 and x_2, since the mass conservation is included as a constraint $x_1+x_2+(x_3)=1$. One may take instead, another two variables: $l_3=x_1$ and $l_2=x_2+l_3$ (since $l_1=1$)[1]. For HVAC duct networks the x_i, defined so, are meaningful since are also the required supply or return flow rates at the grilles. In any case, eq.(9) can be written as:

$$(F\circ g)(x_1,x_2,\ldots)=F(\psi_1(x_1,x_2,\ldots),\ldots,\psi_{nsect}(x_1,x_2,\ldots)) \tag{10}$$

[1]Notice the linear relationship between the variables x and the loop ones l

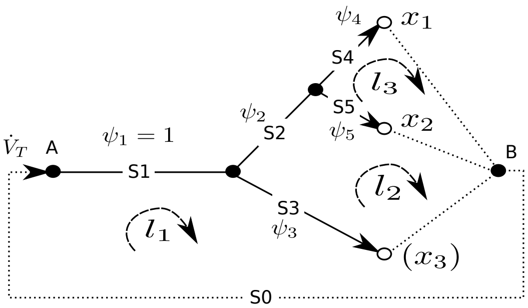

Figure 2. HVAC tree-shaped duct network. *A*: supply or return fan. *B*: room air. Dotted lines indicate *pseudo-loops*. White circles mean air grilles.

where g is the linear map $g : \vec{x} \to \vec{\psi}$. For instance, in figure (2) the map is given by:

$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \end{bmatrix} = \begin{bmatrix} 1\,1\,1 \\ 1\,1\,0 \\ 0\,0\,1 \\ 1\,0\,0 \\ 0\,1\,0 \end{bmatrix} \cdot \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) \tag{11}$$

The stationarity condition of the scalar equation (10) with respect $\vec{x}$, is written by using the chain rule as:

$$\mathcal{D}(F \circ g) = (\mathcal{D}F \circ g) \cdot \mathcal{D}g = \vec{0} \tag{12}$$

For the network of figure (2), $\mathcal{D}g$ can be easily obtained from equation (11) and it is shown in eq.(13).

$$\mathcal{D}g = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{13}$$

But, what about the other term $(\mathcal{D}F \circ g)$?. Next section gives a brief answer.

Energy minimisation and the fixed point problem[2]

It is clear that the information in g is just topological and contains the mass conservation, while F contains also, the way the net dissipates. Intuitively, one may see that each column of eq.(13) takes into account the energy balance for two independent loops (see figure (3))[2]. If by $\widehat{\varphi}_{Sj}$ we mean the dissipation within section j, then each component of equation (12) has the form:

$$\begin{aligned} \widehat{\varphi}_{S2} + \widehat{\varphi}_{S4} - \widehat{\varphi}_{S3} = 0 \\ \widehat{\varphi}_{S2} + \widehat{\varphi}_{S5} - \widehat{\varphi}_{S3} = 0 \end{aligned} \tag{14}$$

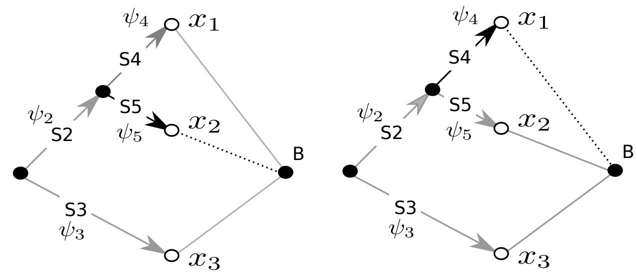

Figure 3. Loop energy balances eqs.(13),(14) as solid lines. Left: first column. Right: second column.

which are just the energy conservation equations (see eq.(7)). However, in order to get this result, *the dissipation function* $\widehat{\varphi}$ *must have a definite form*. In other words, the equations (14) are obtained from equation (12) if and only if the following is fulfilled:

$$\begin{aligned} &\widehat{\varphi}_j = \widehat{K}_j \cdot |\psi_j|^m \\ &\widehat{K}_j > 0, \text{ and constant} \\ &m \in \{\mathbb{R} - [-1, 0]\} \text{ and must be the same for all the network.} \end{aligned} \tag{15}$$

To our knowledge, Niven [3] was the first to deduce this. Unfortunately Niven discarded it as a generally applicable result. However in [1] we proved that, in fact, this is a more far-reaching outcome which, in our opinion, deserves a careful look. Moreover, if the previous conditions are fulfilled, then the stationary point of eq.(12) is a minimum[2].

Therefore equation (9) must have the form (note: $\psi_j > 0$):

$$F(\psi_1, \ldots, \psi_{nsect}) = (F \circ g)(x_1, \ldots, x_n) = \sum_{j=1}^{nsect} \widehat{K}_j \cdot \psi_j^{m+1} > 0 \tag{16}$$

After the maths: what is the physical interpretation of $\widehat{K}_j$?. To gain insight, let us take the *simplest* (and most studied) network component: a straight conduit. Figure (4) shows at the top, the well-known Moody's chart for the Darcy's friction factor f_D (extended for non-monotonical cases [10]). The f_D is represented versus the Reynolds number Re and the curves are parametrised by the relative roughness ε/D. The bottom curve in the same fig.(4), does represent the *same* Moody's chart at its top (see details in [11] by Afzal)(although in a non-logarithmic scale). The difference is that the x-axis is now what Afzal called *roughness Reynolds*, $Re_\phi = Re/\phi$. He called the bottom curve, the *universal relation* for the friction factor in pipes. The ϕ was named *roughness scale* and for completely smooth conduits, i.e. $\varepsilon = 0$, has value: $\phi = 1$. In short, ϕ indicates how rough is seen the conduit by the flow. The higher the Re the more rough the conduit. The universal curve can be adjusted very accurately, for a wide range of Re_ϕ, to a power-law (for instance, $Re \in [15 \cdot 10^3, 6 \cdot 10^5]$, $K_{fit} = 0.185$, $m_{fit} = -0.20$, $R_{squared} = 0.998$):

$$f_D = K_{fit} \cdot Re_\phi^{m_{fit}} = \frac{K_{fit}}{\phi^{m_{fit}}} \cdot Re^{m_{fit}} \tag{17}$$

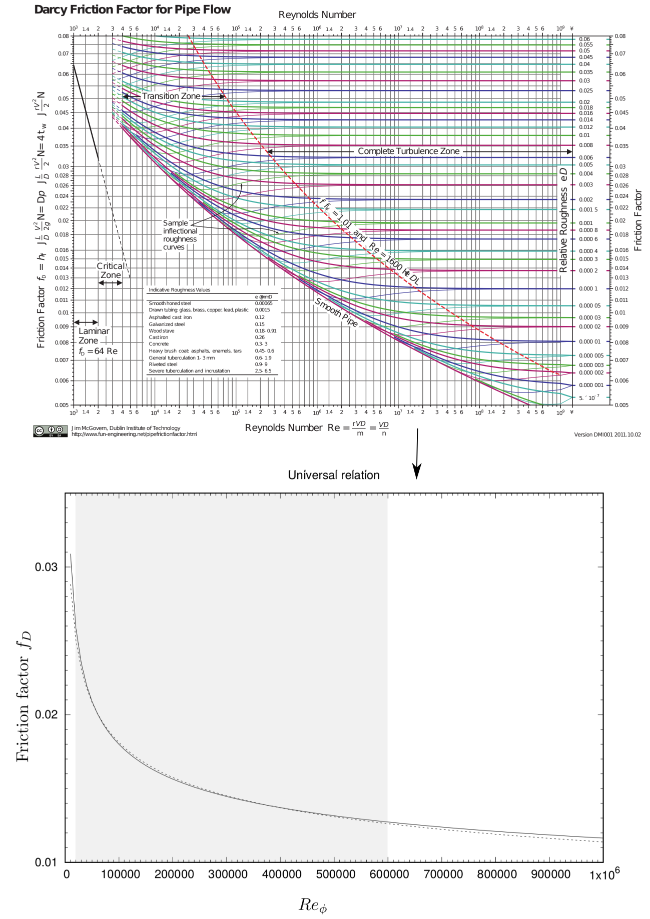

Figure 4. Darcy's friction factor f_D. Top: Modern Moody's chart by [10]. Bottom: Universal relation by Afzal [11].

Therefore equation (5) , assuming $\bar{v} \cdot (\pi D)/4 = \dot{V}$, turns into:

$$\widehat{\varphi}_{12} = \frac{K_{fit}}{\phi^{m_{fit}}} \cdot \left(\frac{4}{\nu \pi D}\right)^{m_{fit}} \cdot \left(\frac{8\rho L}{\pi^2 D^5}\right) \cdot \dot{V}_T^{(2+m_{fit})} \cdot |\psi|^{(2+m_{fit})} \quad (18)$$

Eq.(18) has the form $\widehat{\varphi} = \widehat{K} \cdot |\psi|^m$. Moreover, if the conduit is smooth then $\widehat{K}$ is also a constant since $\phi = 1$. This means that, for a certain fluid, in order to achieve a certain $\widehat{K}$ we may take the geometry as an *external* control $\{L, D\}$. Therefore for a flow system made up of completely smooth conduits, minimisation using eq.(18) would provide the flow distribution through the net. However if the conduit is rough, then $\widehat{K}$ has also another *internal* control ϕ which depends, in turn, on the flow rate. In practice, it is not possible to keep $\widehat{K}$ constant by using L, D and ϕ. Therefore the original minimisation problem turns into a fixed-point one as follows: 1) A vector $(\widehat{K}_1, \ldots, \widehat{K}_{nsect})$ is assumed. 2) Proceed with a minimisation of the dissipation function eq.(16). Its output is a flow distribution through the network. 3) If the previously assumed $\widehat{K}_j$ were not compatible or coherent with the obtained flow rates then the vector $(\widehat{K}_1, \ldots, \widehat{K}_{nsect})$ should be recomputed for the new flow rates. The process is repeated from step 2) until this vector does not change[2]. The solution to the fixed-point problem, corresponds to the minimum of the dissipation function where each dissipation point is computed using the assumed flow distribution at that point.

Another way to see the aforementioned, lies in changing the *point of view*. Notice that if the power-law fit eq.(17) to the Afzal's universal relation, was not used, then:

$$\widehat{K}_{12} = f_D \cdot \left(\frac{8\rho L}{\pi^2 D^5}\right) \cdot \dot{V}_T^2 \quad , \quad \widehat{\varphi}_{12} = \widehat{K}_{12} \cdot |\psi|^2 \qquad (19)$$

and in this case even for the case of smooth conduits, the energy dissipation minimisation would become a fixed value problem. However we know, by using the fitting, that the solution corresponds to the minimum of the dissipation function.

Therefore, the role of eq.(16) seems to be that of a measure. This measure can be calibrated or tuned by the exponent m. If the physical dissipation mechanism through all the network components, can be accurately tuned to a concrete value m^*, then the steady-state flow distribution follows from just a minimisation step. Otherwise the problem becomes a fixed-point one.

Next subsection generalises the previous result for general flow networks.

General flow networks

Instead of using a general formulation, we will use an example network like the one shown in figure (5). It has four independent loops.

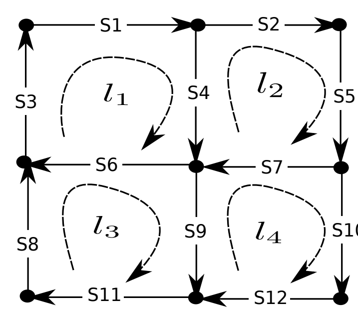

Figure 5. Example network.

Now they are going to be used as independent variables x instead of the outlet/inlet flows at the grilles of a duct network. The matrix M which maps column vector $\vec{l}$ to $\vec{\psi}$ is :

$$M^T = \begin{bmatrix} 1\,0\,1 & 1\,0 & 1 & 0\,0 & 0\,0\,0\,0 \\ 0\,1\,0 & -1\,1 & 0 & 1\,0 & 0\,0\,0\,0 \\ 0\,0\,0 & 0\,0 & -1 & 0\,1 & 1\,0\,1\,0 \\ 0\,0\,0 & 0\,0 & 0 & -1\,0 & -1\,1\,0\,1 \end{bmatrix} \qquad (20)$$

Let us assume that the flow in section $S6$ is imposed and equal to $\dot{V}_6 = \dot{V}_T$. That means that there exists a constraint between the

[2]For the details the reader is referred to [1]

volume flow rates of the loops: $\dot{V}_6 = \dot{V}_{l_1} - \dot{V}_{l_3}$. Divinding by $\dot{V}_T$ we get $1 = l_1 - l_3$. Therefore:

$$\begin{bmatrix} \psi_1 \\ \cdots \\ \psi_6 = 1 \\ \cdots \\ \psi_{12} \end{bmatrix} = M \cdot \begin{bmatrix} 1+l_3 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix} = M \cdot \left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} l_3 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix} \right) \tag{21}$$

Now equation (21) can be rewritten, in a reduced form, as:

$$\vec{\psi}_{(6)} = M_{(6)} \cdot \begin{bmatrix} l_2 \\ l_3 \\ l_4 \end{bmatrix} + A_{(6)} \tag{22}$$

By $\vec{b}_{(6)}$ we mean that component 6 of vector $\vec{b}$ has been removed. Similarly, in case of matrix M the subscript (6) means that the 6-row has been removed. Equation (22) explicitly has the form:

$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_7 \\ \psi_8 \\ \psi_9 \\ \psi_{10} \\ \psi_{11} \\ \psi_{12} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_2 \\ l_3 \\ l_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

The second column of the matrix in equation (23) is obtained by adding columns 1 and 3 of matrix M (eq. (20)). Taking $l_2 = x_1$, $l_3 = x_2$ and $l_4 = x_3$ as independent variables, equation (23) is now the linear map $g : \vec{x} \rightarrow \vec{\psi}$. Its derivative $\mathcal{D}g$, similarly, shows that now the minimisation of the dissipation (if conditions (15) are valid) is equivalent to applying the energy conservation to the three loops shown on the left in figure (6).

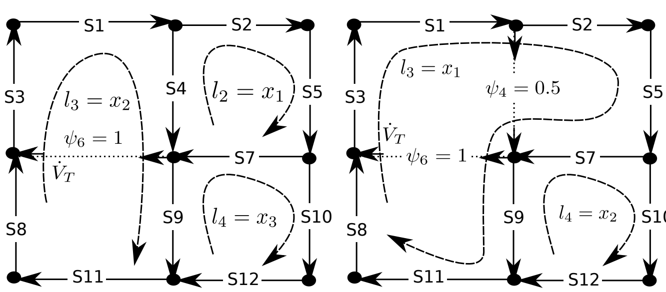

Figure 6. Left: figure (5) but $\dot{V}_6 = \dot{V}_T =$ constant. Right: additionally $\dot{V}_4 = 0.5\dot{V}_T$.

Let's extend this. It is not necessary to fix just one volume flow rate. For instance, if we additionally force that $\psi_4 = 0.5$, (i.e., $\dot{V}_4 = 0.5 \cdot \dot{V}_T = 0.5 \cdot \dot{V}_6$) then we must proceed as before. Now the new constraint is $l_3 - l_2 + 1 = \psi_4 = 0.5$ (see eq. (23)). By substituting $l_2 = l_3 + 0.5$ into equation (23), g becomes:

$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_5 \\ \psi_7 \\ \psi_8 \\ \psi_9 \\ \psi_{10} \\ \psi_{11} \\ \psi_{12} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_3 \\ l_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0.5 \\ 1 \\ 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{24}$$

On the right of figure (6) the two independent loops are shown. The dissipation minimisation with respect $x_1 = l_3$ and $x_2 = l_4$ is equivalent to the energy conservation applied to both loops. As a final remark, another map h is needed $h : \vec{\psi} \rightarrow (|\psi_1|, \dots)$. The general form for eq.(16) thus becomes eq.(25):

$$(F \circ h \circ g)(x_1, \dots, x_n) = \sum_{j=1}^{nsect} \widehat{K}_j \cdot |\psi_j|^{m+1} > 0 \tag{25}$$

The latter corrects automatically a sign change of any term in the energy conservation equations, when any flow sense is reversed. For instance the first row in eq.(14) would become:

$$\text{sign}(\psi_2)\widehat{\varphi}_{S2} + \text{sign}(\psi_4)\widehat{\varphi}_{S4} - \text{sign}(\psi_3)\widehat{\varphi}_{S3} = 0 \tag{26}$$

NUMERICAL EXAMPLE

Let us take the example on the right of figure (6). All sections are straight conduits with the same roughness. The fluid is air: $\nu = 1.49 \cdot 10^{-5} [m^2/s]$ $\rho = 1.21 [kg/m^3]$. The $\dot{V}_6$ is fixed so that $\bar{v}_6 = 7[m/s]$. Table (1) shows the sizes of the network shown in figure (7). Notice that the effect of the branched junctions on the flow is neglected.

Sj: Section index j	$L[m]$
1, 2, 3, 5, 8, 10, 11, 12	10.0
7, 9	14.2
Sj: Section index j	$D[m]$
1, 3, 8, 11	0.6
2, 5, 10, 12	0.4
7, 9	0.3

Table 1. Sizes of the duct network shown in figure (7)

The minimisation method employed was the Nelder-Mead's algorithm (with a tolerance in the search space of 10^{-9}). The results are shown in table (2) Notice that since the dissipation of a straight duct is symmetrical with respect to the flow sense, reversing the sense of the flows has the same solution. The top of table (2), shows the dissipation $\widehat{\varphi}_T$ and the total dissipated power P_T, under the imposed constraints. The results have been obtained as a fixed-point problem. The exponent m used in the dissipation function does not actually matter since the solution is always the same. Figure (8) shows a 3D plot of the specific dissipation $\widehat{\varphi}_T$ as a function of x_1 and x_2. It can be seen that the dissipation of the network has a minimum at the solution of the fixed-point problem.

$\varepsilon = 0.14$ $\quad x_1 = -0.4125468$, $x_2 = -0.1562525$ $\quad \widehat{\varphi}_T = 271.48[Pa]$, $P_T = 537.315[W]$

Section index j	ψ_j	Re
1	0.587453	163806.0
2	0.087453	36880.6
3	0.587453	165160.0
5	0.0874532	36880.6
7	0.243706	137034
8	-0.412547	115986
9	-0.256294	144112
10	-0.156252	65894.6
11	-0.412547	115986
12	-0.156252	65894.6

Table 2. Steady-state distribution of the flow for the air duct network of fig.(7)

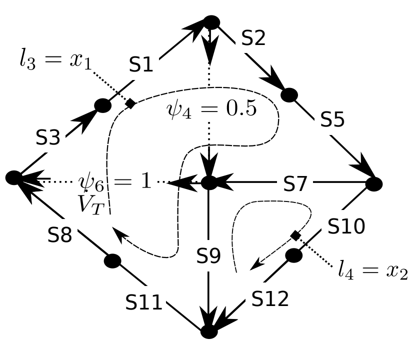

Figure 7. Network made up of straight ducts exclusively.

CONCLUSION

A previous result [1] for tree-shaped networks has been extended to general flow networks. The new ideas of the method have been introduced and, in our opinion, they could be of practical use. In the near future, it will be shown how the branched junctions, which triggered this research, fit into this MinEP method.

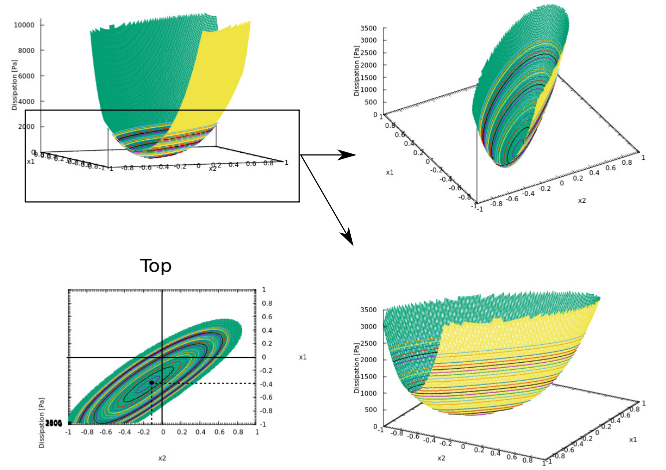

Figure 8. Example. Views of $\widehat{\varphi}_T$ as a function of x_1 and x_2.

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