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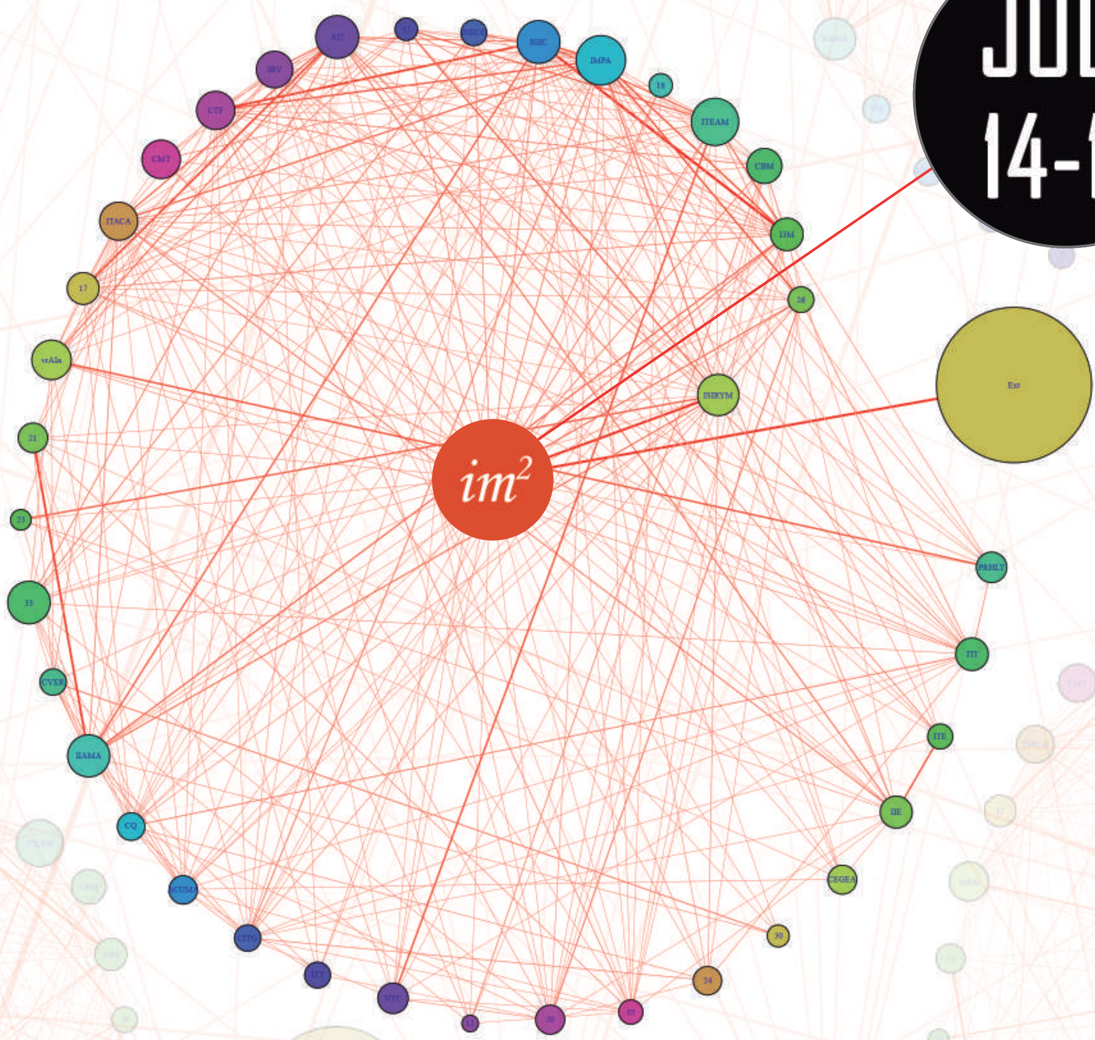

Edited by

Juan Ramón Torregrosa Juan Carlos Cortés Antonio Hervás Antoni Vidal Elena López-Navarro

UNIVERSITAT POLITÈCNICA DE VALÈNCIA

im2 Instituto Universitario de Matemática Multidisciplinar

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Edited by: I.U. de Matemàtica Multidisciplinar, Universitat Politècnica de València.
J.R. Torregrosa, J-C. Cortés, J. A. Hervás, A. Vidal-Ferràndiz and E. López-Navarro

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Contents

Multiobjective Optimization of Impulsive Orbital Trajectories

J. Tatay-Sangüesa[a], J.A. Moraño[b1], E. Vega-Fleitas[c] and S. Moll-Lopez[d]

([a]) TU Delft - Aerospace Engineering (Spaceflight)
([b]) Instituto Universitario de Matemàtica Multidisciplinar,
([c]) Universitat Politècnica de València,
([d]) Departamento de Matemática Aplicada
Universitat Politècnica de València
Camí de Vera s/n, Valencia, Spain.

1 Introduction

Space exploration is becoming one of the fastest developing research areas in recent times. However, the cost of sending a rocket with meaningful payload into interplanetary flights is so overwhelming that demands the reduction of the margin of error to the maximum.
Launch vehicles are responsible for lifting the satellites outside Earth's atmosphere and carrying them to orbit. After the separation of the spacecraft from the launch vehicle, there are some operations that the satellite must be able to perform, such as orbit transfers, orbit maintenance, attitude control and de-orbiting maneuvers. According to the literature [3], orbit transfer is the operation that consumes up to 70% of the total Δv needed to perform the mission. Therefore, the minimization of orbit transfer propellant requirements would have very positive consequences in satellite deployment missions. Nonetheless, other variables such as the time of flight (TOF) taken to complete the transfer, should be considered when designing a space mission.
Classical orbit transfers are only able to provide optimal results at very specific conditions. Hence, the aim of this paper is to develop and implement a program able to provide optimal solutions, in terms of propellant and time of flight, for any orbit transfer in the near-Earth region.

2 Methods

2.1 Optimal Orbit Transfer Problem

The optimal orbit transfer problem can be stated as follows: given an initial orbit, described by the keplerian elements $a_0, e_0, i_0, \Omega_0, \omega_0$ and a final orbit, described by $a_f, e_f, i_f, \Omega_f, \omega_f$, find the region of the optimal solutions that minimize total Δv and TOF. As we are using the ε-constraint approach [4], Δv will be treated as the main objective, whereas TOF will be the constrained one. The transfer problem will be solved using Lambert arcs [5, 6] in order to reduce the dimension of the optimization problem, and employing the iterative method introduced in [7] to ensure a feasible solution, even if not the optimal one.

[1]jomofer@mat.upv.es

In addition, the problem is solved applying up to four impulses. In the two-impulse case, no additional variables are needed. For the case of more impulses, the intermediate impulse locations are not restricted, therefore four more variables are needed for each new impulse. Hence, the impulse locations and the transfer time between them are defined as variables. A more detailed diagram can be seen in Figure 1.

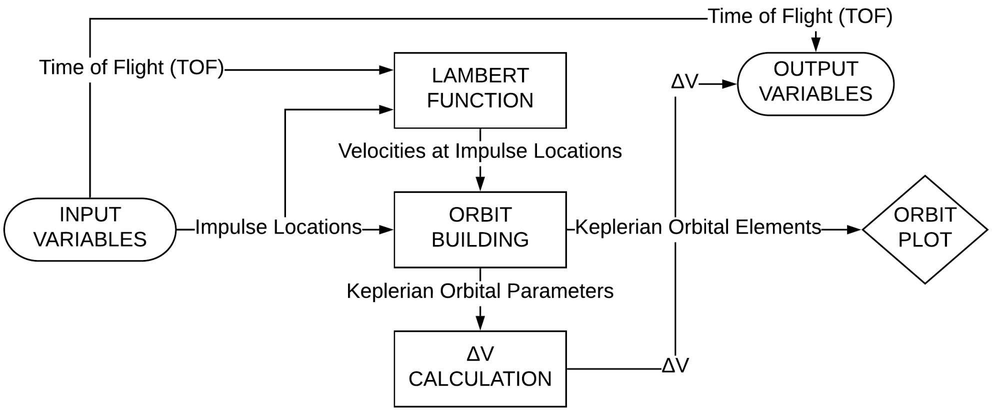

Figure 1: Objective Function Diagram

2.2 Genetic Algorithm

The genetic algorithm (GA) tries different variable combinations until the stopping criterion decides that the optimum has been found. This stopping criterion is the Bit String Affinity (BSA), which compares the individual's chromosomes and stops the search when they are considerably similar (over 90%).
With this method, the solution space must be bounded and its accuracy comes determined by the number of bits allocated to each variable. In this sense, increasing the solution space or the number of bits will increase the algorithm accuracy, but increasing also the computational cost. The r bound distance for the impulse location variables depends on the TOF ε-constraint limit. In order to minimize the solution space limitation, the maximum distance away from the Earth has been taken equal to twice the semi-major axis of such orbit (considering it to be very eccentric). A 10% extra was added to avoid being too restrictive. Following the literature recommendations [8], the population size is four times the total number of bits, whereas the mutation probability is obtained in terms of the number of bits. Since the genetic algorithm cannot handle constraints, a penalty needs to be added to the objective function. If any of the constraints is broken, a quantity proportional to the constraint violation is added to the Δv result, artificially worsening the result and forcing the algorithm to search for other solutions.
Finally, due to the random nature of this method, the algorithm has been executed multiple times to increase the possibilities to find the optimal point. In this paper, up to 25 consecutive runs of the genetic algorithm will be performed. The genetic algorithm MATLAB program used was obtained from the literature [8].

3 Results

In this paper, a transfer from a Molniya to a GEO orbit will be optimized. The orbital parameters that define each of the orbits are shown in Table 1.

Table 1: Initial and Final Orbit Parameters

Initial Orbit (Molniya)		**Final Orbit (GEO)**	
Parameter	**Value**	**Parameter**	**Value**
a_0	26600 km	a_f	42164 km
e_0	0.74	e_f	0
i_0	63.4º	i_f	0º
Ω_0	0º	Ω_f	0º
ω_0	280º	ω_f	0º

In order to find the Pareto frontier plot, the TOF ε-constraint limits need to be discussed. To obtain a reference value from where to choose the limits, a TOF-unconstrained optimization run was performed. This was achieved by establishing a very large TOF-limit so the overall minimum Δv corresponding time of flight for the two-impulse case was computed. The unconstrained optimization results gave 10.7h as the reference time of flight. Hence, it was decided to analyze the following time of flight values to obtain a representation of the Pareto frontier: 500h, 50h, 25h, 15h, 12h, 10h, 8h, 5h, 3h, 1h.

3.1 Pareto Frontier

The Pareto frontier can be found in Figure 2, where the different regions of the plot correspond to different number of impulses.

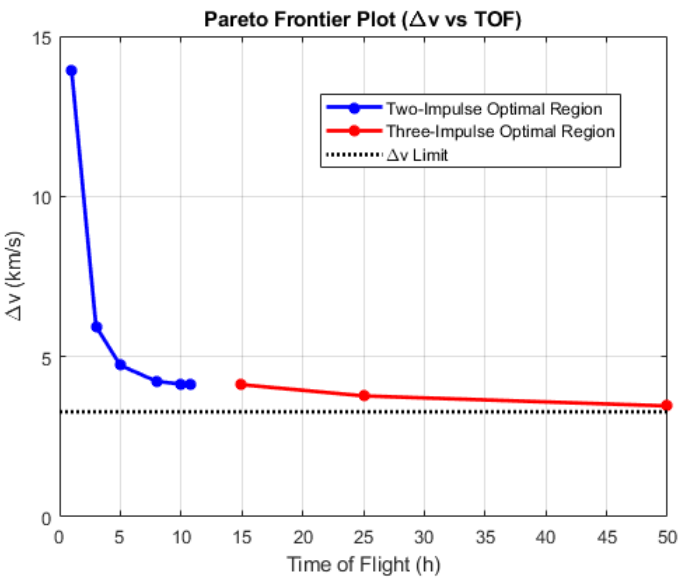

Figure 2: Pareto Frontier Results

The blank area between the two and three-impulse optimal regions is due to the existence of a non-optimal region: the two-impulse region starts increasing from the TOF = 10.7h point and the three-impulse does not improve Δv value until close to the TOF = 15h point. The four-impulse case did not produce any of the optimal points, and thus, is not represented.

3.2 Transfer Orbits

It could be seen on Figure 2 that the TOF = 500h, represented by the dotted line, did not produce a meaningful improvement in the propellant consumption. Hence, we have chosen two different solutions to contemplate different cases regarding the mission priorities.

Time Of Flight = 10h

This option represents the best compromise between propellant and TOF. It requires a total Δv of 4.14 km/s, which is a high value due to the plane change requirements. It is also performed using only two impulses, which increases the mission reliability. The transfer data needed is shown in Tables 2 and 3.A plot of the transfer orbit can be seen on Figure 3.

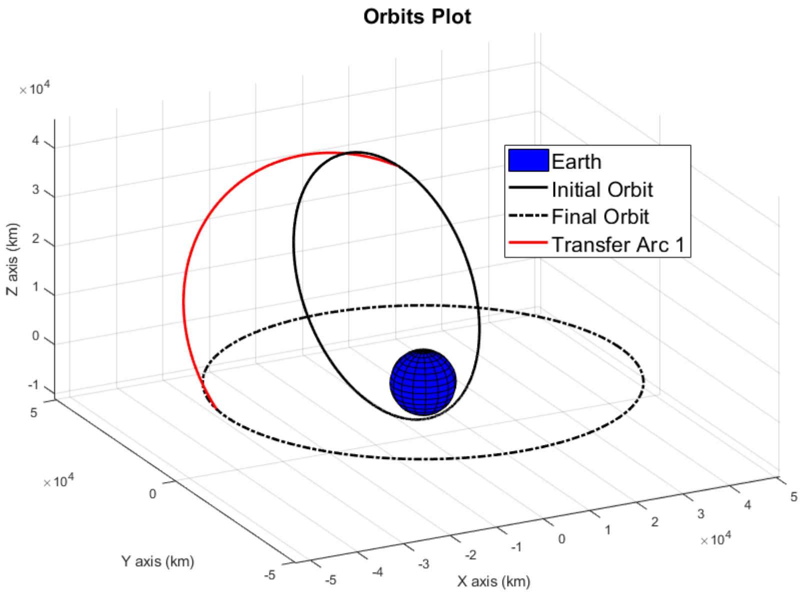

Figure 3: TOF = 10h Orbit Plot

Table 2: TOF = 10h Transfer Orbit Data

Impulse Location		Impulse Coordinates		TOF
1 (º)	**2 (º)**	**1 (km/s)**	**2 (km/s)**	**1 (h)**
164.1	173.3	-0.8857	-1.4199	10.0
		0.3094	-1.7637	
		0.0191	2.2664	

Time Of Flight = 50h

This case represents a situation in which TOF is not relevant for the mission, but as much propellant as possible needs to be saved, employing a three-impulse orbit. It can be seen in Figure 4 how the transfer arc separates from Earth and performs the maneuver near the periapsis, where the velocities are smaller. Therefore, the Δv values are also reduced, as seen in Table 4. The keplerian elements of the transfer arc can be seen in Table 5.

Table 3: TOF = 10h Transfer Orbit Parameters

Parameter	Value
a_t	36304.0 km
e_t	0.4520
i_t	62.95º
Ω_t	-6.693º
ω_t	314.16º
θ_d^*	133.0º
θ_a^*	225.8º

Table 4: TOF = 50h Transfer Orbit Data

Impulse Location			**Impulse Coordinates (km/s)**			**TOF (h)**	
1 (º)	**2 (km)**	**3 (º)**	**1**	**2**	**3**	**1**	**2**
152.2	-102618	332.5	-1.1764	-0.3633	-0.3095	28.5	21.5
	15433		0.7686	-0.9088	-0.5313		
	-1389.7		0.4533	0.9299	-0.1523		

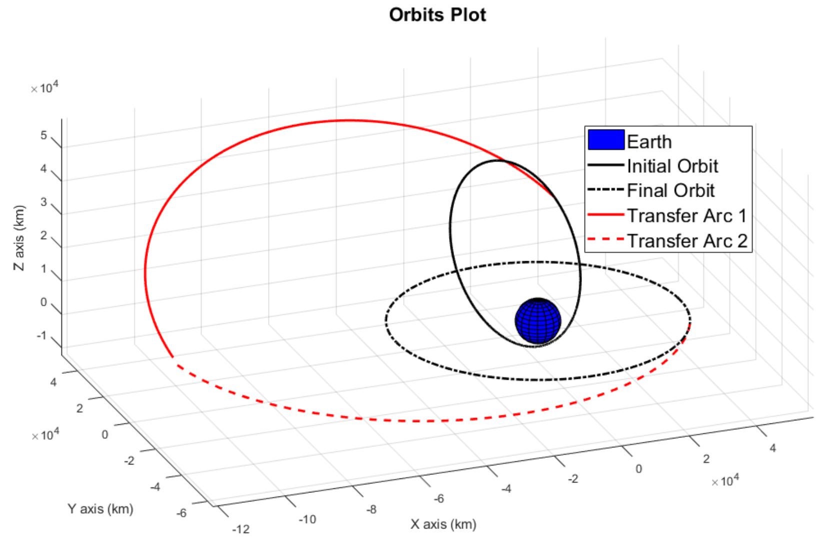

Figure 4: TOF = 50h Orbit Plot

4 Conclusions

The optimizer presented is more versatile and more accurate than the ones similarly implemented in [9–11], that either are very problem-specific, or are limited to the co-planar or the two-impulse case. It has proven to be able to obtain the optimal result with a relative error at least one order of magnitude lower than previous works.
Moreover, the computational cost is acceptable: at maximum 5 minutes per time of flight value, and it can be reduced to under a minute in the two-impulse case. This includes all the repetitive

Table 5: TOF = 50h Transfer Orbit Parameters

Transfer Arc 1		**Transfer Arc 2**	
Parameter	**Value**	**Parameter**	**Value**
a_{t1}	66075.9 km	a_{t2}	75555.9 km
e_{t1}	0.6900	e_{t2}	0.4420
i_{t1}	61.16º	i_{t2}	2.36º
Ω_{t1}	-8.975º	Ω_{t2}	-27.476º
ω_{t1}	-14.14º	ω_{t2}	-1.49º
$\theta^*_{a_{t1}}$	90.5º	$\theta^*_{a_{t2}}$	200.4º
$\theta^*_{d_{t1}}$	195.1º	$\theta^*_{d_{t2}}$	1.49º

calculations needed to ensure the genetic algorithm convergence. Furthermore, this problem settings allow the program to always come up with a solution, that even if not the optimal one, it will be a close one. The Pareto frontier obtained is a useful tool in the mission design as it visually represents the compromises between the different options.

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