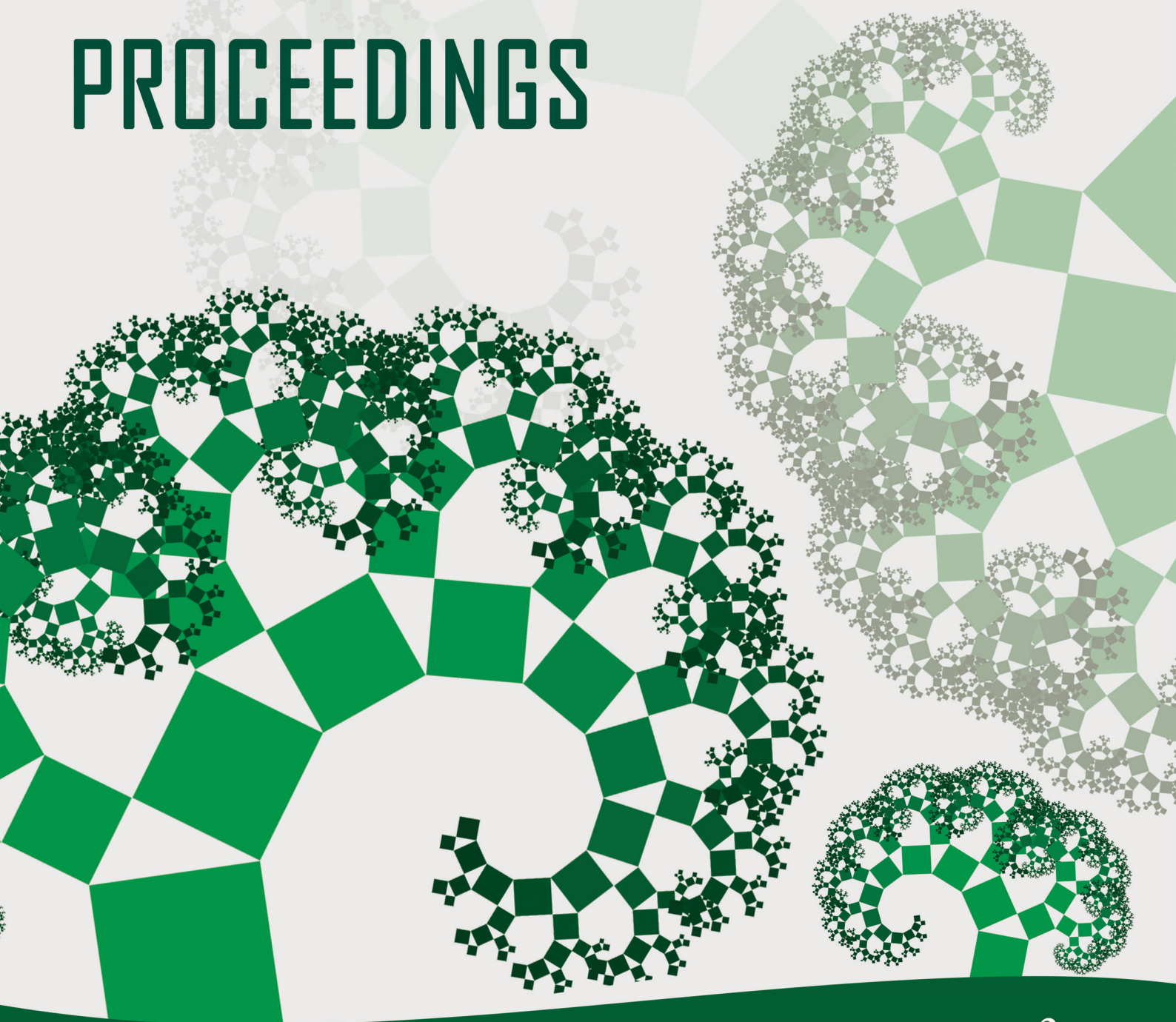


MODELLING FOR ENGINEERING & HUMAN BEHAVIOUR 2022 PROCEEDINGS



Edited by

Juan Ramón Torregrosa
Juan Carlos Cortés
Antonio Hervás

Antoni Vidal
Elena López-Navarro

im²

Instituto Universitario
de Matemática Multidisciplinar



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J.R. Torregrosa, J-C. Cortés, A. Hervás, A. Vidal-Ferràndiz and E. López-Navarro

im²

Instituto Universitario
de Matemática Multidisciplinar

Contents

Developable surface patches bounded by NURBS curves <i>L. Fernández-Jambrina</i>	1
Impact of antibiotic consumption on the dynamic evolution of antibiotic resistance: the colistin-resistant <i>Acinetobacter baumannii</i> case <i>Carlos Andreu-Villarraig, Juan-Carlos Cortés, Rafael-Jacinto Villanueva</i>	7
Relative research contributions towards the application and materialization of Wenner four-point method on concrete curing <i>L. Andrés, J. H. Alcañiz, T. P. Real, P. Suárez</i>	12
Application of mathematical models and multicriteria analysis to establish an optimized priorization for the maintenance of bridges in a network <i>L. Andrés, F. Ribes, E. Fernández, J. Maldonado</i>	18
Probabilistic analysis of scalar random differential equations with state-dependent impulsive terms via probability density functions <i>V. Bevia, J.C. Cortés, M. Jornet and R.J. Villanueva</i>	24
Combined and reduced combined matrices <i>R. Bru, M.T. Gassó and M. Santana</i>	30
Random Fractional Hermite Differential Equation: A full study un mean square sense <i>C. Burgos, T. Caraballo, J. C. Cortés and R. J. Villanueva</i>	36
Mathematical modelling of frailty and dependency in basic activities of daily living in general population aged 70 years <i>S. Camacho Torregrosa, C. Santamaría Navarro and X. Albert Ros</i>	41
Jordan structures of an upper block echelon matrix <i>B. Cantó, R. Cantó and A.M. Urbano</i>	47
Modeling inflammation in wound healing <i>A. Patrick and B. Chen-Charpentier</i>	53
The geometry behind PageRank rankings <i>Gonzalo Contreras-Aso, Regino Criado and Miguel Romance</i>	59
Surveillance model of the evolution of the plant mass affected by <i>Xylella fastidiosa</i> in Alicante (Spain) <i>José Juan Cortés Plana, María Teresa Signes Pont, Joan Boters Pitarch and Higinio Mora Mora</i>	64
New tools for linguistic pattern analysis and specialized text translation: hypergraphs and its derivatives <i>A. Criado-Alonso, D. Aleja, M. Romance and R. Criado</i>	70

On an accurate method to compute the matrix logarithm <i>E. Defez, J.J. Ibáñez, J. M. Alonso and J.R. Herráiz</i>	76
A distribution rule for allocation problems with priority agents using least-squares method <i>J.C. Macías Ponce, A.E. Giles Flores, S.E. Delgadillo Alemán, R.A. Kú Carrillo and L.J.R. Esparza</i>	85
A reaction-diffusion equation to model the population of <i>Candida Auris</i> in an Intensive Care Unit <i>C. Pérez-Diukina, J.-C. Cortés López and R.J. Villanueva Micó</i>	91
Relative research contribution towards railways superstructure quality determination from the vehicles inertial response <i>E. Gómez, J. H. Alcañiz, G. Alandí and F. E. Arriaga</i>	97
Computational Tools in Cosmology <i>Màrius Josep Fullana i Alfonso and Josep Vicent Arnau i Córdoba</i>	102
Dynamical analysis of a family of Traub-type iterative methods for solving nonlinear problems <i>F.I. Chicharro, A. Cordero, N. Garrido and J.R. Torregrosa</i>	107
Multidimensional extension of conformable fractional iterative methods for solving nonlinear problems <i>Giro Candelario, Alicia Cordero, Juan R. Torregrosa and María P. Vassileva</i>	113
Application of Data Envelopment Analysis to the evaluation of biotechnological companies <i>B. Latorre-Scilingo, S. González-de-Julián and I. Barrachina-Martínez</i>	119
An algorithm for solving Feedback Nash stochastic differential games with an application to the Psychology of love <i>Jorge Herrera de la Cruz and José-Manuel Rey</i>	125
Detection of border communities using convolution techniques <i>José Miguel Montañana, Antonio Hervás, Samuel Morillas and Alejandro Méndez</i>	133
Optimizing rehabilitation alternatives for large intermittent water distribution systems <i>Bruno Brentan, Silvia Carpitella, Ariele Zanfei, Rui Gabriel Souza, Andrea Menapace, Gustavo Meirelles and Joaquín Izquierdo</i>	139
Performance analysis of the constructive optimization of railway stiffness transition zones by means of vibration studies <i>M. Labrado, J. del Pozo, R. Cabezas and A. Arias</i>	143
Can any side effects be detected as a result of the COVID-19 pandemic? A study based on social media posts from a Spanish Northwestern-region <i>A. Larrañaga, G. Vilar, J. Martínez and I. Ocarranza</i>	149
Probabilistic analysis of a cantilever beam with load modelled via Brownian motion <i>J.-C. Cortés, E. López-Navarro, J.I. Real Herráiz, J.-V. Romero and M.-D. Roselló</i> ...	155
Pivoting in ISM factorizations <i>J. Mas and J. Marín</i>	160
Relative research contributions towards the characterization of scour in bridge piers based on operational modal analysis techniques <i>S. Mateo, J. H. Alcañiz, J. I. Real, E. A. Colomer</i>	166

Short-term happiness dynamics as a consequence of an alcohol or caffeine intake <i>Salvador Amigó, Antonio Caselles, Joan C. Micó and David Soler</i>	172
First Order Hamiltonian Systems <i>Joan C. Micó</i>	177
Dynamical analysis of a new sixth-order parametric family for solving nonlinear systems of equations <i>Marlon Moscoso-Martínez, Alicia Cordero, Juan R. Torregrosa and F. I. Chicharro</i>	185
Higher order numerical methods for addressing an embedded steel constitutive model <i>J.J. Padilla, A. Cordero^b, A.M. Hernández-Díaz and J.R. Torregrosa</i>	191
A generalization of subdirect sums of matrices <i>F. Pedroche</i>	198
A new iterative inverse display model <i>M.J. Pérez-Peñalver, S.-W.Lee, C. Jordán, E. Sanabria-Codesal and S. Morillas</i>	202
Application of polynomial algebras to non-linear equation solvers <i>J. Canela and D. Pérez-Palau</i>	208
A Linear Quadratic Tracking Problem for Impulsive Controlled Stochastic Systems. The Infinite Horizon Time Case <i>V. Drġan, I-L. Popa and I.Ivanov</i>	213
A new model for the spread of cyber-epidemics <i>E. Primo, D. Aleja, G. Contreras-Aso, K. Alfaro-Bittner, M. Romance and R. Criado</i>	218
Bifurcation analysis in dryland vegetation models with discrete and distributed delays <i>I. Medjahdi, F.Z. Lachachi, M.A. Castro and F. Rodríguez</i>	224
Relative research contribution towards railways superstructure quality determination from the vehicles inertial response <i>J. R. Sánchez, F. J. Veá, G. Muínelo and G. Mateo</i>	230
Probabilistic analysis of the pseudo- n order adsorption kinetic model <i>C. Andreu-Vilarroig, J.-C. Cortés, A. Navarro-Quiles and S.-M. Sferle</i>	236
Approximating Fixed Points by New and Fast Iterative Schemes <i>Puneet Sharma, Vinay Kanwar, Ramandeep Behl and Mithil Rajput</i>	242
Higher-order multiplicative derivative iterative scheme to solve the nonlinear problems <i>G. Singh, S. Bhalla and R. Behl</i>	247
A Seventh Order Steffensen type Iterative Method for Solving Systems of Nonlinear Equations and Applications <i>Sana Sultana and Fiza Zafar</i>	251
Modifying Kurchatov's method to find multiple roots <i>A. Cordero, N. Garrido, J.R. Torregrosa and P. Triguero-Navarro</i>	257
The Relativistic Anharmonic Oscillator within a Double-Well Potential within a Double-Well Potential <i>Michael M. Tung and Frederic Rapp</i>	263

Real-valued preconditioners for complex linear systems arising from the nuclear reactor noise equation <i>A. Vidal-Ferràndiz, A. Carreño, D. Ginestar and G. Verdú</i>	268
Mathematical model for heat transfer and stabilization of LED lamps for measurements in a laboratory <i>Carlos Velásquez, M. Ángeles Castro, Francisco Rodríguez and Francisco Espín</i>	274
A Seventh Order Family of Jarratt Type Iterative Methods for Solving Systems of Nonlinear Equations and Applications <i>Saima Yaseen and Fiza Zafar</i>	279
A Seventh Order Jarratt type Iterative Method for Solving Systems of Nonlinear Equations and Applications <i>Fiza Zafar, Alicia Cordero, Husna Maryam and Juan R. Torregrosa</i>	285
A Multiplicative calculus approach to solve nonlinear equations <i>S. Bhalla and R. Behl</i>	291
Basins of Convergence in a Modified CR3BP <i>D. Villalibre, A. Herrero, J.A. Morano and S. Moll</i>	294
SPECIAL SESSION: STUDENT'S PROJECTS	301
Predicting PDA battery health using machine learning methods <i>Alberto Bono Monreal, Irene Cánovas Vidal, Eduardo Gómez Fernández, Rubén Marco Cabanes, Andrea Pérez López, Alejandra Sánchez Torres and Marta Valero Buj</i>	303
Solving a Crime with Graph Theory <i>Elsa Blasco Novell and Lucía López Ribera</i>	313
Harry Potter and the relics of the graphs <i>D. Vañó Fernández and R. Fornas Sáez</i>	318
Evaluating the sudden change in flight cancellations in Paris during November 2015 <i>D. Romero, C. Gallego, R. Gironés, J. Grau, L. Lillo and A. Losa</i>	325

Probabilistic analysis of scalar random differential equations with state-dependent impulsive terms via probability density functions

V. Bevia ^{b,1} J. C. Cortés ^b M. Jornet [#] and R.J. Villanueva ^b

(b) Instituto de Matemática Multidisciplinar (Imm),
Universitat Politècnica de València (UPV)
Camí de Vera s/n, València, Spain.

(#) Departament Matemàtiques,
Universitat de València (UV),
Burjassot (València), Spain.

1 Introduction

Most phenomena observed in nature can be described as a function changing smoothly over time. However, sometimes the system suddenly changes its state, requiring special mathematical tools to model its dynamics correctly. This is common in biology, medicine, or engineering when determining the effectiveness of specific impulsive-type control strategies. In ecology, these are known as harvesting models [5, 9, 10].

In this contribution, we will study, from a probabilistic standpoint, the following random Initial Value Problem (IVP):

$$\begin{cases} \frac{dX(t, \omega)}{dt} = g(X(t, \omega), t, \mathbf{A}(\omega)) - \sum_{k=1}^N \Gamma_k(\omega) \delta(t - t_k) X(t, \omega), & t > t_0, \\ X(t_0, \omega) = X_0(\omega). \end{cases} \quad (1)$$

Here t_0 denotes a real number; $X_0(\omega)$, $\mathbf{A}(\omega) := (A_1(\omega), \dots, A_m(\omega))$ and $\{\Gamma_k(\omega)\}_{k=1}^N$ are assumed to be independent absolutely continuous random variables defined on the Hilbert space $L^2(\Omega, \mathbb{R})$, whose elements are real-valued random variables with finite variance and $(\Omega, \mathcal{F}, \mathbb{P})$ denotes a complete probability space [6]; $\delta(t - t_k)$ stands for the Dirac delta function [4] acting at the prefixed time instants $t = t_k$, $k = 1, \dots, N$ and g is known as the (*scalar*) *field function* satisfying certain conditions that will be specified later. Finally, $X(t, \omega)$ denotes the solution of the random IVP (1)

¹vibeas@doctor.upv.es

2 Methods and Results

2.1 Pathwise solution

The deterministic Laplace transform [2] and the usual conditions required for the existence and uniqueness of ODEs have allowed us to construct a right-continuous pathwise solution of the random IVP (1), given by

$$X(t, \omega) = X_0(\omega) + \int_{t_0}^t g(X(s, \omega), s, \mathbf{A}(\omega)) ds - \sum_{k=1}^N \Gamma_k(\omega) X(t_k, \omega) \mathbf{H}(t - t_k), \quad t \geq t_0, \quad (2)$$

$$X(t_k, \omega) = \frac{X(t_k^-, \omega)}{1 + \Gamma_k(\omega)} = X(t_k^+, \omega), \quad \omega \in \tilde{\Omega}. \quad (3)$$

2.2 Probability Density Function evolution

RDEs verifies a *probability conservation property*; that is, the total probability in the phase space is conserved through time. This fact gives an evolution PDE which is verified by its 1-PDF. The theorem can be stated as follows

Theorem 1. [1] *Let $b(\cdot, t) : \mathbb{R} \rightarrow \mathbb{R}$ be a Lipschitz-continuous function for all $t \in (t_0, \infty)$, and continuous in t . Let $X(t, \omega)$, $t \geq t_0$, $\omega \in \Omega$ be the stochastic process verifying the following RDE in the almost-surely or mean square sense:*

$$\begin{cases} \frac{dX(t)}{dt} = b(X(t), t), & t > t_0, \\ X(t_0) = X_0 \in L^2(\Omega, \mathbb{R}). \end{cases} \quad (4)$$

Let \mathcal{D} be a set such that $\{X([t_0, \infty), \omega)\}_{\omega \in \Omega} \subset \mathcal{D}$. Then, the 1-PDF of the stochastic process $X(t)$, denoted by $f = f_{X(t)}$, verifies the Liouville PDE:

$$\begin{cases} \partial_t f(x, t) + \partial_x [b f](x, t) = 0, & x \in \mathcal{D}, \quad t > t_0, \\ f(x, t_0) = f_0(x), & x \in \mathcal{D}, \\ \partial_x f(x, t) = 0, & x \in \partial \mathcal{D}, \quad t \geq t_0, \end{cases} \quad (5)$$

where f_0 is the PDF of $X_0 = X_0(\omega)$.

When the RDE has random parameters, IVP (5) becomes a family of deterministic PDE problems indexed in the realizations, \mathbf{a} , of the random parameter vector, $\mathbf{A} = \mathbf{A}(\omega)$,

$$\begin{cases} \partial_t f(x, t | \mathbf{a}) + \partial_x [b(x, t, \mathbf{a}) f(x, t | \mathbf{a})] = 0, & x \in \mathcal{D} \subseteq \mathbb{R}, \quad t > t_0, \\ f(x, t_0 | \mathbf{a}) = f_0(x), & x \in \mathcal{D}. \end{cases} \quad (6)$$

The PDF of the RDE solution (independent of parameter realizations) is obtained by marginalizing the joint PDF of both the solution and the parameter vector \mathbf{A} , which, using the conditional PDF can be written as:

$$f(x, t) = \int_{\mathbb{R}^m} f(x, t | \mathbf{a}) f_{\mathbf{A}}(\mathbf{a}) d\mathbf{a} = \mathbb{E}_{\mathbf{A}}[f(x, t | \mathbf{A})], \quad (7)$$

where $f_{\mathbf{A}}$ is the parameters' joint PDF and \mathbb{E} denotes the expectation operator. This shows that the PDF can be obtained by solving (6) for all realizations \mathbf{a} of \mathbf{A} and then computing its mean.

However, *a priori*, global-in-time existence of a solution to the Liouville equation can only be assured when the field function $b(\cdot, t)$ is Lipschitz continuous, uniformly in t . The field of the RDE class under study, $b(x, t) = g(x, t) - \sum_k \gamma_k \delta(t - t_k)x$, does not verify this hypothesis at the impulse times $\{t_k\}_{k=1}^N$. We want to obtain a condition such as (3), but for the PDF. This will allow the computation of the evolution of f_0 , accurately capturing the discontinuities at the impulse times.

Let us turn back to the set of conditions in (3), which are identities between random variables. We are going to make use of the RVT theorem, which can be written as follows:

Theorem 2. [2, 8] Let $\mathbf{X}, \mathbf{Y} : \Omega \rightarrow \mathbb{R}^M$ be two random vectors with PDFs $f_{\mathbf{X}}$ and $f_{\mathbf{Y}}$, respectively. Assume that there is a one-to-one, C^1 function \mathbf{h} such that $\mathbf{X} = \mathbf{h}(\mathbf{Y})$. Then, denoting \mathbf{h}^{-1} as the inverse mapping of \mathbf{h} ,

$$f_{\mathbf{X}}(\mathbf{x}) = f_{\mathbf{Y}}(\mathbf{h}^{-1}(\mathbf{x})) \left| \frac{\partial \mathbf{h}^{-1}(\mathbf{x})}{\partial \mathbf{x}} \right|, \quad (8)$$

where $\left| \frac{\partial \mathbf{h}^{-1}(\mathbf{x})}{\partial \mathbf{x}} \right|$ denotes the absolute value of the determinant of the Jacobian matrix.

Now, the Liouville equation describes the evolution of the PDF between impulse times, whereas at any impulse time, applying the RVT theorem, we obtain:

$$f(x, t_k) = \mathbb{E}_{\Gamma_k} [f(x(1 + \Gamma_k), t_k^-) | 1 + \Gamma_k|], \quad \forall x > 0, \quad k = 1, \dots, N, \quad (9)$$

because of the relations at (3).

3 Results

In this contribution, we are going to deal with the impulse-harvest generalized logistic model with a finite number of captures, say N ,

$$\begin{aligned} X'(t, \omega) &= \alpha(t) r(\omega) X(t, \omega) \left(1 - \left(\frac{X(t, \omega)}{K(\omega)} \right)^{\nu(\omega)} \right) - \sum_{n=1}^N \Gamma_n(\omega) \delta(t - t_n) X(t, \omega), \\ X(t_0, \omega) &= X_0(\omega), \end{aligned} \quad (10)$$

where $t \geq t_0$ and $\omega \in \tilde{\Omega}$. As usual, t is interpreted as the time, the parameter r is the growth ($r > 0$) or decay ($r < 0$) rate, and K is the carrying capacity. The differential equation is generalized by adding two terms: a positive, monotonically growing function $\alpha(\cdot)$ and a constant positive term ν . The first term, $\alpha(\cdot)$, allows controlling the so-called *lag phase*, which is the growth phase in which the population under study has not yet achieved a fully exponential growth. In particular, we have chosen [7]:

$$\alpha(t) := \frac{q(\omega)}{q(\omega) + e^{-m(\omega)t}}, \quad q, m > 0 \text{ a.s.}$$

The latter, ν , is a power that controls how fast the carrying capacity K is approached and is known as *deceleration* term. When $\nu = 1$, the classical logistic differential equation is obtained. And when ν tends to 0, the Gompertz equation is given. The incorporation of both the function $\alpha(\cdot)$ and the power ν allows for more flexible *S-shaped* curves to model growth phenomena over time.

3.1 Tumor removal

Radiotherapy, chemotherapy, and direct retrieval of a fraction of a tumor mass are some of the main techniques used to treat cancer. The first two treatments have a prolonged effect of tumor destruction, whereas the latter can be modeled via a delta-type impulse function because of the sudden extraction of the tumor mass with respect to the total treatment. Let us model this problem as (10), where the parameter vectors are chosen as follows:

- The initial tumor size $X_0 \sim N|_{(0,1)}(0.15, 0.01)$, where $N|_{(0,1)}$ is a normal distribution truncated on the interval $(0, 1)$.
- Variables q and m will be given the same deterministic values as in the previous example: $q = 1$ and $m = 4$.
- We consider $r \sim N|_{(0,1)}(0.15, 0.0075)$, $\nu \sim \text{Unif}(1, 1.25)$ and $K \sim \text{Unif}(0.9, 1)$.
- We are going to consider 5 removals with equally distributed intensity given by $\Gamma \sim N|_{\mathbb{R}^+}(2, 0.01)$, at times $T_{\text{Tumor}} = \{t_1 = 15, t_2 = 25, t_3 = 35, t_4 = 45, t_5 = 55\}$.

Figure 1, shows the mean and 95%-confidence intervals according to the prefixed parameters and removal times. It is seen how, after each removal, the tumor size grows according to the un-removed size of the tumor. Interestingly, the confidence interval amplitude before each removal is higher than the uncertainty after the removal. Indeed, since all removals are distributed as $\Gamma \sim N|_{\mathbb{R}^+}(2, 0.01)$, it is easily seen that each removal takes away half of the tumor (in average), thus reducing the uncertainty after each removal time. This case allows having a long-time prediction with a reduced level of uncertainty while still considering random impulses. This is further seen in Figure 2, where the PDF given as the solution of the Liouville equation in this particular problem setting is shown in every simulated time in T_{Tumor} .

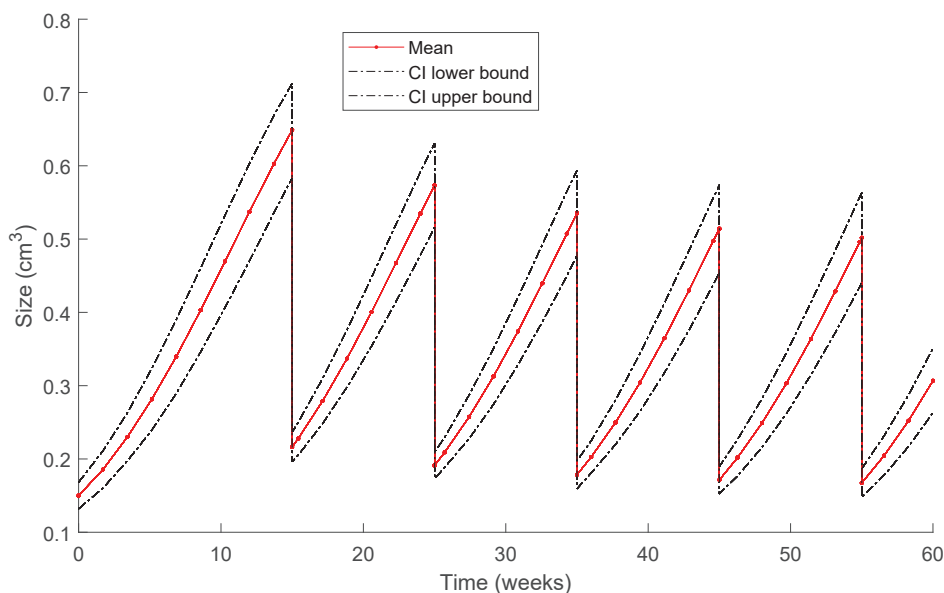


Figure 1: Time evolution of the mean tumor size and a 95% confidence interval with several extractions.

4 Conclusions

In this contribution, we have obtained a pathwise solution to a general random differential equation with a finite number of random-intensity, state-dependent, impulsive terms, with the usual assumptions on the regularity of the field function. Furthermore, we have determined the evolution

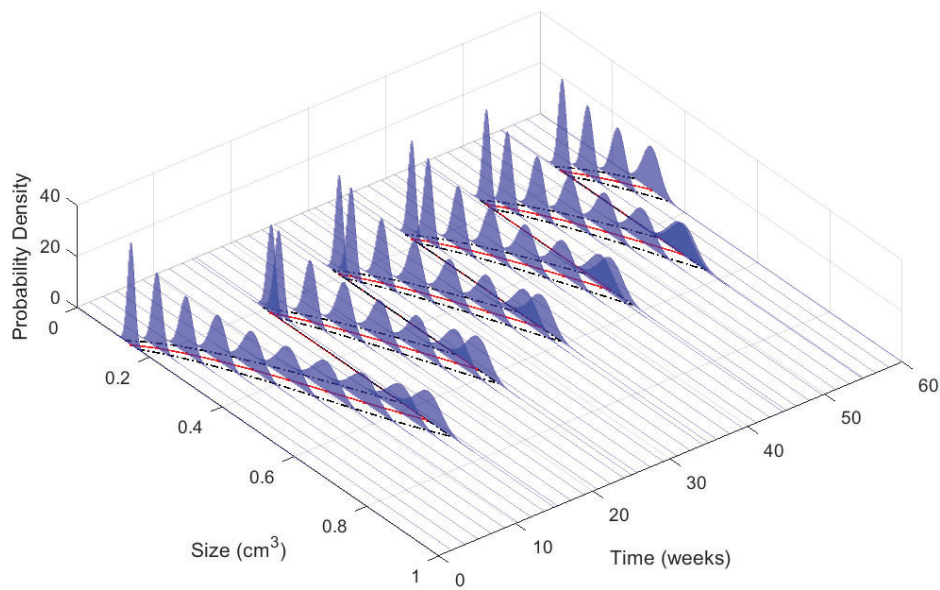


Figure 2: Full view of the PDF evolution simulations at the corresponding time values in T_{Tumor} , together with the mean (red) and 95% confidence intervals (dashed, black). Compare with Figure 1.

of the first probability density function of the solution stochastic process by combining the Liouville equation and the Random Variable Transformation theorem. We have applied our general theoretical findings to a mathematical model emerging from the generalized logistic model with natural growth altered by impulsive terms acting contrarily to its natural dynamics.

As a final note, the work on which this presentation was based has been submitted to a scientific journal as a research article.

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