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Fixed points which belong to the set of unit values of a suitable function on fuzzy metric spaces

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Abstract

In this paper, we introduce the notion of fuzzy $(F,\varphi,\beta\text{-}\psi)$-contractive mappings in fuzzy metric spaces and utilize the same to prove some existence and uniqueness fuzzy φ-fixed point results in both M-complete and G-complete fuzzy metric spaces. The obtained results extend, generalize and improve some relevant results of the existing literature. An illustrative example is utilized to demonstrate the usefulness and effectiveness of our results.

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1. Introduction

The existing literature on fuzzy sets and systems contains several definitions of fuzzy metric spaces (see more details in [3, 4, 13]). The most popular definition of fuzzy metric spaces is essentially due to Kramosil and Michalek [14] in 1975. Afterward, Grabiec [7] defined the notion of a complete fuzzy metric

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space (now known as a G-complete fuzzy metric space) and of a compact fuzzy metric space, respectively. Moreover, he also proved fuzzy versions of two famous fixed point results: the Banach fixed point theorem and Edelstein fixed point theorem. Later, the idea of Cauchy sequences in fuzzy metric spaces was modified by George and Veeramani [5] because even $\mathbb{R}$ is not complete with the completeness due to Grabiec [7]. Furthermore, they slightly modified the concept of fuzzy metric spaces initiated by Kramosil and Michalek [14] and also defined a Hausdorff and first countable topology. This modification allows many natural examples of fuzzy metrics, particularly those constructed from metrics and fuzzy metrics. In this new sense, fuzzy metrics appear more appropriate for studying induced topological structures. Note that the fuzzy metric of two points in a fuzzy metric space is measured by the degree of the nearness of two points concerning one parameter $t > 0$. For instance, if we travel from Thailand (x) to India (y) by aircraft, we can measure the degree of the nearness of x and y concerning a factor (t) related to this travel, such as time or fuel consumption with aircraft of different fuel efficiency (see in Fig. 1).

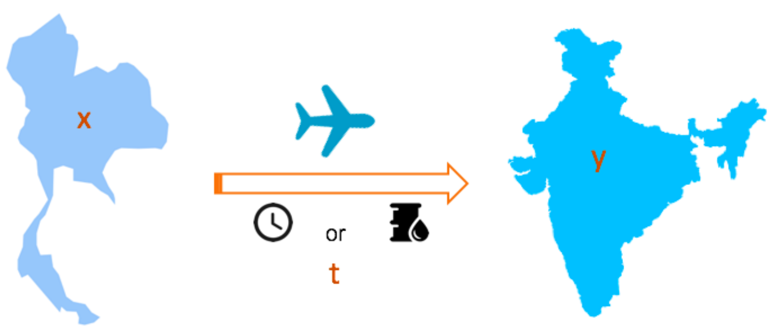

FIGURE 1. Illustrated example of the degree of nearness of x and y with respect to t

Like other areas in mathematics, fuzzy metric fixed point theory is also flourishing and by now there exists a considerable literature on fuzzy metric fixed point theory (cf. [25, 26, 23, 21, 1, 18, 30, 6]).

Nowadays, the complete fuzzy metric space in the sense of Kramosil and Michalek is known as an M-complete fuzzy metric space. It is well-known that the topology induced by a fuzzy metric space in each sense of Kramosil and Michalek, and in the sense of George and Veeramani, is metrizable and thus Hausdorff. It brings to the fact that any compact fuzzy metric space is complete in the sense of George and Veeramani [5]. Based on this fact, the following natural question arises: is a compact fuzzy metric space complete in the sense of Grabiec [7]? In 2012, Tirado [28] answered this question in the negative. In the same continuation, Gregori and Sapena [9] introduced the notion of fuzzy contractive mappings and proved a fuzzy version of the Banach contraction prnciple for such mappings in G-complete fuzzy metric spaces in the sense of George and Veeramani. Thereafter, Mihet [19] generalized the concept of fuzzy contractive mappings by introducing the concept of fuzzy ψ-contractive mappings and proved a fixed point result which in turn generalizes the Banach contraction principle in M-complete non-Archimedean fuzzy metric spaces in the sense of Kramosil and Michalek.

On the other hand, the notion of φ-fixed points was initiated by Jleli et al. [12]. The authors in [12] also introduced the notion of (F, φ)-contractive and proved some φ-fixed point results in the setting of metric spaces. For more results, in this direction we refer the reader to [15, 11, 20, 10, 16, 17]. Inspired by Jleli et al. [12], Sezen et al.[27] introduced the concepts of fuzzy φ-fixed points and (F, φ)-fuzzy contractive mappings, and established some existence and uniqueness fuzzy φ-fixed point results in fuzzy metric spaces.

In this paper, we initiate the concept of fuzzy $(F,\varphi,\beta\text{-}\psi)$-contractive mappings which enlarge and unify some classes of contractive mappings specially, those introduced in [19, 27]. The introduced notion used to prove some fuzzy φ-fixed point results in the setting of both M-complete and G-complete fuzzy metric spaces. The presented results extend and improve the corresponding results obtained in [19, 27].

2. Preliminaries

In order to have a self-contained presentation, we recall the relevant background material from the theory of fuzzy metric spaces, which are needed to prove our results.

Definition 2.1 ([24])**.** A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is said to be a t-norm if, for all $r_1, r_2, r_3 \in [0, 1]$, the following assumptions are fulfilled:

$(T1)$ $r_1 * r_2 = r_2 * r_1$;
$(T2)$ $r_1 * (r_2 * r_3) = (r_1 * r_2) * r_3$;
$(T3)$ $r_1 * r_2 \leq r_3 * r_4$ whenever $r_1 \leq r_3$ and $r_2 \leq r_4$;
$(T4)$ $r_1 * 1 = r_1$.

Three basic examples of t-norms are $*_1, *_2, *_3 : [0, 1] \times [0, 1] \to [0, 1]$ defined by $r_1 *_1 r_2 = r_1 \cdot r_2$, $r_1 *_2 r_2 = \min\{r_1, r_2\}$ and $r_1 *_3 r_2 = \max\{r_1 + r_2 - 1, 0\}$ for all $r_1, r_2 \in [0, 1]$ which known as product, minimum and Lukasiewicz t-norms, respectively.

Definition 2.2 ([5])**.** Let X be a non-empty set, $*$ is a continuous t-norm and $\mathfrak{M} : X^2 \times (0, \infty) \to [0, 1]$ is a fuzzy set. An ordered triple $(X, \mathfrak{M}, *)$ is said to be a fuzzy metric space (in short, FMS) in the sense of George and Veeramani if the following assumptions are fulfilled for all $x, y, z \in X$ and $t, s > 0$:

$(G1)$ $\mathfrak{M}(x, y, t) > 0$;
$(G2)$ $\mathfrak{M}(x, y, t) = 1$ iff $x = y$;
$(G3)$ $\mathfrak{M}(x, y, t) = \mathfrak{M}(y, x, t)$;

$(G4)$ $\mathfrak{M}(x,z,t) * \mathfrak{M}(z,y,s) \leq \mathfrak{M}(x,y,t+s)$;
$(G5)$ $\mathfrak{M}(x,y,\cdot) : (0,\infty) \to [0,1]$ is continuous.

In Definition 2.2, $(G4)$ is a fuzzy version of the triangle inequality. In addition, if the condition $(G4)$ is replaced by the following one:

$(G4)'$ $\mathfrak{M}(x,z,t) * \mathfrak{M}(z,y,s) \leq \mathfrak{M}(x,y,\max\{t,s\})$ for all $x,y,z \in X$ and $s,t > 0$,

then the fuzzy metric space $(X, \mathfrak{M}, *)$ is said to be a non-Archimedean FMS.

The example given below shows that a FMS can be constructed by a metric space.

Example 2.3 ([5])**.** Let (X,d) be a metric space and $* : [0,1]^2 \to [0,1]$ be a product t-norm (or a minimum t-norm). Define $\mathfrak{M} : X^2 \times (0,\infty) \to [0,1]$ by

$$\mathfrak{M}(x,y,t) = \frac{t}{t + d(x,y)}$$

for all $x,y \in X$ and $t > 0$. Then $(X, \mathfrak{M}, *)$ is a FMS, called a standard FMS induced by the metric d.

Now, we give some examples of FMSs due to Gregori et al. [8].

Example 2.4 ([8])**.** Let X be a nonempty set, $f : X \to (0,\infty)$ be a one-to-one function and $g : (0,\infty) \to [0,\infty)$ be an increasing continuous function. Define $\mathfrak{M} : X^2 \times (0,\infty) \to [0,1]$ by

$$\mathfrak{M}(x,y,t) = \left(\frac{(\min\{f(x), f(y)\})^\alpha + g(t)}{(\max\{f(x), f(y)\})^\alpha + g(t)} \right)^\beta$$

for all $x,y \in X$ and $t > 0$, where $\alpha, \beta > 0$. Then $(X, \mathfrak{M}, *)$ is a FMS, where $*$ is the product t-norm.

Example 2.5 ([8])**.** Let (X,d) be a metric space and $g : (0,\infty) \to [0,\infty)$ be an increasing continuous function. Define $\mathfrak{M} : X^2 \times (0,\infty) \to [0,1]$ by

$$\mathfrak{M}(x,y,t) = e^{\left(-\frac{d(x,y)}{g(t)}\right)}$$

for all $x,y \in X$ and $t > 0$. Then $(X, \mathfrak{M}, *)$ is a FMS, where $*$ is the product t-norm.

Example 2.6 ([8])**.** Let (X,d) be a bounded metric space, i.e. $d(x,y) < k$ for all $x,y \in X$, where k is a fixed constant in $(0,\infty)$, and $g : (0,\infty) \to (k,\infty)$ be an increasing continuous function. Define $\mathfrak{M} : X^2 \times (0,\infty) \to [0,1]$ by

$$\mathfrak{M}(x,y,t) = 1 - \frac{d(x,y)}{g(t)}$$

for all $x,y \in X$ and $t > 0$. Then $(X, \mathfrak{M}, *)$ is a FMS, where $*$ is a Lukasiewicz t-norm.

Definition 2.7 ([7, 5])**.** Let $(X, \mathfrak{M}, *)$ be a FMS.

(1) A sequence $\{x_n\} \subseteq X$ is said to be convergent to $x \in X$ if
$$\lim_{n\to\infty} \mathfrak{M}(x_n, x, t) = 1$$
for all $t > 0$.

(2) A sequence $\{x_n\} \subseteq X$ is said to be an M-Cauchy sequence if for each $\epsilon \in (0,1)$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that
$$\mathfrak{M}(x_m, x_n, t) > 1 - \epsilon$$
for all $m, n \geq n_0$.

(3) A sequence $\{x_n\} \subseteq X$ is said to be an G-Cauchy sequence if
$$\mathfrak{M}(x_n, x_{n+p}, t) = 1$$
for all $p \in \mathbb{N}$ and $t > 0$.

(4) The FMS $(X, \mathfrak{M}, *)$ is said to be M-complete (G-complete) if every M-Cauchy (G-Cauchy) sequence in X converges to a point of X.

Lemma 2.8 ([7, 5])**.** *Let $(X, \mathfrak{M}, *)$ be a FMS. Then the following assertions hold:*

(1) *the mapping $\mathfrak{M}$ is continuous on $X^2 \times (0,\infty)$;*
(2) *for each $x, y \in X$, $\mathfrak{M}(x, y, \cdot)$ is non-decreasing function on $(0,\infty)$;*
(3) *the limit of a convergent sequence in $(X, \mathfrak{M}, *)$ is unique.*

Definition 2.9 ([9])**.** Let $(X, \mathfrak{M}, *)$ be a FMS. A mapping $T : X \to X$ is said to be a fuzzy contractive mapping if there exists $\lambda \in (0,1)$ such that
$$\frac{1}{\mathfrak{M}(Tx, Ty, t)} - 1 \leq \lambda \left(\frac{1}{\mathfrak{M}(x, y, t)} - 1 \right)$$
for all $x, y \in X$ and $t > 0$.

Definition 2.10 ([9])**.** Let $(X, \mathfrak{M}, *)$ be a FMS. A sequence $\{x_n\}_{n\in\mathbb{N}}$ is said to be fuzzy contractive if there exists $\lambda \in (0,1)$ such that
$$\frac{1}{\mathfrak{M}(x_{n+1}, x_{n+2}, t)} - 1 \leq \lambda \left(\frac{1}{\mathfrak{M}(x_n, x_{n+1}, t)} - 1 \right)$$
for all $t > 0$.

Let Ψ be the family of all functions $\psi : (0,1] \to (0,1]$ such that ψ is non-decreasing, left continuous function and $\psi(r) > r$ for all $r \in (0,1)$.

Lemma 2.11 ([29])**.** *If $\psi \in \Psi$, then $\psi(1) = 1$.*

Lemma 2.12 ([29])**.** *If $\psi \in \Psi$, then $\lim_{n\to\infty} \psi^n(t) = 1$ for all $t \in (0,1)$.*

Using the mapping $\psi \in \Psi$, Mihet [19] introduced the following concept of fuzzy ψ-contractive mappings and proved a fuzzy fixed point theorem in M-complete non-Archimedean FMSs.

Definition 2.13 ([19]). Let $(X, \mathfrak{M}, *)$ be a FMS. A mapping $T : X \to X$ is said to be a fuzzy ψ-contractive mapping if there exists $\psi \in \Psi$ such that

$$\mathfrak{M}(Tx, Ty, t) \geq \psi(\mathfrak{M}(x, y, t))$$

for all $x, y \in X$ and $t > 0$.

Inspired by the appearance of the idea of α-admissble mappings of Samet et al. [22], Gopal and Vetro [6] employ the following idea in FMSs:

Definition 2.14 ([6]). Let $(X, \mathfrak{M}, *)$ be a FMS and $\beta : X \times X \times (0, \infty) \to (0, \infty)$ be a given mapping. A mapping $T : X \to X$ is said to be β-admissible if the following condition holds:

$$x, y \in X \text{ and } t > 0 \text{ with } \beta(x, y, t) \leq 1 \Longrightarrow \beta(Tx, Ty, t) \leq 1.$$

From now on, $Fix(T)$ denotes the set of all fixed points of a self-mapping T on a non-empty set X, that is, $Fix(T) = \{x \in X : x = Tx\}$, and U_φ stands for the set of all ones of the function $\varphi : X \to (0, 1]$, that is, $U_\varphi = \{x \in X : \varphi(x) = 1\}$.

Let $\mathcal{F}$ be the set of all functions $F : (0, 1]^3 \to (0, 1]$ satisfying the following conditions:

($F1$) $\min\{a, b, c\} \geq F(a, b, c)$ for all $a, b, c \in (0, 1]$;
($F2$) $F(1, 1, 1) = 1$;
($F3$) F is continuous.

Example 2.15. Let $F_1, F_2 : (0, 1]^3 \to (0, 1]$ be functions defined for each $a, b, c \in (0, 1]$ by

- $F_1(a, b, c) = a \cdot b \cdot c$;
- $F_2(a, b, c) = \min\{a, b\} \cdot c$.

Then F_1 and F_2 belong to $\mathcal{F}$.

Sezen et al.[27] introduced the concepts of fuzzy φ-fixed points and (F, φ)-fuzzy contractive mappings as follows:

Definition 2.16 ([27]). Let X be a non-empty set and $\varphi : X \to (0, 1]$ be a given function. An element $z \in X$ is said to be a fuzzy φ-fixed point of a mapping $T : X \to X$ if and only if it is a fixed point of T and $\varphi(z) = 1$, that is, $z \in Fix(T) \cap U_\varphi$.

Definition 2.17 ([27]). Let $(X, \mathfrak{M}, *)$ be a FMS, $F \in \mathcal{F}$ and $\varphi : X \to [0, 1)$ be a function. A mapping $T : X \to X$ is said to be a (F, φ)-fuzzy contractive mapping if there exists $\psi \in \Psi$ such that

$$F(\mathfrak{M}(Tx, Ty, t), \varphi(Tx), \varphi(Ty)) \geq \psi(F(\mathfrak{M}(x, y, t), \varphi(x), \varphi(y))$$

for all $x, y \in X$ and $t > 0$.

Based on the above definitions, the authors in [27] proved the following existence and uniqueness result in the setting of G-complete FMSs:

Theorem 2.18 ([27]). *Let $(X, \mathfrak{M}, *)$ be a G-complete FMS, $F \in \mathcal{F}$ and $\varphi : X \to (0, 1]$ be a continuous function. Suppose that $T : X \to X$ is a (F, φ)-fuzzy contractive mapping. Then T has a unique fuzzy φ-fixed point.*

3. Main result

In this section, we first enlarge the class of functions $\mathcal{F}$ by replacing the condition $(F1)$ by the following one:

$(F1)'$ $\min\{a, b\} \geq F(a, b, c)$ for all $a, b, c \in (0, 1]$.

Let $\mathcal{F}_{\mathcal{H}}$ denotes the class of all functions $F : (0, 1]^3 \to (0, 1]$ satisfying the conditions $(F1)', (F2)$ and $(F3)$.

Remark 3.1. Since $\min\{a, b\} \geq \min\{a, b, c\}$ for all $a, b, c \in (0, 1]$, we have $\mathcal{F} \subseteq \mathcal{F}_{\mathcal{H}}$, but the converse is not in general true as shown in the next example.

Example 3.2. Let $F : (0, 1]^3 \to (0, 1]$ be a function defined for each $a, b, c \in (0, 1]$ by

- $F(a, b, c) = a \cdot b \cdot e^{c-1}$.

It is clear that F belong to $\mathcal{F}_{\mathcal{H}}$ but not to $\mathcal{F}$ because the condition $(F1)$ is not satisfied (for instance, take $a = 1$, $b = 0.9$ and $c = 0.1$).

Next, let us introduce the notion of fuzzy $(F,\varphi,\beta\text{-}\psi)$-contractive mappings as follows:

Definition 3.3. Let $(X, \mathfrak{M}, *)$ be a FMS, $F \in \mathcal{F}_{\mathcal{H}}$ and $\varphi : X \to (0, 1]$ be a function. A mapping $T : X \to X$ is said to be a fuzzy $(F,\varphi,\beta\text{-}\psi)$-contractive mapping if there exist two functions $\beta : X \times X \times (0, \infty) \to (0, \infty)$ and $\psi \in \Psi$ such that

$$\beta(x, y, t)F(\mathfrak{M}(Tx, Ty, t), \varphi(Tx), \varphi(Ty)) \geq \psi(F(\mathfrak{M}(x, y, t), \varphi(x), \varphi(y)) \tag{3.1}$$

for all $x, y \in X$ and $t > 0$.

Remark 3.4. By choosing the essential functions β, F, ψ and φ suitably in Definition 3.3, one can deduce some known contractions as demonstrated under.

(a) Setting $\beta(x, y, t) = 1$ for all $x, y, z \in X$ and $t > 0$, we obtain Definition 2.17.
(b) Taking $F(a, b, c) = a \cdot b \cdot c$ for all $a, b, c \in (0, 1]$, $\beta(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$, and $\varphi(z) = 1$ for all $z \in X$, we deduce Definition 2.13.
(c) Putting $F(a, b, c) = a \cdot b \cdot c$ for all $a, b, c \in (0, 1]$, $\beta(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$, $\varphi(z) = 1$ for all $z \in X$, and $\psi(r) = \frac{r}{r+k(1-r)}$ for all $r \in (0, 1]$, where $k \in (0, 1)$, we deduce Definition 2.9.

Definition 3.5. Let $(X, \mathfrak{M}, *)$ be a FMS and $\beta : X \times X \times (0, \infty) \to (0, \infty)$ be a mapping. A mapping $T : X \to X$ is said to be fuzzy β^*-admissible if the following conditions hold:

(β^*1) for each $x, y \in X$ and $t > 0$ with $\beta(x, y, t) \leq 1$, we have $\beta(Tx, Ty, t) \leq 1$;
(β^*2) for each $x \in X$ with $x = Tx$, we have $\beta(x, x, t) = 1$ for all $t > 0$.

Example 3.6. Let $X = [0, \infty)$. Define two mappings $T : X \to X$ and $\beta : X \times X \times (0, \infty) \to (0, \infty)$ by

$$Tx = \sqrt{x} \text{ for all } x \in X \text{ and } \beta(x, y, t) = \begin{cases} e^{\frac{y-x}{t}} & x \geq y, \\ 2 & x < y. \end{cases}$$

Then the mapping T is β^*-admissible.

Example 3.7. Let $X = (0, \infty)$. Define two mappings $T : X \to X$ and $\beta : X \times X \times (0, \infty) \to (0, \infty)$ by

$$Tx = \ln x \text{ for all } x \in X \text{ and } \beta(x, y, t) = \begin{cases} \frac{1}{t} & \text{if } x \geq y \text{ and } t \geq 1, \\ 2 & \text{otherwise.} \end{cases}$$

Then the mapping T is β^*-admissible.

Theorem 3.8. *Let $(X, \mathfrak{M}, *)$ be a G-complete FMS, $F \in \mathcal{F}_{\mathcal{H}}$, and $\varphi : X \to (0, 1]$ be a continuous function. Suppose that $T : X \to X$ is a fuzzy (F,φ,β-ψ)-contractive mapping satisfying the following conditions:*

(a) *T is β^*-admissible;*
(b) *there exists $x_0 \in X$ such that $\beta(x_0, Tx_0, t) \leq 1$ for all $t > 0$;*
(c) *either T is continuous or*
if $\{x_n\}$ is a sequence in X such that $\beta(x_n, x_{n+1}, t) \leq 1$ for all $n \in \mathbb{N}$ and $t > 0$ and $\lim_{n\to\infty} x_n = x \in X$, then $\beta(x_n, x, t) \leq 1$ for all $n \in \mathbb{N}$ and $t > 0$.

Then $Fix(T) \subseteq U_\varphi$ and T has a fuzzy φ-fixed point.

Proof. Let $x \in Fix(T)$, that is, $x = Tx$. Applying (3.1) with $x = y$ and using the condition (β^*2), we obtain

$$F(1, \varphi(x), \varphi(x)) = \beta(x, x, t)F(1, \varphi(x), \varphi(x)) \geq \psi(F(1, \varphi(x), \varphi(x))). \tag{3.2}$$

From (3.2) and taking into account that $\psi(t) > t$, for all $t \in (0, 1)$, we get

$$F(1, \varphi(x), \varphi(x)) = 1. \tag{3.3}$$

Using (3.3) and $(F1)'$, we have

$$\varphi(x) \geq F(1, \varphi(x), \varphi(x)) = 1,$$

which implies that $\varphi(x) = 1$, and hence $Fix(T) \subseteq U_\varphi$.

Next, let x_0 be an arbitrary point in X such that $\beta(x_0, Tx_0, t) \leq 1$ for all $t > 0$. Define the sequence $\{x_n\}$ in X by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}$. If $x_{n_0} = x_{n_0+1}$ for some $n_0 \in \mathbb{N}_0$, then x_{n_0} is a fixed point of the mapping T, and hence a fuzzy φ-fixed point (as $Fix(T) \subseteq U_\varphi$). Assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}_0$. Since T is β-admissible, we have

$$\beta(x_0, x_1, t) = \beta(x_0, Tx_0, t) \leq 1 \implies \beta(x_1, x_2, t) = \beta(Tx_0, Tx_1, t) \leq 1.$$

By induction, we get

$$\beta(x_n, x_{n+1}, t) \leq 1, \text{ for all } n \in \mathbb{N} \text{ and } t > 0. \tag{3.4}$$

Applying the contractive condition (3.1) with $x = x_{n-1}$ and $y = x_n$, and using (3.4), we have

$$\begin{aligned} F(\mathfrak{M}(x_n, x_{n+1}, t), \varphi(x_n), \varphi(x_{n+1})) &\geq \beta(x_{n-1}, x_n, t) F(\mathfrak{M}(x_n, x_{n+1}, t), \varphi(x_n), \\ &\qquad \varphi(x_{n+1})) \\ &\geq \psi(F(\mathfrak{M}(x_{n-1}, x_n, t), \varphi(x_{n-1}), \varphi(x_n))) \end{aligned}$$

for all $n \in \mathbb{N}$ and $t > 0$. By induction, we get

$$F(\mathfrak{M}(x_n, x_{n+1}, t), \varphi(x_n), \varphi(x_{n+1})) \geq \psi^n(F(\mathfrak{M}(x_0, x_1, t), \varphi(x_0), \varphi(x_1)))$$

for all $n \in \mathbb{N}$ and $t > 0$. From the above inequality and $(F1)'$ we obtain

$$\mathfrak{M}(x_n, x_{n+1}, t) \geq \psi^n(F(\mathfrak{M}(x_0, x_1, t), \varphi(x_0), \varphi(x_1))) \tag{3.5}$$

and

$$\varphi(x_n) \geq \psi^n(F(\mathfrak{M}(x_0, x_1, t), \varphi(x_0), \varphi(x_1))) \tag{3.6}$$

for all $n \in \mathbb{N}$. Now, we will show that $\{x_n\}$ is a G-Cauchy sequence in X. Let $m, n \in \mathbb{N}$ such that $m > n$ and on making use of $(G4)$ and (3.5), we get

(3.7)

$$\begin{aligned} \mathfrak{M}(x_n, x_{n+m}, t) \geq \mathfrak{M}\left(x_n, x_{n+1}, \frac{t}{m}\right) * \mathfrak{M}\left(x_{n+1}, x_{n+2}, \frac{t}{m}\right) * ... * \\ \mathfrak{M}\left(x_{n+m-1}, x_{n+m}, \frac{t}{m}\right) \\ \geq \psi^n(F(\mathfrak{M}(x_0, x_1, t), \varphi(x_0), \varphi(x_1))) * \psi^{n+1}(F(\mathfrak{M}(x_0, x_1, t), \\ \varphi(x_0), \varphi(x_1))) * \cdots * \psi^{n+m-1}(F(\mathfrak{M}(x_0, x_1, t), \varphi(x_0), \varphi(x_1))) \\ = *_{i=0}^{i=m-1} \psi^{n+i}(F(\mathfrak{M}(x_0, x_1, t), \varphi(x_0), \varphi(x_1))), \end{aligned}$$

which in view of Lemma 2.12 gives rise to

$$\lim_{n\to\infty} \mathfrak{M}(x_n, x_{n+m}, t) = 1$$

for all $m, t > 0$, and hence $\{x_n\}$ is a G-Cauchy sequence in X. Therefore, the G-completeness of the fuzzy metric space $(X, \mathfrak{M}, *)$ insures the existence of a point z in X such that

$$\lim_{n\to\infty} \mathfrak{M}(x_n, z, t) = 1. \tag{3.8}$$

Observe that from (3.6) and Lemma 2.12, we have

$$\lim_{n\to\infty} \varphi(x_n) = 1. \tag{3.9}$$

Using (3.8), (3.9) and the upper semi-continuity of φ, we get

$$1 \geq \varphi(z) \geq \lim_{n\to\infty} \sup \varphi(x_n) = 1,$$

which implies that

$$\varphi(z) = 1. \tag{3.10}$$

Now, assume that (c) holds, that is, the mapping T is continuous. Then it follows that

$$\lim_{n\to\infty}\mathfrak{M}(x_{n+1},Tz,t)=\lim_{n\to\infty}\mathfrak{M}(Tx_n,Tz,t)=1$$

for all $t>0$. By the uniqueness of the limit, we get $Tz=z$. Therefore, we conclude that z is a fixed point of T, and hence a fuzzy φ-fixed point of T (due to (3.10)). Otherwise, in view of (3.4) and (3.8), we have

$$\beta(x_n,z,t)\leq 1 \text{ for all } n\in\mathbb{N} \text{ and for all } t>0. \tag{3.11}$$

Applying (3.1) with $x=x_n$ and $y=z$, and using (3.11), we obtain

$$\begin{aligned} F(\mathfrak{M}(x_{n+1},Tz,t),\varphi(x_{n+1}),\varphi(Tz)) &\geq \beta(x_n,z,t)F(\mathfrak{M}(x_{n+1},Tz,t),\varphi(x_{n+1}),\\ &\qquad\qquad\qquad\qquad\varphi(Tz))\\ &\geq \psi(F(\mathfrak{M}(x_n,z,t),\varphi(x_n),\varphi(z)))\end{aligned}$$

for all $n\in\mathbb{N}$ and $t>0$. Taking the limit to the both sides of the above inequality, and using (3.8), (3.9), (3.10) and the continuity of F and ψ, we get

$$F(\mathfrak{M}(z,Tz,t),1,\varphi(Tz))\geq\psi(F(1,1,1))=1,$$

which in view of $(F1)'$ we must have $\mathfrak{M}(z,Tz,t)=1$. Therefore, z is a fuzzy φ-fixed point of T. Thus, the proof is completed. ☐

In order to examine the uniqueness of the fuzzy φ-fixed point, we will take into account the following condition:

- (h) for all $x,y\in Fix(T)$ and $t>0$, there exists $w\in X$ such that $\beta(x,w,t)\leq 1$ and $\beta(y,w,t)\leq 1$.

Theorem 3.9. *In addition to the hypotheses of Theorem 3.8, assume that the condition (h) is satisfied. Then the fuzzy φ-fixed point of T exists and is unique.*

Proof. Theorem 3.8 insures the existence of a fuzzy φ-fixed point of T. Suppose that z_1 and z_2 are two fuzzy φ-fixed points of T, that is, $z_1,z_2\in Fix(T)$ and $\varphi(z_1)=\varphi(z_2)=1$. From the condition (h), there exists $w\in X$ such that

$$\beta(z_1,w,t)\leq 1 \quad\text{and}\quad \beta(z_2,w,t)\leq 1 \tag{3.12}$$

for all $t>0$. Using the condition (β^*1) (as T is β^*-admissible) and (3.12), we obtain

$$\beta(z_1,T^nw,t)\leq 1 \quad\text{and}\quad \beta(z_2,T^nw,t)\leq 1 \tag{3.13}$$

for all $n\in\mathbb{N}$ and $t>0$. Applying (3.1) with $x=z_1$ and $y=T^{n-1}w$, and using (3.13), we have

$$\begin{aligned} F(\mathfrak{M}(z_1,T^nw,t),1,\varphi(T^nw)) &= F(\mathfrak{M}(Tz_1,T(T^{n-1}w),t),\varphi(Tz_1),\varphi(T(T^{n-1}w)))\\ &\geq \beta(z_1,T^{n-1}w)F(\mathfrak{M}(Tz_1,T(T^{n-1}w),t),\varphi(Tz_1),\\ &\qquad\qquad\qquad\qquad\varphi(T(T^{n-1}w)))\\ &\geq \psi(F(\mathfrak{M}(z_1,T^{n-1}w,t),\varphi(z_1),\varphi(T^{n-1}w)))\\ &= \psi(F(\mathfrak{M}(z_1,T^{n-1}w,t),1,\varphi(T^{n-1}w)))\end{aligned}$$

for all $n \in \mathbb{N}$ and $t > 0$. This inductively implies that

$$F(\mathfrak{M}(z_1, T^n w, t), 1, \varphi(T^n w)) \geq \psi^n(F(\mathfrak{M}(z_1, w, t), 1, \varphi(w)))$$

for all $n \in \mathbb{N}$ and $t > 0$. From the above inequality and in view of the condition $(F1)'$, we have

$$\mathfrak{M}(z_1, T^n w, t) \geq \psi^n(F(\mathfrak{M}(z_1, w, t), 1, \varphi(w)))$$

for all $n \in \mathbb{N}$ and $t > 0$. Taking the limit to the both sides of the above inequality, and using (3.13) and Lemma 2.12, we obtain

$$\lim_{n\to\infty} \mathfrak{M}(z_1, T^n w, t) = 1$$

for all $t > 0$. Similarly, one can prove that

$$\lim_{n\to\infty} \mathfrak{M}(z_2, T^n w, t) = 1$$

for all $t > 0$. Therefore, the uniqueness of the limit insures that $z_1 = z_2$ for all $t > 0$, and hence the φ-fixed point of T is unique. This completes the proof. □

Remark 3.10. Setting $\beta(x, y, t) = 1$ for all $x, y, z \in X$ and $t > 0$, Theorem 3.9 reduces to Theorem 2.18.

In the above proved results (Theorems 3.8 and 3.9), the G-completeness of the fuzzy metric space $(X, \mathfrak{M}, *)$ was assumed, and it is known from the definitions of G-Cauchy and M-Cauchy that are different.

Moreover, it is well-known that the G-completeness is a too restrictive notion of the completeness. For instance, the standard FMS is not G-complete for the usual metric space (see [5] for more details), and there exist compact FMSs that are not G-complete. Based on the mentioned fact, the obtained results concerning M-complete FMSs are more interesting than the corresponding ones for G-complete FMSs. Therefore, the following interesting open question will arise:

- **Do Theorems 3.8 and 3.9 remain true if we replace the G-completeness of the FMS by the M-completeness?**

Next, we are going to answer the above question by proving Theorems 3.8 and 3.9 with M-completeness of the fuzzy metric space $(X, \mathfrak{M}, *)$ under the following situations:

- replacing $\mathcal{F}_{\mathcal{H}}$ by $\mathcal{F}_{\mathcal{S}}$, which is the class of all functions $F : (0, 1]^3 \to (0, 1]$ satisfying $(F1)'$, $(F3)$ and $F(a, 1, 1) = a$ for all $a \in (0, 1]$;
- adding one more condition on the function β (the condition (d) in Theorem 3.11).

Theorem 3.11. *Let $(X, \mathfrak{M}, *)$ be an M-complete FMS, $F \in \mathcal{F}_{\mathcal{S}}$, and $\varphi : X \to (0, 1]$ be a continuous function. Suppose that $T : X \to X$ is a fuzzy $(F,\varphi,\beta\text{-}\psi)$-contractive mapping satisfying the following conditions:*

(a) T is β^*-admissible;
(b) there exists $x_0 \in X$ such that $\beta(x_0, Tx_0, t) \leq 1$ for all $t > 0$;

(c) *either T is continuous or*
if $\{x_n\}$ is a sequence in X such that $\beta(x_n, x_{n+1}, t) \leq 1$ for all $n \in \mathbb{N}$ and $t > 0$ and $\lim_{n\to\infty} x_n = x \in X$, then $\beta(x_n, x, t) \leq 1$ for all $n \in \mathbb{N}$ and $t > 0$;

(d) *for each sequence $\{x_n\}$ in X such that $\beta(x_n, x_{n+1}, t) \leq 1$ for all $n \in \mathbb{N}$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that for all $m, n \in \mathbb{N}$ with $m > n \geq k_0$, we have $\beta(x_n, x_m, t) \leq 1$ for all $t > 0$.*

Then $Fix(T) \subseteq U_\varphi$ and T has a fuzzy φ-fixed point.

Proof. The frame of the proof is the same as in Theorem 3.8. So for arbitrary point $x_0 \in X$, it follows from (3.5) and Lemma 2.12 that

$$\lim_{n\to\infty} \mathfrak{M}(x_n, x_{n+1}, t) = 1 \text{ for all } t > 0. \tag{3.14}$$

Now, we need to show only that the sequence $\{x_n\}$ is an M-Cauchy sequence, on contrary, we assume that it is not an M-Cauchy sequence, then by [2, Proposition] and the condition (d), there exist $\epsilon \in (0, 1), t_0 > 0$ and $k_0 \in \mathbb{N}$ such that for each $k \in \mathbb{N}$ with $k \geq k_0$, there exists $m_k, n_k \in \mathbb{N}$ such that $m_k > n_k \geq k$ and

$$\mathfrak{M}(x_{n_k}, x_{m_k}, t_0) \leq 1 - \epsilon, \tag{3.15}$$

with

$$\lim_{n\to\infty} \mathfrak{M}(x_{n_k}, x_{m_k}, \frac{t_0}{2}) = 1 - \epsilon \tag{3.16}$$

and

$$\beta(x_{n_k}, x_{m_k}, \frac{t_0}{2}) \leq 1. \tag{3.17}$$

From $(F1)'$, we have

$$\mathfrak{M}(x_{n_k+1}, x_{m_k+1}, \frac{t_0}{2}) \geq F(\mathfrak{M}(x_{n_k+1}, x_{m_k+1}, \frac{t_0}{2}), \varphi(x_{n_k+1}), \varphi(x_{m_k+1})). \tag{3.18}$$

Making use of the equations (3.1), (3.15), (3.17), (3.18) and $(G4)$, we get

$$
\begin{aligned}
1-\epsilon &\geq \mathfrak{M}(x_{n_k}, x_{m_k}, t_0) \\
&\geq \mathfrak{M}(x_{n_k}, x_{n_k+1}, \frac{t_0}{4}) * \mathfrak{M}(x_{n_k+1}, x_{m_k+1}, \frac{t_0}{2}) * \mathfrak{M}(x_{m_k+1}, x_{m_k}, \frac{t_0}{4}) \\
&\geq \mathfrak{M}(x_{n_k}, x_{n_k+1}, \frac{t_0}{4}) * F(\mathfrak{M}(x_{n_k+1}, x_{m_k+1}, \frac{t_0}{2}), \varphi(x_{n_k+1}), \varphi(x_{m_k+1}))* \\
&\qquad\qquad\qquad\qquad \mathfrak{M}(x_{m_k+1}, x_{m_k}, \frac{t_0}{4}) \\
&\geq \mathfrak{M}(x_{n_k}, x_{n_k+1}, \frac{t_0}{4}) * \beta(x_{n_k}, x_{m_k}, \frac{t_0}{2}) F(\mathfrak{M}(Tx_{n_k}, Tx_{m_k}, \frac{t_0}{2}), \varphi(Tx_{n_k}), \\
&\qquad\qquad\qquad\qquad \varphi(Tx_{m_k})) * \mathfrak{M}(x_{m_k+1}, x_{m_k}, \frac{t_0}{4}) \\
&\geq \mathfrak{M}(x_{n_k}, x_{n_k+1}, \frac{t_0}{4}) * \psi(F(\mathfrak{M}(x_{n_k}, x_{m_k}, \frac{t_0}{2})), \varphi(x_{n_k}), \varphi(x_{m_k}))* \\
&\qquad\qquad\qquad\qquad \mathfrak{M}(x_{m_k+1}, x_{m_k}, \frac{t_0}{4}).
\end{aligned}
$$

Taking $n \to \infty$ to the both sides of the above inequality, using (3.14), (3.16) and the continuity of F and ψ, we obtain

$$1-\epsilon \geq 1 * \psi(F(1-\epsilon, 1, 1)) * 1$$

which in view of $(T4)$ and the fact that $\psi(t) > t$ for all $t \in (0,1)$ and $F \in \mathcal{F}_{\mathcal{S}}$ gives rise

$$1-\epsilon \geq \psi(1-\epsilon) > 1-\epsilon,$$

which is a contradiction, and hence $\{x_n\}$ is a M-Cauchy sequence in X. The rest of the proof follows as in the proof of Theorem 3.8. □

The following theorem ensures the uniqueness of the fuzzy φ-fixed point.

Theorem 3.12. *In addition to the hypotheses of Theorem 3.11. Assume that the condition (h) is satisfied. Then the fuzzy φ-fixed point of T exists and is unique.*

Proof. Theorem 3.11 ensures the existence of a fuzzy φ-fixed point of T. The proof of the uniqueness is the same as in Theorem 3.9, and hence omitted. □

Remark 3.13. Taking $F(a,b,c) = a \cdot b \cdot c$ for all $a,b,c \in (0,1]$, $\beta(x,y,t) = 1$ for all $x, y \in X$ and $t > 0$ and $\varphi(z) = 1$ for all $z \in X$ in Theorem 3.12, we deduce Theorem 3.1 in [19].

To support our result, we provide an illustrative example. Precisely, we show that Theorem 3.12 can be used to cover this example, while Theorem 3.1 in [19] is not applicable.

Example 3.14. Let $X = \mathbb{N}$, the set of all positive integers, $*$ is a minimum t-norm and $\mathfrak{M}$ be a fuzzy set on $X^2 \times (0,\infty)$ given by $\mathfrak{M}(x,y,t) = e^{\frac{-|x-y|}{t}}$ for all $x, y \in X$ and all $t > 0$. Then $(X, \mathfrak{M}, *)$ is an M-complete FMS. Consider the mapping $T : X \to X$ defined by

$$Tx = \begin{cases} 1 & \text{if } x \in A, \\ 2 & \text{if } x \in B, \end{cases} \tag{3.19}$$

where $A = \{2n-1 \mid n \in \mathbb{N}\} \cup \{2\}$ and $B = \{2n+2 \mid n \in \mathbb{N}\}$. Define two essential functions $F : (0,1]^3 \to (0,1]$ and $\varphi : X \to (0,1]$ by

$$F(a,b,c) = a \cdot b \cdot c \text{ for all } a,b,c \in (0,1] \qquad \text{and} \qquad \varphi(x) = e^{1-x} \text{ for all } x \in X.$$

It is obvious that $F \in \mathcal{F_S}$ and φ is a continuous function. Consider the function: $\beta : X \times X \times (0,\infty) \to (0,\infty)$ defined by

$$\beta(x,y,t) = \begin{cases} 1 & \text{if } x,y \in A \textit{ or } x,y \in B, \\ x^y + y^x & \text{otherwise.} \end{cases}$$

Let $x, y \in X$ such that $\beta(x,y,t) \leq 1$. Then either $x, y \in A$ or $x, y \in B$ and by the definition of T, in both cases, we have $Tx = Ty \in A$, and hence $\beta(Tx, Ty, t) = 1$ for all $t > 0$. Moreover, the condition (β^*2) holds. Therefore, T is β^*-admissible mapping. Also, $2 \in X$ and $\beta(2, T2, t) = \beta(2, 1, t) = 1$. Further, let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} x_n = x$ with $k_0 = 1$ and $\beta(x_n, x_{n+1}, t) \leq 1$, for all $n \in \mathbb{N}$. From the definition of β, it follows that either $x_n \in A$ for all $n \in \mathbb{N}$ or $x_n \in B$ for all $n \in \mathbb{N}$. In the case of $x_n \in A$ for all $n \in \mathbb{N}$, if we assume that $x \in B$, then we get

$$\mathfrak{M}(x_n, x, t) = e^{\frac{-|x_n - x|}{t}} \leq e^{\frac{-1}{t}} < 1 \text{ for all } t > 0,$$

which is a contradiction to the assumption that $\lim_{n\to\infty} x_n = x$. Thus, we have $x \in A$. Therefore, $\beta(x_n, x, t) \leq 1$ and $\beta(x_n, x_m, t) \leq 1$ for all $m, n \in \mathbb{N}$ and $t > 0$. Similarly, we get the same thing in the case of $x_n \in B$.

Finally, we will show that T is fuzzy $(F,\varphi,\beta\text{-}\psi)$-contractive mapping, where $\psi : (0,1] \to (0,1]$ is defined by $\psi(t) = \sqrt{t}$ for all $t \in (0,1]$. To do so, we consider three cases.

Case I: If $x, y \in A$, then (as $\beta(x,y,t) = 1$) we have

$$\begin{aligned} \beta(x,y,t)[F(\mathfrak{M}(Tx,Ty,t), \varphi(Tx), \varphi(Ty))] &= \mathfrak{M}(1,1,t)\varphi(1)\varphi(1) \\ &= 1 \\ &\geq \sqrt{e^{\frac{-|x-y|}{t}} e^{1-x} e^{1-y}} \\ &= \sqrt{F(\mathfrak{M}(x,y,t), \varphi(x), \varphi(y))}. \end{aligned}$$

Case II: If $x, y \in B$, then (as $\beta(x,y,t) = 1$) we have

$$\begin{aligned}\beta(x,y,t)[F(\mathfrak{M}(Tx,Ty,t),\varphi(Tx),\varphi(Ty))] &= \mathfrak{M}(2,2,t).\varphi(2).\varphi(2) \\ &= e^{-1}e^{-1} \\ &\geq \sqrt{e^{\frac{-|x-y|}{t}}e^{1-x}e^{1-y}} \\ &= \sqrt{F(\mathfrak{M}(x,y,t),\varphi(x),\varphi(y))}.\end{aligned}$$

Case III: If $x \in A$ and $y \in B$, then (as $\beta(x,y,t) = x^y + y^x$) we have

$$\begin{aligned}\beta(x,y,t)[F(\mathfrak{M}(Tx,Ty,t),\varphi(Tx),\varphi(Ty))] &= (x^y + y^x)\mathfrak{M}(1,2,t)\varphi(1)\varphi(2) \\ &= (x^y + y^x)e^{\frac{-1}{t}}e^{-1} \\ &\geq \sqrt{e^{\frac{-|x-y|}{t}}e^{1-x}e^{1-y}} \\ &= \sqrt{F(\mathfrak{M}(x,y,t),\varphi(x),\varphi(y))}.\end{aligned}$$

Hence, from all cases, we can conclude that T is a fuzzy (F,φ,β-ψ)-contractive mapping. Therefore, all the hypotheses of Theorem 3.9 are satisfied. Hence, T has a unique fuzzy φ-fixed point (namely $x = 1$).

However, T is not a fuzzy ψ-contractive mapping. On contrary, let us assume that T is a fuzzy ψ-contractive mapping, that is, there exists $\psi \in \Psi$ such that

$$\mathfrak{M}(Tx,Ty,t) \geq \psi(\mathfrak{M}(x,y,t))$$

for all $x, y \in X$ and $t > 0$. Choosing $x = 3$, $y = 4$ and $t = \frac{1}{4}$, we have

$$e^{-4} = \mathfrak{M}\left(T3, T4, \frac{1}{4}\right) \geq \psi\left(\mathfrak{M}\left(3, 4, \frac{1}{4}\right)\right) = \psi(e^{-4}) > e^{-4},$$

which is a contradiction, and hence T is not a fuzzy ψ-contractive mapping.

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