Automatic identification and evaluation of Fibonacci retracements: Empirical evidence from three equity markets

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Abstract

We examine the empirical performance of using Fibonacci retracements as a tool in technical analysis. To this end, we propose a novel methodological approach of constructing zones around Fibonacci Support and Resistance levels, in an attempt to address the inherent subjectivity associated with drawing and assessing these levels on a chart. Our empirical results provide no support for the use of Fibonacci levels in technical analysis, when applied on three main equity markets. In particular, we find that prices are equally likely to bounce on Fibonacci levels as they are to bounce on non-Fibonacci levels. Importantly, a trading rule based on Fibonacci levels fails to outperform a strategy based on randomly selected non-Fibonacci levels. We also report a positive relationship between the width of a Fibonacci zone and the probability of identifying a price bounce, although trading performance remains poor irrespective of the width selected.

Keywords: Fibonacci retracements; Technical analysis; Support and Resistance levels; Algorithmic trading; Subjectivity
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1 Introduction

Academics and investment practitioners have long debated the merits of technical analysis. The investment community has generally held the belief that studying the history of prices and volumes in technical analysis can lead to meaningful, and by extension profitable, inferences about the future evolution of prices. As a result, investment managers and traders have traditionally used various technical indicators such as support and resistance levels, filter rules, channel breakouts, moving averages, and candlestick charts, in order to detect patterns in stock markets.[1] For instance, Kavajecz & Odders-White (2004) argue that "virtually all investment banks and trading firms are employing some technical trading strategies".

At the other end, much of the academic literature has historically been based on the hypothesis of efficient markets. In this context, technical analysis cannot consistently lead to profitable investment opportunities, since current prices already incorporate all available information. If technical analysis could provide any incremental information about future prices, this would effectively lead to investors earning excess profits at minimal risk, a situation that is impossible to reconcile with traditional equilibrium models of asset pricing. In the current paper, we contribute to this on-going debate about the place of technical analysis in investment decisions, with particular emphasis on a specific technical analysis tool, namely Fibonacci retracements.

Out of this debate between academics and practitioners, a substantial literature has emerged that examines empirically whether technical analysis can successfully predict future prices and, ultimately, whether technical trading strategies offer returns in excess of their exposure to well-recognized sources of risk. Given the large number of different technical trading rules that have been examined, as well as the different periods and markets that have been considered, the empirical evidence is relatively mixed. For example, Allen & Karjalainen (1999), Ratner & Leal (1999), Jegadeesh (2000), Detolleneare & Mazza (2014), and Psaradellis et al. (2019) examine a large set of technical trading rules across multiple markets and find that they do not consistently offer abnormal profits. At the other end, Menkhoff (2010), Moskowitz et al. (2012), Han et al. (2013), and Avramov et al. (2018) provide evidence of trading rules offering abnormal returns that are statistically and economically significant. Other studies hold the relative middle ground, arguing that technical analysis can lead to abnormal profits in specific markets and under certain conditions (see, for example, Hsu et al. 2010, Marshall et al. 2017).

Support and Resistance (S&R) levels represent one of the most commonly used tools in technical analysis. These S&R levels refer to certain price thresholds at which a previous trend is expected to stop and reverse, at least temporarily (Pring 2014). For instance, when prices follow a downward trend and reach a support level, then the price finds "support" and it is expected to revert to a higher level instead of continue falling. Similarly,

[1] See Sullivan et al. (1999) for an in-depth discussion of a large universe of 7,846 trading rules.

when prices follow an upward trend and reach a resistance level, then the price encounters "resistance" and it is expected to revert to a lower level rather than continue moving upwards and breaking through that threshold.

Several ways have been proposed to identify Support and Resistance levels in technical analysis. Arguably the most common approach is to determine S&R levels by using local minima or maxima that were realized during a preceding time interval. Trading rules that are based on these S&R levels are typically referred to as "trading range breakouts", and several studies have examined their performance in various markets, with relatively mixed results (Brock et al. 1992, Hudson et al. 1996, Sullivan et al. 1999, Kavajecz & Odders-White 2004, Marshall et al. 2008*a*,*b*). Zapranis & Tsinaslanidis (2012) expand this framework and propose a rule-based method that identifies each support or resistance level by jointly considering multiple historical locals instead of just one. Another approach refers to applying simple arithmetic rules to establish S&R levels, such as the 50% rule (bisection), and the 33.3%-66.6% rule (trisection) (Pring 2014). Finally, an alternative approach used to establish Support and Resistance levels is motivated by recent research in behavioral science. For example, Osler (2000) highlights a preference for round numbers, with firms being significantly more likely to publish S&R levels ending in 0 or 5 than if they had been chosen at random, while Doucouliagos (2005) reports evidence of certain price levels acting as psychological barriers.

In this paper, we focus on a particular approach for determining Support and Resistance levels, namely Fibonacci retracements. These S&R levels describe by how much prices have retraced after a preceding significant trend by dividing the distance between two extreme price points on a chart with key Fibonacci ratios, which are based on the well-known sequence of Fibonacci numbers. A hypothetical ratio of 0% represents the start of the retracement, while a ratio of 100% represents the extreme case where prices have fully reverted to their original level from before the trend had started. In our empirical analysis, we explore the performance of trading rules that are based on four key Fibonacci ratios, namely 23.6%, 38.2%, 61.8%, and 100.0%.

We contribute to the literature in two main ways. First, we provide a thorough evaluation of Fibonacci retracements as a tool in technical analysis. A limited number of previous studies have examined the performance of trading rules based generally on Support and Resistance levels. However, these studies have typically examined S&R levels that are drawn based on other approaches, usually triggered by prices reaching local lows or highs. This absence of empirical evidence on Fibonacci retracements is somewhat surprising, given that they have been commonly used by investment practitioners. We attempt to fill this gap in the literature by providing an empirical analysis of the probability of prices bouncing on Fibonacci vs non-Fibonacci levels using logistic regressions with bootstrapped standard errors. We also evaluate the performance of a technical trading rule that is based on Fibonacci retracements against the benchmark of trading on non-Fibonacci S&R levels.

Second, we contribute to the literature by proposing a new methodological approach for determining Support and Resistance using Fibonacci *zones* instead of levels. This novel approach is motivated by the inherent subjectivity of the process of identifying price trends and, consequently, drawing S&R levels on a chart. Different traders are likely to identify different starting and ending points for a preceding trend, even if they are looking at the same price chart. Therefore, the resulting S&R levels will be characterized by some level of subjectivity and vary across different traders, even though they are all constructed based on the same predetermined Fibonacci ratios. We argue that establishing a zone around a Fibonacci level can address some of this inherent subjectivity, by allowing for a range of prices that represent Support and Resistance levels. In addition, although our empirical application is based on Fibonacci levels, the general approach of constructing zones around levels can be applied to any other type of S&R levels.[2]

We examine the effectiveness of using Fibonacci retracements as a technical trading rule across stocks trading in three main indices, namely the Dow Jones and NASDAQ indices in the US, and the DAX index in Germany. Overall, our empirical results provide no support for the use of Fibonacci retracements in technical analysis, consistent with the large stream of the literature that finds no evidence of technical trading rules consistently producing abnormal profits. We find that estimating a logistic regression of bounce probabilities against a dummy for Fibonacci zones results in statistically insignificant slopes across the three sample equity markets. In other words, the probability of prices bouncing on a Fibonacci zone is statistically indistinguishable from the probability of prices bouncing on any other non-Fibonacci zone. This empirical finding casts substantial doubt on the main argument behind the use of Fibonacci levels, which states that prices are especially likely to bounce back when they reach these specific threshold levels. In the end, the fact that prices are not more likely to find support or resistance on Fibonacci levels compared to any other arbitrary selected price level makes it unlikely that Fibonacci S&R levels can help traders forecast the future evolution of prices.

More importantly, we find that a trading rule that is based on Fibonacci S&R zones fails to outperform a benchmark trading rule that is based on random non-Fibonacci S&R zones. In this sense, an investor who trades based on Fibonacci zones would not necessarily have performed poorly relative to the overall market, but they would have done equally well by simply trading based on randomly selected non-Fibonacci zones. Although this finding is not unexpected, given that prices had been previously shown to be equally likely to bounce on Fibonacci and non-Fibonacci zones, it offers further grounds to reject the use of Fibonacci retracements in technical analysis based on their actual trading performance.

Finally, our results provide support for the hypothesis that constructing wider Fibonacci zones results in higher probabilities of identifying price bounces. More specifi-

[2] Our approach of constructing zones around Fibonacci levels is similar, albeit not identical, to the use of a fixed percentage band filter in Sullivan et al. (1999).

cally, we estimate a logistic regression of bounce probabilities against dummy variables for different levels of the fixed percentage used in order to construct zones around Fibonacci levels. After estimating this model separately for stocks trading in each of the three equity indices in our sample, we find that the resulting slopes are consistently positive, statistically significant, and monotonically increasing across the zone width. This finding confirms the simple intuition that the wider the zone that has been constructed around a Fibonacci level, the more likely one is to detect price bounces. We interpret this finding in terms of wider zones reflecting greater analyst subjectivity and uncertainty about the “true” price trend. Nevertheless, trading on Fibonacci S&R zones ultimately fails to produce abnormal profits, irrespective of the zones’ width, due to the failure of Fibonacci levels to reflect genuine thresholds where prices are more likely to bounce compared to any other random non-Fibonacci levels.

The rest of the paper is organized as follows. Section 2 discusses the data used in the empirical analysis and presents the methodology for constructing Fibonacci S&R levels and zones. Section 3 presents the empirical results, and Section 4 concludes.

2 Data and Methodology

We examine a large dataset of individual stocks trading in three main equity indices, namely the Dow Jones (30 stocks), the German DAX index (30 stocks), and NASDAQ (100 stocks). Our dataset essentially consists of large “blue chip” stocks trading in the US and Europe, in line with the existing empirical literature on technical analysis that predominantly focuses on stocks that are the most likely to be followed by a large number of analysts. The sample period runs from January 1968 to March 2019, for a total in excess of 50 years. All stock market data was obtained from Bloomberg.

The dataset contains, among other fields, the daily prices of individual stocks trading in the three indices. Hereafter the following notation will be used: for a given stock, let $\texttt{Open}[t]$, $\texttt{Close}[t]$, $\texttt{High}[t]$ and $\texttt{Low}[t]$ be the open, close, high and low price at day $t = 1, \ldots, T$.

The first step in our methodology is the identification of local peaks and bottoms. Similarly to Dempster & Jones (2002), we apply a rolling window where each observation j is compared against a window defined by the ω_L preceding observations and the ω_R subsequent observations. As a result, our rolling window is not necessarily centered around the particular observation that is being assessed. Let p_i and b_i be the days that local peaks and bottoms appear respectively. For each sample stock, we identify the i^{th} local peak on day p_i if the high price of that day is greater than the prices that have been observed during the previous ω_L days and at least as high as the prices observed during the subsequent ω_R days, satisfying

$$\texttt{High}[p_i] > \max_{p_i - \omega_L \leq j < p_i - 1} \texttt{High}[j], \tag{1}$$

and

$$\texttt{High}[p_i] \geq \max_{p_i+1<j\leq p_i+\omega_R} \texttt{High}[j]. \tag{2}$$

Similarly, we identify a local minimum on day b_i if the low price of that day is lower than the prices that have been observed during the previous ω_L days, and at least as low as the prices observed during the subsequent ω_R days, satisfying

$$\texttt{Low}[b_i] < \min_{b_i-\omega_L\leq j<b_i-1} \texttt{Low}[j], \tag{3}$$

and

$$\texttt{Low}[b_i] \leq \min_{b_i+1<j\leq b_i+\omega_R} \texttt{Low}[j]. \tag{4}$$

The use of a non-fixed rolling window $[\omega_L, \omega_R]$ allows for a more flexible framework to identify local maxima and minima, compared to using a fixed-length rolling window. In this sense, the specific window length can be amended from one period to the other in order to identify trends that are shorter- or longer-term. In addition, the $[\omega_L, \omega_R]$ window does not necessarily need to be symmetric. For instance, one can select a post-day window that is narrower than the pre-day window (i.e. $\omega_L > \omega_R$) in order to place a greater emphasis on the analysis of trading signals shortly after the confirmation of the immediately previous trend.[3]

The next step in our methodolgy involves using these previously identified locals in order to determine the trend that precedes a Fibonacci retracement level. To do this, we adopt the definition of the market-technical trend proposed by Maier-Paape (2015), who defines an up-trend as a series of prices with monotonically increasing lows and strictly monotonically increasing highs. Following this principle, we identify an up-trend when at least the last two consecutive lows as well as the last two consecutive highs have been rising. More specifically, to identify an up-trend we search for a sequence of four interchangeable locals (b_1^*, p_1^*, b_2^* and p_2^*) that satisfy the following conditions

$$\texttt{High}[p_2^*] > \texttt{High}[p_1^*] \tag{5a}$$

$$\texttt{Low}[b_2^*] \geq \texttt{Low}[b_1^*] \tag{5b}$$

$$b_1^* < p_1^* < b_2^* < p_2^* \tag{5c}$$

Similarly, we identify a down-trend when at least the last two consecutive lows as well as the last two consecutive highs have been falling. In other words, we search for a sequence (p_1^*, b_1^*, p_2^* and b_2^*) such that

[3] Tsinaslanidis & Zapranis (2016) provide a detailed discussion of several alternative methods for identifying local peaks and bottoms.

$$\texttt{High}[p_2^*] \leq \texttt{High}[p_1^*] \tag{6a}$$
$$\texttt{Low}[b_2^*] < \texttt{Low}[b_1^*] \tag{6b}$$
$$p_1^* < b_1^* < p_2^* < b_2^* \tag{6c}$$

Once we have identified the preceding upward or downward trend, we continue by computing the respective Fibonacci Support and Resistance (S&R) levels.[4] In our empirical analysis, we consider the four most commonly adopted Fibonacci retracement levels, namely $\{L_k\}_{k=0}^{3} = \{100\%, 61.80\%, 38.20\%, 23.61\%\}$[5] and a rolling window $[\omega_L = 20, \omega_R = 10]$.[6] For each up-trend (defined by the four locals b_1^*, p_1^*, b_2^* and p_2^*), we transform the respective lows/highs and map them on to the four Fibonacci levels. More specifically, we compute the transformed lows and highs (R_L and R_H, respectively) as follows

$$R_{\text{L},t} = \frac{\texttt{High}[p_2^*] - \texttt{Low}[t]}{\texttt{High}[p_2^*] - \texttt{Low}[b_1^*]}, \tag{7}$$

and

$$R_{\text{H},t} = \frac{\texttt{High}[p_2^*] - \texttt{High}[t]}{\texttt{High}[p_2^*] - \texttt{Low}[b_1^*]}. \tag{8}$$

In order to avoid the issue of look-ahead bias, these calculations are based on prices that are observed after the end of the rolling window that had been previously used to identify the locals ω_R. In other words, equations (7) and (8) are estimated for time $t > p_2^* + \omega_R$. It is also worth noting that there is an upper limit for t too. For a given preceding trend, equations (7) and (8) are not calculated until the end of the time series. Rather, the algorithm stops when one of the following conditions has being met; (i) the entire candle moves to a negative area ($R_{\text{L},t} < 0$) which signals the continuation of the preceding

[4]The Fibonacci sequence is a series of numbers such that each number is given as the sum of the previous two numbers, i.e. $F_n = F_{n-1} + F_{n-2}, \forall n \in \mathbb{N}_{>1}$. In particular, the Fibonacci numbers are given by the sequence $\{F_n\}_{n=0}^{\infty} = \{0, 1, 1, 2, 3, 5, \ldots\}$. The analytical expression of the Fibonacci sequence can be written as

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n, \forall n \in \mathbb{N}_0.$$

[5]The four Fibonacci retracement levels result from the following expression

$$\lim_{n\to\infty} \frac{F_n}{F_{n+k}}, \quad \texttt{for} \quad k = 0, 1, 2, 3$$

[6]Results obtained with different values for ω_L and ω_R are qualitatively the same and, thus omitted for brevity but available upon request.

uptrend or (ii) the entire candle is above 120% ($R_{\text{H},t} > 1.2$) which signals a retracement beyond 100% and thus a trend reversal.[7]

The mirrored process can be used for the case of the preceding downward trend which is characterised by the four locals p_1^*, b_1^*, p_2^* and b_2^*. More specifically, in this case (7) and (8) become

$$R_{\text{L},t} = \frac{\texttt{Low}[t] - \texttt{Low}[b_2^*]}{\texttt{High}[p_1^*] - \texttt{Low}[b_2^*]}, \tag{9}$$

and

$$R_{\text{H},t} = \frac{\texttt{High}[t] - \texttt{Low}[b_2^*]}{\texttt{High}[p_1^*] - \texttt{Low}[b_2^*]}, \tag{10}$$

respectively, for time $t > b_2^* + \omega_R$ and until ($R_{\text{H},t} < 0$) ($R_{\text{L},t} > 1.2$).

We use the Fibonacci retracement levels as price zones rather than as precise price points (see also Zapranis & Tsinaslanidis 2012). On this issue, Bulkowski (2003) argues that *"Support and resistance are not individual price points, but rather thick bands of molasses that slow or even stop price movement"*. More specifically, we convert the individual Fibonacci price levels $\{L_k\}$ to price zones $\{Z_k\} \equiv \left[\mathcal{Z}_k^-, \mathcal{Z}_k^+\right)$ by adding and subtracting a fixed percentage ζ, i.e. $\mathcal{Z}_k^- = L_k - \zeta$ and $\mathcal{Z}_k^+ = L_k + \zeta$.[8] Overall, this approach results in the construction of a set of intervals of the form $\left[\mathcal{Z}_k^-, \mathcal{Z}_k^+\right) \Rightarrow \{x \in \mathbb{R} : \mathcal{Z}_k^- \leq x < \mathcal{Z}_k^+\}$. These intervals are constructed so that no two intervals have an interior point in common. To ensure no overlap between Fibonacci price zones, the fixed percentage ζ is constrained to take positive values that are lower than 7.295%.[9]

After having constructed the Fibonacci price zones, the last step of our methodology involves identifying hits. In general, a *hit* describes the case where prices enter a Fibonacci zone. Once this happens, we then classify this case as a *bounce* or as a *failure* depending on whether subsequent prices retrace back or not, respectively. For instance, consider first the case where a Fibonacci retracement level has been preceded by an up-trend. In order to evaluate the performance of $K = 4$ Fibonacci levels, we define $2K + 1 = 9$ price intervals (bins) $\{\mathcal{B}_j\}_{j=1}^{2K+1}$ by using $2K + 2$ edges $\{\mathcal{E}_j\}_{j=1}^{2K+2}$. In particular, for the specific

[7] The additional 20% in condition (ii) was set to allow for the assessment of the price behaviour on the 100% Fibonacci zone.

[8] For example, consider the specific case of $\zeta = 1\%$. In this case, the respective Fibonacci price zones are given as

$$\begin{aligned} \{Z_k\} &= \{[L_k - 1\%, L_k + 1\%)\} \\ &= \{100\% \pm 1\%, 61.80\% \pm 1\%, \ldots, 23.61 \pm 1\%\} \\ &= \{[99\%, 101\%), [60.8\%, 62.8\%), \ldots, [22.61\%, 24.61\%)\} \end{aligned}$$

[9] The upper bound of 7.295% for the fixed percentage ζ is determined by the minimum distance between any two consecutive Fibonacci levels. When we consider the first four Fibonacci levels, this minimum distance appears between L_2 and L_3 and is $L_2 - L_3 = 38.20\% - 23.61\% = 14.59\%$. Therefore, in order to avoid any overlap, the fixed percentage needs to satisfy $\zeta < 14.59\%/2 = 7.295\%$.

case of four Fibonacci retracement levels and a fixed percentage $\zeta = 1\%$, the bin edges are given by

$$\begin{aligned}\{\mathcal{E}_j\}_{j=1}^{10} &= \{-\infty, \mathcal{Z}_3^-, \mathcal{Z}_3^+, \mathcal{Z}_2^-, \mathcal{Z}_2^+, \mathcal{Z}_1^-, \mathcal{Z}_1^+, \mathcal{Z}_0^-, \mathcal{Z}_0^+, +\infty\} \\ &= \{-\infty, 22.61\%, 24.61\%, \ldots, 99\%, 101\%, +\infty\}\end{aligned} \tag{11}$$

We then group the lower and upper retracement levels $R_{\text{L},t}$ and $R_{\text{H},t}$ from (7) and (8) (hereafter collectively referred to as R_t) into these bins as follows

$$\mathcal{B}_j \ni R_t \begin{cases} \mathcal{E}_j \leq R_t < \mathcal{E}_{j+1}, & j \neq 1 \\ \mathcal{E}_j < R_t < \mathcal{E}_{j+1}, & j = 1 \end{cases} \tag{12}$$

For a given up-trend (where $\mathcal{Z}_k^-$ is above $\mathcal{Z}_k^+$), a support hit is identified when low prices penetrate from above the lower limit point $\mathcal{Z}_k^-$ of the Fibonacci zone $\{Z_k\}$.[10] Also, given the specific way in which the bins have been constructed, $\mathcal{B}_j$ would correspond to a Fibonacci zone only when j is an even number. To illustrate this, Table 1 presents the mapping of the main mathematical expressions used in defining hits onto the specific example of an up-trend with $K = 4$ and $\zeta = 1\%$.

[Table 1 about here.]

Furthermore we define the variables $B_{\text{L},t}$ and $B_{\text{H},t}$ to index the bin that contains $R_{\text{L},t}$ and $R_{\text{H},t}$ respectively, i.e. $\mathcal{B}_j \ni R_{\text{L},t} \Rightarrow B_{\text{L},t} = j$ and $\mathcal{B}_j \ni R_{\text{H},t} \Rightarrow B_{\text{H},t} = j$. It is worth noting that prices may hit more than one Fibonacci zone on the same day, especially if these zones are relatively close to one another (such as Z_2 and Z_3). For example, it is possible for prices to breach Z_3 and then hit Z_2, or to even breach both zones.[11] Therefore, in order to capture these cases of multiple hits on the same day, when $B_{\text{L},t} > B_{\text{L},t-1}$ a hit is assigned to every Fibonacci zone that corresponds to $\mathcal{B}_{j*}$, on day t when $B_{\text{L},t} \geq j^* > B_{\text{L},t-1} : j^*$ is even.

Finally, bounces and failures are defined as follows. After each hit, we find (i) the first time (if any) that the low price belongs to a lower bin and (ii) the first time (if any) that the high price belongs to a higher bin than the one that corresponds to the Fibonacci zone under evaluation. Then, a bounce refers to the case where event (i) occurs first, while a failure refers to the case where event (ii) occurs first.

Figure 1 presents an illustrative example of our methodology, applied on data for the stock of Home Depot (HD), which is listed in the Dow Jones index. A preceding up-trend, defined by a sequence of four locals (b_1^*, p_1^*, b_2^* and p_2^*), can be easily seen in the Figure. In particular, four Fibonacci levels and their corresponding zones (see Table 1) are plotted

[10] Figure 1 depicts an example of such a case, where $\mathcal{Z}_3^- = 22.61\%$ is located above $\mathcal{Z}_3^+ = 24.61\%$.

[11] For example, consider a low price at time $t-1$ that falls into bin $\mathcal{B}_1$ (i.e. $B_{\text{L},t-1} = 1$), and $R_{\text{L},t} \in \mathcal{B}_4$ (i.e. $B_{\text{L},t} = 4$). In this case, a hit is assigned to both Fibonacci zones Z_3 and Z_2. The assessment of whether prices bounced or penetrated these zones is carried out separately.

on the same graph. Figures 2, 3 and 4 focus on the 'A', 'B' and 'C' areas of Figure 1, and they depict with green triangles the cases where prices bounce on a Fibonacci zone and with red triangles the cases of failures. The direction of the triangle shows the direction prices follow after a hit. For instance, an upward green triangle shows a case where prices bounce on a support level, while a red upward triangle shows a case where prices penetrate a resistance level (failure). Similarly, green downward triangles depict bounces on resistance levels, while red downward triangles depict failures on support levels.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

3 Empirical results

3.1 Bounces on Fibonacci vs non-Fibonacci zones

We begin the empirical analysis by examining whether, for a given value of ζ, prices are more likely to bounce on a Fibonacci zone $\{Z_k\}$ compared to the probability of bouncing on a non-Fibonacci zone (hereafter denoted by $\{Z^*\}$). To this end, we estimate the following logit model

$$\ln\left(\frac{P_i}{1-P_i}\right) = \texttt{logit}\, P_i = \theta_1 + \theta_2 D_{\texttt{Fib}} + u_i \tag{13}$$

where P_i is the probability that, after a hit occurs, prices will bounce on the corresponding zone, and $D_{\texttt{Fib}}$ is a dummy variable that takes the value of one when prices hit on a Fibonacci zone and the value of zero otherwise. Given how the logit model has been structured, the reference category refers to cases where hits occur in a non-Fibonacci zone. The intercept θ_1 reflects the log-odds in favor of the price bouncing on any non-Fibonacci zone, while the slope θ_2 reflects how the log-odds in favor of a bounce change when prices bounce on a Fibonacci zone. Our null hypothesis is that prices are equally likely to bounce on a Fibonacci zone as they are to bounce on a non-Fibonacci zone. Therefore, a statistically significant θ_2 would constitute evidence against this null hypothesis, indicating a statistically different behavior of prices in $\{Z_k\}$ relative to $\{Z^*\}$.

One challenging aspect of estimating the logit model in (13) refers to the fact that, while Fibonacci zones are specific and finite, possible non-Fibonacci zones are infinite. As a result, identifying non-Fibonacci cases to use as an input in (13) is far from straighforward. To address this issue, we adopt the following bootstrap approach. First, we construct a sample of price hits where half of these hits are on the Fibonacci zones and the other half are on non-Fibonacci zones. To construct this sample, we randomly choose with replacement cases of preceding trends from all stocks composing the market index under consideration. For every randomly selected case, we consider the four Fibonacci zones $\{Z_k\}_{k=0}^{3}$ plus one non-Fibonacci zone Z^*, such that $L^* \sim \mathcal{U}(0,1)$ and $\mid L^* - L_k \mid \geq 2\zeta$, for $k = 0, 1, 2$ and 3. The latter condition is set to avoid Z^* overlapping with any Z_k, so as to ensure that it is indeed a non-Fibonacci zone. After recording the number of price bounces and failures in the constructed sample, the process is repeated until we have created a sample of size n that is composed by at least $n/2$ hits on Z_k and $n/2$ hits on Z^*. Then, we randomly select and discard hits from both categories in this sample until the final sample is composed of exactly $n/2$ hits on Z_k and exactly $n/2$ hits on Z^*. This final sample is used to estimate the logit regression in (13). Finally, this procedure is repeated 500 times, and the distribution of the estimated slopes θ_2 is used to construct bootstrapped 95% confidence intervals (CIs).

Figures 5, 6 and 7 present the bootstrapped 95% CIs for θ_2 with respect to stocks in the DOW, DAX and NASDAQ indices, respectively. The estimated CIs are based on various sample sizes (ranging from 500 to 10,000), with 500 bootstrap repetitions. Furthermore, we consider the special case of $\zeta = 0\%$, which corresponds to Fibonacci levels rather than zones, as a simple starting point before we turn our attention later to the effect of different levels of ζ on our empirical results. For robustness, we use three alternative ways of constructing CIs, namely the basic percentile approach, the studentized CI approach, and the normal approximation approach.

[Figure 5 about here.]

[Figure 6 about here.]

[Figure 7 about here.]

As can be seen from these Figures, the resulting CIs are qualitatively similar under all three approaches. More importantly, we cannot reject the null hypothesis of equal probabilities at the 5% significance level. Although increasing the sample size results in narrower confidence intervals due to the lower dispersion of the θ_2 distribution, all the reported 95% CIs still contain the null value of zero. In other words, the probability of prices bouncing on Fibonacci levels is statistically indistinguishable from the probability of bouncing on non-Fibonacci levels.

Moving from Fibonacci levels to zones, Table 2 reports the results of estimating the logit model in (13), but across different values of the fixed percentage ζ (namely 0%, 2%, 4%, and 6%). Bootstrapped confidence intervals are obtained using a sample size of 10,000 observations, across 500 repetitions. Similarly to the previous results on Fibonacci levels, prices appear to be equally likely to bounce of Fibonacci zones as they are to bounce on non-Fibonacci zones when we look at the first three values for ζ (0%, 2%, 4%). In particular, bootstrapped 95% CIs for θ_2 consistently contain the null of zero, indicating statistically indistinguishable log-odds of price bounces on Fibonacci zones relative to non-Fibonacci zones when we set ζ equal to either 0%, 2%, or 4%. Interestingly, we find a statistically significant decrease in the log-odds when we set ζ equal to the higher value of 6%. This finding constitutes even stronger evidence against using Fibonacci zones since it implies that, when wider S&R zones are considered, prices are in fact significantly more likely to bounce on non-Fibonacci zones compared to Fibonacci zones. We further explore the impact of setting the fixed percentage ζ at different levels in subsection 3.3.

[Table 2 about here.]

3.2 Trading rule profitability

Our previous empirical results showed that prices are at best as likely, if not actually less likely, to bounce on a Fibonacci zone compared to a non-Fibonacci zone. Therefore, these findings appear to cast substantial doubt on the merits of using Fibonacci levels or zones to predict the future movements of stock prices. In order to get a better understanding of the informational content of Fibonacci zones, we proceed by investigating the performance of a Fibonacci-based trading rule.

More specifically, we evaluate the performance of a trading rule that is based on Fibonacci zones against the benchmark of a trading rule that is based on non-Fibonacci zones. Under both trading rules, once a hit on a respective zone (Fibonacci or non-Fibonacci) has been identified on day $t-1$, a trading position is opened at the opening price offered on the following day t. The trading position is long when prices hit on a support zone, and short when they hit on a resistance zone. We evaluate performance by computing returns for multiple holding periods of $j+1$ days ($j = 0, \ldots 20$), assuming that positions for each holding period are closed at the closing price of that period's last day. Then, the $[j+1]$-period return $R_{[j+1]}$ is computed as

$$R_{[j+1]} = \mathcal{H}\left(\frac{\texttt{Close}[t+j] - \texttt{Open}[t]}{\texttt{Open}[t]}\right) \tag{14}$$

where $\mathcal{H}$ takes the value of 1 or -1 when the opening position of the trade is long or short, respectively. Similarly to Section 3.1, 500 samples of 10,000 hits each are randomly created from the components of the market index under consideration. Half of these hits are on Fibonacci zones with the other half on non-Fibonacci zones. Then, we compute the difference between the mean holding-period return of the Fibonacci and the non-Fibonacci trading rules across multiple holding periods.

Figure 8 plots the mean differences in the returns of the two trading rules across multiple holding periods when applied on stocks listed in the Dow Jones index. Each subplot depicts separately the mean return differences under four different values of ζ, namely 0%, 2%, 4%, and 6%.[12] Unsurprisingly, mean returns and return variance seem to be higher when we consider longer holding periods. Also, higher values of ζ are associated with slightly higher returns, although differences are statistically insignificant and even less evident when we convert total-period returns to equivalent mean daily returns in Figure 9.

More importantly, the differences in the mean returns of trading rules based on Fibonacci vs non-Fibonacci zones are statistically insignificant at the 5% level, as evidenced by bootstrapped CIs that consistently contain the null value of zero. In other words, an investor who trades according to hits on Fibonacci zones would have earned statistically the same mean return as another investor who simply trades based on hits on random non-Fibonacci zones. This finding of trading on Fibonacci zones failing to outperform the benchmark of trading on non-Fibonacci zones is consistent with our previously reported findings of prices generally being equally likely to bounce on either type of zone. Taken together, these empirical results seem to provide strong evidence for the rejection of Fibonacci zones as a profitable tool in technical analysis.

[Figure 8 about here.]

[Figure 9 about here.]

3.3 Analyst subjectivity

Our previous results cast substantial doubt on the practice of using Fibonacci levels, or zones, in technical analysis, considering how prices seem to be equally likely to bounce on non-Fibonacci zones and that a trading rule based on non-Fibonacci zones performs equally well as a Fibonacci-based trading rule. Notwithstanding this strong evidence against the use of Fibonacci zones in general, our results also show that performance depends on how wide these Fibonacci zones have been selected to be, which is determined by the choice for the fixed percentage ζ around the respective Fibonacci level.

We argue that ζ can be interpreted as a reflection of analyst (i) inaccuracy when drawing Fibonacci *levels* on a chart and (ii) subjectivity when assessing prices vis-a-vis Fibonacci levels. This visual process is bound to involve a significant level of inaccuracy and/or subjectivity from the analyst's side. More precisely, defining starting and ending points of the preceding trend inaccurately will produce, in turn, inaccurate Fibonacci levels and hence unreliable inferences. Furthermore, an accurate drawing of Fibonacci levels is not necessarily followed by an accurate and objective assessment of the price behaviour on these levels. For example, consider the case where prices retrace as they approach a given Fibonacci level (say 61.80%). An analyst may subjectively attribute

[12] Results are qualitatively the same when the trading rules are applied on stocks listed in the DAX 30 and NASDAQ 100 indices. These results have been omitted for brevity, but they are available upon request.

this behaviour to the Fibonacci level, even if prices actually retraced at the (relatively close, but non-Fibonacci) level of 61%. A similar statement in support of our argument can be found in, Abouloula et al. (2019) where the authors argue that *"The Fibonacci sequence is ideal mathematically, but this method releases noise when included in the trading graphics"*.

Then, the fixed percentage ζ when constructing zones around Fibonacci levels can serve as a measure of the subjectivity, or inaccuracy, inherent in an analyst's visual identification and assessment of a Fibonacci retracement level. In this sense, a tighter zone around a Fibonacci level would reflect a case where the analyst attaches a relatively high degree of certainty on the identification of the preceding trend and the assessment of price behaviour around Fibonacci levels. In contrast, a wider Fibonacci zone would be indicative of greater uncertainty and subjectivity around the visual identification and assessment of a Fibonacci retracement level.

We explore the effect of ζ on the probability that prices are bouncing on a support or resistance zone by estimating the following logit model

$$\ln\left(\frac{P_i}{1-P_i}\right) = \texttt{logit}\, P_i = \beta_1 + \beta_2 D_{0.02} + \beta_3 D_{0.04} + \beta_4 D_{0.06} + u_i \tag{15}$$

where P_i is the probability that, after a hit has occured, prices will bounce on the specific Fibonacci zone that is constructed using the respective value for ζ. When estimating (15), the reference category is the case where $\zeta = 0$. Furthermore, the dummy variables $D_{0.02}$, $D_{0.04}$, and $D_{0.06}$ take the value of one when ζ is equal to 0.02, 0.04, and 0.06, respectively, and the value of zero otherwise. In this setting, the intercept β_1 corresponds to the log-odds in favor of prices bouncing on a S&R zone when $\zeta = 0$ (i.e. the special case where zones and levels are identical). The slope coefficients β_2, β_3, and β_4 measure how the log-odds in favor of a bounce change when we use a fixed percentage ζ equal to 0.02, 0.04, and 0.06, respectively, when constructing Fibonacci zones, compared to the reference category of using Fibonacci levels (i.e. when $\zeta = 0$).[13]

We estimate equation (15) separately across the three sample markets (DOW, DAX, and NASDAQ). Table 3 reports the probabilities P_i that prices will bounce after a hit has occurred, for different values of ζ. The bounce probability P_i is given as

$$P_i = \begin{cases} e^{\beta_1} / \left(1 + e^{\beta_1}\right), & \zeta = 0 \\ e^{\beta_1+\beta_i} / \left(1 + e^{\beta_1+\beta_i}\right), & \zeta \neq 0 \end{cases} \tag{16}$$

The results presented in Table 3 suggest a positive relationship between ζ and the probability of prices bouncing on a level/zone. When considering Fibonacci levels (i.e.

[13]The fixed percentage ζ is obviously a continuous variable, taking values in the range $(0, 7.295\%)$. Nevertheless, in our empirical analysis we use four distinct values for ζ (namely 0, 0.02, 0.04, and 0.06) for simplicity, since analysts would be unlikely to consider an infinite number of values when selecting their preferred level of ζ.

when $\zeta = 0$), the probabilities of a bounce are equal to 49.08%, 51.89%, and 49.84% for stocks trading in DOW, DAX and NASDAQ, respectively. The slope coefficients that capture the change in probabilities from setting ζ to values different than the reference case of $\zeta = 0$ (i.e. the coefficients β_2, β_3 and β_4) are all found to be positive and statistically significant at any meaningful significance level. Therefore, moving from Fibonacci levels to Fibonacci zones seems to result in higher log-odds of prices bouncing on a S&R level. Another interesting finding is that, in all markets considered, the slope coefficients are monotonically increasing across ζ (i.e. $\beta_1 < \beta_2 < \beta_3 < \beta_4$), consistent with wider Fibonacci zones being associated with higher bounce probabilities.

[Table 3 about here.]

An explanation of these findings can be provided by considering the specific approach that we use to identify hits, bounces and failures (Section 2). For example, consider the case of a support hit. The minimum requirement for a support hit to be identified is that low prices breach downwards the upper bound of the zone on a graph. Once this breach has been observed, a bounce takes place if the low price retraces back to exceed the upper bound of the zone. Symmetrically, a failure is identified if the high price (i.e. the entire candle) moves below the lower bound of the zone. For each hit, only one outcome is recorded (either a bounce or a failure), whichever occurs first. In this setting, a larger value of ζ will correspond to a wider support zone. Therefore, after a support hit where low prices are below the upper bound of the zone, but still above the corresponding level, a bounce will require a smaller price movement than a failure to be identified. In other words, the probability of a bounce is greater than the probability of a failure in such a case. Figure 4 depicts a characteristic example of this case.

Overall, our empirical results seem to provide strong support for the intuitive hypothesis that the probability of identifying a bounce increases significantly when we move from Fibonacci levels to Fibonacci zones, and even more so when we increase the width of these zones by using a higher value for the fixed percentage ζ. However, this higher probability of identifying bounces does not necessarily result in a higher profitability from trading on these bounces, as was highlighted by our previous empirical findings. Ultimately, analysts are naturally more likely to identify what they perceive to be profitable trading opportunities as they consider wider Fibonacci zones that reflect higher subjectivity/inaccuracy, but at no significant improvement in their subsequent trading performance.

4 Conclusion

In this paper, we empirically examine the performance of Fibonacci retracements, a widely applied tool in technical analysis. In order to account for the inherent subjectivity associated with analysts identifying price trends and manually drawing Fibonacci levels on a chart, we propose a new algorithmic approach of constructing zones around Fibonacci levels. Overall, our empirical results cast substantial doubt on the merits of using Fibonacci retracements as a stand-alone technical trading tool.

Using a sample of three main equity indices, we document that stock prices are equally, if not even less, likely to bounce on Fibonacci Support and Resistance levels compared to randomly selected non-Fibonacci levels. When we consider wider Fibonacci zones, reflecting a greater degree of analyst subjectivity or inaccuracy, the probability of price bounces increases, but it is still statistically indistinguishable from the probability of prices bouncing on non-Fibonacci zones. More importantly, we find that a trading rule that is based on Fibonacci zones cannot outperform a simple banchmark rule that trades based on randomly selected non-Fibonacci zones. Taken together, these empirical results provide strong support against the use of Fibonacci Support and Resistance levels.

The finding that a greater degree of subjectivity, in terms of adopting wider Support and Resistance zones, is more likely to result in identifying perceived bounces potentially explains why Fibonacci zones are still widely used by practitioners, even in the absence of empirical evidence that validates their merits. In this sense, our paper contributes to the long-standing debate among academics and practitioners on whether technical trading rules can lead to efficient price forecasts and, by extension, profitable trading opportunities. In this debate, our results provide further support for the extensive stream of research arguing that technical trading rules fail to consistently offer profits in excess of their respective exposure to well-known sources of risk.

We hope that our results will motivate further research into the performance of technical trading tools. For instance, an interesting avenue for future research could be to explore the performance of alternative algorithms for identifying locals and/or defining a preceding trend when establishing S&R levels. Furthermore, our proposed approach for constructing zones around Support and Resistance levels is flexible and it can be applied to other technical rules, allowing for a better understanding of the role of analyst subjectivity in the performance of technical trading tools.

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Table 1: Notation mapping: uptrend case, $K = 4$ and $\zeta = 1\%$.

$\{L_k\}$	$\{Z_k\} = \left[\mathcal{Z}_k^-, \mathcal{Z}_k^+\right)$	$\{\mathcal{E}_j\}_{j=1}^{2K+2}$	$\{\mathcal{B}_j\}_{j=1}^{2K+1}$
$L_0 = 100\%$	$Z_0 = [99\%, 101\%)$	$\{\mathcal{E}_8, \mathcal{E}_9\}$	$\mathcal{B}_8$
$L_1 = 61.80\%$	$Z_1 = [60.8\%, 62.8\%)$	$\{\mathcal{E}_6, \mathcal{E}_7\}$	$\mathcal{B}_6$
$L_2 = 38.20\%$	$Z_2 = [37.2\%, 38.2\%)$	$\{\mathcal{E}_4, \mathcal{E}_5\}$	$\mathcal{B}_4$
$L_3 = 23.61\%$	$Z_3 = [22.61\%, 24.61\%)$	$\{\mathcal{E}_2, \mathcal{E}_3\}$	$\mathcal{B}_2$

Notes: This Table presents the mapping of the main mathematical expressions used in defining hits onto the specific example of an up-trend with $K = 4$ Fibonacci levels and a fixed percentage $\zeta = 1\%$ applied when constructing Fibonacci zones. The variable L_k denotes a Fibonacci level, while Z_k denotes the respective Fibonacci zone. $\mathcal{B}_j$ denotes the price intervals (bins) used to evaluate these Fibonacci zones, while $\mathcal{E}_j$ denotes the respective bin edges.

Table 2: Bounce probabilities on Fibonacci vs non-Fibonacci zones

Panel A: DOW				
ζ	$\overline{\theta}_1^*$	95% CIs	$\overline{\theta}_2^*$	95% CIs
0.00	-0.027720	$[-0.09367, 0.033603]$	-0.01575	$[-0.09213, 0.064036]$
0.02	0.442787	$[0.385504, 0.514240]$	-0.03770	$[-0.12375, 0.043438]$
0.04	0.775993	$[0.709090, 0.848250]$	-0.05201	$[-0.14229, 0.041220]$
0.06	1.075306	$[0.993608, 1.154874]$	-0.10782	$[-0.20144, -0.01350]$
Panel B: DAX				
ζ	$\overline{\theta}_1^*$	95% CIs	$\overline{\theta}_2^*$	95% CIs
0.00	0.095707	$[0.026402, 0.157122]$	-0.02201	$[-0.1034, 0.056819]$
0.02	0.535797	$[0.463314, 0.611136]$	-0.02687	$[-0.10614, 0.054861]$
0.04	0.861315	$[0.790786, 0.937528]$	-0.04294	$[-0.12535, 0.041686]$
0.06	1.150082	$[1.064765, 1.226469]$	-0.10839	$[-0.20294, -0.00313]$
Panel C: NASDAQ				
ζ	$\overline{\theta}_1^*$	95% CIs	$\overline{\theta}_2^*$	95% CIs
0.00	0.018314	$[-0.04321, 0.080043]$	-0.02206	$[-0.10083, 0.054405]$
0.02	0.476861	$[0.412971, 0.539947]$	-0.04202	$[-0.12430, 0.037871]$
0.04	0.822207	$[0.755611, 0.894413]$	-0.07847	$[-0.16984, 0.020357]$
0.06	1.124646	$[1.044929, 1.213966]$	-0.13801	$[-0.24347, -0.03946]$

Notes: This Table reports the results from estimating a logit regression of bounce probabilities in Fibonacci and non-Fibonacci zones. The results presented refer to estimating the following logit model

$$\ln\left(\frac{P_i}{1-P_i}\right) = \texttt{logit}P_i = \theta_1 + \theta_2 D_{\texttt{Fib}} + u_i$$

where P_i is the probability that, after a hit occurs, prices will bounce on the corresponding zone, and $D_{\texttt{Fib}}$ is a dummy variable that takes the value of one when prices hit on a Fibonacci zone and the value of zero otherwise. The logit regression is estimated separately for different values of the fixed percentage ζ, namely 0.00, 0.02, 0.04, and 0.06. The Table reports the mean intercept, $\overline{\theta}_1^*$, and the mean slope, $\overline{\theta}_2^*$, across all the logit estimations, as well as the associated bootstrapped 95% confidence intervals (CIs). Panels A, B, and C present the results when considering stocks in the Dow Jones, DAX, and NASDAQ indices, respectively. The sample period runs from January 1968 to March 2019.

Table 3: Bounce probabilities across different values for ζ

	Estimate	SE	t-stat	p-value	$P_i(\%)$
Panel A: DOW					
(Intercept)	-0.0370	0.0092	-4.033	0.000	49.08
β_2	0.4705	0.0130	36.162	0.000	60.67
β_3	0.7987	0.0132	60.709	0.000	68.17
β_4	1.0434	0.0134	77.618	0.000	73.23
χ^2-stat vs. constant model: 7.02e+03, p-value = 0					
Panel B: DAX					
(Intercept)	0.0755	0.0121	6.258	0.000	51.89
β_2	0.4760	0.0172	27.751	0.000	63.45
β_3	0.8021	0.0175	45.935	0.000	70.63
β_4	1.0342	0.0179	57.900	0.000	75.21
χ^2-stat vs. constant model: 3.93e+03, p-value = 0					
Panel C: NASDAQ					
(Intercept)	-0.0064	0.0061	-1.0476	0.295	49.84
β_2	0.4733	0.0087	54.326	0.000	61.46
β_3	0.7894	0.0088	89.379	0.000	68.63
β_4	1.0372	0.0090	115.170	0.000	73.71
χ^2-stat vs. constant model: 1.54e+04, p-value = 0					

Notes: This Table reports the results of a logit regression of the probability of prices bouncing against dummy variables for different values for the fixed percentage ζ when constructing Fibonacci zones and using $\omega_L = 20$, $\omega_R = 10$. The results presented refer to estimating the following logit model

$$\ln\left(\frac{P_i}{1-P_i}\right) = \texttt{logit}P_i = \beta_1 + \beta_2 D_{0.02} + \beta_3 D_{0.04} + \beta_4 D_{0.06} + u_i$$

where P_i is the probability that, after a hit has occured, prices will bounce on the specific Fibonacci zone that is constructed using the respective value for ζ. The reference category is the case where $\zeta = 0$. The dummy variables $D_{0.02}$, $D_{0.04}$, and $D_{0.06}$ take the value of one when ζ is equal to 0.02, 0.04, and 0.06, respectively, and the value of zero otherwise. The Table reports the estimated intercept and slope coefficients, their standard errors, t-statistics, p-values, and the expected probability P_i. We also report the model's χ^2-statistic (and its p-value) against the constant model. Panels A, B, and C present the results when considering stocks in the Dow Jones, DAX, and NASDAQ indices, respectively. The sample period runs from January 1968 to March 2019.

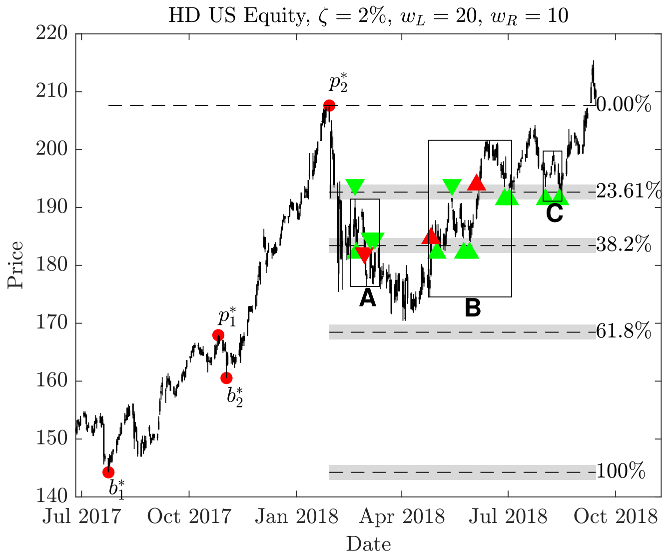

Figure 1: An illustrative example of bounces and failures on Fibonacci zones identified after a preceding uptrend defined by the sequence of b_1^*, p_1^*, b_2^* and p_2^*. Frames '**A**','**B**' and '**C**' are illustrated separately in Figures 2, 3 and 4.

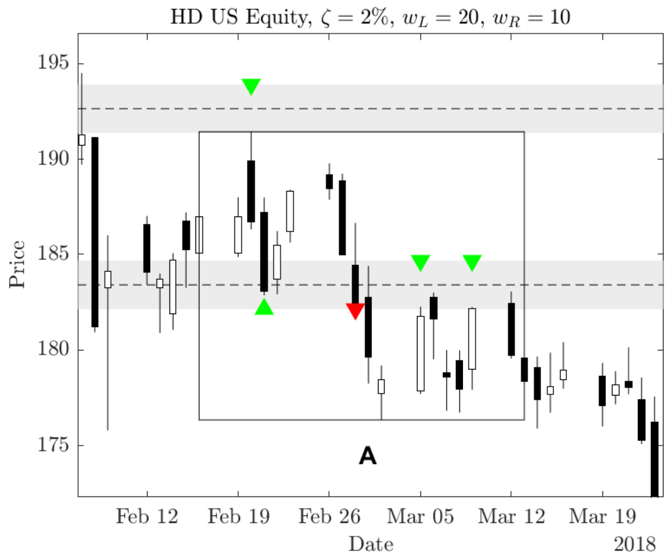

Figure 2: Frame '**A**' of Figure 1. Green triangles indicate bounces, while red triangles indicate failures. The direction of the triangle shows the direction prices follow after hitting a Fibonacci zone. In this frame, prices bounce one and three times on the 23.61% and the 38.2% Fibonacci zones, respectively. In addition, prices failed to bounce the 38.2% Fibonacci zone one time.

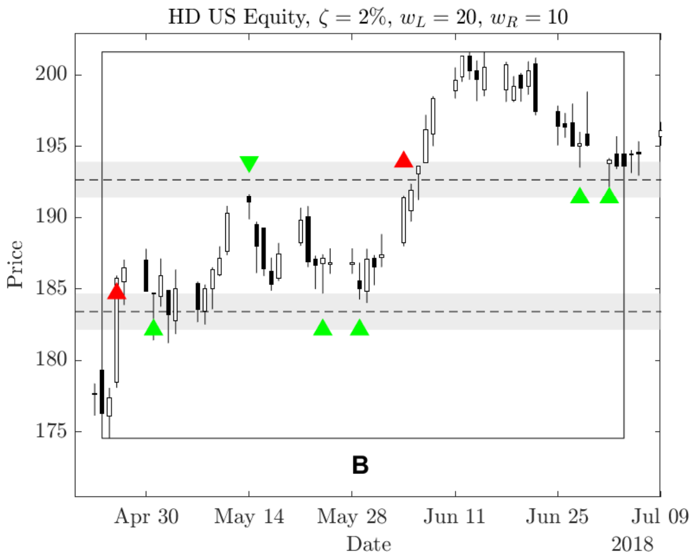

Figure 3: Frame '**B**' of Figure 1.Details as for Figure 2.

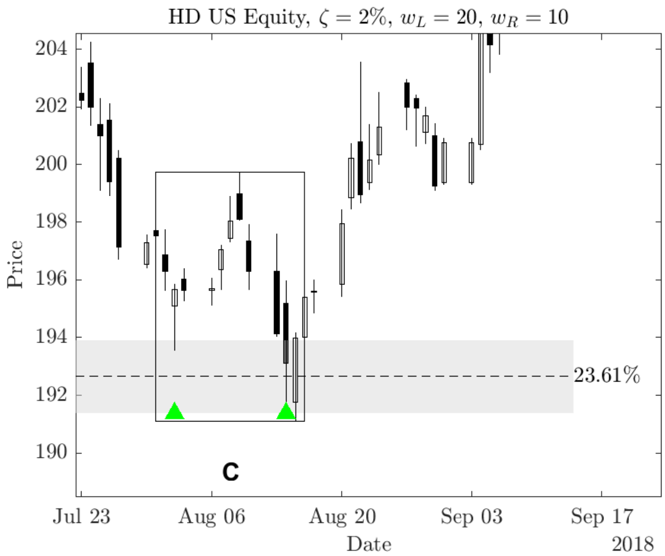

Figure 4: Frame '**C**' of Figure 1. Details as for Figure 2.

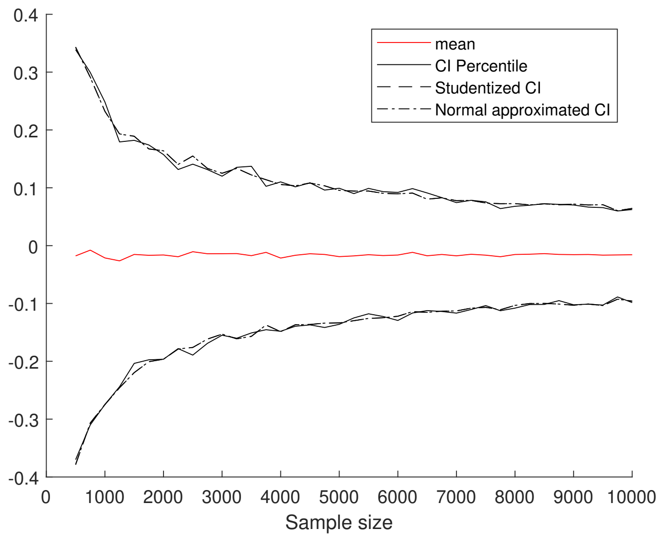

Figure 5: Bootstrapped 95% CIs for θ_2. For various sample sizes (500 to 10,000) of hits, model in (13) is estimated 500 times and mean values of θ_2 along with the corresponding 95% confidence intervals are illustrated. Results presented here correspond to price hits from stocks listed in Dow Jones index.

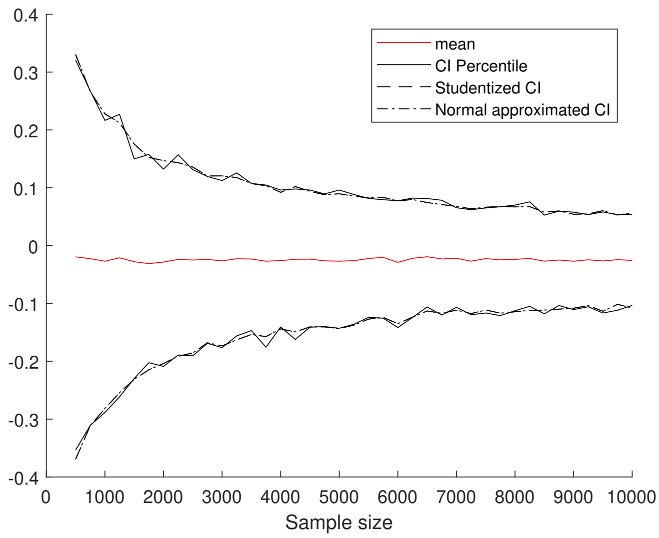

Figure 6: Bootstrapped 95% CIs for θ_2. Details as for Figure 5 but for stocks listed in DAX index.

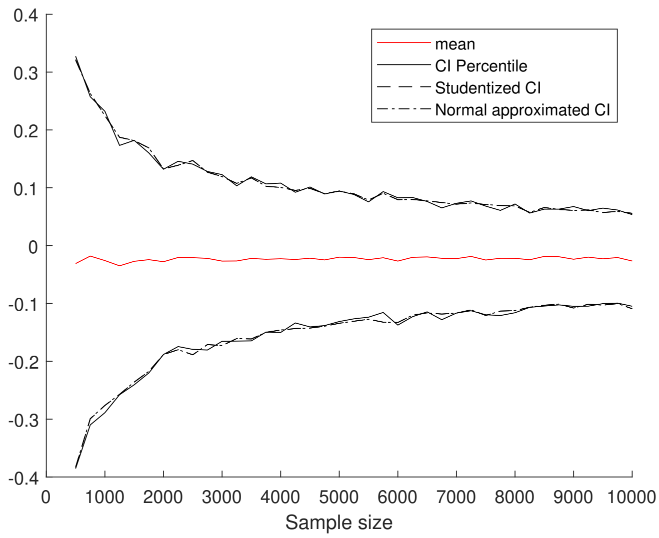

Figure 7: Bootstrapped 95% CIs for θ_2. Details as for Figure 5 but for stocks listed in NASDAQ index.

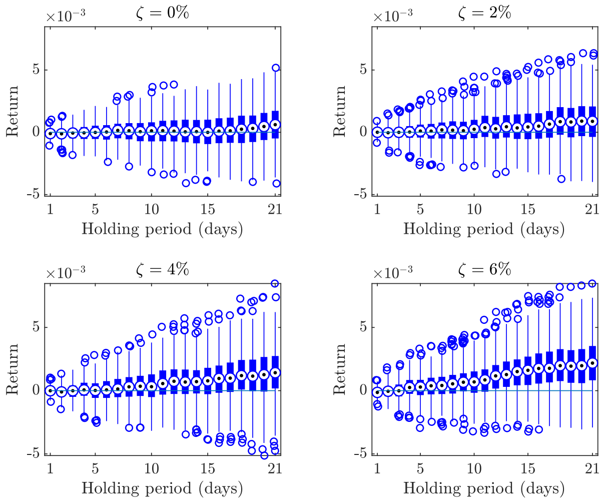

Figure 8: The effect of ζ on multi period returns for different holding periods. Each subplot presents the distributions (as boxplots) of mean differences between the returns obtained from trading on Fibonacci zones against those obtained from trading on non-Fibonacci zones. Each boxplot corresponds to a different holding period and each subplot to a different ζ value.

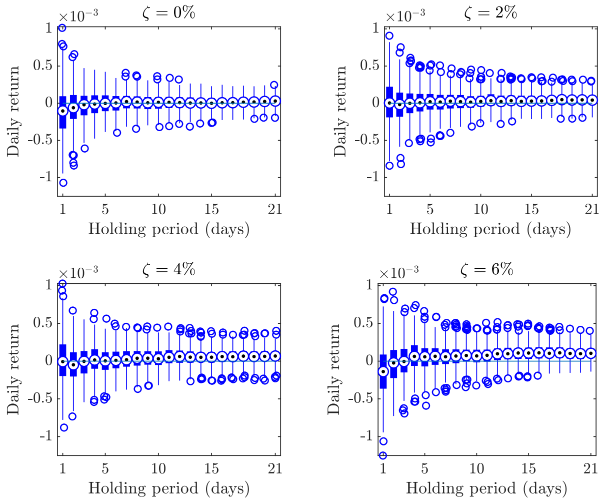

Figure 9: The effect of ζ on daily equivalent returns for different holding periods. Details as for Figure 9 but total-period returns are converted to equivalent mean daily returns.