Recovering the Vestiges of the Old Meridian Arc of Paris in the Valencian Community of Spain

Suzanne V. Débarbat[1], Luis García-Asenjo[2], Sergio Baselga, M.ASCE[3*], Pascual Garrigues[4] and Xavier Navarro[5]

Affiliations:

[1]Honorary Titular Astronomer, Observatoire de Paris SYRTE, 61 Av de l'Observatoire, 75014 Paris, France; Titular Member, Bureau des longitudes, 23 quai de Conti, 75006 Paris, France. Email: Suzanne.Debarbat@obspm.fr

[2]Associate Professor, Universitat Politècnica de València, Camino de Vera s/n, 46022 València, Spain, Email: lugarcia@cgf.upv.es

[3]Associate Professor, Universitat Politècnica de València, Camino de Vera s/n, 46022 València, Spain, Email: serbamo@cgf.upv.es ([*]Corresponding author)

[4]Associate Professor, Universitat Politècnica de València, Camino de Vera s/n, 46022 València, Spain, Email: pasgarta@cgf.upv.es

[5]Director, Institut Cartogràfic Valencià, C/ Democracia, 77, 46018, València, Spain, Email: navarro_xav@gva.es

Abstract

The paper describes the research carried out both to obtain the location and to identify possible vestiges of the old geodetic benchmarks used by the expedition formed by French geodesists Arago and Biot and Spanish geodesists Chaix and Rodríguez, to carry out the prolongation of the meridian arc of the Paris Observatory in the Valencian Community in the early 19th century to define the meter as the base unit of length of the International System of Units. Geodetic, historical and field work methodologies were integrated in the study in order to obtain scientifically sound results, which include the determination of coordinates of the original geodetic benchmarks as well as successful identification of several remains. An outstanding typical error of 0.4" for the old angular measurements can also be deduced from the analysis. Contrariwise, several old altitudes were found to have errors or considerably lower accuracies than those expected, around 1 m.

Keywords: History of geodesy, Meter, Metrology, Meridian arc, Geodetic networks, Reference systems.

Introduction

After the pioneering works in geodetic triangulation by Snellius (1580-1626) in the Netherlands, and Picard (1620-1682) in France, enabling them to determine the length of meridian arcs, the expeditions of 1736 to Lapland, led by Maupertuis, and to Ecuador, led by La Condamine, measured lengths of meridian arcs and firmly concluded the Earth's spheroidal shape with polar flattening. In 1791 the French National Assembly approved the definition of the length unit of the new International System of units, the meter, as the length of one ten-millionth of the quadrant of the Earth along the meridian of the Paris Observatory. The realization of this definition, however, had to be done with the information available at the time: the required knowledge of the Earth's gravitational field and the deflection of the vertical were not yet available (Vaníček and Foroughi, 2019; Alder 2003).

Méchain and Delambre were commissioned with the measurement campaigns to define the meter, whose details were published in three volumes: Delambre (1806, 1807 and 1810). The measurements by Méchain in the southern part, from Rodez to Barcelona, and those by Delambre in France crystallized in the definition of the "Final meter", established in 1799. A few years later, from 1806 to 1809, the measurements were extended southward in Spain to Formentera –an island in the western Mediterranean Sea– by Biot and Arago aided by Chaix and Rodríguez (Biot and Arago, 1821), so that the meridian arc measured was symmetrical with respect to 45º (Bigourdan 1900, Ten 1996); roughly, from latitude 51º in Dunkerque to latitude 39º in Formentera.

The angular observations were done by using a newly devised instrument, the Borda's repeating circle constructed by Lenoir, Fig. 1. This circle, having a diameter of only one foot (32.5 cm), was easier to use than the English theodolite. According to Débarbat (2004) the instrument, which had two glasses, one attached to the limbus the other serving as an alidade, presented the definite advantage of possible repetition of angular measurements leading to an improvement of precision by a factor up to ten.

Note that in Fig. 1 the instrument is prepared to measure horizontal angles. In addition, one can see in Fig. 2 the same instrument in disposition to measure vertical angles, after controlling verticality by the use of a plumb, as well as other representation of the instrument in Débarbat (2004), which enabled the user to measure angles in any desired plane. In the French expeditions to determine the length of the meter it was also used for latitude determination after measuring the zenith angles of a set of stars when crossing the meridian.

In the present paper we are aimed at finding the location of the geodetic benchmarks, as well as their possible remains, used in the expedition led by Arago and Biot to prolong the Paris meridian in the Valencian Community, including Formentera and the rest of the Balearic Islands as well.

The available documentation, most notably Biot and Arago (1821) and Arago (1859), includes the measurements taken, as well as the adjusted angles for the triangles. Fig. 3, taken from Arago (1859), shows these triangles, although Arago himself eventually discarded the triangle number 17 to Mallorca (in the bottom-right part of the plot) due to poor geometry.

Knowing the present location of several of these geodetic marks would permit to obtain the rest of the marks conforming the network. However, none of these locations is known with certainty at present. This situation is even more complicated since questions like the deflection of the vertical, or the distinction between the geoid and the ellipsoid, were not known at the time. The relatively poor geometric configuration, with little redundancy, the substantial length of observations, subject to the effects of atmospheric refraction are additional hampering factors.

In views of these limitations, we pursued the following strategy:

1. Geodetic calculations based on several different assumptions (to be described in the next section) for the determination of initial ETRS89 coordinates with presumable accuracy of a few meters that allow the field reconnaissance and documentation work of possible remains of the old geodetic marks, both for those located in the Valencian Region (object of this research) and for those framing these triangles (stations in Balearic Islands, to the east, and stations in Catalonia, to the north).
2. Stakeout of the ETRS89 coordinates obtained and exhaustive reconnaissance within a radius of about five meters. Each potential vestige located in this phase, whether in the form of a mark, a

cluster of stones or semi-buried cut stones, was surveyed using GNSS techniques. Additionally, they were photographically documented and, for those stations in the Valencian Region, an aerial photogrammetric survey using RPAS (Remotely Piloted Aircraft System, commonly referred to as drone) was performed. Where possible, information was also gathered from local authorities, chroniclers, or groups familiar with certain details of the geodesic expedition to complement or shed some light on the search.

3. Each of the possible remains or indicative clues of the possible situation of any old observation point led to a new readjustment of the network. Eventually a final solution was obtained, which is estimated to have an accuracy of around 1 m, as concluded from the accuracy obtained for the old measurements and the good agreement with the locations of the vestiges found.

The resulting coordinates may be employed in the future for the installation of commemorative plaques by authorities and the commandment of excavation works where deemed appropriate. In any case, these actions are outside the scope of this paper.

The following sections of the manuscript are dedicated to the development of the strategy outlined above as well as the discussion of the results obtained.

Initial set of coordinates

The first objective is to obtain a set of ETRS89 coordinates that allows for the field reconnaissance of possible vestiges of the old geodetic marks within a few meters. We searched for these coordinates before going to the field by using the following assumptions:

1. We can use the final adjusted angles of the observed spherical triangles and old mean sea level (MSL) heights published in Arago (1859). Only planimetric calculations will be carried out while the old MSL heights will be used only as check values for the analysis or the search of possible vestiges. The geodetic coordinates published therein can also be of use, although they refer to an old reference system with uncertain definition.

2. We assume that some of the ancient vertices were possibly reused in the posterior geodetic network (still preserved today). If there were at least two common vertices with the present geodetic network, that would make it possible to calculate the ETRS89 coordinates of the rest of the old marks by determining the four degrees of freedom of a two-dimensional coordinate system transformation (latitude and longitude coordinates of the origin, scale and network orientation). Alternatively, the only possibility to succeed in the endeavor would come from the unequivocal determination of vestiges of at least two geodetic marks of the old network during field reconnaissance.
3. The posterior visual inspection of the locations obtained must corroborate their possible validity. Otherwise, if inconsistencies appear with respect to the actual terrain, the coordinates will not be accepted.

It is important to mention that the expedition only observed a chain of triangles (recall previous Fig. 3), so that it was not necessary to have good visibility in all directions, but only in those in which they needed to observe. In other words, it was not necessary to locate the vertices on the ridges, where it is usually windy, but a few meters below, where the measurements could be carried out under better conditions. In general, the vertices located on the Peninsula should be on slopes facing east, while those located on the islands should face west.

As previously mentioned, we also considered to be quite likely that, out of a total of 18 vertices, at least two coincided or were within a few meters of those used in the posterior networks: it seems much probable that in the late 19th century the first Spanish geodetic network by Ibañez de Ibero, first president of the International Geodetic Association (Soler 1997 and Soler and Ruiz-Morales 2006), inherited some of these locations, and some of them still lingered into its densification in the 20th century leading to the National Geodetic Network (Instituto Geográfico Nacional 2020a).

These assumptions established, we followed different tentative approaches to obtain the first set of approximate coordinates. The first of these approaches consisted in a crude Helmert similarity transformation between the coordinates published in Arago (1859) (Table 1) and those of possibly coincident or nearby points belonging to the present geodetic network (Table 2). It is worth noting that the original names given by the French expedition are used in Table 1, even if they do not match the current names or spelling.

The coordinates of Tables 1 and 2 are not directly comparable: first, the old longitudes are referred to the Paris meridian (whose longitude in the ETRS89 system is 2º20'13.95"); second, they are assumed to be referred to Delambre 1810 ellipsoid (of semi-major axis 6376985 m and flattening 1/308.64, Mugnier 2001) whose orientation with respect to the current GRS80 is not known. In order to establish a possible correspondence between the old and current vertices, all the old longitudes were shifted, preserving the latitudes, to later check which of the old points were less than 1000 m away of the current ones. The subsequent Helmert transformation produced quite ill-determined results (e.g. uncertainties of the order of 25 m for shifts), not conclusive enough for the task of identification of vestiges, so we opted for a slightly different strategy.

The second approach is based on an initial arbitrary adjustment followed by the use of Helmert similarity transformations. Hence, the adjusted horizontal angles published in Arago (1859) were used in a least squares adjustment on the GRS80 ellipsoid fixing the ETRS89 coordinates of two arbitrarily (in the sense that they may not have been reused) chosen vertices: Montserrat and La Mola. These were chosen because they are located in the network extremes (Montserrat to the North, La Mola to the Southeast), so that in case they had not been reused the corresponding displacements would produce an error in the scale and orientation of the network inferior to the one that would have been obtained for vertices located more to the center of the network. It is important to mention that the resulting standard error for the adjusted angles was only 0.42" leading to positional uncertainties from 0.4 to 3.5 m. Then a Helmert transformation was performed and after iterative elimination of the points with higher residuals and computation of new Helmert transformations a final transformation was found with uncertainties below 1 m for shifts and residuals in the range of 0.00 to -0.90 m with an average value of 0.5 m, which maintained 7 (Lleberia), 13 (Espadan), 16 (Campvey) and 17 (Formentera) as common points with the present network. Visual inspection of the resulting locations by using aerial images from the National Aerial Images Program (Instituto Geográfico Nacional 2020b) confirmed their plausibility. The resulting coordinates, to be used in the subsequent field reconnaissance are shown in Table 3, where the ellipsoidal heights have been obtained as the old MSL heights plus the geoid undulation obtained from the official model in Spain, EGM08-REDNAP (Instituto Geográfico Nacional 2020c).

A third approach, completely different from the former two, was also followed. The results of this brute-force approach, easy to implement in a computer, were perfectly consistent with the former ones leading to Table 3. For the sake of brevity, this third approach is only briefly outlined here with no numeric results given. As previously assumed, several old geodetic benchmarks coincide with the present ones. We take first the (unrealistic) assumption that all the old marks are coincident with the present geodetic benchmarks and perform a least squares adjustment of the angles published in Arago (1859) fixing the current ETRS89 coordinates of the presumed coincident benchmarks. The results are clearly unsatisfactory, so we iteratively eliminate one (then two, three, etc.) fixed points, computing the corresponding least squares adjustments for all possible combinations, until a clear feasible solution emerges: the one retaining points 7 (LLEBERIA), 13 (ESPADAN), 16 (CAMPVEY) and 17 (FORMENTERA) as common points between the old and the present networks. The resulting coordinates, with discrepancies below 1 m with respect to those in Table 3, were considered perfectly equivalent for practical purposes, which reinforced our confidence in these coordinates, reliable enough within an error margin of a few meters to start the field reconnaissance phase.

Field reconnaissance

Mongo was the first location to be inspected for several reasons. First, it is a point that during the first phase did not participate neither as a fixed point in the adjustments nor as a known point in the transformation, being therefore a good control point to evaluate the quality of the coordinates determined for field recognition. Second, it is located more than 500 m away from the current geodetic benchmark in use and outside the busiest route for hikers, so there might be hopes for finding some vestiges. Third, it is located at the southwest end of the network, so provided the point was located, the MONGO – FORMENTERA axis would allow for the knowledge of some representative initial values for the scale and orientation of the network. Fig. 4 shows the first stakeout of the old Mongo location with the purpose of photogrammetric recording. The pole with a yellow ball at its end indicates the point whose coordinates were determined in the previous phase as the most probable situation of the observation point. About a meter away, a mound of stacked stones can be seen. About two meters to the right of the stone mound there is also an old landmark of unknown origin. In the left part of the image the remains of the stone hut that served as a refuge during the measurements, according to Biot and Arago (1821), which is still named as Ca Biot (i.e., Biot's home) in local maps, can also be seen.

Incidentally, it has to be mentioned that since this was the first location visited and the aerial photogrammetric survey using RPAS was not done at the time, the location was revisited after computation of the (almost) final set of coordinates. Then the RPAS flight was performed and the computed coordinates were surveyed with the unequivocal result that the pile stone was indeed the location of the old observation site. These coordinates were also held fixed for the final adjustment.

Once the location of the old MONGO point was recognized, it was decided to visit the sites located in the Balearic Islands, starting with FORMENTERA as it is the best documented of them all. This point was believed to be coincident in planimetric coordinates with the one in the present network although there was a significant height discrepancy between them. Soon was evident the reason for this difference: in 1868 Ibañez de Ibero built MOLA benchmark for the primitive Spanish geodetic network as accurately as possible on the same vertical where a small millstone had been found buried, which was the buried witness used here by the French expedition in case the observing site had to be reused (Ruiz Morales 2018). The planimetric location was retained for the present geodetic network when a geodetic pillar was built but, since the place was inside a private property, indeed inside a house, the geodetic pillar was built on the roof of the house. Therefore, the present geodetic point MOLA is considered to be located in the same planimetric location as the old FORMENTERA (LA MOLA) and its current official ETRS89 coordinates can be considered fixed for the subsequent adjustments.

Also in the Balearic Islands, CLOP DE GALAZO was visited next: the reconnaissance coordinates resulted to be close to the wall of the remains of a stone hut similar to the one found in MONGO and no additional vestiges of interest were found. It seems plausible that the observing site was inside the hut (observing through a window) due to the usually quite strong winds in the place as well as the observed disposition of windows in the directions of FORMENTERA and DESIERTO DE LAS

PALMAS/MONT-SIA. In CAMPVEY a pile of stones like the one in MONGO was found some 3 m off the current geodetic pillar. The pile of stones is placed over an artificial ground whereas the current geodetic benchmark is over the natural ground. The visual inspection and the similarity with MONGO, however, led us to assume that the pile of stones was indeed the original observation site, which was compatible with the coordinates computed for the reconnaissance.

Regarding the locations in Catalonia, one place resulted of singular interest: LLEBERIA. First observed by Méchain in 1803 and later re-observed in the geodetic prolongation to the South by Arago in 1807, we can read in Biot and Arago (1821) – our translation – first, written by Méchain: "The tent was placed in the highest part of the mountain, at the edge of a rung situated to the north; west of the station, and a short distance away, there is a hole in which there was a fire, and several deep crevices" and, second, four years later written by Arago: "On leaving the station of Lleberia, Mr. Mechain had planted a stake in the ground, the center of which corresponded vertically to that of the station [...] had also dug on large stones which are part of the ground, crosses whose distance from the center of the station is equal to 4 feet 11 inches 4 lines; the first was in the direction of San Carlos; the second in the direction of the hole where the fire was made, the two others were diametrically opposed to these first". During our field reconnaissance we were able to see the hole where Méchain made the fire as well as the deep crevices, we also saw some of the crosses Arago mentions and the stones dug in the ground (Fig. 5).

We expected this location to be coincident with the current geodetic pillar; however, being only some two meters away from the current mark (Fig. 6) this also perfectly agrees with the precision of our initial set of coordinates. Its elevation 917.8 m was also quite close (one meter off) to the old MSL height determined by the French expedition (Arago 1859).

From all points in Table 3, the only ones not visited were BOSCH DE L'ESPINA, impossible due to the weather and light conditions in the day of the visit, and MONT-SERRAT, since the day of the visit was crowded by tourists, the same as PUY DE LA MORELLA a location which we indeed visited during the same dates and surveyed with difficulties due to the number of hikers, Fig. 7, finding a circular mark chiseled in a rock next to the current geodetic benchmark, Fig. 8, which is not mentioned in the literature.

At any rate, these stations belong to Catalonia and are not of primary interest for the Valencian Region part of the network, which is the object of this research.

The old MONT-JOUY, also in Catalonia, was excluded from the calculations since it does not belong to the main triangulation, being linked to it by secondary triangulations whose measurements do not offer full guarantee (Alder 2003) and offers no help for strengthening the determination of the observing sites in the Valencian Region, the main object of this research.

From the rest of the stations in Catalonia, it is worth mentioning that in MONT-SIA we found one stone hut, Fig. 9, as well as one (but only one) of the possible crosses mentioned in Biot and Arago (1821) to relocate the station, Fig. 10, whose remains could not be found. No clear evidence was found in the rest of Catalonia stations, some of them having suffered a substantial alteration of the environment, for example in SAINT-JEAN after the construction of bunkers and trenches in the Spanish civil war.

Regarding the stations in the Valencian Region, ESPADAN had been pointed in the previous computation phase as a common point between the old and the current network. We found no additional clues during the field reconnaissance and deemed it possible that the old mark lies below the cylindrical structure of several bodies that constitute the current geodetic benchmark (Fig. 11).

No vestiges were found either for the rest of the locations, finding sometimes a substantially altered environment due to human action, such as in DESIERTO DE LAS PALMAS (Fig. 12).

Final network adjustment

With LLEBERIA (No.7) and FORMENTERA (LA MOLA) (No.17) held fixed, and recalling that MONT-JOUY (LA TOUR) (No.4) was eliminated since it does not belong to the main triangulation but to an internal subnetwork, we obtain the following set of adjusted coordinates with positional uncertainties ranging from 0.4 to 3.4 m (Table 4).

The standard error of the angular observations, 0.4”, with a maximum value reaching only 0.8” (for ESPADAN to MONGO) confirmed the excellent measurement precision of the French expedition.

We can see that the initial coordinates were already globally correct in orientation and scale, although some of the points – MONTAGUT, MONT-MATAS, MONT-SIA, CLOP DE GALAZO and ARES – especially in the network extremes, seem to be not in the initially estimated area of within some 3 meters.

Also, as previously said, we observe that the final coordinates of MONGO and CAMPVEY coincide with the pile of stones found and those of ESPADAN lie below the present geodetic benchmark.

Conclusions and future work

As initially expected, the task of finding vestiges of the old observation sites used by the French expedition has proved to be considerably difficult after more than 200 years. Some locations have undergone significant alterations due to human action (SAINT-JEAN, DESIERTO DE LAS PALMAS and MONT-SERRAT), while in others the vegetation has remarkably changed (CAMPVEY, MONT-MATAS, CULLERA, MONTAGUT). Despite this, we consider having located the original observation site in LLEBERIA and conclude that FORMENTERA (LA MOLA) has the same coordinates, MONGO and CAMPVEY have remained as stone mounds and ESPADAN is under the present geodetic benchmark. For the rest of the stations, the coordinates obtained permit to know the location of the original sites within 3 m or less, which could be of interest for the future installation of commemorative plaques.

From the technical point of view, our current assessment of precision after the error analysis of the least squares adjustment permits to conclude the excellent precision of the old angular measurements: typically 0.4” per horizontal angular direction. We have also noticed that the old altitudes, however, do not have the accuracy that was initially supposed (around 1 m) and in some cases – CULLERA, ARES, MONTAGUT and CLOP DE GALAZO – the errors are significantly larger, reaching up to 10 m.

Finally, it has to be mentioned that Barbeta (2019) began the creation of a geoportal for the dissemination and maintenance of the work carried out in this research. It can be found at http://arcmeridia.webs.upv.es/ though it is currently in its initial development stage. It will serve as a forum for the exchange of ideas and experiences between scientists, historians and amateurs, as well as for a better management and preservation of heritage and tourism exploitation.

Data Availability Statement

All data, models, and code that support the findings of this study (i.e., geodetic coordinates and programming code) are available from the corresponding author upon reasonable request.

Acknowledgments

This research has been funded by the Institut Cartogràfic Valencià. The authors are grateful to the editor and the three anonymous reviewers for their valuable comments that helped to improve the manuscript.

References

Alder, K. 2003. *The Measure of All Things: The Seven-Year Odyssey and Hidden Error That Transformed the World.* New York, NY: Free Press.

Arago, F. 1857. *Astronomie populaire*, Paris, France: Gide Éditeur.

Arago, F. 1859. *Oeuvres de François Arago. Memoires Scientifiques. Tome Second. Vitesse du son, Mesure de la Méridienne, Forme et Constitution des Planètes, Électricité Atmosphérique, Pouvoirs Réfringents.* Paris, France: Gide Éditeur.

Barbeta, V. 2019. *Desarrollo de herramientas geomáticas para la difusión de los trabajos geodésicos para la recuperación de los vestigios del Meridiano de París en el Antiguo Reino de Valencia*, Dissertation for the Geomatics and Surveying Engineering degree, València, Spain: Universitat Politècnica de València. https://riunet.upv.es/handle/10251/124460. Accessed 28 April 2020.

Bigourdan, M.G. 1900. "La Prolongation de la Méridienne de Paris, de Barcelone aux Baléares". *Bulletin Astronomique*, XVII, 348–480.

Biot, J.B., and Arago, D.F.J. 1821. *Recueil d'observations géodésiques, astronomiques et physiques, exécutées par ordre du Bureau des longitudes de France, en Espagne, en France, en Anglaterre et en Écosse, pour déterminer la variation de la pesanteur et des degrés terrestres sur le prolongment du Méridien de Paris; faisant suit au troisième volume de la Base du Système métrique*. xxx+588 p., 2 fold. pl., Paris, France: Ve. Courcier.

Débarbat, S. 2004. "Méchain géodésien". *Revue XYZ*, 100(3), 63–70.

Delambre, J.B. 1806. *Base du système métrique décimal, ou Mesure de l'arc du méridien compris entre les parallèles de Dunkerque et Barcelone , exécutée en 1792 et années suivantes*. Tome Premier. Paris, France: Institut National.

Delambre, J.B. 1807. *Base du système métrique décimal, ou Mesure de l'arc du méridien compris entre les parallèles de Dunkerque et Barcelone , exécutée en 1792 et années suivantes*. Tome Second. Paris, France: Institut National.

Delambre, J.B. 1810. *Base du système métrique décimal, ou Mesure de l'arc du méridien compris entre les parallèles de Dunkerque et Barcelone, exécutée en 1792 et années suivantes*. Tome Troisième. Paris, France: Institut National.

Instituto Geográfico Nacional. 2020. Red Geodésica Nacional. ftp://ftp.geodesia.ign.es/Red_Geodesica/. Accessed 26 April 2020.

Instituto Geográfico Nacional. 2020. Plan Nacional de Ortofotografía Aérea (PNOA). https://pnoa.ign.es. Accessed 26 April 2020.

Instituto Geográfico Nacional. 2020. Modelo de geoide EGM08-REDNAP. ftp://ftp.geodesia.ign.es/geoide/. Accessed 6 May 2020.

Mugnier, C.J. 2001. "The French Republic, Grids and Datums". *Photogramm. Eng. Rem. S.*, 67(1), 33–35.

Ruiz Morales, M. 2018. *El Enlace Geodésico de las Islas Baleares con el Continente (1867-1885).* 26 pp, personal communication.

Soler, T. 1997. “A profile of General Carlos Ibáñez e Ibáñez de Ibero: first president of the International Geodetic Association.” *J. Geod.* 71, 176–188.

Soler, T., and Ruíz-Morales, M. 2006. “Letters from Carlos Ibáñez e Ibáñez de Ibero to Aimé Laussedat: new sources for the history of nineteenth century geodesy.” *J. Geod.*. 80, 313–321. http://doi.org/10.1007/s00190-006-0069-1

Ten, A. 1996. *Medir el metro. La historia de la prolongación del arco de meridiano Dunkerque-Barcelona, base del Sistema Métrico Decimal.* Cuadernos Valencianos de Historia de la Medicina y de la Ciencia, IL, serie A Monografías, Universitat de València - C.S.I.C. https://digital.csic.es/bitstream/10261/92150/1/IL_Medir_el_metro.pdf. Accessed 27 April 2020.

Vaníček, P., and Foroughi, I. 2019. “How gravity field shortened our metre.” *J. Geod.* 93, 1821–1827. https://doi.org/10.1007/s00190-019-01257-7

Tables

Table 1. Geodetic coordinates and elevations (above sea level).

No.	Latitude	Longitude	Elevation (m)	Name
01	41°36′21.52″	−0°31′35.44″	1237.200	MONT-SERRAT
02	41°30′34.18″	−0°04′11.40″	468.900	MONT-MATAS
03	41°24′25.92″	−0°54′56.04″	953.300	MONTAGUT
04	41°21′50.17″	−0°17′08.53″	204.800	MONT-JOUY (LA TOUR)
05	41°17′49.07″	−0°23′21.20″	598.200	MORELLA (PUY DE LA)
06	41°08′03.31″	−0°59′05.30″	92.600	SAINT-JEAN
07	41°05′34.32″	−1°28′22.29″	919.400	LLEBERIA
08	40°52′47.83″	−1°58′38.76″	1179.000	BOSCH DE L’ESPINA
09	40°43′23.33″	−2°12′41.46″	1393.400	TOSAL DE ENCANADE
10	40°36′51.94″	−1°48′28.20″	763.800	MONT-SIA
11	40°28′00.79″	−2°28′13.41″	1318.700	ARÈS
12	40°05′10.50″	−2°18′27.30″	727.900	DESIERTO DE LAS PALMAS
13	39°54′21.48″	−2°43′11.35″	1040.200	ESPADAN
14	39°10′37.13″	−2°35′23.29″	221.700	CULLERA
15	38°48′27.02″	−2°12′50.86″	713.100	MONGO
16	39°03′36.06″	−0°59′04.04″	397.700	CAMPVEY (IVIZA)
17	38°39′56.80″	−0°48′11.26″	187.900	FORMENTERA (LA MOLA)
18	39°37′18.00″	0°12′16.00″	969.000	CLOP DE GALAZO

Source: Data from Arago (1859).
Note: Latitudes and longitudes are referred to an old reference system with origin in Paris. Names follow the writing in Arago (1859).

Table 2. Geodetic coordinates and heights of present geodetic benchmarks in the ETRS89 system.

No.	Latitude	Longitude	Orthometric height (m)	Ellipsoidal height(m)	Pillar height	Name
39196	41°36′19.25584″	1°48′40.77087″	1236.158	1285.793	1.000	MONSERRAT
39340	41°30′16.81887″	2°15′59.89629″	485.052	534.418	1.120	MATAS
41884	41°24′24.72077″	1°25′20.17502″	963.518	1013.293	1.200	MONTAGUT
42111	41°21′48.79092″	2°09′58.79759″	191.089	240.157	1.200	MONTJUIC(V)
99999	41°21′48.83080″	2°09′58.92220″	191.089	240.157	1.200	MONTJUIC(N)
44837	41°17′47.45549″	1°54′55.77110″	596.444	646.822	1.000	MORELLA
47215	41°05′32.53608″	0°51′46.37657″	919.121	968.575	1.200	LLAVERIA
49662	40°52′45.66151″	0°21′39.27402″	1180.966	1231.119	1.200	ESPINA
52093	40°43′23.24058″	0°07′36.83121″	1393.780	1444.801	1.200	ENCANADE
54716	40°36′48.92994″	0°31′49.06972″	764.053	813.796	1.200	MONTSIA
57018	40°28′06.23391″	−0°07′42.22499″	1321.617	1373.263	1.200	ARES
61665	40°05′07.40254″	0°01′51.66555″	736.131	786.629	1.200	DESIERTO
64044	39°54′21.48288″	−0°22′52.43148″	1043.488	1094.790	1.200	ESPADAN
74780	39°10′33.96373″	−0°15′03.77126″	233.986	285.072	1.200	CULLERA
82298	38°48′11.62570″	0°07′45.73181″	751.806	801.125	1.200	MONTGO
77263	39°03′34.88486″	1°21′15.22480″	400.777	451.233	1.200	CAMPVELL
85000	38°39′56.26015″	1°32′08.26625″	196.764	246.530	1.200	MOLA
85030	38°39′57.22712″	1°32′07.20125″	201.779	250.333	0.000	TORRE MOLA
69797	39°37′31.31641″	2°26′45.91122″	926.888	976.054	1.200	ESCLOP

Source: Data from Instituto Geográfico Nacional (2020a).

Table 3. Geodetic coordinates and heights for field reconnaissance in the ETRS89 system.

No.	Latitude	Longitude	Ellipsoidal height(m)	Name
01	41°36′19.5443″	1°48′41.6257″	1286.834	MONT-SERRAT
02	41°30′33.0111″	2°16′05.6234″	518.274	MONT-MATAS
03	41°24′24.3063″	1°25′21.2692″	1002.075	MONTAGUT
04	41°21′48.4461″	2°03′08.3287″	241.020	MONT-JOUY (LA TOUR)
05	41°17′47.6552″	1°54′56.0917″	642.572	MORELLA (PUY DE LA)
06	41°08′01.6331″	1°21′12.2427″	134.828	SAINT-JEAN
07	41°05′32.5370″	0°51′46.3946″	967.553	LLEBERIA
08	40°52′45.5155″	0°21′39.1859″	1228.151	BOSCH DE L’ESPINA
09	40°43′22.8475″	0°07′36.6734″	1443.018	TOSAL DE ENCANADE
10	40°36′49.6332″	0°31′49.9154″	812.042	MONT-SIA
11	40°27′58.1029″	−0°07′55.1461″	1369.255	ARÈS
12	40°05′07.7618″	0°01′51.3682″	776.898	DESIERTO DE LAS PALMAS
13	39°54′21.4775″	−0°22′52.4229″	1090.002	ESPADAN
14	39°10′33.7683″	−0°15′03.7879″	269.686	CULLERA

15	38°48′23.5620″	0°07′28.8020″	761.218	MONGO
16	39°03′34.9043″	1°21′15.1804″	445.654	CAMPVEY (IVIZA)
17	38°39′56.2454″	1°32′08.2846″	231.753	FORMENTERA (LA MOLA)
18	39°37′28.5701″	2°26′46.4273″	1015.166	CLOP DE GALAZO

Table 4. Adjusted coordinates (ETRS89) and error ellipses (semimajor axis *a*, semiminor axis *b* and azimuth of semi-major axis).

No.	Latitude	Longitude	a(m)	b(m)	θ_a (deg)	Name
01	41°36′19.6396″	1°48′41.5363″	2.072	0.709	41.5	MONT-SERRAT
02	41°30′33.1113″	2°16′05.5459″	2.721	0.946	58.1	MONT-MATAS
03	41°24′24.3921″	1°25′21.1771″	1.185	0.419	39.1	MONTAGUT
05	41°17′47.7464″	1°54′56.0124″	1.892	0.671	64.4	MORELLA (PUY DE LA)
06	41°08′01.7134″	1°21′12.1561″	0.810	0.287	68.4	SAINT-JEAN
07	41°05′32.6093″	0°51′46.3003″	-	-	-	LLEBERIA
08	40°52′45.5791″	0°21′39.0936″	0.784	0.255	34.3	BOSCH DE L′ESPINA
09	40°43′22.9065″	0°07′36.5826″	1.088	0.348	27.4	TOSAL DE ENCANADE
10	40°36′49.6968″	0°31′49.8314″	0.868	0.340	5. 4	MONT-SIA
11	40°27′58.1570″	-0°07′55.2332″	1.485	0.432	19.2	ARÈS
12	40°05′07.8129″	0°01′51.2900″	1.620	0.448	4.2	DESIERTO DE LAS PALMAS
13	39°54′21.5228″	-0°22′52.5045″	1.877	0.535	2.1	ESPADAN
14	39°10′33.7919″	-0°15′03.8592″	1.600	0.558	158.4	CULLERA
15	38°48′23.5760″	0°07′28.7470″	1.189	0.540	142.1	MONGO
16	39°03′34.9313″	1°21′15.1534″	0.597	0.302	175.2	CAMPVEY (IVIZA)
17	38°39′56.2601″	1°32′08.2662″	-	-	-	FORMENTERA (LA MOLA)
18	39°37′28.6217″	2°26′46.4368″	2.492	0.866	50.3	CLOP DE GALAZO

Figures

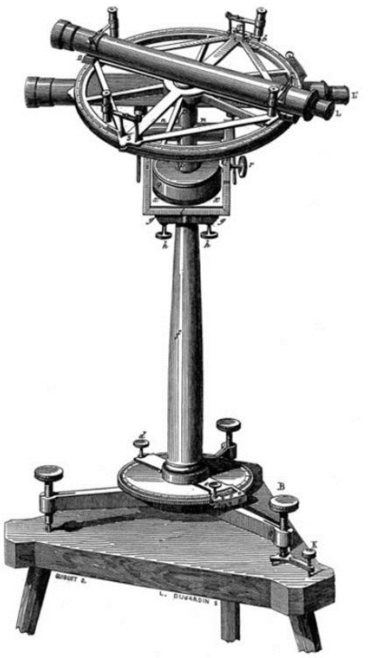

Fig. 1. Drawing representing the circle of Borda for azimuthal observation. (Reproduced from Arago 1857.)

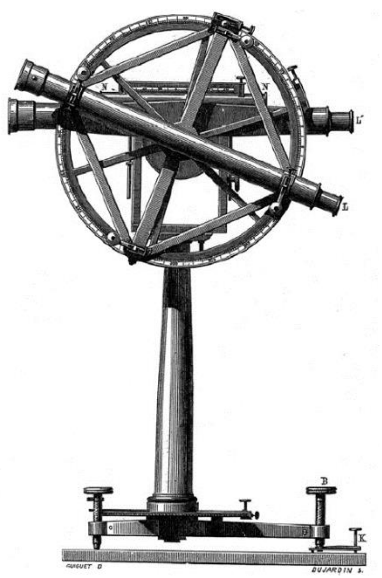

Fig. 2. Drawing representing the circle of Borda for zenith angle observation. (Reproduced from Arago 1857.)

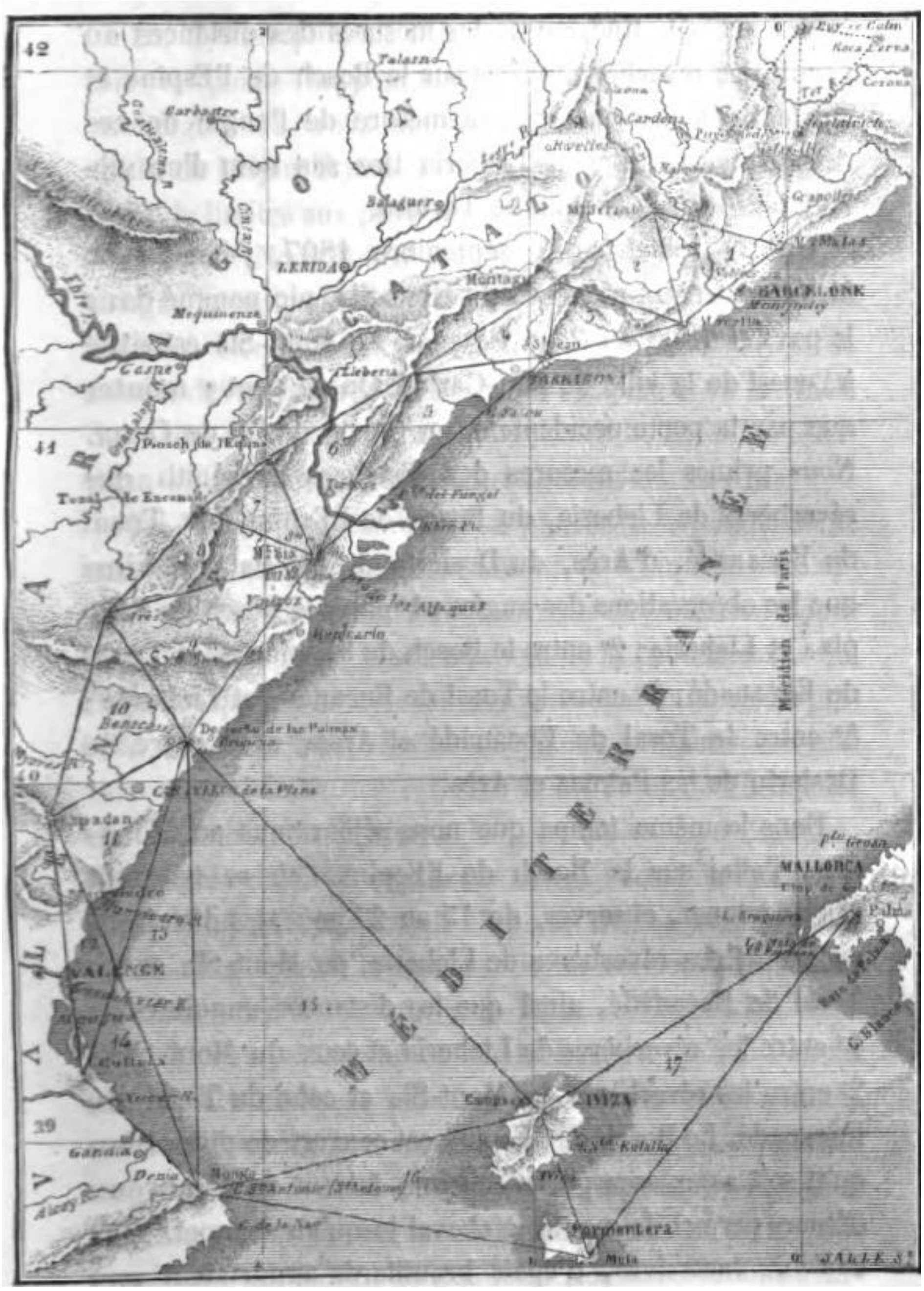

Fig. 3. Biot and Arago extension to Formentera of the geodetic network to determine the Paris meridian. (Reproduced from Arago 1857.)

Fig. 4. MONGO. The pole with a yellow ball on its end (center of the image) marks the position from the initial set of coordinates. (Image by Luis García-Asenjo.)

Fig. 5. LLEBERIA. Stones dug in the ground by the French expedition. (Image by Luis García-Asenjo.)

Fig. 6. LLEBERIA. General view of the stones dug in the ground by the French expedition and the present geodetic benchmark (on the back). (Image by Sergio Baselga.)

Fig. 7. PUY DE LA MORELLA. Surveying preparation for the photogrammetric record. (Image by Luis García-Asenjo.)

Fig. 8. PUY DE LA MORELLA. Circular mark chiseled in a rock. (Image by Luis García-Asenjo.)

Fig. 9. MONT-SIA. Stone hut possibly used by the French expedition. (Image by Luis García-Asenjo.)

Fig. 10. MONT-SIA. Possible cross from the French expedition as mentioned in Biot and Arago (1821). (Image by Luis García-Asenjo.)

Fig. 11. ESPADAN. Present geodetic benchmark. (Image by Luis García-Asenjo.)

Fig. 12. DESIERTO DE LAS PALMAS. The coordinates of the old mark indicate approximately one leg of the communications tower. The current geodetic benchmark can also be seen on the back. (Image by Luis García-Asenjo.)