



Octagonal Layouts: Project Genesis of the Cathedral of Valencia

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Abstract

Octagonal layouts are widely used in medieval architecture. In Spain, the Cathedral of Valencia is an exceptional example because of its compositional arrangement and early date of construction. The study of this cathedral serves not only to propose the regulatory layout and geometric process to develop the transept, the ambulatory, and the dome but allows the empirical establishment of the complex properties and characteristics of the octagon. By surveying the planes with a 3D laser scanner, the graphic procedures used in the original design of the temple were verified, as were the geometric theories pertaining to the figure of the octagon in architecture, set by the golden ratio.

Keywords Spanish Gothic · Proportional analysis · “Faborum” geometry · Golden ratio · Octagonal relationships

Introduction

When King James I captured the city of Valencia from the Muslims in 1238, one of his first decisions was to convert the main mosque into a Christian cathedral. During the first years after the conversion, worship took place in the building that had previously

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housed the mosque. In 1257, Pope Alexander IV granted indulgences to those who visited the cathedral (Magro Moro 1985: 439), and on March 25, 1249, King James I prohibited the construction of arches, cantilevers, or buildings in the circuit of the cathedral (García Edo 1988: 73 and 127). In 1262, the works of the new Trecentista building began under the supervision of Bishop Andrés Albalat, with Arnaldo Vitali (Arnau Vidal) in charge, according to a 1267 document in the Archive of the Crown of Aragon of Barcelona, which specified that “A. Vitali magistri operis eclesie sancte Marie, Valentie” (Sanchis Sivera 1933: 3–24). This *magistri operis* can be credited for the elaboration of a determined plan for the construction of the Cathedral, on which he worked as a master builder between 1268 and 1273 (Magro Moro 1985: 438).

The work began with the chevet (Dalmases and Pitarch 1988) and, at the end of the thirteenth century, the temple had fourteen chapels in addition to the main altar, seven of which were at the ambulatory (Robres Lluch 1973: 125–126). Following common religious practice, construction started with the apse as the epicentre of worship, celebration and is the most sacred place of the church (Hani 1978: 69).

Vitali was succeeded in 1303 by Nicolás de Ancona, an Italian architect hired by Bishop Raimundo Despont, who had previously been the pontifical governor of Ancona (Zaragoza Catalán 1995: 16–55). He is credited with the door of the Apostles, completed in 1354, the lateral naves, the dome, and the old bell tower.

The architecture of the nave presents simple layouts and austerity in its ornamentation, typical of Cistercian architecture. Recall that during the thirteenth century, Cistercian architecture was deeply rooted in southern France and the Crown of Aragon, with King James I himself entering the Cistercian order shortly before his death (García Edo 1989: 280). Likewise, Bishop Andrés de Albalat belonged to the Order of Preachers (Dominicans), and therefore, a more suitable space for sermons was designed. The proposal consisted of square sections with a greater height in the central nave and rectangular spaces with a lower height in the lateral naves to illuminate the central nave (Soler Verdú 1996: 2).

It is a very simple design, with no complex use of geometry. However, there are two elements in the interior of the cathedral in which the octagon forms the genesis of the design: the ambulatory and the dome. These are the two most significant, complex, and admired elements of the cathedral.

To date, no monographic study has been carried out based on data collection with a 3D terrestrial laser scanner (TLS) to obtain a precise survey of the octagonal layouts that govern the design of these two elements. In this sense, the research present here is innovative. Its main objective, based on a survey of plans with a TLS, is to study the geometry, morphology, and proportions of the spaces that make up the ambulatory and dome of the Cathedral of Valencia as a continuation of the principles proposed by Soler Sanz (2014).

Object of Study

The Ambulatory

In contrast to the sober architecture of the nave, which shows no notable innovations, it is worth observing the structure of the apse, which incorporates an ambulatory in the French Gothic style, one of the first examples of this style in the Crown of Aragon. The church of the Cistercian monastery of Poblet (Tarragona), built during the twelfth century, offers the only earlier example of semicircular chapels built around an ambulatory, stands out as an exceptional case, since the Cisterian monasteries and churches built for the Crown of Aragon usually have straight or polygonal apses with one or more lateral apses. It is, therefore, a significant and extraordinary element. However, a similar solution for the chevet of the Cistercian order was found in the abbey of Royaumont (France), which was founded in 1228 and demolished in 1791 (Leroux-Dhuys 2006: 314).

According to Sanchis Sivera 1933: 3–24), ambulatory chapels were constructed in the last years of the thirteenth century. This arrangement is very innovative with respect to other examples since the ambulatory sections are pentagonal, as each section has two radial chapels instead of the usual one. The chapels also fit in octagonal layout. This design can be considered the counterpoint to the Languedoc influence reflected in the Romanesque portal and the Cistercian style of the nave.

This solution served as an example for other churches: Also in Valencia, the Church of Santa Catalina, whose chancel was built in the first half of the fourteenth century, presents a similar solution in which two radial chapels correspond to each section of the ambulatory. However, in this case, the solution is developed in a simpler way because the chapels are square. Additionally, the Collegiate Church of Santa María de Cervera (Lérida) and the Cathedral of Vic resolve the ambulatory by means of radial chapels, where two chapels correspond to each section of the ambulatory, as in the Valencian cathedral; these are the only churches in the entire Crown of Aragon that use this solution. Construction on both began at the beginning of the fourteenth century, after the ambulatory of the Cathedral of Valencia was already completed. Other examples in the Crown of Aragon of churches with apsidal solutions using ambulatory methods, such as Santa María del Mar (1329) and the Cathedral of Santa Creu de Barcelona (1298), are resolved with radial chapels in which each section of the ambulatory corresponds to a single radial chapel. In the cathedral of Murcia (1388), this same solution is repeated, but its chapels have a trapezoidal plan (Zaragoza Catalán 1995: 16–55).

The Valencian Gothic cathedral initially had exterior buttresses, according to Julián Esteban Chapapriá (Zaragoza Catalán 2000: 68). However, from its beginnings, sacristies were located between the buttresses and chapels on the outside, partially concealing the extrados of the ambulatory. Subsequently, with the construction of the Loggia of the Canons in 1556 (Oñate Ojeda 2012: 37), part of the chancel was hidden, and in 1816, in a project by Vicente Marzo, a closing

Fig. 1 Pentagonal vaults of the ambulatory. Image: authors



Fig. 2 Ambulatory and ambulatory chapels. Image: authors



wall was raised that completely hid the upper part of the buttresses. Finally, in 1826, the construction of the chevet was completed with the extension of the sacristy by a chapel for relics, a project by Joaquín Tomás y Sanz (Pingarrón Esain-Seco 2013) (Figs. 1 and 2).

The Dome

The ambulatory is not the only element of the cathedral with octagonal layouts; the dome is also governed by this regulatory layout.

The dome undoubtedly counts as the most remarkable element of the temple (Street 1865: 259–268). Its structure, composition, elegance, slenderness, and beauty have been admired over the centuries by multiple authors (Escolano 1878: I,893; Orellana 1924: 556; Teixidor 1895: 227; Street 1865: 284). The mathematician and architect Tomás Vicente Tosca includes it in his "Treatise of the Montea y Cortes de Cantería" in the chapter dedicated to "forming a vault with cross arches on any polygon, of the square above, which is supported by its own weight, without further abutments" [formar una bóveda con arcos cruceros sobre cualquier polígono, del cuadrado arriba, que se mantenga con su propio peso, sin más estribos] (Tosca 1712).

Despite the richness of documents in the archives of the cathedral, the exact date of its construction remains unknown. Most authors agree that a dome existed at the beginning of the fourteenth century (Street 1865) and attribute it to Nicolás de Ancona (Segura de Lago 1971), based on documentary evidence that a dome was existent in 1356 (Magro Moro 1985: 96). However, the original dome was partially destroyed on several occasions due to winds (1383, 1392 and 1432) and an earthquake (1392). In 1404, significant reconstruction was carried out under the direction of Martín Llobet, resulting in the dome as we see it today. It underwent several subsequent maintenance interventions over the years, the most important of which took place in 1731, when the stained-glass windows were replaced by alabaster plates. It was also necessary to perform structural interventions on the pillars that support it (1660 and 1774), reinforce the roof with a concrete perimeter band and reinforce the foundation of the pillars (1976) (Llopis Pulido et al. 2016: 4) (Figs. 3 and 4).

The dome rests on four pillars and four pointed arches, which were covered with marble and stucco after an academic intervention of the interior of the cathedral by Antonio Gilabert dating to 1769 (Cortés Meseguer 2014: 8). The transition from a square to an octagonal plan is solved by conical horns. The dome is composed of two similar levels, with the upper level of greater dimension in height than the lower level. Each side in each of the levels comprises a large, pointed window divided into six lobed arches by five intermediate mullions and two mullions at the ends. These large holes increase the feeling of lightness. The tracery of the arches occupies the entire ogive, providing elegance to the composition. There are two geometric tracery patterns located alternately on the sides of the octagon and between the two levels. This composition provides movement to the whole. These characteristics, together with the absence of abutments, make this element a singular masterpiece.

The general dimensions, according to the geometric survey with TLS, are as follows: The inner diameter is 4.93 m; the outer diameter measures 6.23 m.

Fig. 3 Exterior view of the dome. Image: authors



Fig. 4 Interior view of the dome. Image: authors



State of the Art

Some authors have focused their analysis on octagonal layouts. Ecochard (1977) focuses on the geometry of Greek, Roman, Byzantine, and Islamic buildings, all of which had octagonal layouts, although they were built in different places and in different periods. His paper analyses the radius of the circle that circumscribes the squares that give rise to the starry octagon that serves as the basis for its design. His experiment verified that in all of the examples, the same design formula is repeated: a circle with a radius of 26.87 m and a 38-m-sided square that, when rotated, forms the octagon. Savvides (2021) discusses a diagram etched in stone at the Galerian complex in Thessaloniki, which may be the first evidence of a conceptual design that explains the geometric patterns derived from the octagon.

Naredi-Rainer (1982: 191-ff) studies harmony in architecture, describing a divine proportion between the radius and the side of a regular decagon (1.61) and stating the same relationship is present in the octagon (1.31), resulting in an irrational number practically equal to that determined for human proportions.

Ghyka (1983: 83-ss) explores the use of different regular polygons in regulatory layouts in architectural designs. Among them, he analyses the octagon.

de la Hoz (1973) studied the relation between the radius and the side of the regular octagon, calling it “Cordovan proportion” and naming it with a ‘c’:

$$1/\sqrt{2 - \sqrt{2}} = 1.3065629 \approx 1.3066.$$

Soler Sanz (2014) carries out in-depth research on regulatory layouts in architecture, with a special emphasis on regular octagons. He studied the geometry of the regular octagon, the properties of its primary elements, the inscribed and circumscribed circumferences, their radii and their diameters, the inside and outside of the octagon, the sides of the star polygon, etc., leading to

the relationship $1 + \theta^2 = 4\lambda^2$, that is, from the algebraic relation between λ and θ (Fig. 5). He also verified that when the radius equals 1, the distance between two parallels equal the silver number ($1 + \sqrt{2} = 2.4142$) and the ratio between the side and the radius equals the cordovan proportion.

Regarding the proportions of the Cathedral of Valencia, its geometry remains scarcely studied. Juan Carlos Navarro Fajardo, in his doctoral thesis (Navarro Fajardo 2003), performed a geometric analysis focused on the study of the geometry of the layouts and mounts of the vaults of the nave. This analysis was elaborated using plans made with traditional methods that were far from the real model, and consequently, the obtained results do not conform to reality.

Soler Sanz (2008) made important advances in the knowledge of the geometry that originated the original regulatory layout of the cathedral. However, these analyses were developed in the theoretical field and applied an ideal geometry.

Subsequently, Jorge Luis García Valldecabres carried out an extensive analysis of the layouts and templates used in the design and construction of the church of San Juan del Hospital de Valencia, comparing them with those used in other *Reconquista* churches, including the Cathedral of Valencia (García Valldecabres 2010).

In 2014, Llopis Pulido (2014) carried out a structural and constructive analysis of the cathedral, particularizing the element of greatest interest throughout history in the cathedral complex, the lantern tower. In parallel, Luis Cortés Meseguer addressed the study of the layouts in the framework of the neoclassical project carried out by Antonio Gilabert between 1769 and 1831 (Cortés Meseguer 2014).

In all these studies, tangential references are made to the proportions and geometric relationships of the cathedral. However, none of them delve into the regulatory layout used for the octagonal elements, which have a more complex resolution, and none of them conducted a plane survey using a laser scanner to guarantee the accuracy of the obtained data.

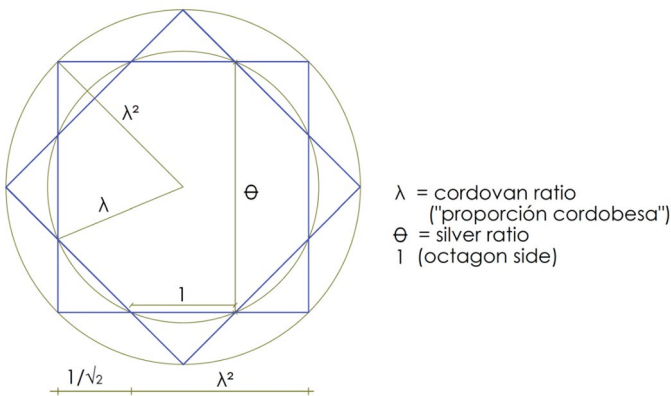


Fig. 5 Octagonal relationships of side 1. Image: authors based on Soler Sanz (2014)

The Survey

Establishing the ideal geometry on which the design of a Gothic building could be based is not an especially complex task. However, the difficulty increases considerably when the goal is to establish the regulatory layout and the applied geometric law based on real and exact measurements of the structure. For this purpose, the 3D laser scanner formed the most appropriate method for this research. The complex morphology of the study objects (the ambulatory and the dome), the difficulty of accessing most of the elements and the profusion of sculptural and ornamental elements made it necessary to use this tool to obtain reliable results. The resulting point clouds provided the decisive data to establish 3D digital models on which geometric analyses could be tested. The results show the potential of this approach and its advantages over the technology that is frequently used today, similar to the way that technology was advantageous in preceding traditional methodologies (Moral Saiz et al. 2021: 4–26). Likewise, Professor Francisco Pinto Puerto, in a recent contribution, establishes a strategy for studying each piece of a building that considers fragments of an architectural unit with their own qualities and values that, in interaction with other fragments, establish new values for the whole unit (Pinto Puerto et al. 2021: 61–83).

For this purpose, we used the Faro Focus X130 scanner. The data were processed and recorded with the 3D Scene program, which facilitates the unification of point clouds, leading to the creation of the complete 3D model and allows sections to be obtained using the Clipping Box tool. Additionally, this program provides scaled orthophotos of the designated views or sections (Figs. 6 and 7).

A separate survey campaign was conducted for each element to be studied. First, the interior of the chevet and transept were scanned. A scanner was placed in each of the chapels of the ambulatory, under each of the sections of the ambulatory, in the

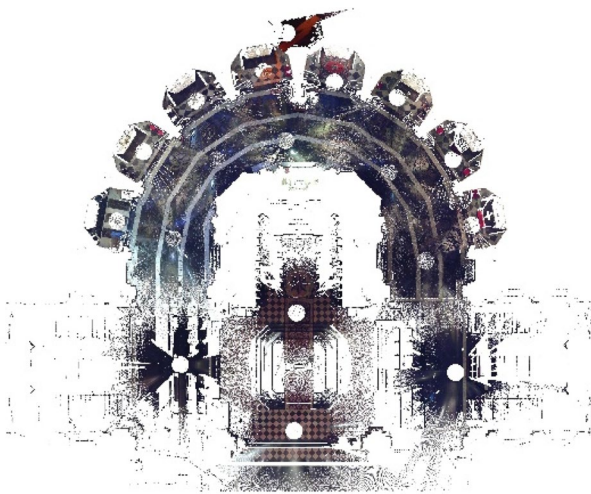


Fig. 6 Orthophoto of the horizontal section of the ambulatory and transept. Image: López González

Fig. 7 Point cloud of the ambulatory. Image: López González



centre of the apse, in the arms of the transept and under the dome to determine the exact geometry of the ambulatory. A total of 18 scans were performed. The mean error was 2.5 mm, and the minimum overlap was 15.80%. Both values perfectly conform to acceptable standards.

In the subsequent campaign, the dome was scanned from the inside by placing scanners in the lower windows that were accessible from the roof of the nave. At the same time, the exterior was scanned, covering the ambulatory and the nave by placing the scanner in front of each of the sides of the dome. In this way, it was possible to join the interior scans with the exterior scans to determine the thickness of the pillars. A total of 10 scans were performed. The mean error was 2.1 mm, and the minimum overlap was 35.50%.

The obtained results were digitized, and plans, elevations and sections were retrieved. It was possible to verify dimensional differences from the plans of the cathedral by using traditional techniques. However, the surveyed measurements hold no meaning if the analysis is not performed according to the metrological system used by the original builder. The layouts and the consequent rethinking of the building were based on a different metrological system from today's one. Therefore, the measurements obtained with the laser scanner had to transfer to the metrological system used in the construction of each of the analysed elements.

Measurements were made using the three metrological systems used in Valencia at the time of the cathedral's construction: The Valencian, established by James I in the Kingdom of Valencia in 1249 (García Edo 1988: 73 & 128), the Aragonese and the Lleidan (Salvador Peláez 2000) (Table 1).

According to the survey performed with the laser scanner, the thickness of the outer walls of the chapels is 46.71 cm. The width of the chapels (the Gothic chapels that are not covered by the neoclassical intervention were measured) is 553 cm. The width of the ribs of the ambulatory is 30.70 cm. The width of the ribs of the vault of the Gothic chapels is 22.70 cm.

The results are shown in Table 2.

Initially, the metrological system of Valencian feet was used as established by James I for the Kingdom of Valencia 13 years before the first stone of the cathedral was placed. The measurements obtained throughout the analysis were compared using all three systems.

Table 1 Equivalences of metrological systems with the metric system decimal

Valencian unit system	Metric system
1 yard	90.60 cm
1 handspan	1/4 yard = 22.65 cm
1 foot	1/3 yard = 30.20 cm
Aragonese metrological system	Lleidan unit system
1 yard = 77.20 cm	1 yard = 77.80 cm
1 handspan = 1/4 yard = 19.30 cm	1 handspan = 1/4 yard = 19.45 cm
1 foot = 1/3 yard = 25.74 cm	1 foot = 1/3 yard = 25.94 cm

The Proportions and Geometry of the Cathedral of Valencia

The study of historical building layouts requires delving into its design and construction process. The graphic expression of the layouts, understood in terms of geometric schemes, has its origin in the conception of the architectural design itself, which manifests and transcends thought as a response to the need to develop a programme of spatial organization prior to the implementation of a building, thereby acquiring the constructive control of the architectural form López González (2012: 231).

The knowledge of these formulas and the geometric rules-of-thumb used by master designers in designing these structures adds considerably to the knowledge of the building, its origins, its historiography and, consequently, its understanding. Layout analysis and metrological studies are of great importance in the drafting of previous studies leading to architectural interventions.

The layouts of the ambulatory of the cathedral were analysed, as previously explained, by Felipe Soler Sanz in plans that were prepared in the mid-twentieth century using traditional and inaccurate means. Continuing the work initiated by Soler, by taking laser scanned data as a reference, a metrological and geometric analysis could determine the original design of the chancel of the cathedral, establishing the geometric layouts of the ambulatory and its direct relationship with those of the dome.

First, the regulatory path that formed the ambulatory was determined, while the layouts of the original design were analysed. The greatest difficulty was the identification of the original Gothic structures underneath the later neoclassical intervention. This limitation was corrected by searching for and measuring the original architectural elements and faces that have been discovered of the walls.

Transept Data

To obtain these data, a section was made through the highest part of the transept where the walls were not covered with the neoclassical intervention. In this way, the measurements of the original construction were obtained.

Table 2 Results obtained in the three old metrological systems

Element	Metric (cm)	Valencian	Aragonese	Lleida
Thickness of the closing walls of the chapels	47.71	2.11 handspans 1.58 ft.	2.48 handspans 1.86 ft.	2.46 handspans 1.84 ft.
Width of ribs of the ambulatory	30.70	1.02 ft.	1.35 handspans 1.02 ft.	
Width of the ribs of the chapels	22.70	1.01 handspans	1.01 handspans 0.76 ft.	
Width of the transept	1288.23	56.88 handspans 42.66 ft.	66.75 handspans 50.05 ft.	66.24 handspans 49.67 ft.

The width of the arms of the transept is 12.88 m (42.50 Valencian feet; 50 Aragonese feet; 49.50 Lleidan feet).

The total length of the arms of the transept from end to end is 53.89 m. This measurement does not result in whole numbers of handspans, feet, or yards, because they are based on the measurement of the width of the arm (Fig. 8).

The following layout hypothesis be established: Starting from a given width of the arms of the transept, squares are formed and folded according to the golden ratio, as indicated in Fig. 8. The arm of the transept on the Gospel side is shorter (22 cm) than the arm on the Epistle side. Presumably a layout error occurred when the outline of the layouts was transferred to the terrain. The inner octagon of the dome is inscribed in a circumference with a diameter equal to the width of the transept.

Juan Carlos Navarro Fajardo studied the layouts of the transept (Navarro Fajardo 2003) and obtained disparate results because his geometric diagrams were supported by a previous survey using traditional means that had considerable measurement errors.

We can establish the square with 12.88 m sides and the circumference inscribed within this square as the regulatory layout of the entire chancel, as shown below (Fig. 9). From the transept, layouts that configure the ambulatory plan are developed. An attempt has been made to relate the compositional schemes of both elements. The axes of the ambulatory ribs were placed on the point cloud (Fig. 9). These axes served as the basis for establishing the geometric relationship between the ambulatory and the transept. It was possible to verify that the geometry of these ribs is perfect in the construction, without deviations from the geometric model.

The centre of the ambulatory is located 3.80 m from the transept (Fig. 10). This distance is determined by the golden rectangle, starting with the square whose side comprises half of the regulating square, as indicated in Fig. 10 in blue. In a simpler way, the centre of the ambulatory is obtained with the circumference of a circle with a radius equal to the distance between the centre and the midpoint of the lateral rectangle.

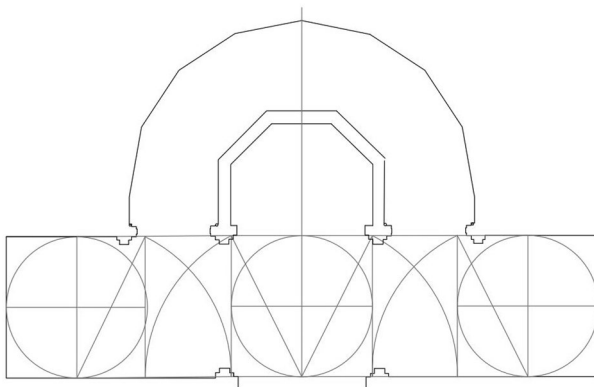


Fig. 8 Layout of the transept and transept crossing. Image: author López González

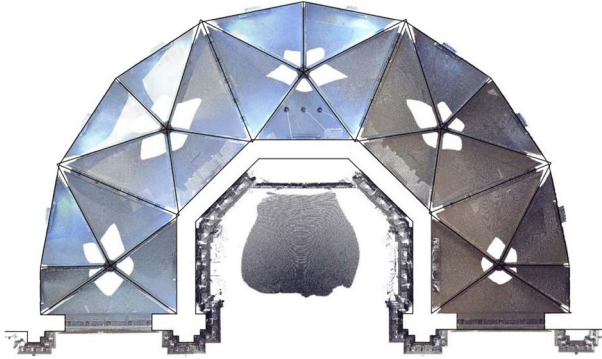


Fig. 9 Axes of the ambulatory's ribs projected on the point clouds. Image: author López González

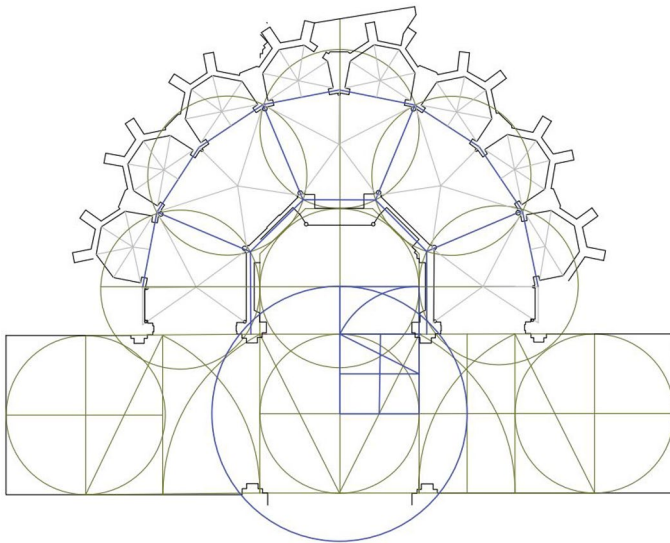


Fig. 10 Relationship between the layout of the transept and that of the ambulatory. In blue, the golden rectangle relating the centre of the transept with that of the apse and ambulatory. Image: author López González

The circumference of the circle with a diameter 12.88 m (the width of the arms of the transept and also width of the nave) that is inscribed within the regulating square determines the sections of the ambulatory: The five sections of the ambulatory are formed by the sequence of regulating circles with their centres on the horizontal, vertical, and 45° radial axes; that is, on the bisector of each side of the octagon of the apse and tangent to it (Fig. 10). The intersection of the regulatory circles in the ambulatory determines the axis that divides each of the sections from the next. It also determines the axis of the access side to each of the

chapels and the axis of the perimeter of the central apse, that is, the intersections of the circumferences delimit the width of the ambulatory (blue color in Fig. 10).

The geometric location of the keys of the vaults of the sections is at the intersection of a circumference of a circle with a 11.60 m radius concentric to the original one that defines the apse, and the radial axes at 0° , 45° , 90° , 135° and 180° (Fig. 11).

The keys of the vaults of the sections of the ambulatory are located in the centre of the circumference of the circle inscribed within the irregular pentagon that forms each section. The diameter of this circumference is equal to the distance between the side of the octagon of the axis of the apse and the vertex of the hexadecagon.

The geometry of the chapels is also obtained with a simple geometric procedure: The surrounding circumference of the regulating circumferences that make up the ambulatory is traced. This circumference intersects with the axes between chapels. The diagonals of the obtained geometric figure are drawn, and the intersection becomes the centre of the circumference in which the octagon of each chapel and key of the vault is inscribed (Fig. 12). The diameter of this circumference coincides with the octagonal side of the apse's axis (Fig. 13).

The circle inscribed by the octagon of the chapels does not repeat a sequence of whole numbers on the radial axis, as Professor Soler Sanz proposed for determining the length of the sides of the octagons in the chapels (Fig. 14). Rather, a difference of 59 cm appears from the survey. In reality, the octagon of the chapels is obtained from the circle that circumscribes it, the diameter of which is equal to the length of the side of the octagon formed by the axis of the apse.

However, when we start laying out the geometry from the centres of the keystone of the vaults of the chapels of the ambulatory a circle results. The circle passes

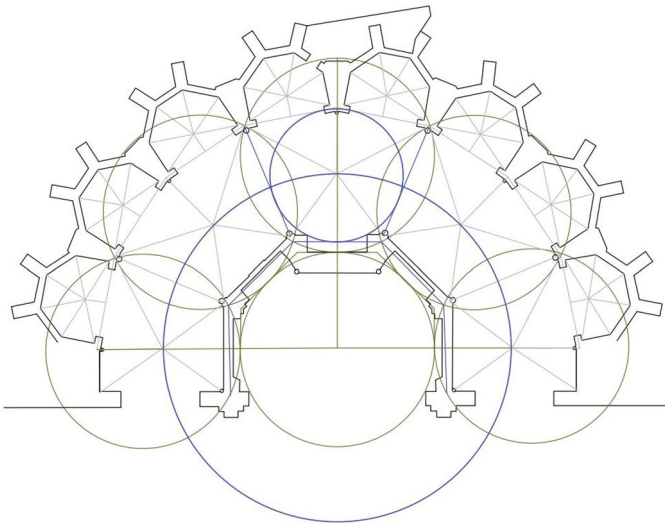


Fig. 11 Delineation of the ambulatory with the regulatory circumference of the transept and apse. Image: author López González

Fig. 12 Determining the centre of the octagon of the chapels.
Image: author López González

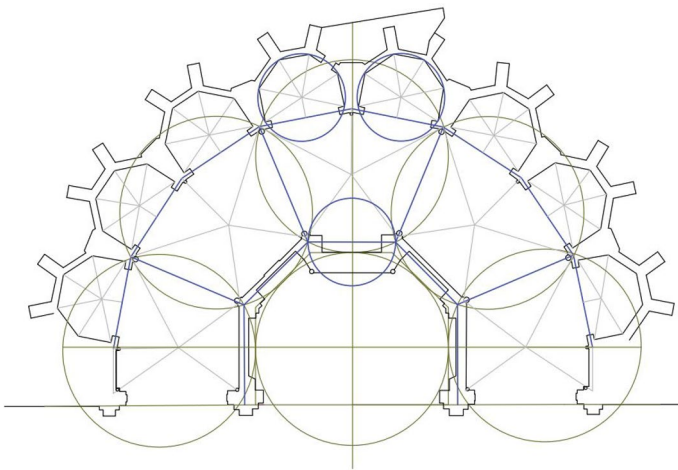
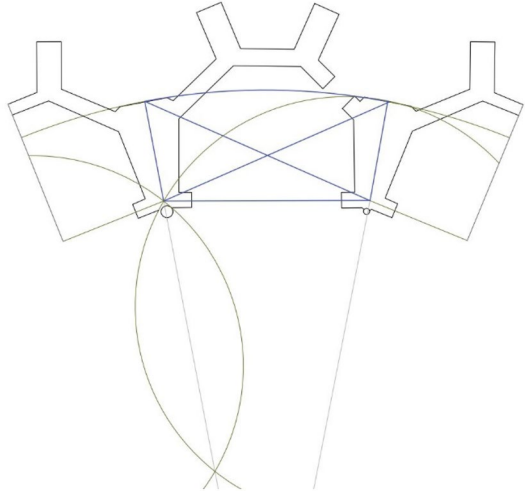


Fig. 13 The diameter of the circumference circumscribed by the octagons of the chapels is equal to the side of the octagon side of the axis of the apse. Image: author López González

through the vertices of the sides of the 16-sided polygon, or hexadecagon, and defines a sequence of three circles tangent to the centre of the apse. Based on this circle, an outer polygon with a radius of 3.40 m results and determines the thickness of the walls (Fig. 15).

The analysis established the Ecochar (1977) constant (circumference of radius 26.87 m). According to Ecochar, this circumference appears in the geometric analyzes of multiple central-plan religious buildings built between the fifth and tenth centuries in Syria, Armenia, Italy and France. It also had an impact in Spain. The dimension of the length of the transept is 53.86 m, which practically coincides with

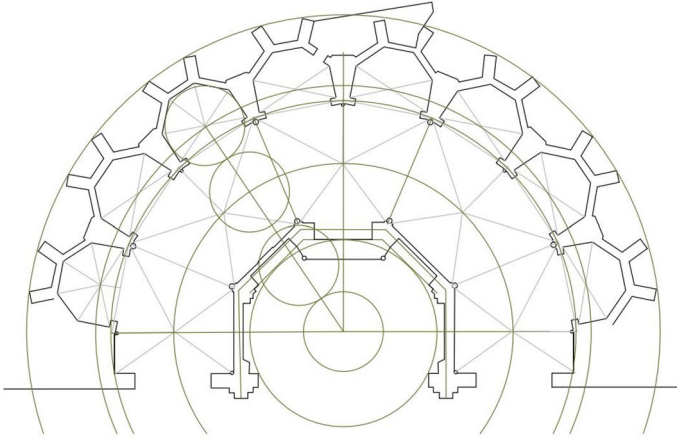


Fig. 14 The circular construction associated with the ambulatory. Image: author López González

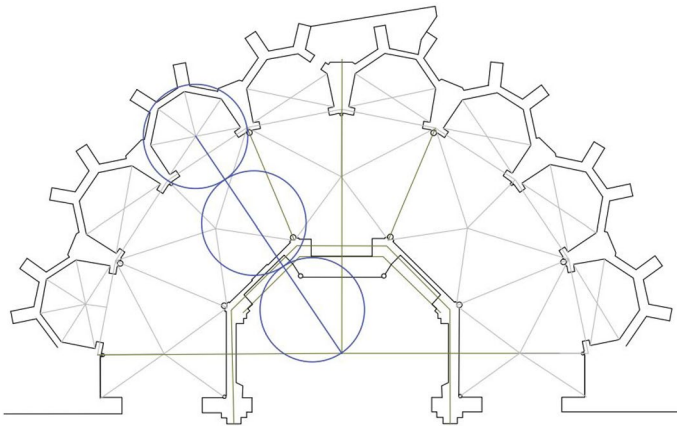


Fig. 15 Construction for defining the thickness of the chapels' walls. Image: author López González

the diameter of the Ecohard circumference (Fig. 16). The inclined squares with sides of 38 m form an inner octagon. The circumference inscribed in this octagon coincides with the surrounding circumference of the five tangent circles that form the ambulatory.

Discussion

The geometric analysis of the chancel and transept of the Cathedral of Valencia show that the geometric process used in its design and the subsequent rethinking of its implementation was as follows:

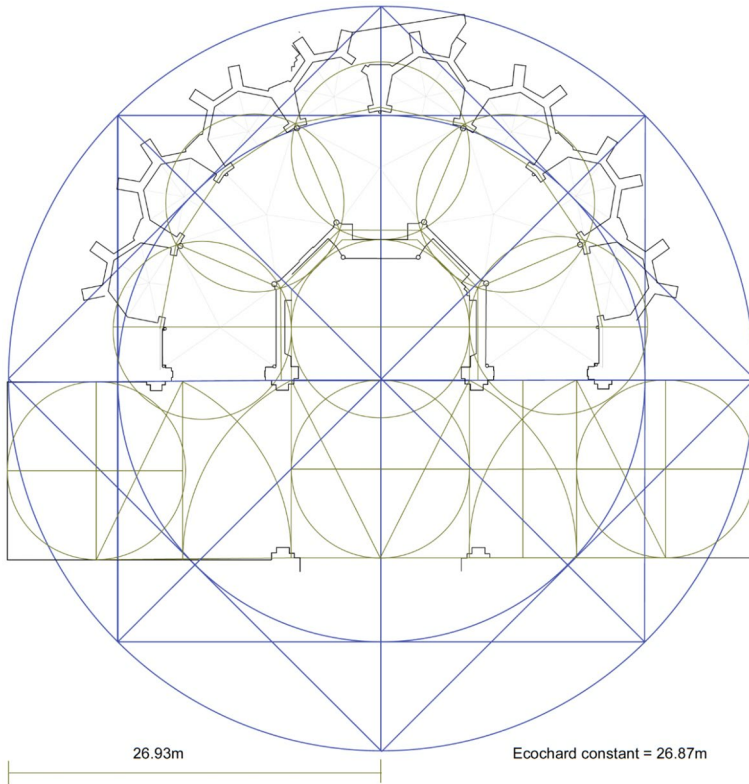


Fig. 16 Relationship with the Ecochard constant. Image: author López González

Similarly, the ratio between the circumferences that inscribe or circumscribe the generating octagons of the chevet have been mathematically verified:

The Radius of the inscribed circumference inside the apse and dome (regulatory circumference) 6.44 m.

Radius of the circumference circumscribed to the perimeter axis of the apse: 7.78 m.

Radius of the circumference circumscribed to the hexadecagon that crosses the axis of the access arches to the chapels: 15.99 m.

Radius of the circumference that circumscribes the octagon of the chapels = side of the octagon formed on the perimeter axis of the apse.

Radius of the circumference that circumscribes the walls of the chapels = $1/3$ radius of the circumference that crosses the center of the chapels.

The relation between the regulatory circumference and the circumference that circumscribes the hexadecagon that defines the chapels is as follows:

We use a polar coordinate system. We take the center of the circumference of the apse as the coordinates of the origin (0, 0) and as polar axis the parallel line

to the transept's direction. We look for the coordinates of points A and B defined by their distance r from the center and the angle with the horizontal β (Fig. 17).

The equation of the circumference with center at the point $(2a, 0^\circ)$ is:

$$r^2 - 2r \cdot 2a \cdot \cos(\beta - 0^\circ) + (2a)^2 = a^2$$

The equation of the line that joins a vertex of the chevet hexahedron (A) with the center of the apse is: $\beta = 22,50^\circ$.

The system of equations is solved by substituting the value of β in the equation of the circumference:

$$r^2 - 2r \cdot 2a \cdot \cos(22,50^\circ - 0^\circ) + (2a)^2 = a^2$$

$$r^2 a^2 = 0$$

Solving for r :

$$r = \frac{1}{2} \cdot \left(3,69a \pm \sqrt{13,65a^2 - 12a^2} \right) = \frac{1}{2} \cdot (3,69a \pm 1,28a)$$

The solutions are:

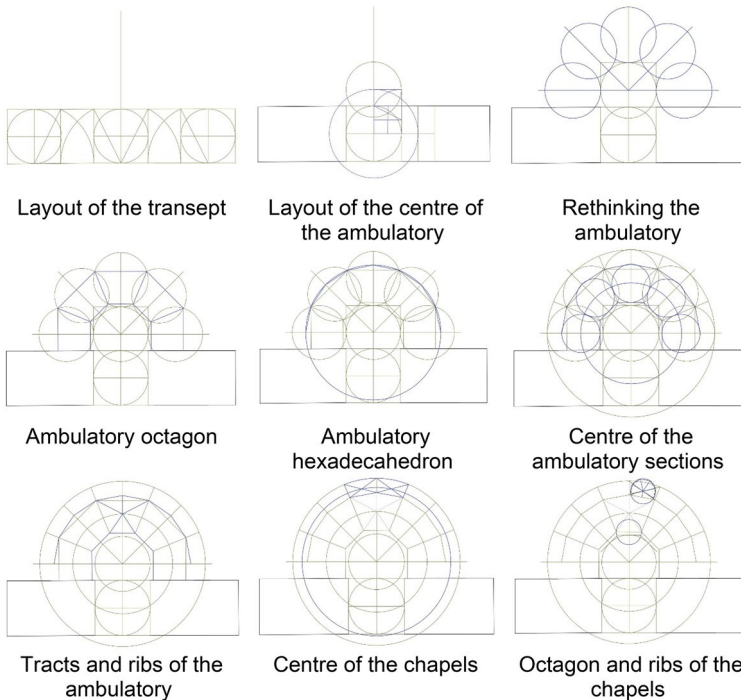


Fig. 17 The geometric process and subsequent interpretation for the chancel's and transept's design. Image: author López González

$$r_1 = 2.49a \quad r_2 = 1.20a \quad r_3 = 3a$$

Coordinates of point A = (2.49a, 22.50°).

Coordinates of point B = (1.20a, 22.50°).

From all this we conclude that the ratio between the radius of a circumference of radius "a" and the radius of the circumference that crosses the intersection points of two tangent circumferences of the same radius is constant for each of the points.

We call this constant with the Greek letter ρ:

$$\rho_A = r_A = 2.49135 \quad \rho_B = r_B = 1.20 \quad \frac{OA}{a} = 2.49 = \rho_A \quad \frac{OB}{a} = 1.20 = \rho_B$$

Therefore, the ratio between the radius of the circumference circumscribing the hexadecagon and the radius of the circumference inscribed in the inner octagon of the apse is 2.49.

Likewise, the ratio between the radius of the circumference circumscribed in the octagon of the axis of the apse and that inscribed inside the apse is 1.2.

Graphically, Fig. 18 verifies that the segment formed by the union of the vertices of the hexadecagon with the diametral axes of the circumferences is equal to the side of a regular pentagon inscribed in the circumference.

In this case the center of the apse O (0, 0) has been taken as origin or pole and as polar axis the line that goes through both, the polar center and point A (Fig. 19).

This result was also mathematically analyzed:

We calculate the distance between the points AE being A (2.49a, 0°), D (2.49a, 45°) and E (3a, 22.50°) for a coordinate center located at point O.

We apply the formula for the distance between two points in polar coordinates:

$$d^2 = r_1^2 + r_2^2 - 2r_1r_2\cos(\varphi_2 - \varphi_1)$$

Substituting the values:

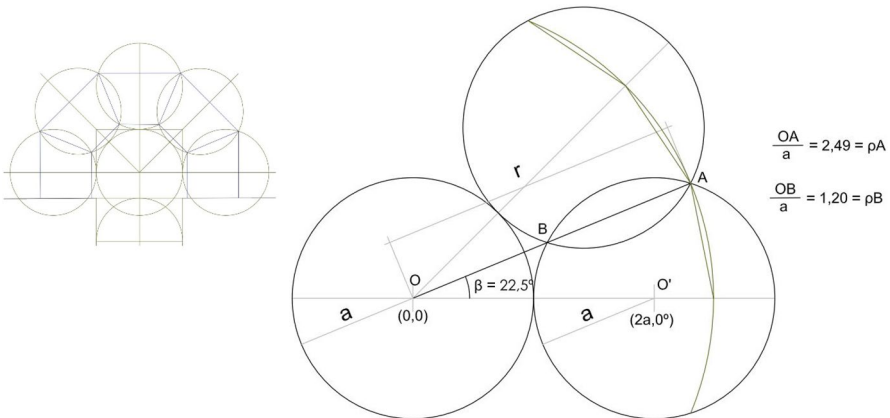
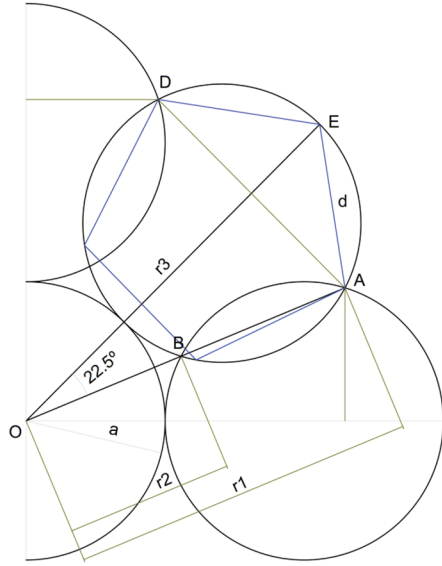


Fig. 18 Ratio between the hexadecagon and the regulatory circumference. Image: author López González

Fig. 19 Ratio between the ambulatory hexadecagon and the pentagon. Image: Cortés Meseguer



$$d^2 = 2.49a^2 + 9a^2 - 2 \cdot 2.49 \cdot 3\cos(22.50^\circ) \cdot a^2$$

$$d^2 = 1.40a^2 \quad d = 1.183a$$

The result coincides with the side of a regular pentagon inscribed on the circumference of radius a . $AE = ED$.

The distance AD is:

$$AD^2 = OA + OD - 2OA \cdot OD\cos 45^\circ$$

$$AD^2 = 2(2.49^2 a^2) - 2(2.49^2 a^2)/\sqrt{2}$$

$$AD = 1.91a$$

$$AD/AE = 1.91a/1.18a = 1.618$$

Consequently, the AD/AE ratio is golden.

Conclusions

The results show the direct relationship between the cathedral's geometric designs and knowledge of Euclidean geometry, as presumably applied in the design and construction of the building. This method is inserted into the tradition of architects.

These master builders designed religious edifices based on previously tested models and generalized working methods but adapted them to the geometric and aesthetic conditions of each case, as proposed by the studies by Ecohard (1977) and Soler Sanz (2014). Additionally, the proposed scheme for the layout of the cathedral based itself on a single regulatory layout that is applied in a simple, direct, and continuous way in both the preliminary design phase with compositional schemes and in the layout of the entire chancel. As verified, it is possible to complete the entire layout with a 6.44-m-long rope.

The module of the regulating layout (the circumference with a diameter of the width of the transept, which equals the width of the central nave) does not only generate the octagon of the apse and the dome but organizes the chapels of the ambulatory and the ambulatory sections. Its unfolding gives rise to the hexadecagon on which the chapels rest. It was confirmed that the layout model used as the generating principle for the composition of the temple corresponds to the geometry of the octogram and with the proportions associated with the silver number $\theta = 1 + \sqrt{2} = 2.41\dots$. It has been found that the relation between the hexadecagon and the regulatory circumference is a constant due to the tangent arrangement of the circumferences that form the ambulatory. This constant can be used in other examples where the same geometric arrangements are set although the measures vary.

The regulatory circumference of any geometric process that generates the ambulatory is based on the diameter and width of the transept (12.88 m = 50 Aragonese feet). A single circumference with a radius 25 Aragonese feet lays out the entire ambulatory: the transept, the apse, the key sections and ribs of the ambulatory and the octagon and ribs of the chapels. The use of a rope equal to 25 Aragonese feet (6.44 m) in length allowed us to rethink the entire chancel of the cathedral and obtain an innovative result that was ahead of its time.

The side chapels of the naves adjust their width to the width of the ambulatory, preventing these widths to adapt to any established proportion. This fact indicates that the chevet was drawn prior to the layout of the nave. Finally, the study concludes that the compositional model facilitates the analysis of construction processes and aids in documentary registration.

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