



Optimal Maintenance Thresholds to Perform Preventive Actions by Using Multi-Objective Evolutionary Algorithms

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Abstract: Maintenance has always been a key activity in the manufacturing industry because of its economic consequences. Nowadays, its importance is increasing thanks to the "Industry 4.0" or "fourth industrial revolution". There are more and more complex systems to maintain, and maintenance management must gain efficiency and effectiveness in order to keep all these devices in proper conditions. Within maintenance, Condition-Based Maintenance (CBM) programs can provide significant advantages, even though often these programs are complex to manage and understand. For this reason, several research papers propose approaches that are as simple as possible and can be understood by users and modified by experts. In this context, this paper focuses on CBM optimization in an industrial environment, with the objective of determining the optimal values of preventive intervention limits for equipment under corrective and preventive maintenance cost criteria. In this work, a cost-benefit mathematical model is developed. It considers the evolution in quality and production speed, along with condition based, corrective and preventive maintenance. The cost-benefit optimization is performed using a Multi-Objective Evolutionary Algorithm. Both the model and the optimization approach are applied to an industrial case.

Keywords: condition-based maintenance; optimization; multi-objective evolutionary algorithms; production systems

1. Introduction

The connected industry is currently a fact [1]. Thanks to the explosion of the Industry 4.0 which promotes automation through computer systems in manufacturing and aims to achieve an intelligent or smart factory [2], Condition-Based Maintenance (CBM) and Predictive Maintenance (PM) have gained importance in the last few years. Communication via cloud and artificial intelligence are facilitators of the explosion of the CBM concept [3]. Data availability coming from the effect of IoT (Internet of Things) devices is an influencing factor pushing research in the field of CBM, as the existing ICT (Information and Communications Technology) solutions simplify the on-field collection of a large amount of data [4]. As a consequence, the amount of papers including "Condition-Based Maintenance" or "Predictive Maintenance" in their title has skyrocketed, as it can be seen in Figure 1.





Figure 1. Amount of papers included in the Web of Science including "Condition-Based Maintenance" or "Predictive Maintenance" in their title.

In addition to what it is shown in Figure 1, in [5], an extensive review of condition-based maintenance research is performed using bibliometric indicators to show the increasing interest the topic is generating. A CBM strategy, where the optimal time to schedule a service visit is forecasted based on the condition of the equipment, is often proposed as an answer to the challenge of increasing the efficiency and reducing the cost of the equipment over their lifecycle. Therefore, CBM techniques are a key factor of maintenance.

The recent literature concerning these terms is properly summarized in specific bibliographic reviews [6–8]. These studies promote implementation frameworks for CBM such as the one proposed in reference [4], which are appropriate for complex production systems based on data mining and machine learning. One of the conclusions extracted from the literature reviewed was that almost no emphasis has been given to the modeling of the relation between the production speed of equipment and its age or deterioration level. It is worth noting that, in the existing papers modelling the production speed loss (e.g., [9]), a linear relation between deterioration and speed is established. Nevertheless, in this research, and based on a different input justifying a nonlinear relationship between speed and aging [10], a new speed model is proposed. Azadeh et al. [11] maintain that most existing literature either discusses CBM optimization of single component systems or focuses on technical issues about condition monitoring equipment and diagnosis. However, in this approach, a multi-component system will be approached.

Therefore, overall, this paper focuses on the problem of the CBM optimization in a manufacturing environment, with the aim of determining the optimal deterioration levels beyond which PM activities should be applied under cost and profit criteria. The developed cost and profit model considers in a relatively simple way the interaction of production, quality and maintenance aspects of a multi-component single machine. The CBM model is applied to a plastic injection machine in a manufacturing plant using a Multiple-objective Optimization Problem (MOP) and optimized using a Multi-Objective Evolutionary Algorithm (MOEA).

This paper is organized as follows: Section 2 introduces the proposed imperfect maintenance model, based on a Proportional Age Reduction (PAR) model, along with the calculation of the reliability parameters of a CBM maintenance strategy considering a PAR model. Section 3 explains the cost and benefit quantification models used in the optimization process, while Section 4 describes the

optimization procedure. Section 5 presents the case optimized and, finally, in Section 6, the conclusions and further guidelines of this research are reported.

2. Imperfect Maintenance Model

The effect of maintenance activities on the state of a piece of equipment assumes three possibilities. One of them is the perfect maintenance, which leaves the equipment "As Good as New" (GAN). On the other side, we find the minimal maintenance that leaves the equipment in "As Bad as Old" (BAO) state. The third possibility is the imperfect maintenance which improves the state of the equipment by some degree, by an effectiveness rate or factor, and it is the one that better represents real-world situations.

Several models have been developed to simulate imperfect maintenance. The Proportional Age Set-back (PAS) model proposed by Martorell et al. [12] and the Proportional Age Reduction (PAR) proposed by Malik [13] are some of them. In this paper, we use an age reduction preventive maintenance model based on PAR. In this approach, each maintenance activity is assumed to shift the origin of time from which the age of the component had its last maintenance intervention. The PAR model assumes that the maintenance activity reduces proportionally by an effectiveness factor ε the age gained from previous maintenance, where ε ranges in the interval $\varepsilon \in [0, 1]$. This model contains the extreme situations BAO and GAN. The PAR model is reduced to a BAO situation when $\varepsilon = 0$, while $\varepsilon = 1$ reduces to a GAN situation.

We will assume that a component is continuously monitored, so that no preventive action is taken until it arrives at a critical age. Let us denote this critical age by w_c . The component is left as BAO when a corrective maintenance (CM) activity is executed. The execution of a preventive maintenance activity (PM) means the application of an effectiveness ε and a PAR aging model. The behavior of the system is shown in Figure 2.

It can be observed that a ε proportion of the age gained since the last PM activity is shifted, and that time between two consecutive PM activities, M, gets shorter. As M tends to zero, it is necessary to fix a limit value, M_{min} , which indicates that the component should be upgraded. The model developed here considers an upgrading of $\varepsilon = 1$ (GAN).



Figure 2. Age vs. chronological time in a Proportional Age Reduction (PAR) model under a Corrective Maintenance (CM) strategy.

Under these conditions, it is possible to model the time between two consecutive maintenance activities, M. The time M_1 between the installation of the component and the execution of the first activity is given by:

$$M_1 = t_1 = w_c. \tag{1}$$

We will denote by w_1^+ the age of the component immediately after the first maintenance:

$$w_1^+ = (1 - \varepsilon)t_1 = (1 - \varepsilon)w_c.$$
⁽²⁾

Let t_2 be the time in which the second maintenance activity is executed, and M_2 the time interval between the first and the second maintenance activities:

$$M_2 = t_2 - t_1 = w_c \varepsilon. \tag{3}$$

We will denote by w_2^- the age of the component immediately before the second maintenance activity:

$$w_2^- = w_1^+ + (t_2 - t_1). \tag{4}$$

In addition, taking into account expressions (1) and (4), the expression for t_2 is obtained:

$$t_2 = w_c + w_c \cdot \varepsilon = w_c \cdot (1 + \varepsilon). \tag{5}$$

Proceeding in a similar way with t_3 (the time in which the third maintenance activity is executed), and M_3 (time interval between the second and the third maintenance activities), we have:

$$\begin{cases} w_2^+ = (1-\varepsilon)t_2 = (1-\varepsilon)(1+\varepsilon)w_c, \\ w_3^- = w_2^+ + (t_3 - t_2), \\ M_3 = (t_3 - t_2) = w_c \cdot \varepsilon^2. \end{cases}$$
(6)

Thus, the generalization to t_m , the time in which the *m*th maintenance activity is executed, and M_m the time interval between the (m - 1)th and the *m*th maintenance activities is given by:

$$\begin{cases} w_{m-1}^{+} = (1-\varepsilon)t_{m-1}, \\ w_{m}^{-} = w_{m-1}^{+} + (t_{m} - t_{m-1}), \\ M_{m} = (t_{m} - t_{m-1}) = w_{c} \cdot \varepsilon^{m-1}. \end{cases}$$
(7)

An overall expression of w_m^+ can be obtained by analyzing the evolution of w_3^+ :

$$w_{3}^{+} = w_{2}^{+} + (w_{c} - w_{2}^{+}) (1 - \varepsilon) =$$

$$= w_{c} (1 - \varepsilon^{2}) + [w_{c} - w_{c} (1 - \varepsilon^{2})] (1 - \varepsilon) =$$

$$= w_{c} (1 - \varepsilon^{2}) + w_{c} (1 - \varepsilon) - w_{c} (1 - \varepsilon^{2}) (1 - \varepsilon) =$$

$$= w_{c} (1 - \varepsilon) (1 + \varepsilon + 1 - 1 + \varepsilon^{2}) = w_{c} (1 - \varepsilon) (1 + \varepsilon + \varepsilon^{2}).$$
(8)

Hence, for w_m^+ , we obtain:

$$w_m^+ = w_c (1-\varepsilon) \sum_{k=0}^{n-1} \varepsilon^k.$$
 (9)

As it has been stated before, a component will be substituted or upgraded into a GAN situation when $M_{m+1} < M_{min}$ is satisfied. Given w_c and M_{min} , the amount of PM activities *e* executed before a component is replaced is calculated as follows:

$$w_{c}\varepsilon^{e-1} \leq M_{min} \Rightarrow (e-1)\ln\varepsilon \leq \ln\left(\frac{M_{min}}{w_{c}}\right)$$

$$\Rightarrow e-1 \leq \frac{\ln\left(\frac{M_{min}}{w_{c}}\right)}{\ln\varepsilon} \Rightarrow e \leq 1 + \ln\left(\frac{M_{min}}{w_{c}}\right)^{1/\varepsilon}.$$
(10)

Under these conditions, it is possible to obtain an age-dependent reliability model in which the induced or conditional failure rate in the period *m* after the maintenance number *m* is given by:

$$h_m(w) = h(w(t,\varepsilon)) + h_0, \quad w \ge w_m^+, \tag{11}$$

where h_0 represents the initial failure rate of the component, that is to say, when it was first installed.

The age of the component after the maintenance number m is given by Equation (9). If we adopt a Weibull model for the failure rate, the expression of the induced failure rate after the maintenance number m can be written as:

$$h_m(w) = \lambda^{\gamma} \cdot \gamma \cdot (w_m(t,\varepsilon))^{\gamma-1} + h_0, \quad w \ge w_{m-1}^+, \tag{12}$$

where λ is the scale parameter, and γ is the shape parameter. For this Weibull distribution, the accumulated failure rate is defined by:

$$H_m(w) = H(w_m) = (\lambda w_m(t,\varepsilon))^{\gamma} + h_0 w_m.$$
(13)

For the specific cases of w_m^- and w_{m-1}^+ , the accumulated failure rate is given by:

$$H_m^- = H_m(w_m^-) = (\lambda w_m^-)^{\gamma} + h_0 w_m^- = (\lambda w_c)^{\gamma} + h_0 w_c$$
(14)

and:

$$\begin{aligned}
H_{m-1}^{+} &= H_{m-1}(w_{m-1}^{+}) = \left(\lambda w_{m-1}^{+}\right)^{\gamma} + h_{0}w_{m-1}^{+} - = \\
&= \left(\lambda w_{c}(1-\varepsilon)\sum_{k=0}^{m-2}\varepsilon^{k}\right)^{\gamma} + h_{0}w_{c}(1-\varepsilon)\sum_{k=0}^{m-2}\varepsilon^{k} = \\
&= \left(\lambda w_{c}(1-\varepsilon)\sum_{k=1}^{m-1}\varepsilon^{k-1}\right)^{\gamma} + h_{0}w_{c}(1-\varepsilon)\sum_{k=1}^{m-1}\varepsilon^{k-1}.
\end{aligned}$$
(15)

Next, we will define the averaged failure rate between $t_m - t_{m-1}$ (h_m^*), which is necessary to calculate unavailability rates:

$$h_{m}^{*} = \frac{1}{w_{m}^{-} - w_{m-1}^{+}} \left(H_{M}^{-} - H_{m-1}^{+}\right) = \frac{1}{w_{c} - \left(w_{c}(1-\varepsilon)\sum_{k=1}^{m-1}\varepsilon^{k-1}\right)} = \frac{\lambda^{\gamma}w_{c}^{\gamma-1}\left[1 - \left((1-\varepsilon)\sum_{k=1}^{m-1}\varepsilon^{k-1}\right)^{\gamma}\right]}{1 - (1-\varepsilon)\sum_{k=1}^{m-1}\varepsilon^{k-1}} + h_{0}.$$
(16)

Finally, using the averaging process given by Equation (16), the averaged failure rate over the lifetime of a component subject to e PM activities can be obtained:

$$h^* = \frac{\sum_{1}^{e} h_e^* M_e}{M_e}.$$
 (17)

3. Cost and Benefit Models

3.1. Maintenance Costs

Maintenance costs of equipment come from condition monitoring (CM), preventive maintenance, corrective maintenance (consequences of idling, minor stops and failure or breakdowns), and costs of upgrading or substitution of components (C_u). The cost associated with condition monitoring is:

$$C_{cmt}(\mathbf{x}) = L \cdot C_{hct},\tag{18}$$

where *L* represents the analysis period and C_{hct} the hourly cost of monitoring.

Preventive maintenance costs are calculated using:

$$C_{pm}(\mathbf{x}) = \frac{L}{M} \cdot d_{pm} \cdot c_{hpm},\tag{19}$$

where d_{pm} and c_{hpm} represent, respectively, the mean time and the average hourly cost of performing preventive maintenance, and x represents the vector of decision variables. For this model, it is worth noting that this cost will be incurred every PM action. In addition, in some preventive actions, it will also be necessary to add the cost of upgrading the component.

The cost related to corrective maintenance is:

$$C_{cm}(\mathbf{x}) = u_r(\mathbf{x}, \mathbf{M}) \cdot \frac{L}{M} \cdot c_{hcm} \cdot d_{cm},$$
(20)

where d_{cm} is the mean time of performing corrective maintenance, c_{hcm} is the average hourly cost of performing corrective maintenance and $u_r(\mathbf{x}, M)$ is the time-dependent unreliability caused by discontinuity:

$$u_r(\mathbf{x}, \mathbf{M}) = \rho + (1 - \rho) \cdot \left(1 - e^{-h^* M}\right), \qquad (21)$$

 ρ being the probability of failure on demand, and h^* the averaged failure rate over the lifetime of a component (17).

Finally, when the amount of maintenance activities allowed before changing a component is exceeded, the cost of upgrading a component (C_u) has to be considered. It is calculated by multiplying the amount of times a component has been changed (n_c) and the cost of upgrading or substituting a single component C_c :

$$C_u = C_c \cdot n_c, \tag{22}$$

being

$$n_c = \frac{L}{\sum_{k=1}^e M_k}.$$
(23)

3.2. Cost Related to the Production Speed Loss Because of Aging

Traditionally, the literature assumes that the production rate or speed of the equipment is predetermined and constant during the equipment's life. However, it is expected that the production speed decreases as a consequence of the aging of the equipment. In [10], both linear and bent deterioration or aging speeds are proposed. As bent relationships between chronological time and age or deterioration level have not been proposed in the literature for modeling equipment speed loss, a quadratic relationship between the age and the production speed is proposed in this research. The production speed after having performed the *m*th maintenance activity can be calculated as:

$$V_m(w) = V_0 - \tau^2 \cdot w_m(t,\varepsilon), \tag{24}$$

where V_0 is the initial production speed (i.e., as per design), τ represents the speed reduction coefficient, and $w_m(t, \varepsilon)$ is the age of the component after maintenance number *m*, which under a PAR model is given by Equation (9).

An average value of the production speed between the activities $(m - 1) (V_{m-1}^+)$ and $m (V_m^-)$ is required in order to determine the cost of the lost production speed. Let V_m^* be this average value and it can be calculated using the following equations:

$$\begin{cases} V_{m}^{-} = V_{0} - \tau^{2} \cdot w_{c}, \\ V_{m-1}^{+} = V_{0} - \tau^{2} \cdot w_{c}(1-\varepsilon) \cdot \sum_{k=1}^{n-1} \varepsilon^{k-1}, \\ V_{m}^{*} = \frac{1}{M_{m}} \int_{w_{m-1}^{-}}^{w_{m}^{+}} v(t) \cdot dt = V_{0} - \tau^{2} \cdot w_{c} \left(\frac{1 + (1-\varepsilon) \cdot \sum_{k=1}^{n-1} \varepsilon^{k-1}}{2}\right). \end{cases}$$
(25)

Similar to the failure rate, it is possible again to get an averaged production speed for the lifetime of a component, V^* :

$$V^* = \frac{\sum_{1}^{e} V_e^* M_e}{M_e}.$$
 (26)

In addition, using expression (26) to calculate the average value of the speed production, it is possible to determine the production time lost t_{sl} related to a reduced speed. Assuming that only a fraction of the production system is available, the production time lost is expressed as:

$$t_{sl}(\mathbf{x}) = \left(1 - A_s(\mathbf{x}) \cdot \frac{V^*}{V_0}\right) \cdot L,$$
(27)

 $A_s(\mathbf{x})$ being the availability of the system obtained as:

$$A_s(\mathbf{x}) = 1 - U_s(\mathbf{x}). \tag{28}$$

 $U_s(\mathbf{x})$ is the unavailability of the system. It is evaluated using the system fault tree and the single component unavailability. These contributions are the time-dependent unreliability $u_r(\mathbf{x})$ given by Equation (21) and the unavailability due to corrective maintenance $u_{cm}(\mathbf{x})$ given by:

$$u_{cm}(\mathbf{x}) = \frac{1}{M} \cdot u_r(\mathbf{x}) \cdot d_{cm}.$$
(29)

The unavailability associated with preventive maintenance in the period *L* is given by:

$$u_{pm}(\mathbf{x}) = \frac{1}{M} \cdot d_{pm}.$$
(30)

The cost related to production speed loss of the equipment (C_{sl}) in the period *L* is proportional to the production time lost and is given by:

$$C_{sl}(\mathbf{x}) = c_{hsl} \cdot t_{sl}(\mathbf{x}), \tag{31}$$

 c_{hsl} being the average hourly cost due to non-produced items and t_{sl} is defined in (27).

3.3. Quality Costs

In Section 2, the PAR model was defined. It assumes that each preventive maintenance activity reduces the age of the equipment, depending on an effectiveness parameter ε . Changing the age of the equipment affects the time distribution of the system swaps to an out-of-control state. As a consequence, the expected number of non-conforming items is also affected.

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In this section, we develop a quality cost model that considers the effects of PM and upgrading activities on component age based on the PAR model. The model is defined under the following assumptions:

- (1) the equipment only produces non-conforming items with constant rate, *α*, while the process is out-of-control,
- (2) the time of the system swaps out-of-control follows a Weibull distribution which depends on the age of the equipment,
- (3) the preventive maintenance and the process inspection are performed simultaneously,
- (4) inspections are error free,
- (5) the process returns to the in-control state when the preventive maintenance activity is performed.

It is necessary to determine the fraction of time in which the process is in Under Control state (UC state):

$$\kappa_m(w) = \int_{w_{m-1}^+}^{w_m^-} w_m \cdot f(w_m) \cdot dw_m,\tag{32}$$

where $f(w_m)$ is the density function, obtained using the conditional hazard function and calculated as:

$$f(w_m) = \lambda \cdot \gamma \cdot (\lambda w_m)^{\gamma - 1} \cdot \exp\left(-\left(\lambda \cdot w_m\right)^{\gamma}\right).$$
(33)

After substituting (33) in expression (32), we obtain:

$$\kappa_{m}(w) = \int_{w_{m-1}^{m-1}}^{w_{m}^{-}} w_{m} \cdot f(w_{m}) \cdot dw_{m} = = \int_{w_{c}}^{w_{c}^{-}} w_{c}} w_{m} \left(\lambda \cdot \gamma \cdot (\lambda w_{m})^{\gamma-1} \cdot \exp\left(-(\lambda \cdot w_{m})^{\gamma}\right)\right) dw_{m}.$$
(34)

Once the time the process is in-control between two maintenance activities is calculated, $(M_m - \kappa_m)$, it is possible to obtain quality costs:

$$C_q = n_c \cdot C_{\alpha} \cdot \sum_{m=1}^{x} V_m^* \cdot (M_m - \kappa_m) \cdot A_s(\mathbf{x}) \cdot \alpha, \qquad (35)$$

where C_{α} is the cost of the non-conforming unit.

3.4. Profit

Besides the costs, benefits also have to be considered to quantify the consequences of a given preventive maintenance action in economic terms. A net profit function P that denotes the benefits from the sale of products is introduced as follows:

$$P = n \cdot \Psi, \tag{36}$$

where *n* is the number of non-defective items produced in the period of analysis *L*, and Ψ is the estimated cost margin of one single product unit.

The number of non-defective items produced during the *L* period can be evaluated by considering the time the process has been in an in-control or out-of-control state. When the process is in an out-of-control state, a percentage of $(1 - \alpha)$ of the products are non-defective, whereas in an in-control state, the 100% of the products are non-defective. Therefore, the profit can be evaluated as:

$$P = n_c \cdot A_s(\mathbf{x}) \cdot \sum_{m=1}^{x} \left[(M_m - \kappa_m) \cdot (1 - \alpha) + \kappa_m \right] V_m^* \cdot \Psi.$$
(37)

4. Mathematical Formulation and Optimization Procedure

A multi-objective problem (MOP) can be stated as follows [9]:

minimize/maximize
$$\mathbf{y} = \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})),$$
 (38)

subject to
$$\mathbf{g}(\mathbf{x}) = (g_1(\mathbf{x}), g_2(\mathbf{x}), \dots, g_m(\mathbf{x})) \le \mathbf{L},$$
 (39)

where $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^k$ is the objective function, $g_i : \mathbb{R}^n \to \mathbb{R}$, i = 1, ..., m, are the constraints, $\mathbf{L} = (l_1, l_2, ..., l_m)$, $\mathbf{x} = (x_1, x_2, ..., x_n) \in X$ is the decision vector (or vector of decision variables), and $\mathbf{y} = (y_1, y_2, ..., y_k) \in Y$ is the objective vector. We denote by X and $Y = \mathbf{f}(X)$, the decision space and the objective space, respectively.

The optimization of the maintenance interval in terms of cost and benefit can be expressed as a MOP. The formulation of both variables has been presented in Section 3. It can be observed that both models depend on w_c and M_{min} which act as decision variables and are encoded in the decision vector **x**. In our problem, the problem is bi-objective (n = 2) $\mathbf{f}(\mathbf{x}) = (C(\mathbf{x}), P(\mathbf{x}))$, and the aim is to minimize the cost function $C(\mathbf{x})$ and to maximize the profit function $P(\mathbf{x})$. In this optimization approach, maintenance managers' and plant managers' purposes are considered jointly: maintenance managers usually try to minimize costs related to equipment inefficiencies $(C(\mathbf{x}))$, whereas plant managers aim at maximizing the profitability of the plant $(P(\mathbf{x}))$. It is widely known that multi-objective optimization should be complementary with respect to measures taken into consideration. In this case, $C(\mathbf{x})$ and $P(\mathbf{x})$ are defined as complementary, since Ψ is defined as a constant value, so the cost terms of $C(\mathbf{x})$ do not have an influence on it. In a real system, Ψ is a consequence of the sale price and all the productive and logistic costs, including the ones formulated in the equipment model. However, if the model had to consider $C(\mathbf{x})$ to obtain Ψ , other costs, such as logistics, management and raw material costs should be considered. It is worth remarking that the same joint $C(\mathbf{x})$ and $P(\mathbf{x})$ optimization approach was applied to the optimization of preventive maintenance in [9]. The cost function $C(\mathbf{x})$ is defined as the sum of the maintenance costs, costs related to production speed loss and quality costs. It is calculated using Equations (18), (19), (20), (22), (31) and (35):

$$C(\mathbf{x}) = C_{cmt} + C_{pm} + C_{cm} + C_u + C_{sl} + C_q.$$
(40)

The profit $P(\mathbf{x})$ is calculated using (37). The vector of constraints, $\mathbf{g}(\mathbf{x})$, is given by

$$\mathbf{g}(\mathbf{x}) = (C(\mathbf{x}), P(\mathbf{x}), U(\mathbf{x})), \qquad (41)$$

which means that m = 3, and $\mathbf{L} = (C_i, P_i, U_i)$, C_i , P_i and U_i being the cost, the profit and the unavailability associated with the initial values of the decision vector, respectively.

Hence, our problem is formulated as follows:

minimize/maximize
$$\mathbf{y} = \mathbf{f}(\mathbf{x}) = (C(\mathbf{x}), P(\mathbf{x}))$$
, (42)

subject to
$$\mathbf{g}(\mathbf{x}) = (C(\mathbf{x}), P(\mathbf{x}), U(\mathbf{x})) \le (C_i, P_i, U_i),$$
 (43)

where **x** represents the vector of the CBM decision variables to be optimized that will be presented in Section 5.1.

Problem (42) has been solved using the Nondominated Sorting Genetic Algorithm (NSGA-II) [14]. Despite NSGA-III [15] being suggested for joint optimization of several objectives, NSGA-II results efficient with bi-objective optimizations. The stopping criteria used in the simulation has been the convergence of the result after five generations.

5. Application Case

In this section, the optimization problem (42) that consists of three continuously monitored components of a simplified injection system is considered. The optimization problem is solved by applying the cost and profit models described in Section 3 and the hybrid algorithms.

The system is installed in a Spanish manufacturing company and consists of three components (C1, C2 and C3) in serial configuration. The effect of component faults is the following: component C1's deterioration influences only unavailability, C2's deterioration affects unavailability and productive speed loss, and C3's deterioration affects unavailability and quality. Three preventive maintenance activities (M1, M2 and M3) are applied to the components to reduce the deterioration level: M1 is applied to C1, M2 to C2 and M3 to C3. This study is oriented to offer a CM alternative.

5.1. Simulation Values of the Equipment

We will consider the optimization criteria formulated for the cost $C(\mathbf{x})$ and profit $P(\mathbf{x})$ in (43) and a year working interval of L = 10. The models for cost and profit depend on the frequencies of the PM activities (M1, M2, and M3), which act as decision variables. The definition of two parameters, w_c and M_{min} , is required for each PM activity. Each w_{c_i} for i = 1, 2, 3 is the level of deterioration of the monitored component when a preventive maintenance (PM) action is performed. M_{min_i} represents the minimum time between two PMs, that is to say, if the maintenance age reduced in a component via PM is small, a replacement of the component will be performed, making the age to tend to zero again. Thus, w_{c_1} , M_{min_1} , w_{c_2} , M_{min_2} , w_{c_3} , M_{min_3} are the components of the chromosome of this problem. Variables w_{c_i} are integer values ranging from 1 to 260 days (average number of working days per year). M_{min_i} is represented by an integer value between 1 and w_{c_i} . No constraints are applied to the objective functions.

Tables 1–4 show the relevant component reliability, preventive maintenance, corrective maintenance and cost data for this case study, respectively.

Component	$\lambda \ 10^{-4}/\mu$	γ
C1	5	2
C2	2	2.9
C3	4	2

Table 1. Reliability data.

Activity	ε	Duration <i>h</i>		
M1	0.9	0.5		
M2	0.9	0.5		
M3	0.9	1		

Table 3. Parameters related to corrective maintenance.

Component	ε	Duration <i>h</i>		
C1	0	0.5		
C2	0	0.5		
C3	0	1		

<i>C</i> _α €/u	τ u/h ²	C _{hsl} €/h	ho 10 ⁻³	α	h_0	V ₀ u/h	c _{hcm} €/h	c _{hpm} €/h	c _u €/u
6	0.0017	25	1	0.03	0	180	45	30	1

u being the product unit.

5.2. Simulation Values of the Algorithms

Based on similar past research [9], we present in Table 5 the simulation values that require less computational cost.

Parameter	Value		
Population size	100		
Selection rate	0.25		
Crossover rate	0.5		
Mutation probability	0.75		

Table 5. Values used in the NSGA-II.

The selection rate represents the percentage of the best individuals of the solution matrix that will be selected to be "potential parents" of the forthcoming generation. The crossover rate is the probability of exchanging genetic material between two parent individuals. Note that in Table 5 we include the mutation probability which indicates the percentage of the cases in which mutation will be applied after having performed crossover. The mutation rate, however, represents the percentage in which mutation is applied. In our case, it has been considered $\frac{1}{n}$ as the mutation rate, *n* being the number of bits in the encoding of the decision variable.

5.3. Results

Figure 3 shows the Pareto Front obtained in the simulation under the aforementioned conditions. The set of solutions satisfies the constraints of the problem and the decision makers can select the solution that fits their preferences.



Figure 3. Optimization results.

6. Conclusions

The digitalization of our assets within the Industry 4.0 context puts an extra stress on the correct maintenance of machinery and devices. The economic optimization of maintenance is challenging. This paper presents a model which can be useful to calculate the profitability of a Condition-Based Maintenance strategy applied to equipment. The model jointly considers unavailability, production speed loss, loss of quality costs, maintenance (corrective, preventive, condition monitoring and component replacement) costs, and the profit related to a maintenance strategy (as the profit margin of sold products), modelled by an imperfect component aging model. The approach presented herein is innovative because the model is based on a nonlinear speed loss relation between the age or deterioration level and the chronological time. Genetic Algorithms (GAs) are likely the most

widely used type of Evolutionary Algorithm. In recent years, there has been a growing effort to apply GAs to general constrained optimization problems, as most of the solutions of engineering optimization problems are constrained by restrictions imposed on the decision variables. In this paper, we have implemented a Multi-Objective Nondominated-Sorting GA and we have used it to perform the constrained optimization of condition monitoring maintenance activities. It has been successfully applied to train and give extra inputs to technicians and managers in their decision-making processes, in order to establish when to perform a maintenance activity. However, further research oriented into the joint optimization of several pieces of equipment within a production line is needed, as e.g., bottleneck equipment cannot be handled in the same way as non-critical equipment.

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Abbreviations

The following abbreviations are used in this manuscript:

- CBM Condition-Based Maintenance
- IoT Internet of Things
- ICT Information and Communications Technology
- PM Predictive Maintenance
- MOP Multiple-objective Optimization Problem
- MOEA Multi-Objective Evolutionary Algorithm
- PAR Proportional Age Reduction
- GAN As Good as New
- BAO As Bad as Old
- PAS Proportional Age Set-back
- CM Corrective Maintenance
- NSGA Nondominated Sorting Genetic Algorithm
- GA Genetic Algorithm

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