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Metamodel-assisted design optimization in the field of structural engineering: A literature review

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ABSTRACT

Metamodel-assisted optimization is a valuable alternative to handle structural design optimization procedures, which are usually quite expensive and sometimes even prohibitive. This paper presents an up-to-date literature review on *metamodel-assisted structural design optimization* (MASDO) in the structural engineering field. The period analyzed is from 2000 to the present, involving 111 publications and 169 case studies. In order to provide practical recommendations on best practices to perform MASDO, eight categorical variables are analyzed, and underlying relationships between them are detected by applying simple and multiple correspondence analysis. Surprisingly, there are fewer published papers on the subject than expected. Most focus on improving or developing metamodeling strategies using simple (benchmark) case studies to validate the proposed methodologies. Consequently, the originality and value of this study lie in the conclusions obtained from the statistical analysis, which serve as a practical guide for incorporating metamodeling strategies in future projects related to structural design optimization.

1. Introduction

One of the most important challenges faced by today's engineers and designers is obtaining optimal structures according to the new challenges posed by the current issue of climate change and the gradual lack of existing resources. It is well known that the construction sector has a significant impact on the environment due to the high consumption of natural resources through the extraction of materials [1], the use of energy, harmful emissions, and waste generation [2]. For this reason, environmental impacts and resource consumption can be reduced by incorporating novel building materials and recycling, but also by the more efficient use of them due to the structural design optimization of constructions [3]. However, these optimization procedures are usually highly computationally expensive.

In general, there are two main methods to perform structural optimization [4]. *Exact methods* are generally based on mathematic programming. On the other hand, *heuristics* consist of artificial intelligence strategies usually imitating natural processes [5]. Heuristic algorithms have proven their efficiency and versatility in solving large-scale and highly nonlinear optimization problems [6], as is usually the case with structural optimization. For this reason, it is usual to find structural optimization problems solved by heuristics. Several heuristic search algorithms belonging to this category are harmony search (HS), simulated annealing (SA), threshold accepting (TA), genetic algorithms (GA), ant colonies (ACO), particle swarm optimization (PSO), tabu search (TS), flower pollination algorithm (FPA), teaching–learning based optimization (TLBO), among others [7]. However, these clever strategies are not always enough to deal with challenges that have been appearing for years.

Although computational availability has proliferated in recent years, engineering systems have become progressively more accurate and, consequently, more complex. Accurate and expensive numerical methods must be the solution for most engineering troubles, usually represented by partial differential or integral equations, e.g., structural finite element analysis (FEA), where a single function evaluation is usually considerably time-consuming [8]. Regular optimization problems involve thousands of single-function evaluations, and if aspects such as design under uncertainties are considered, the problem becomes almost prohibitive.

For this reason, *metamodel-assisted optimization* has arisen as a valuable methodology to handle such complex engineering optimization problems [9]. The most straightforward methodology consists of

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creating a group of design vectors belonging to the design space, for which high fidelity (or physics-based model) simulations (see Fig. 1(a)) are carried out. Then, regression or interpolation models (also called *metamodels* or *surrogate models*) are constructed, and they can be analyzed by, e.g., optimization algorithms [8]. Thus, in their most basic form, metamodels are black-box functions that relate input variables × to an output *Y(x)*, permitting cheap evaluations of *Y(x)* in function of the values of × (see Fig. 1(b)) [10].

As mentioned, a metamodel approximates an accurate simulation model, i.e., the so-called "model of a model". Consequently, when metamodels are used to replace expensive high-fidelity simulation during structural optimization processes, this is denominated *metamodel-assisted structural design optimization* (MASDO). The mathematical implementation (type of metamodel) appropriate for a particular estimation can vary depending on the intended use or the underlying physics that the model needs to simulate. Conventionally, metamodels have been simple polynomials, but other that better reflect complex interactions are gaining acceptance [12]. Kriging, Neural Networks (NNs), or Radial Basis Functions (RBFs) are among the most used metamodels in MASDO.

On the other hand, each metamodel needs a specific dataset for its proper construction. Depending on the number of variables and the size of the solution space, the number of high-fidelity simulations required will directly influence the correct construction of the metamodel. In addition, the metamodel selection depends on the formulated problem and the case study or structure to be optimized. Therefore, knowing the relationships between the type of metamodels and other categorical variables related to structural design optimization would be of great interest. In this context, this paper presents a comprehensive review of the up-to-date literature involving metamodels to support the optimization of civil engineering structures.

Similar studies have been previously developed. [13] conducted a review on surrogate modeling for sustainable building design concerning applications in the conceptual design stage of buildings. A similar research was carried out by [11], where the scope was focused on the application of Neural Networks for building performance simulation. These two papers present an architectural point of view and not a structural one. [14] and [15] provide a review on the artificial intelligence applied to a wide range of structural engineering applications. The former is more focused on machine learning techniques. For their part, [16] and [17] emphasize their review of surrogate-assisted optimization considering uncertainties: reliability-based and robust optimization, respectively. On the other hand, [18] presents a systematic literature review on metamodel-based simulation optimization, considering a wide range of applications and focusing primarily on bibliometric results. Finally, [19] presents a review on machine learning techniques applied to construction, specifically in the areas of concrete technology, retaining wall design, pavement engineering, tunneling, and construction management. As can be seen, and based on the point of view and scope of this research, these studies have two main drawbacks. Firstly, they are devoted to a particular type of metamodeling techniques or a specific type of formulation. Secondly, the implemented case studies cover a wide range of fields, even some focusing on architectural rather than structural design. Consequently, there is a research gap in the practical use of metamodels to aid the design optimization of, e.g. a particular building, in function of some other aspects such as the type of formulation (deterministic or considering uncertainties), the objective function or the variables of the optimization problem. That is to say, a researcher that wants to propose strategies to support a certain structural design process could search previous studies and select alternatives based on them. However, this information may be not enough to choose proper strategies. In order to fill this gap, eight categorical variables related to MASDO problems are selected to be analyzed. Fig. 2 shows them and their relationship with the other ones. Consequently, with this review, we intend to determine the best practices for implementing proper metamodeling strategies in structural optimization problems by giving practical recommendations. It is expected to lead the way for the application of metamodeling in current and future design optimization projects in structural engineering. These recommendations are obtained through a statistical analysis to detect subjacent relationships among the categorical variables. Therefore, these are based on what has been done in this field. It can be the first step to designing an experiment in which metamodeling strategies are applied and tested on specific problems.

To reach previous goals, the organization of this paper is as follows. Section 2 is dedicated to explaining the literature search strategy and corresponding results. In section 3, a general overview of metamodeling strategies is presented. Section 4 presents the analysis of the selected categorical variables, summarizing the most relevant reviewed research framework and providing shallow relationships between metamodeling techniques and highly related variables such as the type of formulation. Section 5 consists of the statistical analysis, using correspondence analysis to go deeper into the subjacent relationships among categorical variables. Section 6 encompasses a discussion of results, summarizes the benefits of incorporating metamodels into structural design processes and proposes future promising lines of research. Finally, in section 7, conclusions are drawn.

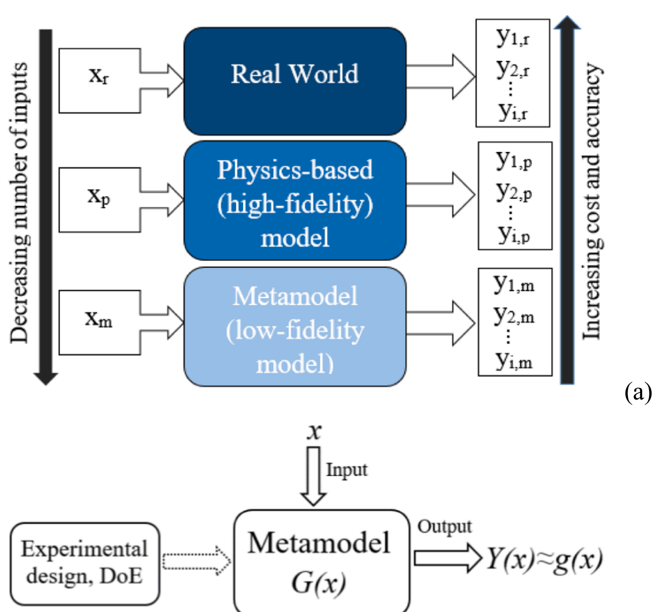

Fig. 1. Concepts related to metamodeling, (a) relationship between accuracy and computational cost for different modeling approaches (adapted from [11]) and (b) generic description of a metamodel as a black-box function (adapted from [10]).

2. Literature search strategy

Several literature searches were performed using the searching engines Google Scholar [20], ScienceDirect [21], and Springer [22]. The keywords used were "structural optimization", in combination with the metamodel-assisted part (e.g. "metamodel assisted", "metamodel aided" "metamodel supported", "surrogate model" or simply the nomenclature of the most used metamodels) and the civil engineering structures part (e.g. "structures", "concrete structures", "steel structures", and so on). For example, the most helpful combination was "metamodel assisted" + "structural optimization" + "structures". Hundreds of documents were filtered by eliminating duplicates and those that did not match the scope of this review. Another search strategy was to check the references of the selected papers. This was very useful for incorporating new documents overlooked in the main search. A total of 111 publications were finally selected, containing 169 case studies.

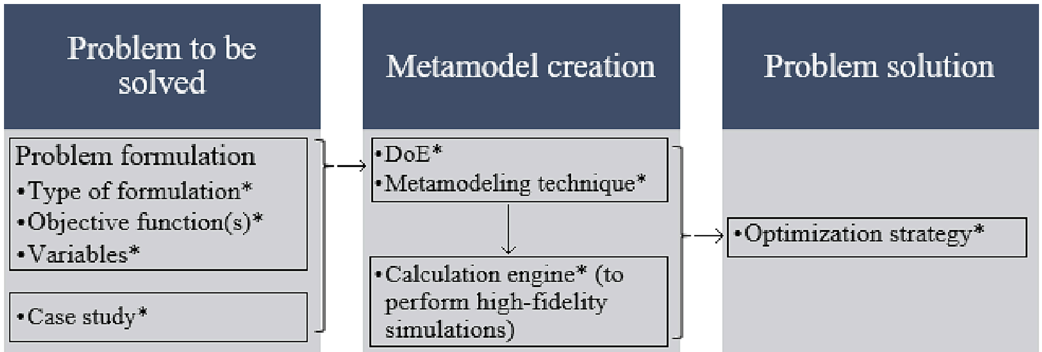

Fig. 2. The eight categorical variables considered (*) and their relationship with the other ones in the MASDO problem.

Fig. 3 shows that the journal with the more publications is, by far, Structural and Multidisciplinary Optimization, and most of the publications are journal papers. However, it can be said that the editorial with the higher number of publications is Elsevier (37% of the total publications), closely followed by Springer (33%) and Tylor & Francis (8%). Finally, China has the highest number of publications (27% of the total), followed by Spain (8%), the USA, Iran, and India (7% each).

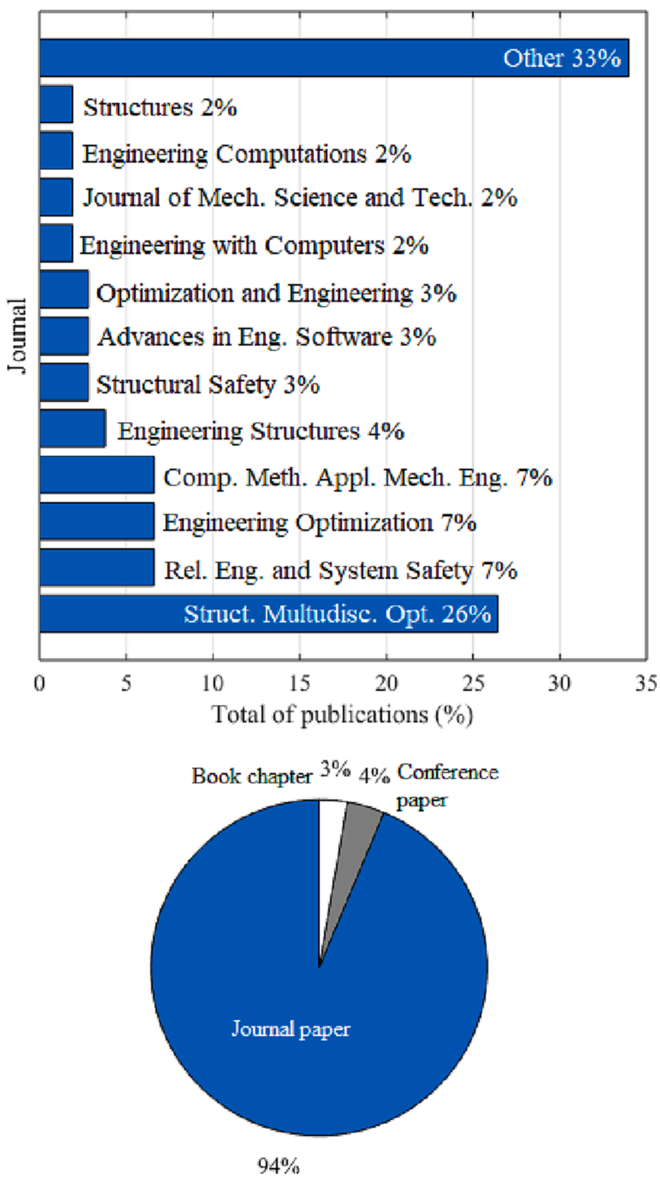

Fig. 3. Analysis of the search results classified by journal and type of publication.

3. Overview on metamodel techniques

The metamodel construction consists of three main parts: (1) obtaining the initial sampling points inside the design space (*DoE*), (2) choosing the *metamodel* technique to build the approximate mathematical model, and (3) choosing the *fitting model* or approach to validate the proposed strategy [23]. There are several alternatives for carrying out these steps [24]. Based on the literature review, the main strategies used to incorporate metamodels into the structural design optimization are shown in Fig. 4. A deep description of these procedures can be found in [12,25,26].

3.1. Design of experiment

The DoE allows for selecting the coordinates of the initial points of the input data in the best possible manner. It is advisable to minimize the quantity of these points to reduce the experimental effort. The location of these points should allow for collecting substantial information about the analyzed system.

DoE can be split into two main groups. *Classic designs* include *factorial or fractional factorial designs*, *central composite designs*, *D-optimal designs*, and others [27]. These designs lean toward placing the sample points near the design space boundaries, and only a few points are located inside it. Consequently, they are mainly used to construct polynomial metamodels. The other group, called *space-filling designs*, is more suitable for building more advanced metamodels. The most popular *space-filling designs* are *Latin Hypercube Sampling*, *Distance-Based designs*, and *Low-Discrepancy Sequences* [23]. A brief description of the most used metamodels for MASDO (see Fig. 4) is further developed.

A design in which the n factors belonging to the system are varied only on a finite number of defined levels l is denominated a factorial design. If a design covers all possible factor-level combinations is then a *full factorial design* (FFD) (see Fig. 5(a)). The number of experimental runs to be performed is obtained by the product of each factor's respective number of discrete levels. An FFD with n factors and l levels has l^n sampling points. The major use of FFDs is for *screening experiments* [26].

FFDs usually provide too many sample points to construct the metamodel, so a reduction of the l^n samplings can be made through *fractional factorial designs*. Therefore, a *fractional factorial design* is a subgroup of an FFD. Thus, the experimental design is represented by l^{n-p}. In FFDs, n factors are analyzed at l levels each. Therefore, the integer p defines the decrease compared to an FFD (see Fig. 5(b)) [26].

On the other hand, a Central Composite Design (CCD) is a two-level factorial design, increased by n_0 center points and two "star" points located at $\pm\, a$ for each factor. An example of a CCD for three factors is

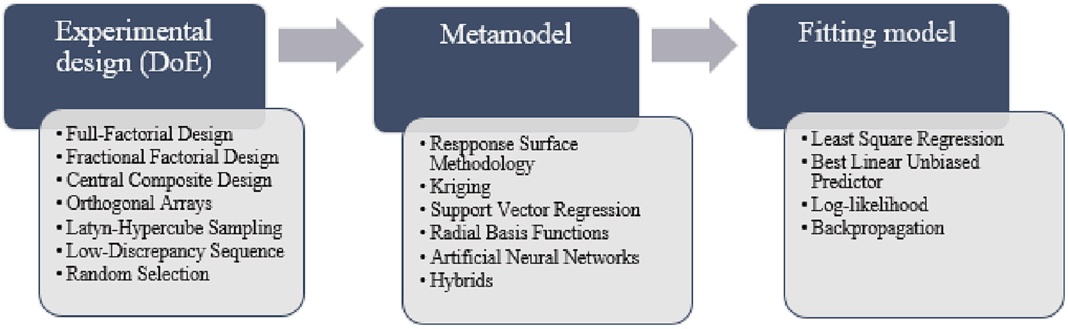

Fig. 4. Classic metamodeling techniques commonly used in MASDO. Extra information can be found in [12,25,26].

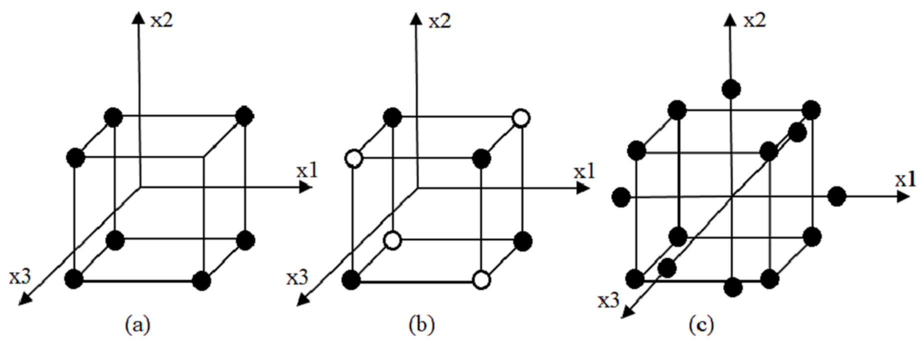

Fig. 5. Elementary three-factor designs: (a) 2^3 full factorial, (b) $2^{3\text{-}1}$ fractional factorial, and (c) composite design [25].

represented in Fig. 5c, where $2\,k + k(k\text{-}1)/2 + 1$ coefficients are estimated by $2^{(k\text{-}p)} + 2\,k + n_o$ total design points. In such cases, setting $a = 1$ means placing the star points at the centers of the cube faces, obtaining a face-centered CCD (CCF) [25].

In the DoE environment, *orthogonality* is referred to designs in which the scalar product of any arrangement of its column vectors is equal to zero, i.e., a design is orthogonal if X^TX describes a diagonal matrix. Full factorial designs of nature 2^n and 3^n and correspondent fractional factorial designs $2^{n\text{-}p}$ and $3^{n\text{-}p}$ are orthogonal. Orthogonal arrays (OAs) focus on assessing the main effects. If interactions between certain factors are of interest, these interactions should be explicitly implemented as independent factors [26].

One of the most important and widespread DoE is *Latin Hypercube Sampling* (LHS). It was proposed by [28]. This method determines the *N* number of non-overlapping intervals for each variable from several design variables (v) and some initial input sample points (N). Then, the design space is distributed in N^v sections. Each point corresponds to a combination of different intervals of the design variable. Thus, one sample point is associated with each interval of each design variable range. Consequently, LHS designs ensure that all design variables are represented with their intervals [23]. LHS can also be combined with OAs to improve DoE designs, e.g., the randomized OA [29] or the OA-based LHS [30]. Fig. 6 shows a comparison between them.

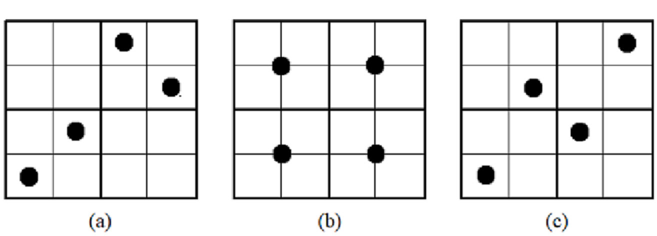

Fig. 6. Evaluation of three space-filling DoEs involving two variables and four sampling points: (a) Median LHS, (b) randomized OA, and (c) OA-based LHS [12].

Finally, the deviation of a uniform distribution is measured by the *discrepancy*. While LHSs are uniform only in a one-dimensional projection, *Low-Discrepancy Sequences* (LDS) are usually more uniform in the design space. Two of the most implemented LDS strategies are *Hammersley sequence sampling* (HSS) [31] and *uniform designs* (UD) [32]. In HSS, with the LDS of Hammersley points, the *k*-dimensional space is conformed. UDs, instead, have matches with LHS. The points are chosen from the center of compartments similar to median LHS (Fig. 6(a)). In addition to the balance of LHS designs, UDs implement k-dimensional uniformity [12].

3.2. Metamodel implementation

After selecting a suitable DoE and performing the correspondent high-fidelity simulations, the next step consists of the metamodel and fitting strategy selection [25]. There are several metamodel techniques, but in this study, we only partially review basic principles of the most predominant in the MASDO literature: Response Surface Methodology, Kriging, Support Vector Regression, Radial Basis Function, and Neural Network.

The *Response Surface Methodology* (RSM) was proposed by [33]. The basic idea of RSM strategies is the establishment of a functional association between the input variables $\times$ and the output value y [26], i.e., given a group of factors $\times$ which influence the value y (the response), the relationship between $\times$ and y could be established by Eq. (1) [25].

$$y = f(x) + e \tag{1}$$

here, e is the random error, which is normally distributed with mean zero and standard deviation σ. Since the latter is unknown, the function $g(x)$ is produced to approximate $f(x)$. The expected values are found by $\hat{y}= g(x)$. Low-order polynomials are the most commonly used response surface approximation functions. For surfaces with low curvature, it is frequent to use first-order polynomials (see Eq. (2)). For substantial curvatures, second-order polynomials such as the one in Eq. (3) are implemented [25].

$$\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i \tag{2}$$

$$\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{j=1, i<j}^{k} \beta_{ij} x_i x_j \tag{3}$$

In Eqs. (2) and (3), the parameters are often found by least-squares regression analysis, and the function approximation is fitted to the real data.

Instead, and based on the work of Daniel G. Krige [34], adapted to handle problems in geostatistics [35], *Kriging models* are today an extensive and very popular global approximation technique [26]. Kriging basic's principle describes the deterministic response $y(x)$ as in Eq. (4).

$$y(x) = f(x) + Z(x) \tag{4}$$

here, $f(x)$ is the function to deal with the approximation. Additionally, in a stochastic procedure using mean zero, variance σ^2, and non-zero covariance, $Z(x)$ is obtained. Moreover, $f(x)$ is analogous to a regression model, as shown in Eq. (5). In such a way, $Z(x)$ also produces local deviations allowing the kriging model to interpolate the dataset points, as in Eq. (6) [23]. There are two main alternatives for $f(x)$. If it is a constant, then we have an *ordinary kriging*. If it is set to 0, it implies that the response $y(x)$ has a mean of zero, then a *simple kriging* strategy is implemented [36].

$$f(x) = \sum_{i=1}^{n} \beta_i f_i(x) \tag{5}$$

$$cov[Z(x_i), Z(x_j)] = \sigma^2 R(x_i, x_j) \tag{6}$$

here, $R(x_i, x_j)$ is the spatial correlation function between two points, and σ^2 represents the variance of such process. In the engineering field, Eq. (7), a Gaussian correlation function, is the most frequently implemented [36]. It can be obtained with the parameter θ, which regulates the area of influence of close points [37]. Low θ values mean a high correlation among all the sample points; therefore, the term $Z(x)$ is similar throughout the design space. If the θ value increases, points with higher correlation are closer, so the Z(x) term fluctuates according to the location in the design space.

$$R(x_i, x_j) = e^{-\sum_{k=1}^{m} \theta \left| x_k^i - x_k^j \right|^2} \tag{7}$$

Alternatively, *Support Vector Regression* (SVR) is originated from the theory of *Support Vector Machines* [38]. SVR metamodels can be defined by the representative mathematical formulation [12]:

$$\hat{y}(x) = b + W^T Q(x) = b + \sum_{m=1}^{M} w_m Q_m(x) \tag{8}$$

hence, a sum of basic functions $Q = [Q_1(\mathbf{x}), \ldots, Q_M(\mathbf{x})]^T$ with weights $\mathbf{w} = [w_1, \ldots, w_M]^T$ are added to a base term b. The parameters b and w_m are estimated differently than the counterparts in other metamodels. In the SVR model, basic functions Q can be seen as a transformation of × into some feature space in which the model is linear.

One of the main particularities of SVR strategies is the imposition of a margin ε (the approximation error). It serves as a threshold for accepting (or not) the differences between the responses of the fitting set and the metamodel prediction, i.e., points located inside the ± ε space (called ε-tube) are discarded. Thus, the metamodel is completely defined by the points located on or outside this region, named *support vectors*. Estimating the unknown parameters of an SVR metamodel is an optimization problem [12]. The objective is to find a function $\hat{y}(x)$ that diverges by at most ε from the experimental output y_i for the regression based on the training data and minimizes the model complexity simultaneously [39].

On the other hand, *Radial Basis Functions* (RBF) are metamodels composed of a polynomial part $\eta(v, \beta)$ and a sum of the radial functions ψ. Eq. (9) defines the Euclidean distance between the approximate point and the respective real point, which is the independent variable.

$$y = \eta(v, \beta) + \sum_{l=1}^{m} \lambda_l \psi(d_l(v)) + \varepsilon \tag{9}$$

here, λ_l and β are the model parameters fitted to the sampled data. The polynomial part of the RBF models is similar to Kriging and provides a global tendency for the response. On the other hand, the interpolation property of the model is guaranteed by the radial function. Several alternatives can be used to define the radial function $\psi(r)$, e.g., linear, cubic, *thin plate spline*, multiquadric, or Gaussian [24].

Finally, *artificial neural networks*, often just *neural networks* (NNs), are designed to respond to stimuli in a scheme analogous to biological nervous systems. One of the attractive features of these structures is their capability to learn about deep relations between sets of input and output data; thus, they can be used as metamodels [26].

Regular NNs are conformed by computing elements named neurons assembled in such a way to build an architecture. Based on the input × = $(x_1, x_2, \ldots, x_k)^T$, the output y_m from a single neuron m is evaluated as:

$$y_m(x) = f\left(b_m + \sum_{i=1}^{k} w_{mi} x_i\right) = f(a) \tag{10}$$

where, f is the transfer or activation function, b_m is the bias value, w_{mi} is the weight of the corresponding input x_i for a neuron m. Thus, a represents the information that "arrives" at the transfer function, which is responsible for producing the output values. The form of the NN is determined by the connection topology of the architecture, the weights, the bias, and the used transfer function. A common architecture is the *multi-layer feed-forward neural network*, where the information is only transmitted forward, and no information is fed backward. Sigmoid functions are usually used as the transfer function [40]. *Radial basis function neural network* is another great class of NN in which RBFs are used as activation functions [26].

3.3. Metamodel validation

Both the type of metamodel and the quality and quantity of the data set from which it is constructed influence their precision. It is then assessed and controlled by fitting models. As mentioned, each metamodel type usually has its linked fitting method. For example, a NN construction is associated with *back-propagation* strategies, RSMs usually employ a *least-squares regression analysis* [25], and Kriging formulations often use the *Best Linear Unbiased Predictor* [23].

On the other hand, the quality of a metamodel cannot be described by only one single measure. Instead, several measures can be used to assess the accuracy of a metamodel implementation. A very useful way to evaluate the precision of a metamodel is through the study of its residuals, i.e., the difference between the high-fidelity value y_i and the value offered by the metamodel $\hat{y}_i$. It is denominated *error measures*, and they can be evaluated by, e.g., *maximum absolute error* (MAE), the *average absolute error* (AAE), the *mean absolute percentage error* (MAPE), the *mean squared error* (MSE), and the *root mean squared error* (RMSE). More information on calculating these errors can be found at [12].

Another way to evaluate the quality of a metamodel and compare it with other ones is called cross-validation (CV). This methodology allows interpolation metamodels to be compared with other approximation metamodels. Unlike the approach used while working with the errors mentioned above (which require the use of data other than the ones used to create the metamodel), in cross-validation (CV), the same dataset is used to fit and validate the model. When simulation time is long and available data is limited (as is often the case in structural optimization), it may be desirable to use the complete data set to fit the metamodels rather than potentially reducing accuracy by omitting part of the set for validation. Main CV strategies are *p-fold CV* and *leave-k-out CV* [12].

During *p-fold CV*, the dataset of n input–output data pairs is partitioned into p distinct subsets. The metamodel is trained p times, with each iteration excluding one of the subsets. The excluded subset is then used to evaluate the performance of the metamodel by measuring the error metrics of interest. A variation of the method is the *leave-k-out* approach, which involves excluding all possible $\binom{n}{k}$ subsets of size k from the dataset and training the metamodel on the remaining data. The error measures are then evaluated at the omitted points for each subset. While this approach is more computationally expensive than a *p-fold CV*, it can be helpful in cases where $k = 1$, also known as a *leave-one-out CV*. This method can estimate the prediction error at a relatively low cost for some metamodels, such as polynomial, Kriging, and RBF models [12]. This type of procedure is also associated with the use of some of the errors described above (e.g., MSE).

4. Metamodel-assisted structural design optimization

As stated, a metamodel is an approximate model involving complex input–output relationships presented by another more complex model, e.g., a physics-based model [32]. Moreover, the formulation can be expressed in a simple analytical form, and its application is much easier to implement. Consequently, metamodels have become increasingly popular in structural optimization due to their excellent performance in conducting research involving many simulations, such as accounting for uncertainties or optimizing challenging real-life problems [11]. The general process of metamodel-assisted structural design optimization is represented in Fig. 7 and consists of the following steps [39]:

1. Problem formulation: the optimization problem is formulated, i.e., the objective(s) function(s), variables, constraints are defined, but also the metamodeling strategy to be used (DoE, metamodel, and validation methods)
2. DoE: A pre-defined strategy is used to select the initial sampling points, e.g., using LHS, which are evaluated through high-fidelity simulations. In structural design optimization, the simulation is often performed by FEA.
3. The features of the optimization issue to be solved determine the selection of the metamodel to be used. In structural optimization, the most used ones are Kriging, NN, RBF, RSM, and SVR. Hybridizations can also be implemented to increase the metamodel precision.
4. Once the low-fidelity (surrogate) model is obtained, several strategies can optimize it. The use of gradient-based and metaheuristic methods are the most common approaches. If the optimization process ends in this step, it is denominated *off-line* optimization.
5. The results of the new designs evaluated by the high-fidelity simulations are incorporated into the existing database to improve the metamodel's precision. This process depends on the selected DoE and corresponding initial samplings. If the stopping criterion is not reached, the process returns to step 3.

As explained, the metamodel training and validation are implemented in *off-line* optimization procedures before optimizing the low-fidelity model. NNs are suitable metamodels to perform *off-line* optimization. Alternatively, the initial surrogate model is constantly improved and optimized in *on-line* optimization. Kriging metamodels are more appropriate to support *on-line* optimization procedures. Deeper explanations about metamodeling strategies are found in further sections.

The eight categorical variables are analyzed in this section according to the reviewed literature. *Type of formulation*, *objective functions*, and *type of variables* are related to the problem formulation. *DoE* and *metamodel techniques* are associated with the metamodel implementation. The other ones have *implemented optimization methods*, *case studies*, and

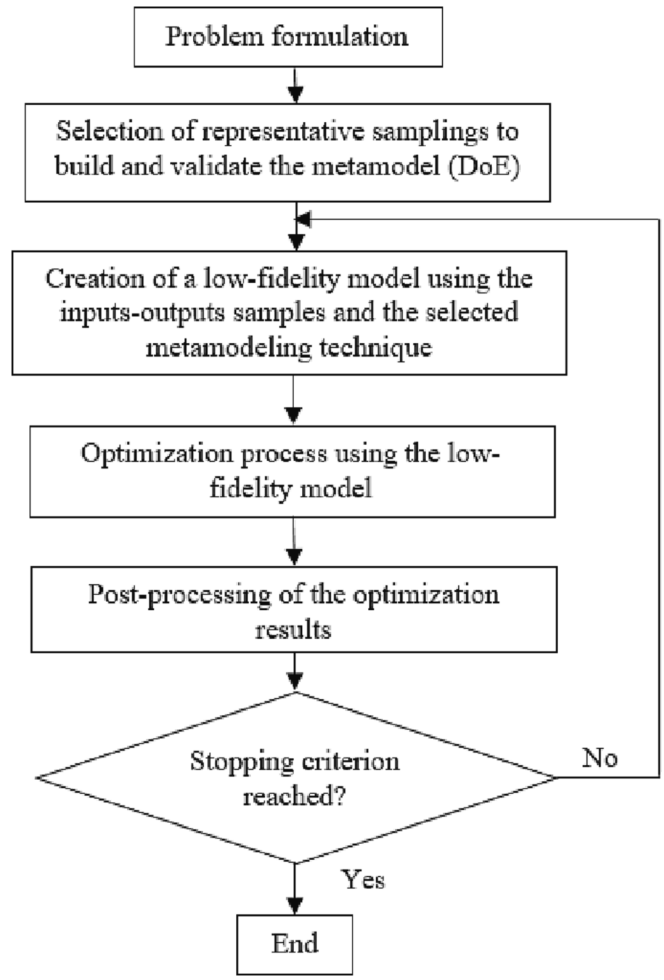

Fig. 7. General flowchart of *on-line* metamodel-assisted structural design optimization. *Off-line* optimization is performed up to the fourth step.

calculation engines used in the high-fidelity simulations to obtain the surrogate model.

4.1. Problem formulation

This subsection is dedicated to analyzing the proposed MASDO problem formulations, considering three categorical variables. *Type of formulation* is about the approach to formulate the problem, i.e., *deterministic* (or conventional), *reliability-based*, *robust*, or a combination of the two previous: *reliability-based robust*. The other is related to the *objective function* (or functions, in the case of multiobjective optimization) implemented for the process, and the final one is about the *formulated variables*.

4.1.1. Type of formulation

The type of formulation is essential in using metamodels to assist the structural design optimization process. While deterministic approaches are expensive since such optimization problems usually need a lot of complex model simulations (e.g., FEA of actual case studies) [41], optimization under uncertainties needs extra simulations to consider them (uncertainties). For this reason, several lines are dedicated to explaining the basic concepts of each formulation approach.

In structural optimization problems, the usual goal is to minimize (or maximize) a given objective(s) function(s) subjected to deterministic behavioral constraints. However, Reliability-based Design Optimization (RBDO) and Robust Design Optimization (RDO) are the principal categories or approaches resulting from encouraging the development of stochastic methods and their application to structural design. The main difference between them is that the fundamental objective of RBDO approaches is the increment of the structure's safety levels concerning the fluctuation of the random design parameters. On the other hand, the minimization of the influence of stochastic fluctuations on the mean design of a structural system is what RDO methodologies search for, i.e., to get the best objective reply with the lowest variation [42]. Since both approaches can be complemented, the probabilistic constraints can be incorporated into the standard RDO formulations. As a result, Reliability-based Robust Design Optimization (RBRDO) has emerged as other alternative to handle uncertainties [43].

4.1.1.1. Deterministic optimization. In a typical Deterministic Optimization (DO) problem, the optimization formulation is defined by objective function(s) and constraints to be met by the design variables. Objective functions are usually defined according to weight, economic or environmental cost, among others (see next section). The constraint functions are determined by prescribed thresholds of the performance indicators, e.g., the limit state methodology (serviceability and ultimate). Consequently, a typical DO can be formulated as [44]:

$$\min F(X) = \sum_{i=1}^{n} F_i(X); s.t.g(X) \leqslant 0; X^l \leqslant X \leqslant X^u \tag{11}$$

where, X are the design variables, $F(X)$ is the objective function, F_i is the objective function value for each i^{th} member, $g(x)$ represents constraint functions, and X^l and X^u are the lower and upper bounds of X, respectively.

Several papers have implemented metamodel-assisted DO (MADO), highlighting novel approaches to formulate DO problems. [45] were among the firsts to handle MASDO (sequential approximation method with NN). It was considered actual characteristics related to the difficulties of the engineering design optimization: (a) the design variables are usually discrete, (b) the design variables cannot be usually considered in analytical expression of the constraint functions and (c), "pass–fail" ("0–1") type binary constraints are frequently found as critical constraints in many applications related to engineering. [46] implemented an Evolutionary Strategy with NN to improve the computational efficiency of large-scale structural optimization problems. [47] combined GA with NN using discrete design variables to handle multiple natural frequency constraints of structures. [48] used a combination of PSO with RBF to formulate a structural optimization problem subjected to time history loading. [49] applied an adaptive metamodel-based optimization combined with RBF to truss structure optimization where sizing, geometry, and typology variables were considered. Several deterministic formulations have been implemented for the design optimization of real-life complex structures, e.g., bridges, using strategies such as a combination of multiobjective HS (MOHS) with NN [50,51] or SA with Kriging [13].

4.1.1.2. Reliability-based design optimization. Unlike the deterministic formulation discussed above, RBDO formulations include probabilistic constraints to account for random parameters and ensure the probability of failure remains within acceptable limits. It imposes the condition that the probability of exceeding the threshold value of a given limit state is less than a particular value. Thus, RBDO problem can be written as [43]:

$$\min F(X,s); s.t.g_j(X,s) \leqslant 0 j = 1, \ldots, m; pf(X,s) \leqslant P_{all} \tag{12}$$

where $F(X,s)$ is the objective to be minimized, X and s are the vectors of the design and random variables, respectively, $g(X, s)$ are the deterministic constraint functions, and $pf(X, s)$ is the probability of design failure that is bounded by an upper allowed probability equal to P_{all}.

Either the introduction of approximations in the analysis of the reliability or the reformulation of the optimization problem are present in all the methods proposed to solve the RBDO problem [17]. According to [52] and [53], there are three main methods to solve them: *two-level* (*nested, double-loop*), *mono-level* (*single-loop*), and *decoupled* approaches.

In order to solve RBDO problems, the two-level approach is considered the straightforward way. It is related to using an appropriate optimization strategy, which is implemented so that the outer nested loop of the two involved explores the design space. At the same time, the inner one performs the reliability analysis. However, the overall cost of this approach is generally prohibitive. For this reason, the *reliability index approach* (RIA) and the *performance measure approach* are the two principal approximation techniques used in the *first order reliability method* (FORM), implemented to avoid this deficiency [17]. The *second order reliability method* (SORM) is another largely implemented method, which is more accurate than FORM when limit state functions are highly nonlinear.

By imposing the optimization conditions proposed originally in [54] and eliminating the reliability analysis as well, *mono-level* approaches are used to solve RBDO problems. Theoretically, and because the probability of failure is not explicitly calculated anymore, the computational cost is reduced [17]. Most widespread mono-level techniques include the single loop single vector and the single loop methodology. They are based on the approximation of the minimum performance objective point incorporating the use of sensitivities of the limit-state function [55–57].

The implementation of the *decoupled approaches* is used as another possibility for the mono-level approaches. They are based on solving the RBDO problem using a deterministic optimization process and *reliability analysis*, i.e., by using information from previous *reliability analyses*, the solution of a deterministic optimization problem is obtained [17]. The *sequential optimization and reliability assessment* (SORA) is the most implemented *decoupled approach*. Probabilistic constraints are adapted for deterministic optimization using the inverse FORM [58].

It is important to note that, in this research, even though it is dedicated to *structural design optimization*, we consider several investigations focused on *reliability-based analysis* (not design). This is because the construction of metamodels in these publications helps incorporate additional information into the study related to RBDO formulations. It could be said that *reliability analysis* consists of the first three steps of the flowchart shown in Fig. 7. However, this process also includes *on-line* optimization strategies, in which the accuracy of the metamodel is improved, but design optimization is not performed. These investigations are reflected in the graph in Fig. 9 as *"Rel Analysis"*.

4.1.1.3. Robust design optimization. The RDO's objective is to minimize the sensitivity to variations in design variables [59]. The example provided in Fig. 8 represents a function with one design variable showing the difference between the optimal solution and the robust optimal solution. It can be seen that the same variation in the design variable (x)

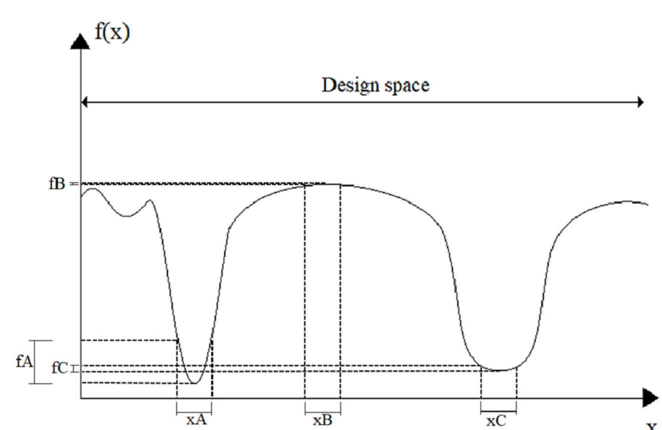

Fig. 8. Example of RDO principles. Adapted from [60].

provides a larger difference in the objective function value of solution *A* (*fA*) than in solution *C* (*fC*). It is a very slight fluctuation in the objective function value of solution B. As a result, point A is the optimal solution, point B is the most robust solution, and point C is the ideal robust optimal solution [60].

The uncertainties of the design variables and the noise factors are considered when the mean and standard deviation of the structural performance is evaluated to achieve a robust design. Based on the optimization viewpoint, there are four classical formulations to approach robust designs: (a) the mean value of the objective function is minimized, and the standard deviation is constrained to an upper bound (b) the standard deviation of the objective function is minimized, and the mean value is constrained to an upper bound, (c) the mean value and standard deviation are formulated as a single robustness cost function by weighting them, and (d) both the mean and standard deviation are minimized using multiobjective optimization [59].

4.1.1.4. Reliability-based robust design optimization. As mentioned, structural RDO problems are formulated as two-objective optimization problems in which the stochastic nature of structural parameters and loading conditions are considered in an auxiliary objective function [61]. The purpose is to minimize the main objective function (e.g., the economic cost) and the variation of the structure's response. In the combination of *reliability* with *robust* design optimization, the consideration of an added group of probabilistic constraints and the deterministic ones enforced by the RDO formulation is analyzed as the probability of failure or constraints violations of the structure. Therefore, the general RBRDO problem could be formulated as in Eq. (13) [62].

$$\min[F(X),\sigma_u(X,r)];s.t.g_j(X)\leqslant 0 j=1,\dots,m;p_{v,\max}(X,r)\leqslant P_{v,all}\,X_i\in R_i^d,i=1,\dots,n \tag{13}$$

Here, the two objectives to be minimized are *F(X)* and $\sigma_u(X,r)$. The vectors of design and random variables are *X* and *r*, respectively; $g_j(X)$ represents the deterministic constraint functions and among them, the maximum violation probability is represented by $p_{v,\max}(X, r)$; *Xi* takes values from the discrete set R_i^d and *Pv,dl* is the admissible possibility of constraints violations.

4.1.1.5. Graphical representation. Fig. 9 shows the behavior of formulations that include metamodels to aim at the structural optimization process through time. It can be appreciated that in the first decade of the XXI century, DO was the main approach to formulate the problems. However, since 2010 there has been an increase in the use of RBDO in conjunction with DO. It is important to highlight that *reliability analysis* has gained attention in the last years due to a tendency to develop strategies to improve the metamodels accuracy without regard to optimization. The pie chart in Fig. 9 shows the final distribution of types of formulations during this century, with a significant predominance of DO and RBDO approaches over the others.

It is remarkable how few RDO and RBRDO formulations are implemented to consider uncertainties in the design. It can be attributed to the simplicity of the formulations, which are often focused on mono-objective optimization of basic criteria such as weight. According to [43], RBDO formulations typically involve adding probabilistic constraints to the standard deterministic formulations. It is usually associated with mono-objective optimization. Thus, these types of formulations are usually related to mono-objective optimization. In contrast, RDO and RBRDO formulations are linked to implementing conflicting objectives derived from real-world optimization problems, which are less considered in MASDO problems.

4.1.2. Objective functions

The type of formulation and objective(s) functions(s) are quite influential on metamodeling technique selection. In structural optimization, objective functions are usually difficult to optimize, with the presence of many local optima. Foe example, in the discrete optimization of reinforced concrete (RC) structures, the configuration of steel bars varies strongly with the cross-sectional dimensions of the elements (variables), producing all of those local optima [63]. Therefore, an appropriate metamodeling technique (DoE, metamodel, and validation) is essential for accurate results.

The objectives used in the reviewed literature vary from simple weight optimization to Lyfe-Cycle Analysis, including several sustainability criteria. It is essential to remember that the objectives pursued in the literature range from the improvement of metamodeling strategies (using simple goals) to metamodel-assisted structural optimization of real-life problems. The latter focuses not on metamodeling strategy development but on formulating rather complex optimization problems.

This paper distinguishes three objectives: *classic*, *sustainability-related*, and *others*. Classic ones are *weight*, *volume*, *cross-sectional area* and *structural behavior*. The first three are mainly related to steel structures. The fourth one is usually used as constraints, but some strategies consider them as objective in multiobjective optimization, i.e., they are usually found as complementary objectives. The group of objectives defining *sustainability criteria* is much more complex. Based on [50,51,64,65], we can distinguish five main groups, broken down in Fig. 10.

The *economic* goal is, by far, the most used one. It can be implemented in a simple way (e.g., initial construction cost) or involving aspects of the Life-Cycle cost related to, for example, risk optimization

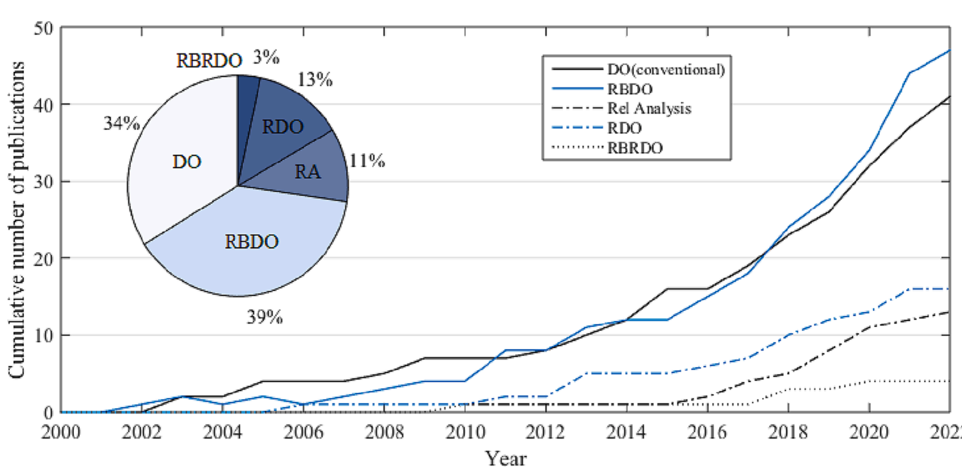

Fig. 9. Cumulative distribution of type of formulation over time (2000 - Present).

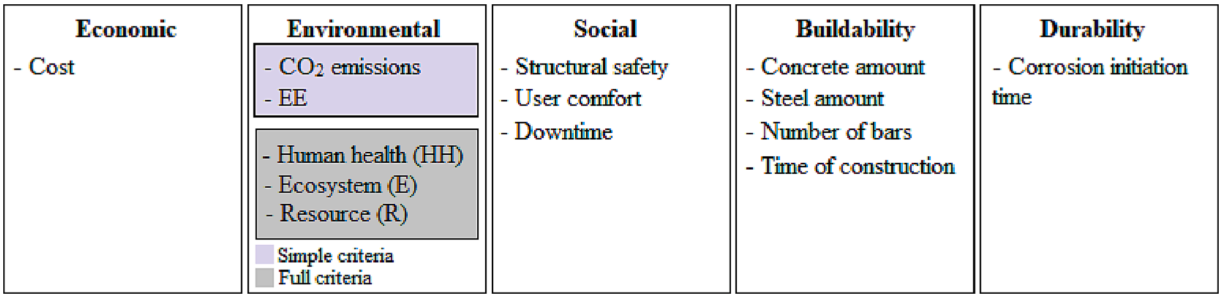

Fig. 10. Objectives that define sustainability criteria used in optimization assisted by metamodels.

problems, defining the total cost as the initial construction cost in addition to the damage repair or replacement cost, produced by an earthquake [66–68] or a windstorm of given intensity [69]. Others consider partial structure life-cycle including *production* and *construction* stages [65] or the whole process taking into account, in addition to the two previous ones, the *use and maintenance* and the *end of life* stages [42,64].

Environmental issues can be directly evaluated by CO_2 emissions [70,71] or embodied energy (EE) [23,51,70]. Nevertheless, to obtain a more profound environmental outline, it is necessary to contemplate other criteria that denote a more detailed environmental assessment [72]. Consequently, [42,64,65] implemented the endpoint approach of the life-cycle impact assessment method ReCiPe [73], complemented by information given by the Ecoinvent database [74] and processed using the OpenLCA software.

Social impact can be measured using the *structural safety* trough, e.g., the overall safety factor [50,51] or the safety coefficient of the ultimate limit state [64]. *User comfort*, measured by the vibration service limit state [64], and *downtime*, obtained by the days that the structure (bridges in this case) is not operational [42,64], can also be used to assess *social* impact. [65] added the four factors of the Social Impacts Weighting Method, using the PSILCA database [75], in conjunction with the Ecoinvent database using an add-on called SOCA [76].

On the other hand, *buildability* is related to the technical aspect of the "easier" construction of structures [64]. It can be implemented by measuring the quantity of concrete and steel or the number of bars [64] and using the number of hours required for the building activity, including construction and transport *on site* and *to the site* [65].

Finally, a usually ignored and very important criteria (especially for structures located in highly aggressive environments, e.g., coastal sites) such as *durability* can be measured using several ways. In this review, this criterion is found to be the corrosion initiation time, i.e., the time it takes for the chloride concentration on the surface of the reinforcing steel to reach a critical threshold value [50,51].

Table 1 shows the authors that performed metamodel-assisted multiobjective structural optimization and the correspondent combined objectives. [78] combined members' cross-sectional areas with elements deflection. Several authors combined weight with maximum vertical displacement of the structure [78] or displacement variance [80], members' maximum stress [79,81], or elements compliance (strain energy) [8,39]. Others, instead *weight*, *combined economic cost* with *structural behavior*: [43] and [62] with the *standard deviation* of a characteristic node displacement, [60] with the structure *vertical deflection*, and [85] with the structure's *shear-bending capacity*. Note that previous combinations are the typical strategy for converting constraints into variables of the optimization problem. On the other hand, the combination of sustainability criteria (previously explained) is usually solved by more complex strategies, including multi-criteria (or multi-attribute) decision-making approaches.

Fig. 11 shows that *classic* objectives are the most used ones, with a significant predominance of *weight*. It is well known that this objective is closely related to steel structures because it loses effectiveness in composite structures, e.g., RC ones, i.e., not always the lightest RC structure is the optimal one. Sixty-two case studies were optimized according to their weight. Also related to *classic* objectives, *structural behavior* is relatively commonly used. It is usually used as a complementary objective in multiobjective structural optimization. In terms of objectives related to sustainability, the *economic cost* is the predominant (31 case studies), followed by *environmental* ones. Generally, it can be inferred that the most common trend in up-to-date MASDO is the development and validation of metamodeling strategies. It uses simple structures as case studies instead of applying existing techniques to real-life problems involving formulations related to sustainable approaches.

Table 1
Combined objectives in multiobjective structural optimization based on metamodels.

Objectives	References
Cross sectional area, structural behavior	[77]
Weight, structural behavior	[8,39,78–82]
Weight, reliability	[83]
Economic (cost) and its standard deviation	[84]
Economic, structural behavior	[43,60,62,85,86]
Economic, environmental	[70]
Economic, environmental, social	[42]
Economic, social, durability	[50]
Environmental, social, durability	[51]
Economic, environmental, social, buildability	[64,65]

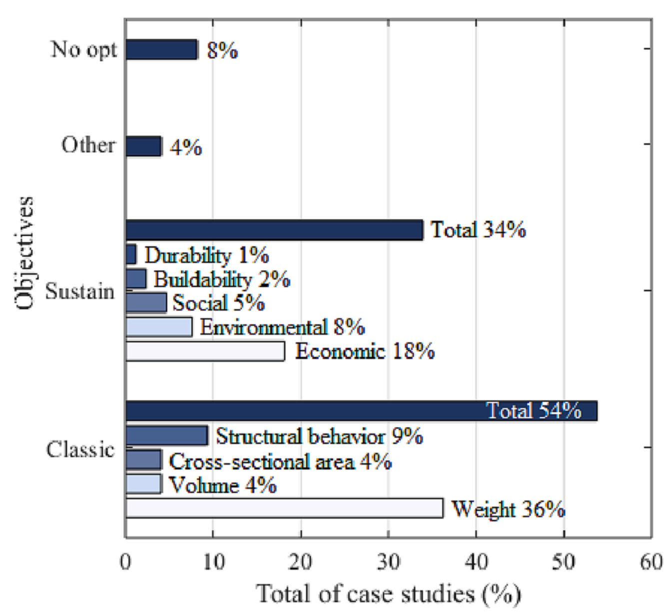

Fig. 11. Distribution of used objectives by case studies. *No optimization* is related to papers focused on reliability analysis.

4.1.3. Variables

According to [39], in formulating structural engineering design optimization problems, main parameterization involves several variables regarding the geometry (sizing of the elements, overall shape, and topology), the building materials, or the support conditions. They can be mathematically classified as:

- *Continuous* variables: real numbers defined inside an interval [x_{min}, x_{max}] ∊ *R*. They are usually used as cross-sectional dimensions (mainly in steel structures) or problems with no practical outputs (RC cross-sectional dimensions with atypical values, e.g., h = 0.4658 m).
- *Discrete* variables: continuous variables only available among a discrete set. For example, the cross-section area of an element profile available in a catalog {*A(1), A(2), …, A(n)*}, or, for reasons of buildability, the constructive dimensions of a RC element, e.g. h = {*0.40, 0.45, …, 0.80*} meters.
- *Integer* variables: strictly belonging to N. Opposing to discrete variables, intermediate values do not have any physical meaning. For instance, the number of girders (beams) conforming to a bridge cross-section can vary from 3 to 10 (3:1:10). Intermediate values between these eight integer numbers do not provide any physical design.
- *Categorical* variables: denote non-numerical factors. From an engineering point of view, they are of great practical interest because they can deal with, for example, the choice of a building material ({*concrete 35 MPa*, *concrete 40 MPa*,…}), the type of connection between elements ({*rigid, semi-rigid, articulated*}) or the shape of a cross-section. They are usually converted into *Integer* variables by encoding–decoding processes.

In this review, two groups of variables were identified: *geometric* and related to *materials*. *Cross-sectional dimensions* and *cross-sectional areas* are found as continuous and discrete variables. *Profile selection* is usually converted from a categorical variable to an integer one, e.g., initially {W_1, W_2, W_3} is converted into {*1, 2, 3*} through an encoding–decoding process, i.e., the optimization algorithm recognizes the vector values as a natural number, but the physics-based model recognizes a profile with its corresponding properties. *Topologic* variable is implemented in [43] as the number of columns distributed in the two structural dimensions. *Reinforcing configuration* is usually defined by discrete spaces. They are related to concrete (reinforced, prestressed or post-tensioned) structures and are usually implemented to measure objectives beyond the economic (buildability, durability). It represents the number and size of the reinforcing bars, and their corresponding placement within the concrete cross-section. Variables related to materials are usually incorporated in the optimization of concrete structures, e.g., the environmental impact of different types of concrete.

Fig. 12 shows that geometric variables are the most used ones, led by *cross-sectional dimensions* and *cross-sectional areas*. The use of *profile selection* is not high, even when many steel structures are used as case studies. It is due to using simple structures instead of real cases implementation (see section 4.4). Additionally, it can be seen how there are a few studies that incorporate reinforcing configuration as variables. It is because of the low use of (1) concrete-made case studies and (2) sustainable approaches in optimization implementation.

4.2. Applied metamodeling techniques

This research focuses on metamodeling techniques used to support structural optimization. Section 3 provides a detailed discussion of the main approaches to implementing these strategies (DoE and metamodeling strategy). This section delves into both categorical variables and highlights the most notable studies in the literature.

Deterministic (or conventional) optimization needs the support of metamodeling to decrease the number of (quite expensive, high-fidelity) simulations necessary to carry on such optimization processes. In section

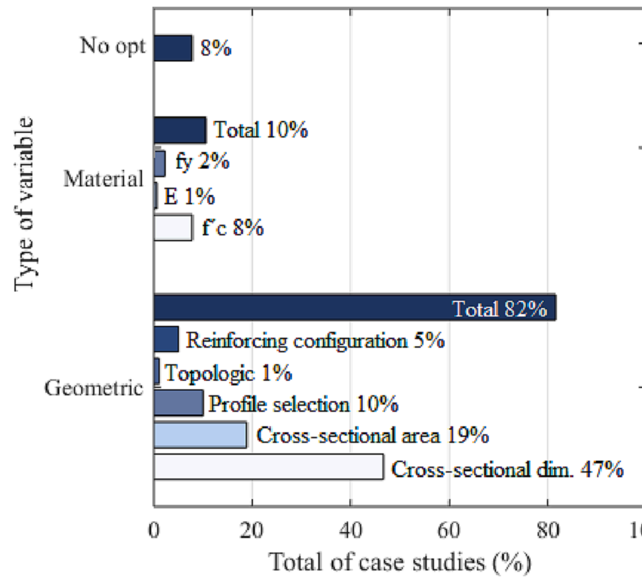

Fig. 12. Distribution of type of variables by case studies. *No optimization* is related to papers focused on reliability analysis.

4, it was stated that DO is usually an off-line practice (flowchart of Fig. 7 until the fourth step). Metamodels such as NN are the most appropriate ones to support problems with the deterministic formulation. Table 2 supports this statement since NN (13% of total usage) and RBF (7%), two very similar approaches, are the most used metamodels to aid DO problems. Additionally, 55% and 70% of implementations of NN and RBF, respectively, are related to deterministic problems. Others, such as Kriging (the most widely used strategy), are only applied in 20% (of the total DO formulations) to deal with deterministic problems.

On the other hand, optimization under uncertainties needs supplementary simulations to handle the special features of these approaches. Reliability-based optimization is the most representative formulation in the reviewed literature (see Fig. 9). It has been mentioned that metamodeling techniques are closely related to the way of solving RBDO problems. According to Table 2, Kriging has been, by far, the most used surrogate model to aid this kind of problem, using, e.g., a two-level - classic approach [132], incorporating FORM and Monte Carlo Simulation (MCS) to handle uncertainties [130]. Others have introduced some variations. [118] formulate the RBDO problem as a non-probabilistic RBDO (NRBDO), building the Kriging metamodel using an innovative importance method based on a practical and functional learning technique. In [112], the most probable point (MPP) is estimated based on a sensitive surrogate model for each point the optimizer analyzes. Thus, the RBDO problem is solved as an iterative deterministic optimization problem. [111] formulate a nested RBDO with an adaptive refinement strategy, where the Kriging metamodeling technique is selected to surrogate the performance functions, genuinely quantifying the surrogate error. The other two main ways to solve RBDO problems in combination with Kriging have been developed to a lesser extent. [140] use a classic decoupled approach based on SORA. [161] propose an alternative to SORA: the threshold shift method (TSM), while [135] propose a novel quantile-based sequential RBDO method using Kriging, integrating an error-controlled adaptive Kriging scheme to derive the accuracy information of surrogate models. Finally, [126] use a Single-Loop Approach (SLA) combined with the Kriging surrogate, where the metamodel is updated efficiently using the MPPs from the last SLA iteration. Alternatively, [68] reformulate the classic bi-objective stochastic optimization problem as a suite of single-objective optimization problems through the ε-constraint approach.

NN has also been used to support RBDO problems, e.g., using *two-level* classic approaches [85] calculating the probability of failure within

Table 2
Relation of metamodel used in function of the type of formulation.

Type of formulation	Metamodel	Number of publications	%	References
DO	Kriging	8	7	[23,65,87–92]
	NN	16	13	[45,46,50,51,67,70,71,86,93–100]
	RBF	9	8	[48,49,77,81,101–105]
	RSM	1	1	[106]
	SVR	2	2	[8,39]
	Hybrid	2	2	[47,107]
	Other	3	2	[108–110]
RBDO	Kriging	33	28	[17,68,69,79,88,111–138]
	NN	11	9	[43,66,85,93,139–145]
	RBF	2	2	[82,100]
	RSM	4	2	[137–140]
	SVR	1	1	[149]
	Hybrid	5	4	[44,78,150–152]
	Other	2	2	[153,154]
RDO	Kriging	11	9	[42,59,60,64,80,89,155–159]
	RBF	2	2	[59,160]
	RSM	3	2	[59,161,162]
RBRDO	Kriging	1	1	[89]
	NN	2	2	[43,62]
	RSM	1	1	[84]

the multi-fidelity framework by MCS, FORM, and SORM [143], or incorporating to SORM a small sample simulation strategy [142]. On the other hand, according to Table 2, 60% of the total number of uses of RSM has been to support RBDO problems. It has been done using a two-level and FORM approach [147] or, as an alternative to FORM, an improved high-order response surface method that includes an effective sampling method [146].

According to Fig. 9, RDO is the second most used approach to consider uncertainties in design procedures. Table 2 shows that Kriging is the most used metamodel to deal with such formulations (70% of the total RDO problems). [42], [64] and [155] employed a classical RDO formulation and corresponding Kriging implementation. In [80], the approximated structural performance is constructed in both the design and the stochastic domain using a Kriging model, in which the uncertainty quantification and the optimization procure are decoupled. In order to update the metamodel concerning the global Pareto front, an infill criterion is used based on the variations of Kriging's predictions. [159] developed an RDO technique based on a global two-layered approximation, in which globally refined Kriging models approximate the response quantity (in the inner layer) and the response statistics computed from the response metamodel (in the outer layer). In [157], the RDO problem is transformed to an equivalent deterministic one by using two adaptive sparse in an advanced Kriging-based computational model. Lastly, [156] built the surrogate models of the mean and the standard deviation for different objective function criteria employing a Kriging-based metamodel.

Finally, the RBRDO problems are the least applied to deal with uncertainties. [89] used a classical RBRDO formulation in combination

Table 3
DoE selection in function of the metamodel type.

Metamodel	DoE	Number of publications	%	References
Kriging	LHS	42	36	[17,23,51,59,60,64,65,68,69,79,80,90,92,113–127,130,132–134,136–138,153–159]
	Random	1	1	[135]
	LDS	2	2	[93,112]
	FFD	2	2	[88,129]
	OA	1	1	[155]
	Other	4	3	[89,90,111,131]
	Not spec	1	1	[128]
NN	LHS	8	7	[43,62,85,99,141–144]
	Random	3	2	[67,98,140]
	LDS	2	2	[93,139]
	FFD	1	1	[70]
	OA	1	1	[45]
	Not spec	11	9	[50,51,66,71,86,94–97,145]
RBF	LHS	8	7	[49,59,77,81,82,102,103,160]
	Random	3	2	[48,104,105]
	Other	1	1	[100]
	Not spec	1	1	[101]
RSM	LHS	3	2	[84,106,147]
	FFD	1	1	[106]
	CCD	3	2	[106,148,162]
	Other	2	2	[117,146]
	Not spec	1	1	[161]
SVR	LHS	2	2	[8,39]
	LDS	1	1	[149]
Hybrid	LHS	4	3	[78,107,150,151]
	LDS	1	1	[44]
	Other	1	1	[152]
	Not spec	1	1	[47]
Other	RS	2	2	[108,110]
	NS	2	2	[86,109]

with a Kriging metamodel to compare structural results with the ones obtained by RBDO and RDO. [43] and [62] used NN in an RBRDO formulation, incorporating probabilistic constraints into the standard RDO formulation, and [84] proposed a new cumulative distribution function (CDF)-based RBRDO approach using a dual RSM framework.

On the other hand, in section 3.1 it was stated the importance of the experimental design and a special relationship with metamodel techniques. Table 3 and Fig. 13(a) show that LHS is the most used DoE strategy, except for NN, where many publications do not specify the initial sampling selection technique. It is straightforward to state that combining LHS with Kriging is the most used to deal with MASDO.

As stated before, Fig. 13(b) and (c) confirm that Kriging is the most used metamodel implemented to aid structural design. It can be seen how NN was the most popular approach in the first part of the analyzed period. However, from 2016 onwards, Kriging has become the most adopted strategy. It seems to be related to the simplicity and ductility of this strategy, which can be used to solve a wide range of problems. Both (Kriging and NN), in combination with LHS, seem to be the most suitable techniques for implementing a good metamodeling strategy to support structural optimization procedures. However, deeper relationships are exposed in section 5.

4.3. Optimization strategies

As mentioned in section 4, the optimization approach can be handled in two main ways: off and on-line procedures. Off-line procedures are the most straightforward strategies. The optimal design is found in the low-fidelity (surrogate) model, and this solution is verified using the high-fidelity numerical simulation [8]. Traditional optimization methods can be easily implemented to solve off-line optimization problems, following traditional criteria to select an appropriate strategy. For example, if the objective function (surrogate function in this case) is highly nonlinear, with many local optima, classical methods (gradient-based strategies, for example) are unsuitable, and *meta*-heuristics could be a good selection. In Fig. 14, it can be seen that traditional optimization procedures are a typical selection to solve metamodel-assisted structural optimization problems. Metaheuristics such as GA (21 uses), PSO (9 uses), or SA (9 uses), but also a classical approach such as

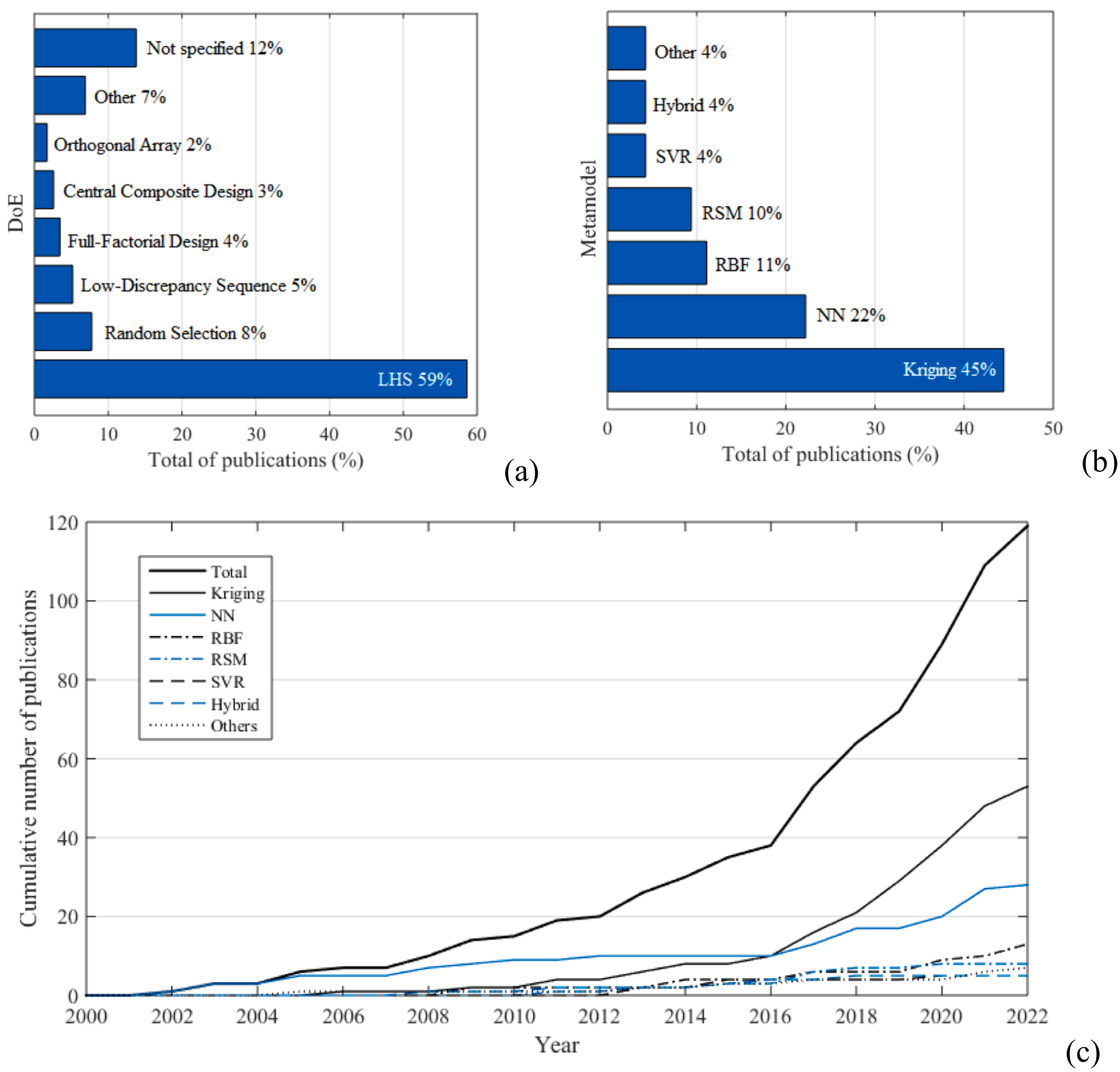

Fig. 13. Results of implemented metamodeling strategies. Distribution of publications regarding (a) the used DoE, (b) the used metamodel and (c) is the cumulative distribution of used metamodels techniques over time (2000 - Present).

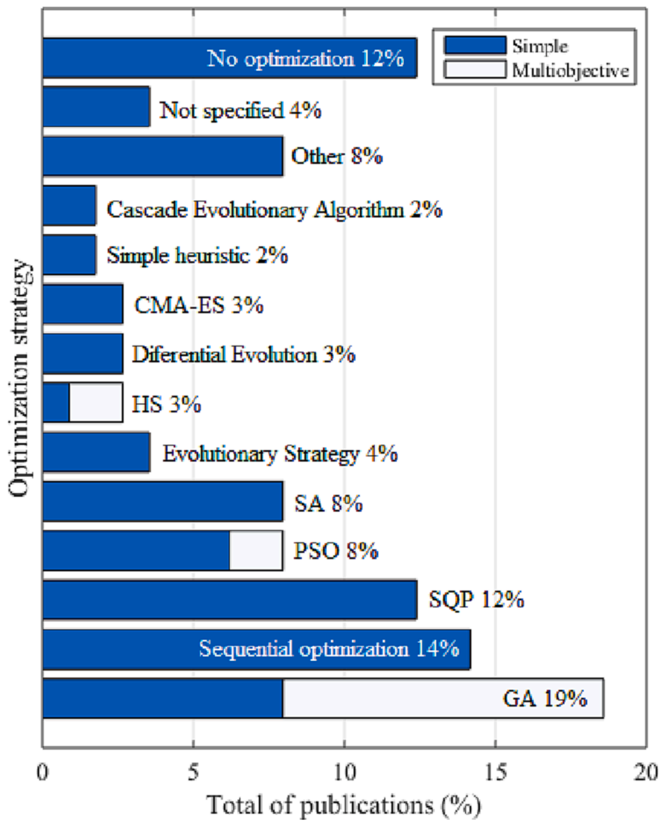

Fig. 14. Distribution of optimization strategies by publications.

Sequential Quadratic Programming (14 uses) are the most used strategies.

However, a group of strategies directly related to on-line-optimization is frequently found as sequential optimization, in which both optimization and metamodel refinement are performed simultaneously. In Fig. 14, it can be checked that several authors have implemented sequential optimization strategies. This process can be implemented with simple search heuristic strategies, e.g., in combination with NN [45], incorporating concepts such as the Aimed Multilevel Sampling method to guide the progression of the simulation into several levels [142] or using the "initial anchor point" to create the new set of combinations to be evaluated [66]. Others have used the MPP method to guide the process and to efficiently improve the metamodel (Kriging in this case) [126], or a variant denominated new local approximation method using the most probable point, also in combination with Kriging [113]. Another previously analyzed strategy is using the TSM method to perform sequential optimization with the SORA method, determining the thresholds for all the constraints by solving a single optimization problem [152]. Other strategies are denominated adaptive metamodeling. [49] presented an adaptive metamodel-denominated sequential radial basis function (SRBF), which distinguishingly features a two-loops searching strategy, where the "inner loop" updates the factors of the augmented Lagrangian's function by searching for reasonable points. In contrast, the "outer loop" updates the RBF model by sequentially providing new additional samples using the improved significant sampling space (ISSS). Additionally, using a new sampling method that permits the refinement of surrogate models either for the deterministic or the probabilistic constraints, [135] proposed an error-controlled adaptive Kriging scheme (a quantile-based sequential RBDO method). Others have developed and implemented the denominated Efficient Global Optimization (EGO) methodology, integrating an adaptive infilling by fuzzy clustering algorithm into a surrogate-based optimization process based on the Kriging model [90] or in combination with the Efficient Global Reliability Analysis (EGRA) to build an effective sequential approximation optimization strategy, based on an adaptive Kriging implementation. Lastly, to solve a time-variant RBDO, [120] implemented a sequential Kriging modeling approach (SKM) built and enhanced through a design-driven adaptive sampling scheme to identify potential instantaneous failure events.

4.4. Case studies

According to the previously discussed results, there seems to be a tendency to use simple case studies with simple problem formulations, focusing on developing and enhancing metamodeling strategies. In contrast, formulating challenging real-life problems involving sustainability aspects is poorly implemented. This affirmation can be verified in Fig. 15, where the use of steel structures is the most common approach (108 case studies), with a predominance of 3D truss structures (36 case studies), 2D truss and frame structures (21 case studies each) and 3D frame structures (11 case studies). Most of them are benchmark problems, used, as stated before, to test and validate new metamodeling strategies. In the pie chart, it can be seen that 70% of the total case studies are benchmark problems. Among the most benchmarking problems it can be found the 10-bars plane truss structure [163] (8 uses), the 23-bars plane truss structure [164] (4 uses), the 25-bars spatial truss transmission tower [163] (9 uses), the 39 [165] (2 uses) and the 1254-bars spatial truss transmission towers [41], and an adaptation with 942 bars [166] (4 uses in total).

Regarding challenging real-life problems, only 44 cases were taken into consideration. Concerning steel structures, it can be found box girders [59,81,113], spatial frames [43] and an industrial building [84], a freestanding-lattice tower [116], a spatial truss structure [159], a wind turbine tower [126], an offshore wind turbine support structure [131], wind turbine foundations [65,86], a spatial truss roof [46], a truss arch bridge [125], crane bridges [123] and steel truss girder structures [44,82,132]. Related to concrete structures, most outstanding applications are RC [70] and post-tensioned slabs [71], prestressed slab beams [94] and concrete roof girders [85], RC foundations [65,91,136], a RC slab bridge and a post-tensioned composite bridge [151], a RC girder bridge [132,151], and post-tensioned concrete box-girder pedestrian [23,42,60,64] and road [50,51] bridges. It is important to highlight that in these six last referenced papers, in addition to using challenging real-life problems as case studies, the problems are formulated from a complex sustainable point of view, as stated in section 4.1.2.

On the other hand, there is a technology called Additive Manufacturing (AM), which is related to producing parts or structures through a layer-by-layer process. Although this research is focused on the design optimization of larger-scale structures, AM is increasingly used in structural engineering. In [167], a novel deep network termed 3DPECP-Net is proposed to address the predicting energy consumption in AM processes, commonly known as 3D printing. In [168], a Gaussian Process is used to build surrogate models of Multiphysics Object-Oriented Simulation Environment MOOSE-based melt pool models to support the AM process. Other authors have used metamodels to optimize selective laser melting (an AM technique), such as the *ensemble of metamodels* (Kriging, RBF, and SVR) [169], or self-learning approaches [170]. Therefore, MASDO is expected to play an essential role in exploiting geometric freedom and improving structural efficiency, especially in topological optimization processes. In this type of optimization, finite element models are usually used, which are associated with high computational consumptions. Therefore, MASDO implementation will be very beneficial in developing this field.

4.5. Calculation engine

As previously stated, it is necessary to perform high-fidelity simulations to obtain the surrogate low-fidelity model. In structural design optimization, these simulations are usually structural FEA, performed by

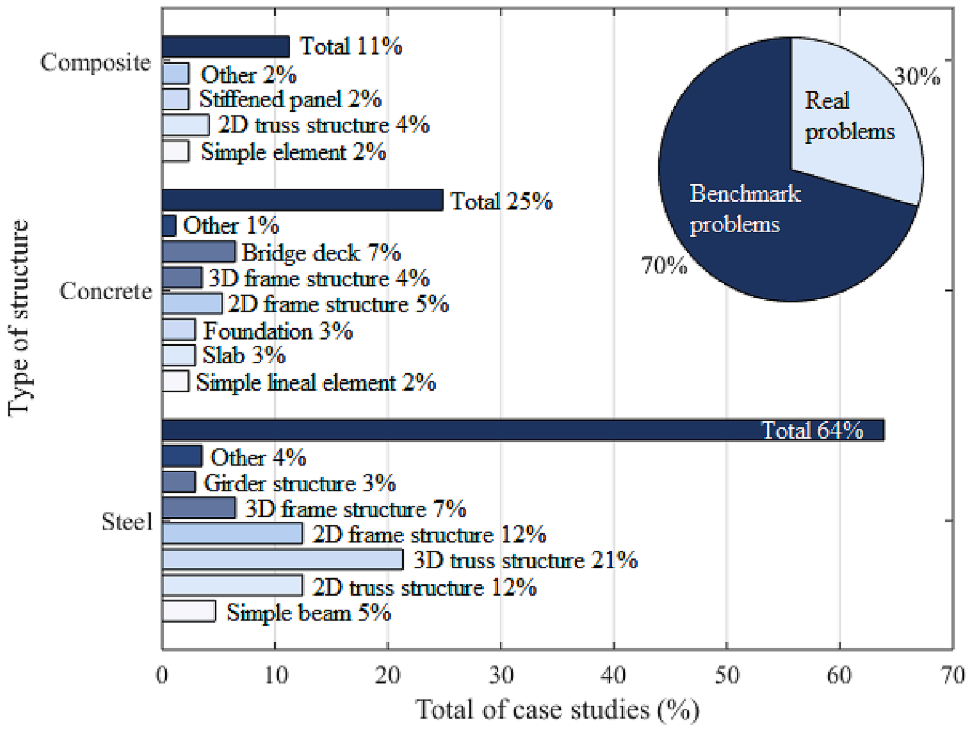

Fig. 15. Distribution of case studies reviewed in the literature.

certain calculation engine. This section discusses the use of such calculation engines according to the reviewed literature.

In Fig. 16, it can be appreciated how most of the high-fidelity simulations have been performed using own codes, i.e., without using any commercial FE software. It is due to the many simple case studies used in the research, as explained above. These codes are usually programmed

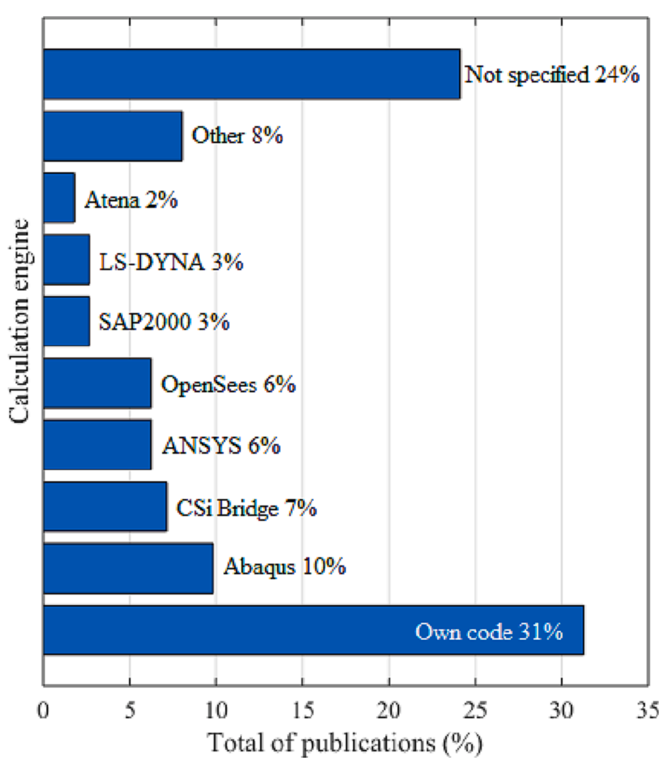

Fig. 16. Distribution of calculation engines used to perform the high-fidelity simulations.

in Matlab [121,124,125,137,171], Python [99,153], Fortran [94], among others. On the other hand, among the FE professional software, the most used one is Abaqus, which is suitable for structures of relatively small size, same as LS-DYNA, following the statement about predominant case studies. Others such as ANSYS, OpenSees, or SAP2000 are most commonly used for modeling, analysis, and structural design of buildings, while CSi Bridge, as its name suggests, is specialized in bridges. It is important to highlight that OpenSees is usually used to handle problems involving the dynamic behavior of structures (e.g., seismic analysis). Many of these commercial software can communicate with common programming languages through, e.g., application programming interface (API) functions. It allows the possibility of using such software (or calculation engines) in fully automated procedures coded in one of the programming languages, using the toolboxes necessary to implement the whole metamodel-assisted structural optimization procedure (e.g., optimization or metamodeling toolboxes).

As for the “not specified”, most of them are related to publications in which relatively complex structures (e.g., plane frame structures with several bays and stories, spatial buildings) are taken as case studies, and there is no specification about the procedure of modelling, analysis, and structural design.

5. Statistical analysis

In order to get insight into the relationships between the most important categorical variables, a deeper statistical analysis is performed. Correspondence analysis is used to reveal all of the information that is difficult to find with the naked eye, i.e., this kind of tool (which is a variation of the Principal Components Analysis with qualitative variables) helps identify subjacent relationships between two (or more) variables [172]. SPSS Statistics 25.0 [173] was used to perform the correspondence analysis.

Five of the categorical variables are going to be analyzed: *type of formulation*, *DoE*, *case studies*, *optimization strategies*, and *used metamodel*, which is the most important one and is going to be correlated with the other four ones. It is important to consider that the graphical results explore the tendency or relationship between two (simple correspondence analysis, SCA) or more variables (multiple correspondence analysis, MCA). In this study, we are going to use three simple rules to interpret such graphics: (a) the closer the variable value (or category) is to the origin, the most common its use is, (b) the tendency or relationship between two variables values (or categories) is more significant in the way each point gets closer to another point and (c) relationship is more exclusive the farther the points are located from the origin.

5.1. Simple correspondence analysis

The SCA will be used to correlate the *implemented metamodel* with the other four categorical variables. Two measures are used to have an idea about the quality of the correspondence. The *level of significance* (*sign*) is a measure of the strength of the evidence that must be present in the sample. Its value, which must be less than 0.05, indicates if the experiment is statistically significant. Additionally, the *cumulative proportion of inertia* (*CPI*) until the first two dimensions (which are the ones represented in the graphic) gives an idea of how information can be extracted from the graphic, i.e. how useful it is. This value must be greater than 0.50 to have a good representation of the data in the two dimensions.

5.1.1. Type of formulation - metamodel

This is a very important combination because, as stated before, the

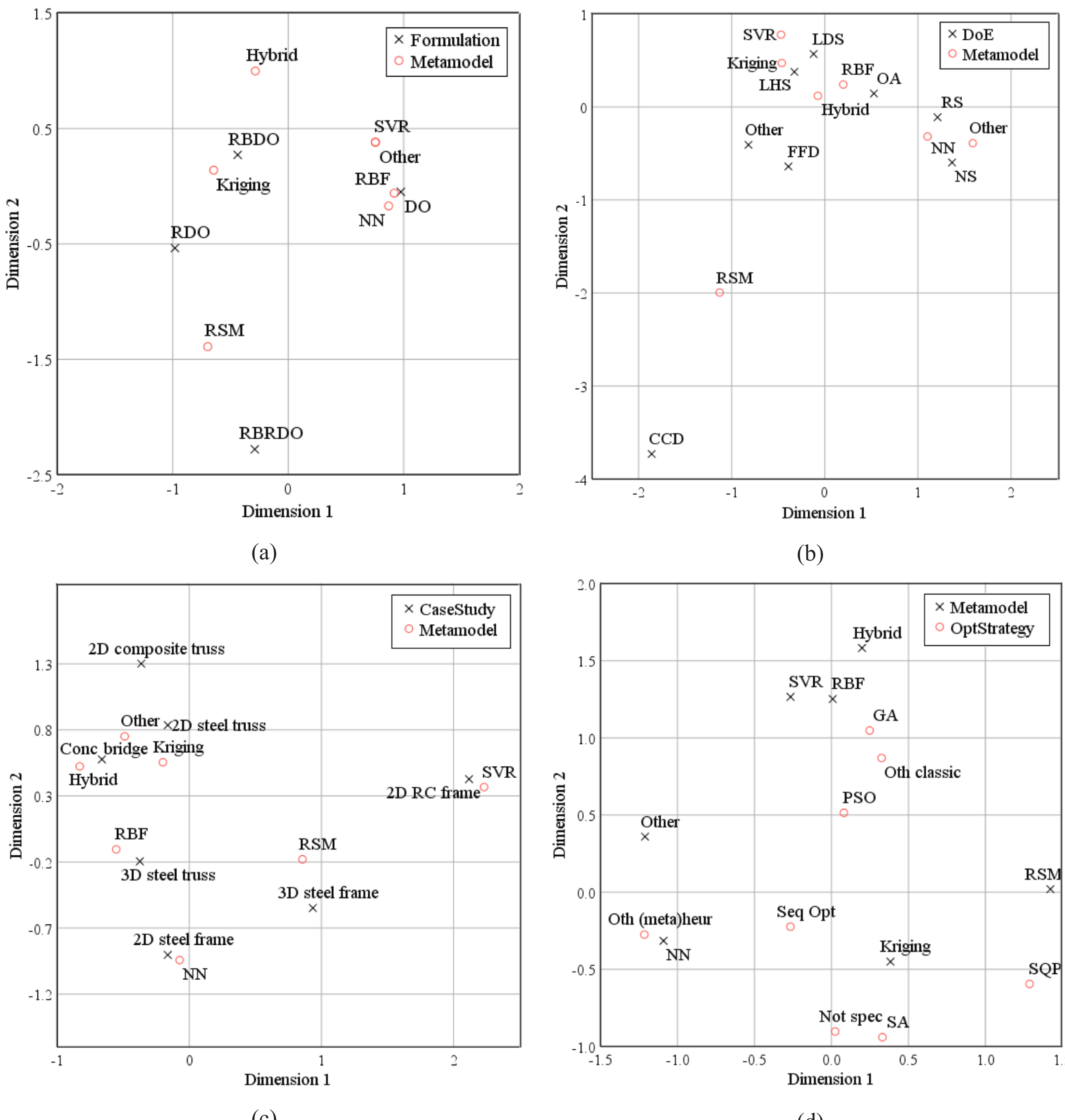

Fig. 17. Simple correspondence analysis (row and column points) for (a) type of formulation-metamodel, (b) metamodel-DoE, (c) case study-metamodel and (d) metamodel-optimization strategy.

problem formulation is closely related to the metamodel selection. Fig. 17(a) shows that DO is strongly associated with RBF and NN, two similar strategies. For its part, RBDO is related to *hybrid* and Kriging-based metamodels, RDO to Kriging, and RBRDO seems to be related to RSM. This last case is based exclusively on RBRDO formulations that are well away from the origin and other metamodel techniques. Generally, the correlation *type of formulation-metamodel* is relatively straightforward and accurate: *sign* = 0.002 and the *CPI* = 0.95.

5.1.2. Metamodel - DoE

In function of the metamodel selected, appropriate initial samplings must be chosen. Fig. 17(b) can be appreciated, first of all, that LHS is the most common DoE for all metamodel techniques due to its closeness with the origin. It can be checked that Kriging (the most used metamodel) is quite strongly related to LHS and, to a lesser extent, to LDS. NN and *Other* are related to RS and *not specified*. It is curious the number of papers that use NN and does not specify the technique to implement the DoE. SVR is related to LDS and LHS, *hybrid* formulations to LHS, and there is a special relationship between CCD and RSM based on exclusivity. The correlation between these two variables is good: *sign* = 0.001 and *CPI* = 0.80.

5.1.3. Metamodel - case study

Fig. 17c shows strong relationships between case studies and metamodels. It is important to highlight that, to decrease the number of elements being analyzed in the *case study* variable, only the seven most commonly used types are included. Concerning steel structures, plane trusses and frames are related to Kriging/others and NN, respectively. This last relationship is quite strong. Spatial frame structures and trusses are related to RSM and RBF, respectively. Regarding concrete structures, plane frames are related to SVR and bridges to hybrid, Kriging, and other metamodels. The correlation between these two categorical variables is not as good as the previous ones: *sign* = 0.003 and *CPI* = 0.74.

5.1.4. Metamodel - optimization strategy

In Fig. 17(d), it can be appreciated that metamodels such as hybrids, RBF, and to a lesser extent, SVR are related to GA and classical methods. Kriging is related to SQP and SA. In addition, several authors that use Kriging metamodels do not specify the implemented optimization strategy. NN is closely related to optimization procedures performed by other heuristic or metaheuristic procedures (e.g., simple heuristic strategies, Cascade Evolutionary Algorithm, HS, among others). Additionally, sequential optimization is suitable for every metamodeling technique due to its closeness to the origin. These two categorical variables hold a very good *sign* (0.001) and an average *CPI* (0.77).

5.2. Multiple correspondence analysis

As stated, MCA involves more than two variables. In this case, the idea is to clarify some conclusions drawn from the SCA by filtering and refining the information, i.e., if more data are involved, more hidden information will be obtained, and relationships will be easier to understand.

In principle, two groups of three variables will be analyzed, considering a logical order. For example, in function of the *type of formulation*, a *metamodel* is chosen, and finally, a *DoE* to complement the *metamodel* selection. Thus, the first group is conformed by *type of formulation – metamodel – DoE*. The second group is selected following this principle: in function of the *case study* to optimize, the *metamodel* and a corresponding *optimization strategy* are selected. However, the correlation among this group was not good because of the amount of discarded data. For example, studies that do not perform optimization, only reliability analysis, are not included. The same happens for the case studies, as stated before. These relationships can be extracted from SCA, previously discussed.

5.2.1. Type of formulation - metamodel - DoE

Fig. 18(a) shows the graphical analysis from MCA involving the formulation, metamodel, and DoE. With this new analysis, several combinations are clearer now. For instance, the best metamodel to support design problems considering uncertainties (RBDO and RBDO) is Kriging and the best DoE to select the initial samplings is LHS. Fig. 18(b) and (c) show the difference between results obtained from SCA and MCA, respectively.

6. Discussion, practical recommendations and future directions

This section aims to discuss and summarize the results found in the review about implementing MASDO in the civil engineering area. The analysis is based on the investigation of each of the eight categorical variables separately, their interaction, and the implementation of the corresponding statistical techniques. Additionally, future directions are drawn based on previous results.

- Regarding the *type of formulation* of optimization problems supported by metamodels, the deterministic approach was predominant in the first decade of the XXI century. However, from here on, reliability analysis and optimization have experienced a boom in their use. Robust optimization (RDO) and its combination with reliability-based optimization (RBRDO) have not been as widely implemented.
- As for the *optimization objectives*, the most implemented are the classic ones, led by weight. Regarding the objectives related to sustainability, the economic approach is the most implemented. On the other hand, multiobjective optimization is mainly applied by combining weight-structural behavior and economic cost-structural behavior. Note that this is the classic strategy of converting constraints (e.g., a characteristic point displacement) into the optimization objective. It can be checked that only a few authors have formulated deeper optimization problems, including sustainability criteria such as environmental or social ones.
- In the case of the *variables*, most are related to cross-sectional dimensions. Very few formulations include the reinforcing configuration or aspects related to the building materials as variables.
- Regarding the use of *metamodels* to assist structural optimization procedures, there has been an increase in their implementation as of 2016, especially those based on Kriging methodologies. Previously, the most implemented metamodeling technique was NN. On the other hand, the most used strategy in the *DoE* is LHS. The Kriging technique has also been widely used to build surrogate models. In addition, these two strategies are by far the most complementary ones. On the other hand, the most used strategy to solve problems with deterministic formulations (DO) is NN. For reliability-based (RBDO) and robust (RDO) formulations, Kriging is the most used surrogate model.
- The most used *optimization strategy* is GA, followed by sequential optimization, SQP, PSO, and SA. Interestingly, even though multiobjective optimization is not the predominant one, more than half of the uses of GA are devoted to solving these problems.
- Regarding the implemented *case studies*, there is a predominance of the use of steel structures over concrete ones. In the former case, most structures consist of 3D truss structures, followed by plane trusses and frames. As for concrete structures, the most used ones are bridge decks. It is important to highlight that 70% of the total case studies are benchmark problems.
- Finally, to perform the high-fidelity simulations to build the metamodel, own programming codes conform the most used *calculation engines*. The most used commercial software are Abaqus, CSi Bridge, ANSYS, and the OpenSees platform, used mainly in problems related to dynamic analysis.

From the separate analysis of each categorical variable, it can be concluded that most MASDO applications focus on creating and

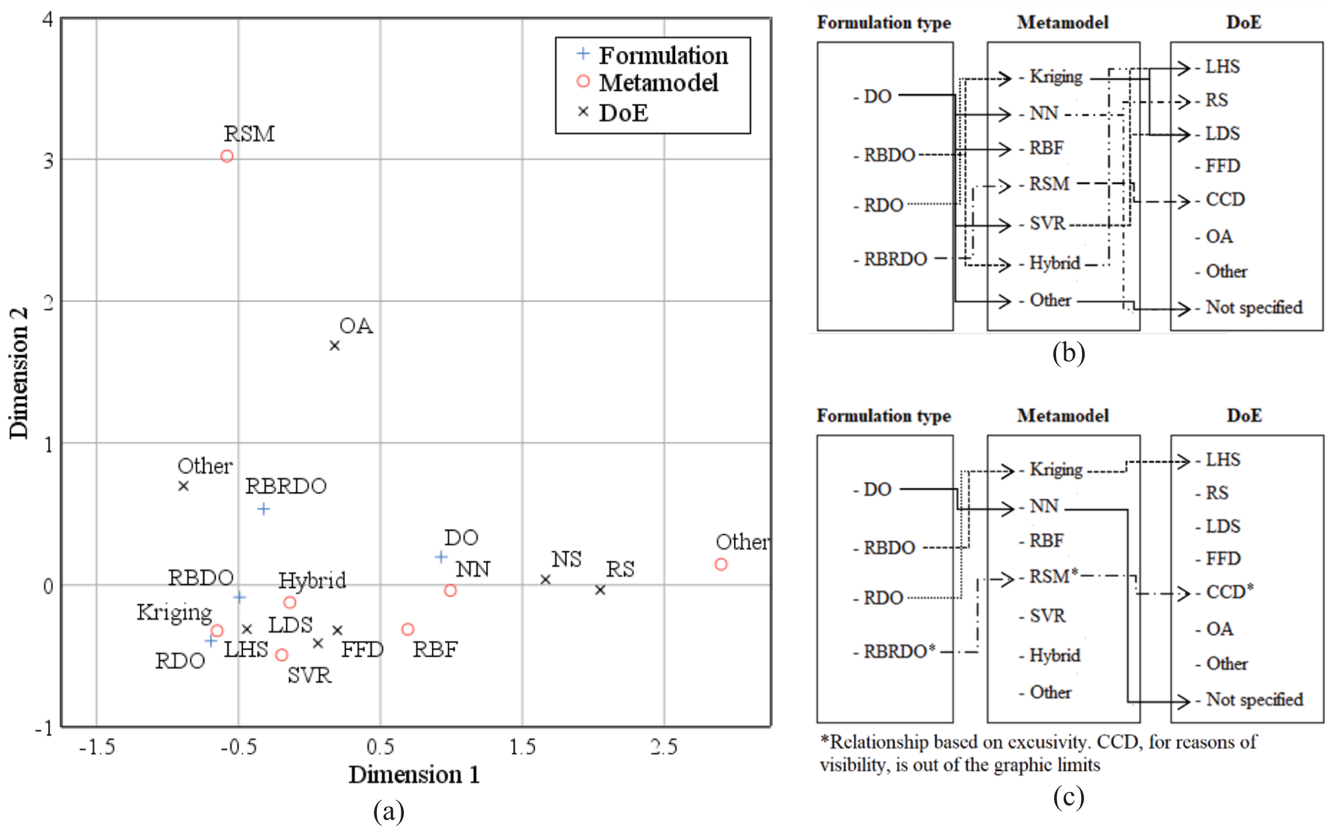

Fig. 18. Multiple correspondence analysis (Formulation-Metamodel-DoE) and its influence, (a) is the graphic of row and column points, (b) is the graphical analysis from simple correspondence analysis (Fig. 17(a)-(b)) and (c) is the graphical analysis from multiple correspondence analysis.

developing new strategies. Simple (benchmark) case studies are used for this. Problems are formulated with fundamental objective functions and variables. The LHS-Kriging combination is the most implemented metamodeling strategy and the use of own programming codes as computational engines to perform high-fidelity simulations is the most common approach. In contrast, using surrogate models to support optimization problems with more encompassing formulations and challenging real-life structures as case studies has been much less developed.

On the other hand, implementing a statistical analysis based on correspondence analysis to detect underlying relationships between categorical variables provides us with practical recommendations on how to proceed based on what has been developed in the field.

- The simple correspondence analysis between the *type of formulation* and the *metamodel technique* used confirms that for deterministic formulations, the most used metamodels are NN and RBF, two similar techniques. On the other hand, RBDO and RDO type formulations are associated with Kriging metamodels. The former is also related to hybrid techniques.
- Regarding selecting a strategy for the *DoE* as a function of the *metamodeling technique*, it is found that LHS is a commonly used approach. Kriging-based strategies are strongly associated with LHS and, to a lesser extent, with LDS. On the other hand, NN is related to RS, and a large number of studies do not specify how to obtain the sample points to construct the metamodel. It may be because, in many cases, the NNs are built on databases created from previous studies. Furthermore, SVR techniques are related to LDS and LHS, hybrid implementations to LHS, and RSM to CCD.
- There is also a relationship between the *case study* and the *metamodeling technique*. Steel structures, such as plane trusses and frames, are related to Kriging and NN, respectively, being the latter a rather strong relationship. In addition, spatial steel frame and truss structures are related to RSM and RBF techniques, respectively. Concerning concrete structures, plane frames are related to SVR and bridges to Kriging and *other* metamodels.
- Finally, the metamodeling techniques are also related to optimization strategies. Hybrid, RBF, and to a lesser extent, SVR methodologies are related to GA and classical methods. Kriging is related to SQP and SA. NN-based metamodels are related to other simple heuristics or metaheuristic procedures, such as Cascade Evolutionary Algorithm or the HS method. Sequential optimization seems to be an appropriate strategy to optimize each surrogate model.

It can be seen that the SCA finds a relationship between two categorical variables. Thus, MCA was also performed to try to go deeper into these subjacent relationships. The problem is that this kind of analysis needs a very good database to be significant. For this reason, it was possible to fit a single relationship among *type of formulation – metamodel – DoE*. Other relationships can be extracted from SCA. This analysis shows that NN metamodels should support DO problems, but there is no clear strategy for performing the DoE. In addition, RBDO and RDO formulations are well supported by Kriging-based metamodels, and LHS is a suitable technique to create the initial sampling. Finally, RBRDO problems seem to have a good relationship with RSM, which has a good relationship with CCD strategies.

Finally, by combining the two statistical analyses (simple and multiple CA), good metamodeling strategies could be designed. For example, to optimize the design of a *planar steel frame* considering two approaches, *deterministic* and *robust*: this type of structures are related to *NN-based* metamodels, as well as *deterministic optimization*. However, *RDO formulations* are connected to *Kriging* metamodels, thus, two metamodeling strategies could be implemented. On one hand, the *NN-based* metamodel can be optimized by a *simple heuristic* (e.g., *HS*), but also by *sequential optimization procedures* (which seem to be proper to deal with any metamodel implementation). On the other hand, as mentioned, *Kriging* metamodels are related to the *SA* methodology. To perform the high-fidelity simulations using the *DO formulation*, it could be use an *own code*, or a commercial software such as *ANSYS*. If the *RDO formulation* is related to dynamic analysis, the OpenSees software could be used.

It should be noted that these strategies are designed based on the interaction of the categorical variables found in the reviewed papers. It is essential to highlight that both the case study and the type of formulation are the starting point for designing the metamodeling approach. In any case, more than one strategy could be designed based on the recommendations obtained from the statistical analysis. It provides a much more solid starting point.

6.1. Benefits of metamodeling supporting structural design optimization

To end this discussion, real practical benefits of performing MASDO in the literature are summarized below. It is important to highlight that the accuracy of a certain strategy can be measured in two ways: the computational savings, but also in the comparison between the results with those obtained by traditional procedures. In the literature, several studies achieve computational savings by maintaining the same results as conventional strategies (or even improving them). However, in the complex optimization of practical problems, it is usually to gain in computational savings but sacrificing accuracy of the results.

In [136], one FLAC^{3D} software simulation takes the same time as about 370 000 Kriging-based simulations. The surrogate modelling technique obtained a value of $R^2 = 0.99$. In [135], the Kriging-based metamodel allows for solving the RBDO problem of the 25-bars spatial truss transmission tower with only 299 finite element runs. [157] proposed an adaptive sparse refined Kriging-based computational model that utilizes between 47.5 and 50.5% of the computational effort compared to the conventional High-Fidelity RDO approach while solving the RDO problem of a plane steel building and the 25-bars spatial truss transmission tower. In [123], the design of a crane bridge is 20 times cheaper using a Kriging-RBDO formulation than a conventional FEA one. Using a Cumulative Distribution Function (CDF)-based RBRDO approach supported by a dual RSM, [84] were capable of optimizing 3D steel and RC buildings, demanding only 600 FEA in both cases. This is significantly inferior to the 200 000 and 300 000 simulations, respectively, needed by a traditional MCS approach. The RDO of the 39-bars spatial truss transmission tower [158] reduced the required CPU time by almost 192 times using a Polynomial Chaos Expansion-Kriging (PCE-KR) hybridization in comparison with a traditional MCS approach. While optimizing a simple plane steel frame, [147] obtained a substantial gain in computational efficacy: the metamodel-aided optimization (using an RSM approach) was 35 times cheaper than a traditional full stochastic implementation. In [159], it was formulated and solved an RDO of a four-legged offshore jacket platform. The proposed RDO framework required only 0.8–1.0% of the CPU time to obtain the optimal solutions compared to conventional MCS-based RDO with parallel processing. [79] optimized the 10-bars plane truss structure using an RBDO formulation, and the metamodel implementation (a hybrid PCE-KR) allowed to reduce the number of FE analysis from 8×10^7 to 1.2×10^3 (in one problem) and to 1.76×10^3 (in the other one). In a similar study, [78] reduced the number of required simulations to optimize a 3D dome structure (RBDO formulation), using a hybrid PCE-Moving Least Square method from 6×10^7 (conventional MCS) to 735. In [50], the implemented NN decreased the number of expensive simulations by between 27 and 37% during the design of post-tensioned concrete road bridges. This is quite significant if the computational time of a high-fidelity simulation is about 375 times a single NN-based simulation. The NN implementation showed values of the coefficient of determination R^2 between 0.912 and 0.999. [47] reduced the computational time of a 3D truss structure optimization between 96 and 98% using a hybridization between NN and RBF for the surrogate model construction. Similarly, in [48], the optimization time of a plane steel structure was reduced by 93% using as a surrogate model an RBF-based metamodel. In [23], the deterministic optimization of a concrete box-girder pedestrian bridge was performed with a computational cost reduction of 91%, aided by a Kriging-based metamodel, keeping an accuracy of over 96%. Finally, to optimize a real wind turbine foundation using a Kriging-based metamodel, [65] obtained a 15% more sustainable design than the original predefined design and around 8% better than the best design obtained by trial-and-error improvement, keeping a low computational effort in the multi-objective optimization procedure. In general terms, and considering that several strategies could be derived from each metamodeling strategy, it is difficult to set an exact computational effectiveness of each approach.

Supported by previous information and based on [26], the general benefits of metamodeling supporting structural design optimization can be summarized as follows:

- It is cheaper to execute low-fidelity simulations using metamodels than complex computer high-fidelity simulations (FEA, for example). In a process in which many function evaluations are required, such as optimization or design under uncertainties, using metamodels leads to significant savings in computational time.
- With metamodels, the entire design space of the system being investigated can be easily traversed, thus deeper understood.
- Aiming to combine information from different sources, e.g., real physical experiments and computer simulations, metamodels can also be used.
- Metamodels help smooth response values in the case of noise in the experiments.

6.2. Future directions

Considering the results obtained by analyzing the categorical variables separately, it can be said that the main obstacle to a broader application of these strategies in the design practice is the simplicity of most of the problems formulated so far. Even when several metamodeling strategies have been proposed, their application to real-world problems still needs to be improved. Concerning the type of formulation, there has been an increase while considering uncertainties in design, especially with the implementation of reliability-based formulations. However, these formulations are usually based on simple objectives. For this reason, one of the most promising future directions is the consideration of sustainability criteria: economic, environmental, social, and constructability. In addition, when defining the optimization objectives, the complete life cycle analysis must be considered: manufacturing, construction, use, maintenance, and end of life. Other issues, such as retrofitting, must also be taken into account. It also will include other variables, such as the concrete cover or the reinforcing configuration (in the case of concrete structures).

Regarding the metamodeling strategy, it should be noted that Kriging and NN are the strategies that have been gaining in popularity in implementing MASDO problems. Therefore, it is expected that they will continue to be exploited in resolving this type of problem, either in their simple form, derived strategies, or hybridizations. With the implementation of more complex problems, more efficient optimization methods must be implemented. Using exact methods (such as SQP) or classical heuristics will be valid options. However, to develop proper metamodeling strategies, sequential optimization procedures appear to be a valuable alternative to more efficiently explore and improve the accuracy of metamodels. Including new and more encompassing objectives will lead to implementing multiobjective optimization procedures, which also leads to the need to develop multi-criteria decision-making procedures. Here, the use of metamodels will play a key role.

On the other hand, due to the imperative need to improve the sustainability indexes of the construction sector, these methodologies must be extended to real case studies. In this context, the use of commercial software for modeling, analysis, and structural design should be exploited to a greater extent. API functions allow us to link these software through the different programming languages with the different metamodel-assisted optimization toolboxes. Additionally, and based on current design trends, interaction with platforms such as BIM can be another alternative for applying metamodels in the design optimization of civil engineering structures.

Another branch in which MASDO is expected to play a fundamental role is Additive Manufacturing. This technique, which is increasingly used in structural engineering, is closely related to the topological optimization of finite element models, which, in turn, are associated with high computational consumption.

7. Concluding remarks

Metamodel-assisted or *surrogate-based optimization* is a growing and very useful topic in civil and structural engineering due to the need to decrease computational resource consumption of structural design optimization procedures, which are sometimes even prohibitive. This paper presents an up-to-date literature review on *metamodel-assisted structural design optimizatio*n (MASDO), analyzing 111 publications, including 169 case studies from 2000 to the present. It was found that there are not as many publications as might be expected, especially studies involving challenging real-life problems as case studies. Most studies are devoted to improving or proposing new metamodeling techniques, using simple case studies or benchmark problems to validate them.

Consequently, the main objective of this research is to provide practical recommendations on best practices to incorporate metamodel strategies whiting structural optimization procedures to make them computationally affordable. In this context, eight categorical variables were considered to carry on the study, led by *metamodel technique*. It was found that the type *of formulation* is closely related to the *metamodel technique* selection. Deterministic optimization (DO) and Reliability-based Design Optimization (RBDO) are the two most implemented approaches, especially from 2010 onwards. There has also been an increase in *reliability analysis* in the last years due to the after-mentioned situation about improving metamodeling strategies. The most used optimization objective is the *weight*, and geometrical variables such as *cross-sectional dimensions/areas* are the most implemented ones. The most used optimization strategies are based on GA, closely followed by *sequential optimization*, a typical strategy of optimizing and improving the metamodel simultaneously (on-line optimization procedures). As mentioned, the case studies related to MASDO are mostly simple steel structures, especially 3D trusses, plane trusses, and plane frames, in that order. Most of these structures are benchmark problems used to test novel metamodeling implementations. In concordance with this, *own codes* are the main way to perform the high-fidelity simulations, being ABAQUS the most used commercial FE software. Finally, the most implemented metamodel and DoE strategies were, by far, Kriging and Latin Hypercube Sampling, respectively.

Additionally, simple and multiple correspondence analyses were performed to find subjacent relationships among the categorical variables. Aimed by the first one, it was found that DO is closely related to NN, SVR, and RBF metamodels, with a closer relationship to the first one. For its part, RBDO is connected to hybrid and Kriging-based metamodels, being the second and most suitable option. RDO is also related to Kriging-based strategies. Regarding case studies, steel structures such as plane trusses and frames are linked to Kriging and NN, respectively, while spatial trusses and frames are related to RBF and RSM, respectively. Concrete structures such as plane frames are related to SVR and bridges to hybrid and Kriging-based metamodels.

On the other hand, hybrid, RBF, and to a lesser extent, SVR metamodels are connected to GA. Kriging is related to SQP and SA, and NN is closely related to other methods such as simple heuristic strategies, Cascade Evolutionary Algorithm, Harmony Search, among others. Finally, it was checked that Kriging is strongly related to LHS and LDS to a lesser extent. NN is associated with RS. SVR is associated to LDS and LHS, *hybrid* formulations to LHS, and there is a special relationship between CCD and RSM based on exclusivity. The implementation of MCA is more challenging to perform. It was possible to perform an analysis among *type of formulatio*n - *metamodel* - *DoE*, which vas valuable to clarify some relationships extracted from the SCA.

Furthermore, the benefits of using metamodels have been proved to support structural design optimization procedures. In general terms, it reduces computational effort, keeping an acceptable accuracy between low and high-fidelity simulations. Apart from creating a baseline, this study was motivated by the need to unify criteria and provide practical recommendations on using metamodels to support the design optimization of civil engineering structures. Such recommendations could be precious in current and future projects related to the structural optimization field.

To conclude, and based on the results obtained in this review, it can be established that the promising lines of research in this field should be directed to extend all the already developed surrogate model-building strategies to solve optimization problems with more encompassing formulations, considering objectives related to environmental, social and constructive aspects. Furthermore, these objectives should not only be limited to mere optimization up to the design stage. Alternatively, they should include the Life-Cycle Analysis of the structures, from the project to demolition, considering aspects related to maintenance and retrofitting. In addition, the construction of metamodels should also support procedures such as multi-criteria decision-making. Finally, these methodologies should be applied to real-life challenging case studies, thus demonstrating the practical potential of using surrogate models in structural design optimization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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