Electric Power Systems Research 227 (2024) 109972

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Algorithm for passive reactive power compensation of an unbalanced three-phase four-wire system using capacitors ensuring minimum line losses

Pedro A. Blasco, Rafael Montoya-Mira, José M. Diez *, Rafael Montoya

Departamento de Ingeniería Eléctrica, Universitat Politècnica de València, Plaza Ferrandiz y Carbonell s/n, Alcoy, Spain

ARTICLE INFO

Keywords:
Compensation
Reactive power
Capacitors
Line losses

ABSTRACT

Currently in three-phase distribution networks, especially low-voltage networks, there are some inefficient powers. These are defined as reactive power, unbalanced power owing to linear loads, power owing to supply unbalanced voltage, and harmonic power. This has given rise to the concept of power quality. To improve power quality, devices called active filters have been developed. These devices eliminate all the inefficient power. However, for most processes, the elimination of reactive power needs to be from a price/performance perspective because the use of active filters is very expensive. Therefore, the use of capacitor banks in any of their versions (single-phase, three-phase, scalable battery, SVC, etc.) is the most economical and sufficient solution. In this study, a calculation algorithm is proposed to obtain compensators for the inductive reactive power of the load, consisting only of single-phase capacitor banks. These capacitors are designed to minimise losses in the supply line, which are lower than those obtained using the minimum loss line (MLL) strategy. The resulting compensator consists of three, two, or one capacitor, depending on the load characteristics.

1. Introduction

The quality of the electrical power supply is one of the most researched topics, especially in three-phase low-voltage systems with three or four wires. Because these are dynamic systems, they always operate under conditions of load imbalance and voltage imbalance. This leads to an increase in the total apparent power with respect to the ideal power of a balanced system, which is characterised by positive-sequence active power. Therefore, these unbalanced powers represent one of the factors that increase the total apparent power of the system [1–4]. These powers and their physical meanings have been widely discussed by the scientific community [5–7].

Technological development has brought another power source due to the use of non-linear loads such as variable speed drives (widely used in industry), arc welding equipment, and switching power supplies for their operation, for example, computers, printers, battery chargers, etc. This nonfundamental power was classified as harmonic power by [8]. This harmonic power has also been widely discussed by the scientific community [9]. This has led to the development of active filters to compensate for the effects of harmonic components [10].

However, this unbalanced power and harmonic power increase the losses in the power lines that feed the loads. A great deal of research has been conducted to compensate for these powers and reduce line losses [11–13]. There are two main areas of research in this field. One line analyses the use and behaviour of passive elements, called reactive power compensators (RPCs), and the other analyses the use and behaviour of active elements, called switching power converters (SPCs), of the advanced compensator or active filter type.

It is evident that SPC-type compensators are much better than RPCs because they have more functions to improve the power quality and are more accurate and faster. However, they are 30–35 % more expensive than RPCs [14]. Furthermore, the use of an RPC compensator provides an acceptable level of power quality for most industrial networks.

The theoretical basis of SPC compensators is basically based on Akagi's p–q theory [15]. In this paper, Akagi proposes two compensation strategies namely "constant instantaneous power control strategy" and the "sinusoidal current control strategy". Both strategies can be applied to RPC compensators and are known as the minimum loss line (MLL) and sinusoidal balancing current (SBC), respectively.

Almost all low-voltage electrical systems encountered in real life are inductive systems. Therefore, in this study, we focus on the compensation of reactive power and asymmetric power in three-phase linear four-

* Corresponding author.
E-mail address: jmdiez@die.upv.es (J.M. Diez).

https://doi.org/10.1016/j.epsr.2023.109972
Received 7 March 2023; Received in revised form 26 June 2023; Accepted 27 October 2023
Available online 4 November 2023
0378-7796/

wire systems with asymmetric voltages and loads using only single-phase capacitor banks. The use of only capacitors, whether they are single-phase banks, scalable banks, or their more advanced form (SVC), is due to the fact that they have already proven their capability in compensating the reactive power consumed by the loads.

It should also be noted that a few authors [16,14] are already working on this line of research. However, in the cited works, they are used for highly inductive loads and do not specify what happens when the loads are not highly inductive.

In 2022, the authors published an article [17] in which a method was developed to compensate for the load reactive power and maximum negative-sequence current in unbalanced three-phase three-wire systems using only capacitors. This results in lower line losses than when the MLL strategy is used. Unlike [17], this work applies to unbalanced four-wire three-phase systems in both voltage and load. Therefore, the zero-sequence current is taken into account, which conditions us to a star configuration for the compensator.

This study proposes a new algorithm for calculating reactive power compensation using capacitors for four-wire linear electrical systems with unbalanced voltages and/or loads. With this procedure, lower line losses were obtained than with the MLL strategy without using coils. This compensator consists of three, two, or one capacitor.

The remainder of the paper is organised as follows. Section 2 develops an algorithm for obtaining the capacitors to be placed in the compensator with two objectives: (1) to compensate for the reactive power of the load and (2) to minimise the losses in the power line. In Section 3, several case studies are analysed to verify the developed calculation algorithm, considering different types of loads. Each of these case studies represents three possible solutions depending on the number of compensation banks used: one, two, or three. The results were compared with those obtained using the MLL reactive power compensation strategy. Finally, Section 4 presents the conclusions of this study.

2. Method of calculating the compensation system

Fig. 1 shows a four-wire electrical system with unbalanced voltages feeding an unbalanced load, whose total reactive power Q_L is inductive. Here, v_{an}, v_{bn} and v_{cn} are the line-to-neutral voltages measured at the load terminals. In contrast, i_{aL}, i_{bL}, i_{cL} and i_{nL} are the line currents in each phase and neutral, respectively. Both the voltages and line currents can be easily measured at the load connection point using a measuring device.

The objective of this study is to compensate for the total reactive power of the system Q_L, using only capacitors, and the optimal solution represents the lowest losses in the line connected to the load. For the compensation elements to be capacitive, it is a prerequisite that Q_L is inductive.

The calculation procedure developed in this study consists of a maximum of three calculations that are performed sequentially depending on the results obtained in such a way that once the optimal solution has been obtained, it is not necessary to continue with the rest of the calculations. First, the calculation of a compensation system using

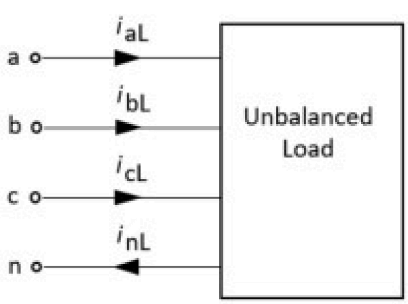

Fig. 1. Unbalanced voltage system feeding an unbalanced load.

three single-phase star-connected compensation banks is discussed. If the results obtained in each bank represent capacitive elements, the optimum solution is obtained directly without the need for subsequent calculations. Throughout this study, a positive value of compensation power is considered to be capacitive, and a negative value of compensation power is considered to be inductive. The second calculation was performed using two single-phase compensation banks. This calculation is performed when the reactive power result obtained from the first calculation is negative in one of the compensation banks; that is, it represents a coil. In this case, because only capacitors are used, the corresponding single-phase compensation bank is cancelled and the compensation system is calculated using two single-phase compensation banks. The third calculation is performed using a single-phase compensation bank. Under the same assumptions as in the previous calculations, this calculation is necessary when using three capacitor banks results in two negative reactive compensation powers or when using two capacitor banks results in one negative reactive compensation power.

The three methods for calculating the aforementioned compensation system are as follows:

- Calculation with three single-phase compensation banks
- Calculation with two single-phase compensation banks
- Calculation with a single-phase compensation bank

2.1. Calculation with three single-phase compensation banks

Suppose that three single-phase banks of wye-connected capacitors are placed in parallel with the load to compensate for the inductive reactive power of the load Q_L, as shown in Fig. 2. The capacitive reactive powers provided by the capacitors are given by Q_{Ca}, Q_{Cb}, and Q_{Cc}, respectively. To compensate for the total reactive power of the load, condition (1) must be fulfilled.

$$Q_{Ca} + Q_{Cb} + Q_{Cc} - Q_L = 0 \tag{1}$$

By including the compensator, the new line currents circulating through each of the phases are given by (2), and the current circulating through the neutral phase by (3).

$$\overrightarrow{I_z} = \overrightarrow{I_{zL}} + \overrightarrow{I_{Cz}} \qquad for \quad z = a, b, c \tag{2}$$

where,

$$\overrightarrow{I_{Cz}} = \frac{Q_{Cz}}{V_{zn}} e^{j\left(\alpha_z + \frac{\pi}{2}\right)} \qquad for \quad z = a, b, c \tag{3}$$

Here, α_z is the angle of the voltage V_{zn}.

In contrast, the current circulating in the neutral conductor is determined by (4).

$$\overrightarrow{I_n} = \sum_{z=a,b,c} \overrightarrow{I_z} \tag{4}$$

As the line-to-neutral voltages are measured at the load terminals, the optimal solution of the compensation system that guarantees the minimum losses on the line is given by the minimum value of the expression $f(x)$ according to (5).

$$f(x) = I_a^2 + I_b^2 + I_c^2 + I_n^2 \tag{5}$$

From (2) and considering (3), the moduli of the currents to be included in (5) are obtained.

$$I_z^2 = \left[Re\left\{\overrightarrow{I_{zL}}\right\} + \frac{Q_{Cz}}{V_{zn}} \cos\left(\alpha_z + \frac{\pi}{2}\right)\right]^2 + \left[Im\left\{\overrightarrow{I_{zL}}\right\} + \frac{Q_{Cz}}{V_{zn}} \sin\left(\alpha_z + \frac{\pi}{2}\right)\right]^2 \tag{6}$$

Therefore,

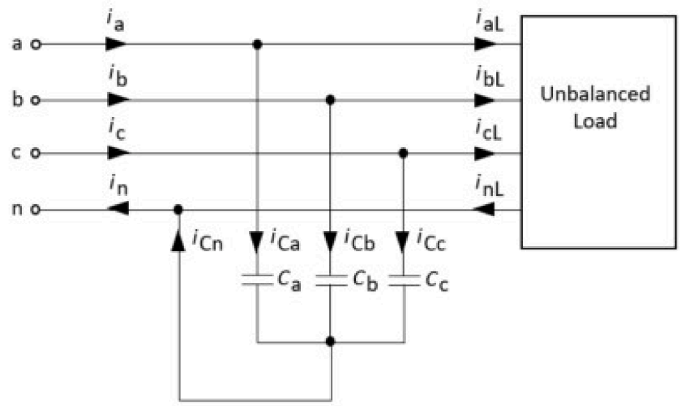

Fig. 2. Compensation using three single-phase capacitors.

$$I_z^2 = \left[I_{zL}\cos\beta_{zL} + \frac{Q_{Cz}}{V_{zn}}\cos\left(\alpha_z + \frac{\pi}{2}\right)\right]^2 + \left[I_{zL}\sin\beta_{zL} + \frac{Q_{Cz}}{V_{zn}}\sin\left(\alpha_z + \frac{\pi}{2}\right)\right]^2 \quad (7)$$

Here, β_{zL} is the current angle of the load I_{zL}.

Considering the fact that $\cos(\alpha_z + \frac{\pi}{2}) = -sin\alpha_z$ and $sin(\alpha_z + \frac{\pi}{2}) = cos\alpha_z$, and substituting in Eq. (7), we obtain (8).

$$I_z^2 = \left[I_{zL}\cos\beta_{zL} - \frac{Q_{Cz}}{V_{zn}}\sin\alpha_z\right]^2 + \left[I_{zL}\sin\beta_{zL} + \frac{Q_{Cz}}{V_{zn}}cos\alpha_z\right]^2 \quad (8)$$

By developing (8) and considering Eqs. (9) and (10), we obtain (11):

$$sin^2(\mathrm{a}) + cos^2(\mathrm{a}) = 1 \quad (9)$$

$$\sin(a)\cos(b) - \cos(\mathrm{a})\sin(b) = \sin(a-b) \quad (10)$$

$$I_z^2 = I_{zL}^2 + \frac{Q_{Cz}^2}{V_{zn}^2} + 2I_{zL}\frac{Q_{Cz}}{V_{zn}}\sin(\beta_{zL} - \alpha_z) \quad (11)$$

The second and third summands in (11) represent the currents that the capacitors should provide to compensate for the reactive power of the system. Substituting $z = a, b, c$, we obtain the expression of the new line currents for each phase of the system as follows:

$$I_a^2 = I_{aL}^2 + \frac{Q_{Ca}^2}{V_{an}^2} + 2I_{aL}\frac{Q_{Ca}}{V_{an}}\sin(\beta_{aL} - \alpha_a) \quad (12)$$

$$I_b^2 = I_{bL}^2 + \frac{Q_{Cb}^2}{V_{bn}^2} + 2I_{bL}\frac{Q_{Cb}}{V_{bn}}\sin(\beta_{bL} - \alpha_b) \quad (13)$$

$$I_c^2 = I_{cL}^2 + \frac{Q_{Cc}^2}{V_{cn}^2} + 2I_{cL}\frac{Q_{Cc}}{V_{cn}}\sin(\beta_{cL} - \alpha_c) \quad (14)$$

Evolving in the same way, the new current that will circulate through the neutral conductor from (4), Eq. (15) is obtained.

$$I_n^2 = \left[\sum_{z=a,b,c}\left(I_{zL}\cos\beta_{zL} - \frac{Q_{Cz}}{V_{zn}}\sin\alpha_z\right)\right]^2 + \left[\sum_{z=a,b,c}\left(I_{zL}\sin\beta_{zL} + \frac{Q_{Cz}}{V_{zn}}cos\alpha_z\right)\right]^2 \quad (15)$$

Developing and regrouping (15), we obtain (16).

$$I_n^2 = \sum_{z=a,b,c} I_{zL}^2 + \sum_{z=a,b,c}\frac{Q_{Cz}^2}{V_{zn}^2} + \sum_{i=1}^{15} f_i \quad (16)$$

where,

$$f_1 = 2I_{aL}\frac{Q_{Ca}}{V_{an}}\sin(\beta_{aL} - \alpha_a) \quad (17)$$

$$f_2 = 2I_{bL}\frac{Q_{Cb}}{V_{bn}}\sin(\beta_{bL} - \alpha_b) \quad (18)$$

$$f_3 = 2I_{cL}\frac{Q_{Cc}}{V_{cn}}\sin(\beta_{cL} - \alpha_c) \quad (19)$$

$$f_4 = 2I_{aL}\frac{Q_{Cb}}{V_{bn}}\sin(\beta_{aL} - \alpha_b) \quad (20)$$

$$f_5 = 2I_{aL}\frac{Q_{Cc}}{V_{cn}}\sin(\beta_{aL} - \alpha_c) \quad (21)$$

$$f_6 = 2I_{bL}\frac{Q_{Ca}}{V_{an}}\sin(\beta_{bL} - \alpha_a) \quad (22)$$

$$f_7 = 2I_{cL}\frac{Q_{Ca}}{V_{an}}\sin(\beta_{cL} - \alpha_a) \quad (23)$$

$$f_8 = 2I_{bL}\frac{Q_{Cc}}{V_{cn}}\sin(\beta_{bL} - \alpha_c) \quad (24)$$

$$f_9 = 2I_{cL}\frac{Q_{Cb}}{V_{bn}}\sin(\beta_{cL} - \alpha_b) \quad (25)$$

$$f_{10} = 2I_{aL}I_{bL}\cos(\beta_{aL} - \beta_{bL}) \quad (26)$$

$$f_{11} = 2I_{aL}I_{cL}\cos(\beta_{aL} - \beta_{cL}) \quad (27)$$

$$f_{12} = 2I_{bL}I_{cL}\cos(\beta_{bL} - \beta_{cL}) \quad (28)$$

$$f_{13} = 2\frac{Q_{Ca}}{V_{an}}\frac{Q_{Cb}}{V_{bn}}\cos(\alpha_a - \alpha_b) \quad (29)$$

$$f_{14} = 2\frac{Q_{Ca}}{V_{an}}\frac{Q_{Cc}}{V_{cn}}\cos(\alpha_a - \alpha_c) \quad (30)$$

$$f_{15} = 2\frac{Q_{Cb}}{V_{bn}}\frac{Q_{Cc}}{V_{cn}}\cos(\alpha_b - \alpha_c) \quad (31)$$

Substituting (11) and (16) into (5) yields (32):

$$f(Q_{Ca}, Q_{Cb}, Q_{Cc}) = 2\sum_{z=a,b,c} I_{zL}^2 + 2\sum_{z=a,b,c}\frac{Q_{Cz}^2}{V_{zn}^2} + 2I_{zL}\frac{Q_{Cz}}{V_{zn}}\sin(\beta_{zL} - \alpha_z) + \sum_{i=1}^{15} f_i \quad (32)$$

Using the Lagrange multipliers method to determine the minimum value of $f(Q_{Ca}, Q_{Cb}, Q_{Cc})$ defined in (32) and considering the constraint of expression (1) as $g(Q_{Ca}, Q_{Cb}, Q_{Cc})$, we obtain (33).

$$\nabla f(Q_{Ca}, Q_{Cb}, Q_{Cc}) = \lambda \nabla g(Q_{Ca}, Q_{Cb}, Q_{Cc}) \quad (33)$$

By calculating the partial derivative of (32) with respect to Q_{Ca}, we obtain (34)

$$\frac{\partial f(Q_{Ca}, Q_{Cb}, Q_{Cc})}{\partial(Q_{Ca})} = a_1 Q_{Ca} + b_1 Q_{Cb} + c_1 Q_{Cc} - \lambda - k_1 \tag{34}$$

where,

$$a_1 = \frac{2}{V_{an}^2} \qquad b_1 = \frac{2\cos(\alpha_a - \alpha_b)}{V_{an}V_{bn}} \qquad c_1 = \frac{2\cos(\alpha_a - \alpha_c)}{V_{an}V_{cn}} \tag{35}$$

$$k_1 = -\frac{1}{V_{an}}\left[4I_{aL}\sin(\beta_{aL} - \alpha_a) + 2I_{bL}\sin(\beta_{bL} - \alpha_a) + 2I_{cL}\sin(\beta_{cL} - \alpha_a)\right] \tag{36}$$

By calculating the partial derivative of (32) with respect to Q_{Cb}, we obtain (37)

$$\frac{\partial f(Q_{Ca}, Q_{Cb}, Q_{Cc})}{\partial(Q_{Cb})} = a_2 Q_{Ca} + b_2 Q_{Cb} + c_2 Q_{Cc} - \lambda - k_2 \tag{37}$$

where,

$$a_2 = \frac{2\cos(\alpha_a - \alpha_b)}{V_{an}V_{bn}} \qquad b_2 = \frac{2}{V_{bn}^2} \qquad c_2 = \frac{2\cos(\alpha_b - \alpha_c)}{V_{bn}V_{cn}} \tag{38}$$

$$k_2 = -\frac{1}{V_{bn}}\left[4I_{bL}\sin(\beta_{bL} - \alpha_b) + 2I_{aL}\sin(\beta_{aL} - \alpha_b) + 2I_{cL}\sin(\beta_{cL} - \alpha_b)\right] \tag{39}$$

By calculating the partial derivative of (32) with respect to Q_{Cc}, we obtain (40)

$$\frac{\partial f(Q_{Ca}, Q_{Cb}, Q_{Cc})}{\partial(Q_{Cc})} = a_3 Q_{Ca} + b_3 Q_{Cb} + c_3 Q_{Cc} - \lambda - k_3 \tag{40}$$

where,

$$a_3 = \frac{2\cos(\alpha_a - \alpha_c)}{V_{an}V_{cn}} \qquad b_3 = \frac{2\cos(\alpha_b - \alpha_c)}{V_{bn}V_{cn}} \qquad c_3 = \frac{2}{V_{cn}^2} \tag{41}$$

$$k_3 = -\frac{1}{V_{cn}}\left[4I_{cL}\sin(\beta_{cL} - \alpha_c) + 2I_{aL}\sin(\beta_{aL} - \alpha_c) + 2I_{bL}\sin(\beta_{bL} - \alpha_c)\right] \tag{42}$$

Generalising in matrix form, the system of equations for the compensation system is determined as follows:

$$(A)\ (X) = (C) \tag{43}$$

where,

$$A = \begin{pmatrix} a_1 & b_1 & c_1 & -1 \\ a_2 & b_2 & c_2 & -1 \\ a_3 & b_3 & c_3 & -1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \qquad X = \begin{pmatrix} Q_{Ca} \\ Q_{Cb} \\ Q_{Cc} \\ \lambda \end{pmatrix} \qquad C = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ Q_L \end{pmatrix} \tag{44}$$

The solution of the system of equations that determines the values of Q_{Ca}, Q_{Cb}, Q_{Cc}, and λ is given by (45).

$$(X) = (A)^{-1}(C) \tag{45}$$

Knowing the values of Q_{Ca}, Q_{Cb} and Q_{Cc}, the values of the reactances corresponding to each phase are as follows:

$$X_{Cz} = \frac{V_{zn}^2}{Q_{Cz}} \qquad for\ z = a, b, c \tag{46}$$

In this case, $X_{Cz} > 0$ represents capacitive reactance, and $X_{Cz} < 0$ represents inductive reactance. Considering the main objective of this study, that is, the reactive power compensation of a system using only capacitors, the following observations should be made, depending on the values of X_{Cz}:

- It is only considered as a valid solution using three single-phase banks of capacitors, one for each phase, when all values of $X_{Cz} \geq 0$ (refer Fig. 3). In this case, the value of the capacitance in farads of each capacitor in the compensator is given by (47).

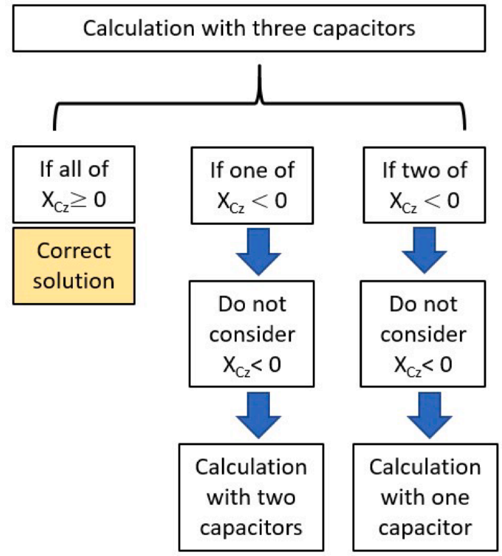

Fig. 3. General procedure with three single-phase capacitors.

$$C_z = \frac{1}{\omega X_{Cz}} \qquad for \quad z = a, b, c \tag{47}$$

- When one of the values of $X_{Cz} < 0$, the compensating element in that phase is a coil. In this case, the phase in which $X_{Cz} < 0$ must be left open circuit, and we will have to reconsider the compensation calculation using two capacitors. The calculation of compensation using the two capacitors, is given in the following sections.
- When there are two values where $X_{Cz} < 0$, as aforementioned, we ignore these values and leave both branches open circuit. In this case, the reactive power compensation of the system is calculated using a single capacitor. This calculation is described in the following sections.
- If it is the case that all values of $X_{Cz} < 0$, it means that the total reactive power of the load is capacitive. The main requirement for applying this method is that the total reactive power of the load must be inductive.

2.2. Calculation with two single-phase compensation banks

As described in the previous section, after calculating the compensation system using three connected banks of capacitors, when the reactance X_{Cz} value of one of the compensation banks is negative, it corresponds to a coil. As the objective of this study is to use only capacitors to compensate for the total reactive power of the system, this compensation bank should be eliminated, and the compensation system is recalculated using the other two single-phase compensation banks. Assume that $X_{Ca} < 0$, therefore, the compensation bank C_a that is connected between phase A and neutral is eliminated, that is, $Q_{Ca} = 0$, as shown in Fig. 4. Therefore, it is necessary to consider the new constraint given by (48).

$$h(Q_{Ca}) = Q_{Ca} = 0 \tag{48}$$

In this case, using Lagrange multipliers and following the same procedure as in the previous section, we have

$$\nabla f(Q_{Ca}, Q_{Cb}, Q_{Cc}) = \lambda \nabla g(Q_{Ca}, Q_{Cb}, Q_{Cc}) + \mu_a \nabla h(Q_{Ca}) \tag{49}$$

where μ_a, the Lagrange multiplier used for the new constraint $Q_{Ca} = 0$ is given by (48). Under these conditions, the matrices defined in (43) are

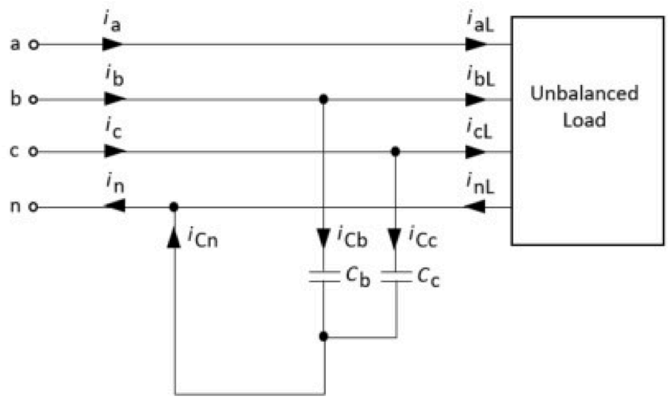

Fig. 4. Compensation using C_b and C_c when $X_{Ca} < 0$.

modified according to (50):

$$A = \begin{pmatrix} a_1 & b_1 & c_1 & -1 & -1 \\ a_2 & b_2 & c_2 & -1 & 0 \\ a_3 & b_3 & c_3 & -1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad X = \begin{pmatrix} Q_{Ca} \\ Q_{Cb} \\ Q_{Cc} \\ \lambda \\ \mu_a \end{pmatrix} \quad C = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ Q_L \\ 0 \end{pmatrix} \tag{50}$$

By following the same procedure, when $X_{Cb} < 0$, C_b must be eliminated (refer Fig. 5). In this case, the additional constraints are $h(Q_{Cb}) = Q_{Cb} = 0$, μ_b is the Lagrange multiplier, and the matrices must be modified according to (51).

$$A = \begin{pmatrix} a_1 & b_1 & c_1 & -1 & 0 \\ a_2 & b_2 & c_2 & -1 & -1 \\ a_3 & b_3 & c_3 & -1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad X = \begin{pmatrix} Q_{Ca} \\ Q_{Cb} \\ Q_{Cc} \\ \lambda \\ \mu_b \end{pmatrix} \quad C = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ Q_L \\ 0 \end{pmatrix} \tag{51}$$

Finally, when $X_{Cc} < 0$, we would eliminate C_c (refer Fig. 6) by replacing the constraint with $h(Q_{Cc}) = Q_{Cc} = 0$, μ_c is the Lagrange multiplier, and the matrices are according to (52).

$$A = \begin{pmatrix} a_1 & b_1 & c_1 & -1 & 0 \\ a_2 & b_2 & c_2 & -1 & 0 \\ a_3 & b_3 & c_3 & -1 & -1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad X = \begin{pmatrix} Q_{Ca} \\ Q_{Cb} \\ Q_{Cc} \\ \lambda \\ \mu_c \end{pmatrix} \quad C = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ Q_L \\ 0 \end{pmatrix} \tag{52}$$

To facilitate the use of the method, based on the results obtained in Section 2.1 when a branch is not considered to be a coil, the connection diagram to be used from those shown in Figs. 4, 5, and 6 is presented in Fig. 7.

When the calculation is performed with two single-phase banks, as shown in Figs. 4, 5 or 6, as in the previous section with three banks, the

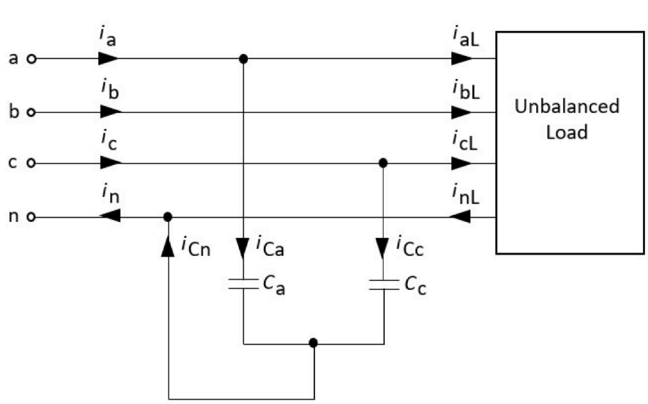

Fig. 5. Compensation using C_a and C_c when $X_{Cb} < 0$.

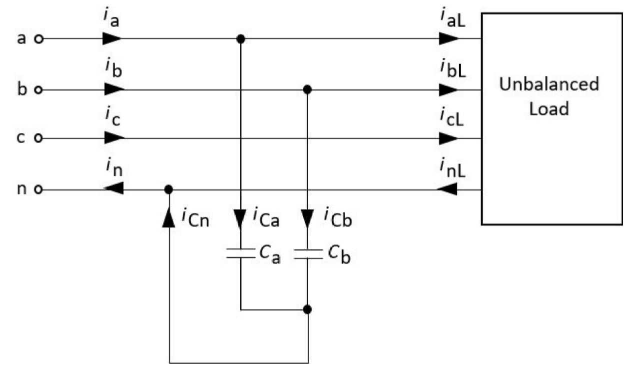

Fig. 6. Compensation using C_a and C_b when $X_{Cc} < 0$.

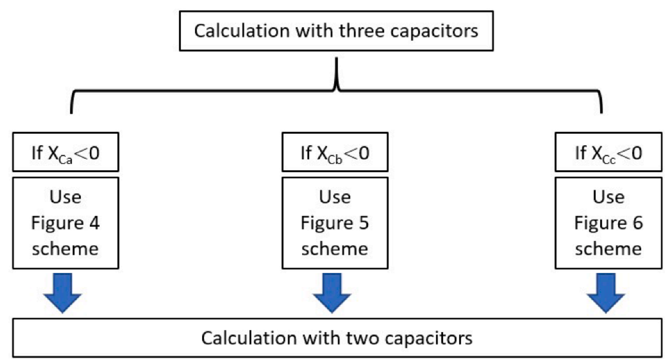

Fig. 7. Choice of connection system using two capacitor banks.

valid solution will be when the two banks are capacitors, that is, when the reactance values are positive. If the result in one branch is a coil, it is not considered, and the calculation must be performed with only one bank to compensate for the reactive power Q_L. Fig. 8 clearly shows the procedure for the ease of understanding.

2.3. Calculation with a single-phase compensation banks

When there are two negative-compensation reactance values using three single-phase banks, or when there is one negative value of compensation reactance using two single-phase banks, it is necessary to implement a compensation system using a single-phase bank. This single-phase bank will have to compensate for the reactive power of the load and will be connected to the phase whose reactance value has been positive in the calculations in the previous sections. For its calculation, it is sufficient to equal the reactive power provided by the single-phase bank Q_{Cz} to the total reactive power of the load Q_L, that is, $Q_{Cz} = Q_L$. Therefore, we have the following:

- $X_{Ca} = \frac{V_{an}^2}{Q_L}$ when the capacitor is connected to Phase A, as shown in Fig. 9.
- $X_{Cb} = \frac{V_{bn}^2}{Q_L}$ when the capacitor is connected to Phase B, as shown in Fig. 10.
- $X_{Cc} = \frac{V_{cn}^2}{Q_L}$ when the capacitor is connected to Phase C, as shown in Fig. 11.

In any of the aforementioned three cases, the value of the compensation bank capacitance is determined by (47).

3. Practical cases

In this section, three case studies are developed to verify the load

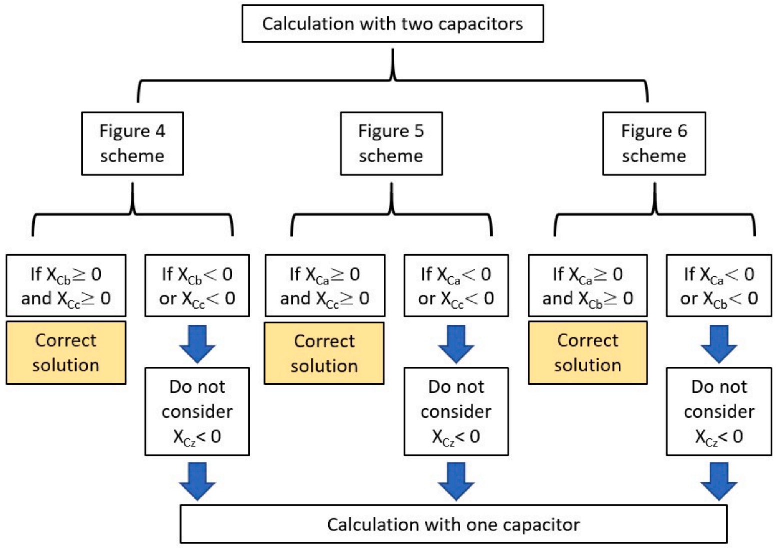

Fig. 8. General procedure using two banks of capacitors.

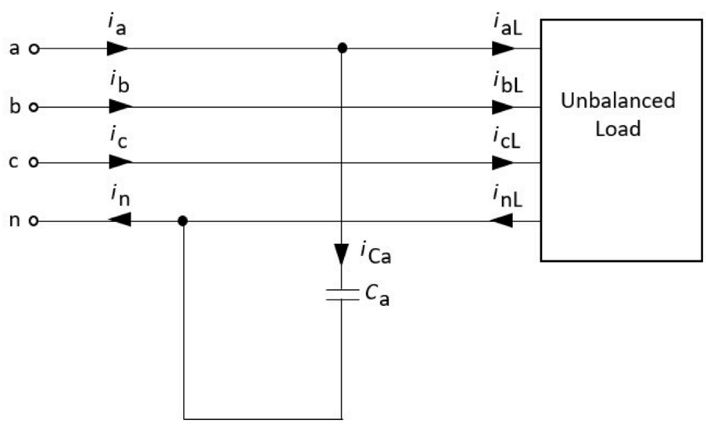

Fig. 9. Single-phase compensation using only C_a.

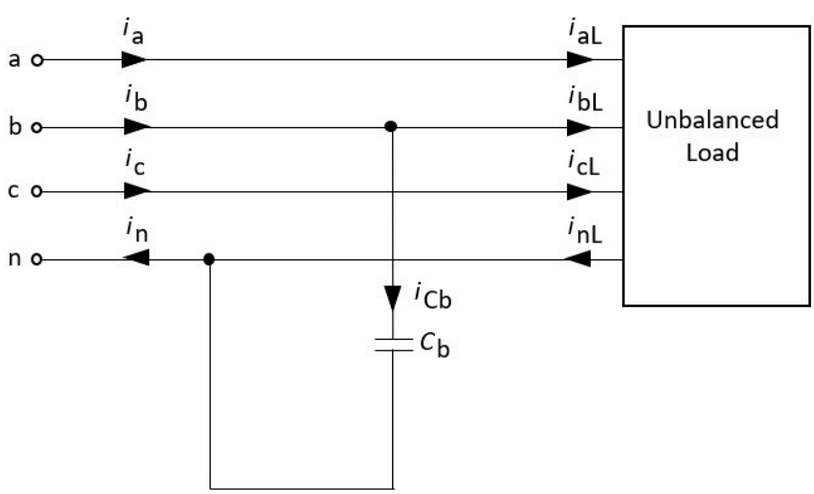

Fig. 10. Single-phase compensation using only C_b.

reactive power compensation calculation algorithm using single-phase capacitor banks proposed in this paper. In all the cases, a four-wire unbalanced inductive load connected to a three-phase system with unbalanced voltages was used, as shown in Fig. 12. The solution for each case is related to previous sections of this paper, and the cases are:

- Case Study 1: Compensation using three single-phase capacitor banks

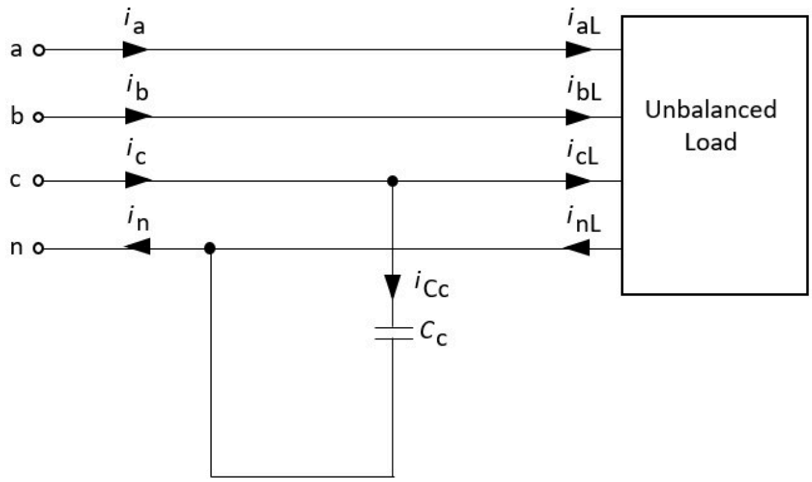

Fig. 11. Single-phase compensation using only C_c.

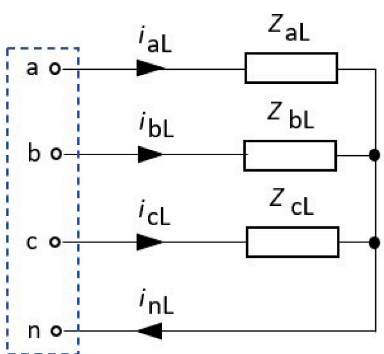

Fig. 12. Model load for case studies.

Table 1
Load impedances for each case study.

N°	Z_{aL} (Ω)		Z_{bL} (Ω)		Z_{cL} (Ω)	
	R	X	R	X	R	X
Case 1	4	2	3	2	2	1
Case 2	4	2	5	2	1	1
Case 3	2	0.5	17	0.5	1	1

- Case Study 2: Compensation using two single-phase capacitor banks
- Case Study 3: Compensation using a single-phase capacitor bank.

In each of the practical cases, the results obtained using the proposed algorithm were compared with those obtained using the MLL method [17]. The reactance values of the compensation banks obtained using the MLL method are those necessary to phase the voltages with their respective current.

Fig. 12 shows a three-phase four-wire unbalanced load connected to a three-phase system, whose line-to-neutral voltages are as follows (53):

$$V_{an} = 230\ e^{j0}\ V \qquad V_{bn} = 232\ e^{-j119}\ V \qquad V_{cn} = 228\ e^{j121}\ V \tag{53}$$

To perform the calculations, the voltage and current data obtained using a network analyser or any other measuring device connected at the PCC will be used. This means that it is not necessary to know the equivalent impedances downstream. However, for the sake of better following the procedure, equivalent impedances will be directly used. The impedance values for each of the cases are shown in Table 1.

Considering the values of the voltages from (53) and the load impedances in each of the cases listed in Table 1, the values of the load currents for each of the practical cases were determined as presented in Table 2.

From the line-to-neutral voltages according to (53) and the line currents shown in Table 2, the electrical power ratio of the load for each case is shown in Table 3.

3.1. Case study 1: compensation using three single-phase capacitor banks

Following the procedure of the algorithm used in this study, the first step was to determine the reactive power compensation of the load using three single-phase capacitor banks, as shown in Fig. 2. For this purpose, the matrix equation defined in (43) is used, and the coefficients of matrices A and C are calculated as follows:

For matrix A in (43), considering the voltages defined in (53), the calculations were performed as follows:

- a_1, b_1 and c_1 by (35)
- a_2, b_2 and c_2 by (38)
- a_3, b_3 and c_3 by (41)

For matrix C of (43), consider the voltages defined in (53) and the currents shown in Table 2. That is,

- k_1 is determined using (36)
- k_2 is determined using (39)
- k_3 is determined using (42)
- The value of Q_L has already been calculated and is presented in Table 3. For Case 1, $Q_L = 23.967\ kVAr$.

Performing different calculations for the coefficients of matrices A and C, the results obtained for this practical case are as follows:

$$a_1 = 7561.4\ x10^{-8} \qquad b_2 = 7431.6\ x10^{-8} \qquad c_3 = 7694.7x10^{-8} \tag{54}$$

$$a_2 = b_1 = -1817.1\ x10^{-8} \qquad a_3 = c_1 = -1964.3\ x10^{-8} \qquad b_3 = c_2 = -1890.5\ x10^{-8} \tag{55}$$

Table 2
Line currents of the load in each of the case studies.

N°	I_{aL} (A) Modulus	Angle	I_{bL} (A) Modulus	Angle	I_{cL} (A) Modulus	Angle	I_{nL} (A) Modulus	Angle
Case 1	*51.43*	*−26.6*	*64.35*	*−152.7*	*101.96*	*94.4*	*52.70*	*111.2*
Case 2	*51.43*	*−26.6*	*43.08*	*−140.8*	*161.22*	*76.0*	*118.08*	*64.1*
Case 3	111.57	−14.0	13.64	−120.7	161.22	76.0	183.08	40.0

Table 3
Active power, reactive power, and power factor.

	P (kW)	Q (kVAr)	PF
Case 1	43.795	23.967	0.877
Case 2	45.852	34.994	0.795
Case 3	54.049	32.309	0.858

$$k_1 = -2272.9\ x10^{-4} \qquad k_2 = 6567.5\ x10^{-4} \qquad k_3 = 4787.0\ x10^{-4} \tag{56}$$

Substituting the values of (54) and (55) in matrix A and the values of (56) in matrix C and solving the matrix equation of (43), the values of the reactive powers of the three compensation banks are as follows:

$$Q_{Ca} = 2.362\ kVAr \qquad Q_{Cb} = 11.876\ kVAr \qquad Q_{Cc} = 9.729\ kVAr \tag{57}$$

By applying (46), the reactance values of the respective compensation banks connected as shown in Fig. 2, are determined by (58).

$$X_{Ca} = 22.394\ \Omega \qquad X_{Cb} = 4.532\ \Omega \qquad X_{Cc} = 5.343\ \Omega \tag{58}$$

The capacitances of single-phase compensation banks can be calculated using (47).

Under these conditions, new currents circulating through the lines feeding the set formed by the load and compensation equipment, were calculated. The values are presented in Table 4. Compared with Table 2 for Case 1, it can be observed that the values of the currents are lower, mainly the current flowing through the neutral conductor, with a reduction of approximately 68% with respect to the initial value.

Table 5 lists the values of the active power, reactive power, and power factor of the proposed system when the compensation equipment is connected. Logically, the reactive power is zero, and the power factor is equal to one.

Considering that the resistance value of the line conductors is 1 Ω, both in the phase and neutral conductors, for both the proposed compensation and that obtained by applying the MLL method, the line losses for both methods are presented in Table 6. Comparing the values obtained by the two methods, it is observed that the compensation algorithm proposed in this study presents lower line losses than the MLL method. Table 6 also shows the losses in the initial system without compensation.

The calculations of the line currents according to MLL method have been calculated as follows:

a) Calculation of the reactive powers of the load in each of the phases.

$$Q_{aL} = V_{an}\ I_{aL}\sin(\alpha_a - \beta_{aL}) = 5.29\ kVAr$$

$$Q_{bL} = V_{bn}\ I_{bL}\sin(\alpha_b - \beta_{bL}) = 8.28\ kVAr$$

$$Q_{cL} = V_{cn}\ I_{cL}\sin(\alpha_c - \beta_{cL}) = 10.40\ kVAr$$

Table 5
Active power, reactive power, and power factor for Case 1 with compensation.

P (kW)	Q (kVAr)	PF
43.795	0.000	1.000

Table 6
Comparison of existing losses in the supply line for Case 1.

	P_{loss} (kW)
No compensation	19.960
MLL method	15.008
Proposed compensation algorithm	13.988

b) Calculation of X_{Cz} according to MLL method.

$$X_{Ca(MLL)} = \frac{V_{an}^2}{Q_{aL}} = 10\ \Omega \qquad X_{Cb(MLL)} = \frac{V_{bn}^2}{Q_{bL}} = 6.5\ \Omega \qquad X_{Cc(MLL)} = \frac{V_{cn}^2}{Q_{cL}} = 5\ \Omega$$

c) Calculation of the currents I_{Cz} considering $X_{Cz(MLL)}$.

$$\overrightarrow{I_{Ca}} = \frac{\overrightarrow{V_{an}}}{X_{Ca(MLL)}} = 30\ e^{j90}\ \text{A}$$

$$\overrightarrow{I_{Cb}} = \frac{\overrightarrow{V_{bn}}}{X_{Cb(MLL)}} = 35.7\ e^{-j29}\ \text{A}$$

$$\overrightarrow{I_{Cc}} = \frac{\overrightarrow{V_{cn}}}{X_{Cc(MLL)}} = 45.6\ e^{-j149}\ \text{A}$$

d) Calculation of the line currents when connecting the three capacitor banks according to MLL method. From these line currents, the line losses are determined.

$$\overrightarrow{I_a} = \overrightarrow{I_{aL}} + \overrightarrow{I_{Ca}} = 46\ e^{j0}\ \text{A}$$

$$\overrightarrow{I_b} = \overrightarrow{I_{bL}} + \overrightarrow{I_{Cb}} = 53.5\ e^{-j119}\ \text{A}$$

$$\overrightarrow{I_c} = \overrightarrow{I_{cL}} + \overrightarrow{I_{Cc}} = 91.2\ e^{j121}\ \text{A}$$

3.2. Case study 2: compensation using two single-phase capacitor banks

Based on the procedure described in Figs. 3 and 7, the compensation system must be calculated using three single-phase banks to determine which of the three possible single-phase banks should not be considered.

Table 4
New line currents with compensation for Case 1.

I_a (A) Modulus	Angle	I_b (A) Modulus	Angle	I_c (A) Modulus	Angle	I_n (A) Modulus	Angle
47.73	−15.5	55.74	−102.8	91.25	119.2	16.65	130.7

Performing these calculations identically to Case 1, and considering the values in Tables 2 and 3 for Case 2, the reactance values of the compensation system are determined by (59).

$$X_{Ca} = -99.209\ \Omega \qquad X_{Cb} = 5.880\ \Omega \qquad X_{Cc} = 1.971\ \Omega \tag{59}$$

It is observed that $X_{Ca} < 0$, therefore, if considered, this compensation bank would be composed of coils. Following the procedure of the proposed algorithm, X_{Ca} should not be considered and the reactive power compensation system of the load is to be recalculated using a compensation system with two single-phase banks. In our case, as X_{Ca} is not considered because it is negative, we used the assembly shown in Fig. 4. In other words, $Q_{Ca} = 0$ was considered, and the system of equations used for the calculation is expressed by (50).

From the system of Eqs. (50), considering the values of the line-to-neutral voltages and the values in Tables 2 and 3 for Case 2, the new coefficients of matrices A and C are as follows:

$$a_1 = 7561.4\ x10^{-8} \qquad b_2 = 7431.6\ x10^{-8} \qquad c_3 = 7694.7x10^{-8} \tag{60}$$

$$a_2 = b_1 = -1817.1\ x10^{-8} \qquad a_3 = \ c_1 = -1964.3\ x10^{-8} \qquad b_3 = c_2 = -1890.5\ x10^{-8} \tag{61}$$

$$k_1 = -7235.1\ x10^{-4} \qquad k_2 = 1926.2\ x10^{-4} \qquad k_3 = 18,679.2\ x10^{-4} \tag{62}$$

Substituting the values in matrices A and C and solving the system of equations, the values of the reactive powers of the two compensation banks are as follows:

$$Q_{Cb} = 8.880\ kVAr \qquad Q_{Cc} = 26.114\ kVAr \tag{62}$$

By applying (46), the reactance values of the two compensation banks connected as shown in Fig. 4, are determined by (63).

$$X_{Cb} = 6.061\ \Omega \qquad X_{Cc} = 1.991\ \Omega \tag{63}$$

Table 7 lists the new currents flowing in the lines feeding the load and compensation equipment. If we compare these values with Table 2 for Case 2, the current flowing through the neutral conductor has been reduced by approximately 73%.

Table 8 lists the values of the active power, reactive power and power factor of the proposed system when the compensation equipment is connected. Logically, the reactive power is zero and the power factor is equal to one.

Considering a resistance of one ohm in the line conductors, both in the phase and neutral conductors, both for the proposed compensation and that obtained by applying the MLL method, the line losses for both methods are shown in Table 9. Comparing the values obtained by both methods, it is observed that, as in Case 1, the proposed compensation algorithm presents lower line losses than the MLL method.

For the calculation according to MLL method, follow the same procedure as described in case study 1.

3.3. Case study 3: compensation using a single-phase capacitor bank

As in the previous cases, the compensation system must first be calculated from the three compensation banks. By performing this calculation considering the values for Case 3 (refer Tables 2 and 3), the following values for the reactances in the three compensation banks are obtained:

$$X_{Ca} = -33.107\ \Omega \qquad X_{Cb} = 98.253\ \Omega \qquad X_{Cc} = 1.558\ \Omega \tag{64}$$

Second, when $X_{Ca} < 0$, the compensation system must be recalculated using two compensation banks. As in the previous case, the two banks will be connected to phases B and C, respectively, as shown in Fig. 4.

$$X_{Cb} = -195.956\ \Omega \qquad X_{Cc} = 1.595\ \Omega \tag{65}$$

In (65), it can be observed that, for this practical case, there is also no valid solution using a compensation system with two single-phase banks, because $X_{Cb} < 0$. Therefore, the only possible solution is to compensate for the total reactive power of the load using a single-phase compensation bank connected to Phase C (refer mounting diagram in Fig. 11).

Under these conditions, where $Q_{Cc} = Q_L = 32.309\ kVAr$, refer Table 3. The reactance value of the compensation bank is determined using Eq. (66).

$$X_{Cc} = \frac{V_{cn}^2}{Q_L} = 1.609\ \Omega \tag{66}$$

Table 10 lists the new currents flowing in the lines feeding the load and compensation equipment. If we compare these values with Table 2 for Case 3, the current flowing through the neutral conductor has been reduced by approximately 74%.

Table 11 lists the values of the active power, reactive power, and power factor of the proposed system when the compensation equipment is connected. Logically, the reactive power is zero, and the power factor is equal to one.

Considering a line conductor resistance of unity value in the phase and neutral conductors, both for the proposed compensation and that obtained by applying the MLL method, the line losses for both methods are listed in Table 12. Comparing the values obtained by both methods,

Table 8
Active power, reactive power, and power factor for Case 2 with compensation.

P (kW)	Q (kVAr)	PF
43.795	0.000	1.000

Table 9
Comparison of existing losses in the supply line for Case 2.

	P_{loss} (kW)
No compensation	44.436
MLL method	21.678
Proposed compensation algorithm	18.710

Table 7
New line currents with compensation for Case 2.

I_a (A) Modulus	Angle	I_b (A) Modulus	Angle	I_c (A) Modulus	Angle	I_n (A) Modulus	Angle
51.43	−26.6	45.78	−89.9	114.0	121.3	31.50	114.5

Table 10
New line currents with compensation for Case 3.

I_a (A) Modulus	Angle	I_b (A) Modulus	Angle	I_c (A) Modulus	Angle	I_n (A) Modulus	Angle
111.57	−14.0	13.64	−120.7	117.3	134.7	48.46	67.2

Table 11
Active power, reactive power, and power factor for Case 3 with compensation.

P (kW)	Q (kVAr)	PF
43.795	0.000	1.000

Table 12
Comparison of existing losses in the supply line for Case 3.

	P_{loss} (kW)
No compensation	72.142
MLL method	34.098
Proposed compensation algorithm	28.745

it is observed that, as in Cases 1 and 2, the proposed compensation algorithm presents lower line losses than the MLL method.

For the calculation according to MLL method, follow the same procedure as described in case study 1.

4. Conclusions

In this study, a calculation method was developed for reactive power compensation of a 4-wire electrical system. The compensation of the system is executed by single-phase banks connected in parallel with the load or a set of loads using only capacitors. Therefore, the main requirement is that the total reactive power of the load be inductive. The method used in this study is also valid for unbalanced loads and voltages. Furthermore, it has been proven that the solution obtained is not only able to compensate for the total reactive power of the system, but also to produce the lowest possible losses in the system's supply lines.

The calculations of the proposed method, depending on whether there is a possible solution using only capacitors, are structured in a maximum of three stages: using three compensation banks, two compensation banks, and a single compensation bank. The results obtained were compared with those of the MLL method, which is one of the most widely used passive compensation methods. From this comparison, it was evident that the solution of the method proposed in this study further reduces the losses in the power lines. Furthermore, Furthermore, to use only capacitors in compensation banks, the total reactive power of the load must be inductive, regardless of the inductive or capacitive character in each phase of the three-phase system. To demonstrate the accuracy of the proposed method and facilitate practical understanding, three case studies were developed with different unbalanced loads fed with unbalanced voltages.

CRediT authorship contribution statement

Pedro A. Blasco: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Resources, Data curation, Visualization. **Rafael Montoya-Mira:** Methodology, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – review & editing, Visualization. **José M. Diez:** Conceptualization, Methodology, Investigation, Writing – original draft, Writing – review & editing, Supervision, Project administration. **Rafael Montoya:** Conceptualization, Methodology, Validation, Investigation, Writing – original draft, Writing – review & editing, Supervision.

Declaration of Competing Interest

No.

Data availability

No data was used for the research described in the article.

Acknowledgments

This work is supported by the Spanish Ministry of Science, Innovation, and Universities (MICINN) and the European Regional Development Fund (ERDF) under Grant PID2021–122835OB-C22.

References

[1] R.C. Dugan, M.F. Mcgranaghan, H.W. Beaty, Electric Power Systems Quality, 2nd edition, McGrawHill, NewYork, NY,USA, 2006.
[2] M.M. Antonio, Power Quality: Mitigation Technologies in a Distributed Environment, Springer, London, UK, 2007.
[3] F.F. Ewald, A.S.M. Mohammad, Power Quality in Power Systems and Electrical Machines, Elsevier Academic Press, London, UK, 2008.
[4] H. Jouini, K. Jemai, and S. Chebbi, "Voltage stability control of electrical network using intelligent load shedding strategy based on fuzzy logic," Math. Probl. Eng., vol. 2010.
[5] J.L. Willems, Reflections on apparent power and power factor in non-sinusoidal and polyphase situations, IEEE Trans. Power Deliv. 19 (2004) 835–840, https://doi.org/10.1109/TPWRD.2003.823182.
[6] P. Pillay, M. Manyage, Loss of life in induction machines operating with unbalanced supplies, IEEE Trans. Energy Convers. 21 (2006) 813–822, https://doi.org/10.1109/TEC.2005.853724.
[7] M.L. Angarita, G.A. Ramos, Power calculations in nonlinear and unbalanced conditions according to IEEE Std 1459–2010, in: Proceedings of the 2013 Workshop on Power Electronics and Power Quality Applications (PEPQA), Bogota, Colombia, 6–7 July 2013, pp. 1–7, https://doi.org/10.1109/PEPQA.2013.6614957.
[8] IEEE Standards Association, IEEE Standard Definittion for the Measurement of Electric Power Quantities under Sinusoidal, Nonsinusoidal, Balanced, Or Unbalanced Conditions. IEEE Std. 1459–2010, IEEE Standards Association, New York, NY, USA, 2010, pp. 1–50.
[9] Giovanni Bucci, Fabrizio Ciancetta, Edoardo Fiorucci, Antonio Ometto, Survey about classical and innovative definitions of the power quantities under nonsinusoidal conditions, Int. J. Emerg. Electric Power Syst. (2017), 20170002.
[10] Marcos B. Ketzer, Cursino B. Jacobina, Antonio M.N. Lima, Shaping control strategies for active power filters, IET Power Electronics 1 (11) (January 2018) 175–181.
[11] R. Langella, A. Testa, A.E. Emanuel, Unbalance definition for electrical power systems in the presence of harmonics and Interharmonics, IEEE Trans. Instrum. Meas. 61 (10) (October 2012) 2622–2631.
[12] P. Salmerón, J.R. Vázquez, R.S. Herrera, S.P. Litrán, Apparent power and power factor in unbalanced and distorted systems. Applications in three phase load compensations, RE & PQJ 1 (5) (March 2007), https://doi.org/10.24084/repqj05.312.
[13] S. Canturk, M.E. Balci, M.H. Hocaoglu, On the definition of apparent power, Electric. Power Qual. Utilizat., J. XVIII (2) (2015). https://www.researchgate.net/publication/297032333.
[14] A. Pană, A. Băloi, F. Molnar-Matei, From the balancing reactive compensator to the balancing capacitive compensator, Energies 11 (2018) 1–24, https://doi.org/10.3390/en11081979.
[15] H. Akagi, E.H. Watanabe, M. Aredes, Instantaneous Power Theory and Applications to Power Conditioning, IEEE Press, John Wiley & Sons, 2007.
[16] Q. Sun, J. Zhou, X. Liu, J. Yang, A novel load-balancing method and device by intelligent grouping compound switches-based capacitor banks shunt compensation, Math. Probl. Eng. (2013), 347361, https://doi.org/10.1155/2013/347361.
[17] Pedro A. Blasco, Rafael Montoya-Mira, José M. Diez, Rafael Montoya, Gabriel Garcerá, Emilio Figueres, Improving the energy efficiency of slightly inductive three-phase three-wire linear systems through single-phase capacitors banks, IET Gener. Transm. Distrib. 16 (2022) 1212–1224, https://doi.org/10.1049/gtd2.12361.