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Clustering and reference value for assessing influence in analytic network process without pairwise comparison matrices: Study of 17 real cases

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ABSTRACT

The analytic network process (ANP) is a well-known multi-criteria decision method that uses pairwise comparison matrices to assess the influence among elements and clusters. This method requires the participation of experts who need to answer a large number of questions. A recent paper proposes using Decision-Making Trial and Evaluation (DEMATEL) scales in ANP to assess influences and suggests the possibility of grouping all elements into a single cluster. This rise the following questions that this paper seek to answer: if no comparison matrices are used in ANP, how similar are the results, whether clusters are used or not, to the original results with ANP using pairwise matrices? Why should or should not one or several groups be used in ANP? How much does the result change when considering multiple groups versus a single group? Does the variation of questions compensate for the variation of the results? How should the evaluation of influences and the use of the scale be approached depending on whether there are one or several groups? For this purpose, published cases solved with ANP have been reviewed and solved without comparison matrices, with the original clustering and with a single cluster, using four different models for each case study. The results show that clustering does influence the results. It should also be noted that the use of clustering helps to identify the elements of the decision problem. Additionally, this work includes the compilation of 17 cases matrices which can be used in further studies

1. Introduction

Analytic network process (ANP) is a well-known method in multi-criteria decision-making (MCDM) proposed by Thomas Saaty in [1] as a generalization of the Analytic Hierarchy Process (AHP). ANP considers that the elements of the decision problem, alternatives and criteria, can influence any other element and, consequently, supports modeling dependencies and feedback between elements in the network model. To measure these influences between elements, Saaty proposes to group the elements of the model into clusters and use pairwise comparison matrices to assess the influence between elements and between clusters. This results in many questions, long questionnaires and a lot of time for the decision makers/experts (DMs) to answer these questionnaires [2]. Therefore, the ANP is often avoided because of its complexity [3]. This last work proposes a new way of integrating Decision Making Trial and Evaluation Laboratory (DEMATEL) with the ANP, using influence measurement scales. They also indicate that with this proposal there is no need to group elements into clusters. Using of measurement scales instead of pairwise comparison matrices is not foreign to AHP/ANP. Saaty himself incorporates them in the so-called Ratings or Absolute measurements [4], but they are only used in the evaluation of alternatives and not in the assessment of importance or influence, since no limit can be set on the number of alternatives and it may not be possible to use comparison matrices. On the other hand, the proposal of not using clusters is challenging for regular users of ANP/AHP because it is necessary to limit the order of pairwise comparison matrices and because it is part of the decomposition process of the problem [5,6]. Although DEMATEL does not require clusters in its model, even in DEMATEL-Based ANP (DANP) proposals it is considered that the influence between clusters should be calculated rather than assuming that they are equally influential to each other [7].

Having verified in [8] that using influence scales instead of pairwise comparison matrices and a single cluster, similar results to the original ones are obtained with ANP, with a significant reduction in the number of questions, we decided to study the proposal of not clustering.

Although it is not necessary to group elements into clusters, from a strictly numerical point of view, should clusters be used or not? After a review of the literature, we did not find any work using ANP or DANP

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that proposed not to use clusters or studied whether clusters should be used or when they should or should not be used.

The objective of this research is to answer the following research questions (RQ): While using a measurement scale to directly measure influence in ANP, RQ1) it is indifferent, better or worse to group all elements into a single cluster or into multiple clusters? RQ2) How much does the result change when considering multiple groups versus a single group? RQ3) Does the variation of questions compensate for the variation of the results?

To answer these questions, published cases have been identified that include the ANP matrices in order to perform the calculations in two ways: on the one hand, considering all the clusters of each model, and on the other hand, grouping the elements in a single cluster. Both results have been compared with the original results with ANP. We consider three comparisons: (1) compare the ordering of the elements; (2) compare the values of the element priorities; (3) compare the priority vector as a whole. For this purpose, the following indicators have been calculated: Spearman's and Kendall's rank correlation coefficients, mean squared error, Saaty's and Garuti's compatibility indexes. Regarding RQ1, a model is considered to be better when it obtains a value of higher correlation/compatibility or lower error with respect to the ANP result. It will be considered indifferent when the correlation/compatibility/error coefficients are similar between both models.

As a consequence of this work, the following additional questions have been posed: When the DMs are asked to evaluate the influence with a measurement scale, RQ4) What should they take as a reference value, the most or the least influential element? RQ5) Does this affect the result depending on how the elements have been grouped, in one or in several clusters?

It is not the purpose of this paper to compare ANP and/or DEMATEL with other MCDM methods, or to justify why to use one method and not others, but if the decision maker has decided to use ANP, we want to contribute to a more efficient and faster use, by collecting less information from the DM, and simpler information, thus needing less time to provide the information needed by the method, with a result as similar as possible to the original ANP.

The Theoretical background section summarizes the ANP and DEMATEL, the new approach, and clustering. The section regarding Method and applied treatments, presents the method used to compare both strategies. The section entitled Case Study an application, presents as an illustrative example of the calculations, one of the cases to which this calculus has been applied. Section Results compiles the results of all the cases analyzed in the Discussion section. Finally, the conclusions and proposals for future works are presented.

Furthermore, all the matrices and results for the 17 cases analyzed can be found in the supplementary material, to be used in further research, and to ensure that this work can be reproduced and verified.

2. Materials and methods

This section presents the ANP and DEMATEL methods, the new DANP approach with its fundamental steps, as well as the clustering technique and a review on clustering.

2.1. The ANP method

The ANP method is a multi-criteria decision making (MCDM) technique initially defined by Saaty in [1], being subsequently referenced in other publications, such as in [9,10]. ANP represents any decision-making problem as a network, presented in a matrix of criteria and/or alternatives (both called elements), grouped in clusters. The elements can be intertwined with each other in any possible way, which makes it possible to include feedback and interdependence connections both within the same cluster and between different clusters. The influence of some elements on other elements of the network are evaluated at the element level and at the cluster level by means of pairwise comparison matrices from which the normalized eigenvectors are obtained as the relative influences sought.

The ANP calculation steps are as follows (for a more detailed explanation regarding its mathematical formulation and execution steps see [8,10–12]):

(1) Clearly define the decision-making problem in order to structure the elements network and thus build a relationship matrix, where each cell takes the value of 0 or 1. It is 0 if the element in the row does not influence the element in the column, and 1 if it does.
(2) Secondly, to obtain the so-called Unweighted Supermatrix, where the influence of the elements of each row with respect to each element of the column is analyzed. The value of each cell is proportional to the influence of that row element with respect to the rest of the elements of the same cluster that influence the column element. Considering that influence is an intangible variable [13], the level of influence is measured by pairwise comparison matrices using Saaty's fundamental scale, a nine-point absolute scale defined by Saaty [11], where 1 to 9 represent a range from "equal influence" to "extreme influence", respectively.
(3) A cluster influence matrix is defined, where the influence of the clusters on each other is evaluated. The value of each cell is the proportion of influence of the cluster in each row with respect to all clusters influencing that cluster column. Pairwise comparison matrices and Saaty's scale are used to assess influence.
(4) To determine the order of precedence between the interdependencies of the system, the Weighted Supermatrix is constructed, which collects the influences of the elements on the elements, considering the influence of the elements and of the clusters. It is calculated by multiplying the influence value of each cell element of the Unweighted Supermatrix and the influence value of its row cluster on the column cluster. Each column of the matrix is then normalized to sum up to 1 if necessary, and thus, a stochastic matrix is obtained.
(5) Finally, the Limit Supermatrix is calculated to obtain the priorities of the elements, considering the accumulated direct and indirect influences. The priority values of the criteria and alternatives will be their weights and ratings, respectively.

If the model has n elements, nxn questions are needed to identify relationships. To assess the influences of n_p items (elements/clusters), with pairwise matrices, $(n_p \times (n_p\text{-}1))/2$ questions are needed. If the number of relationships detected between elements is high, the DM have to answer a very large number of questions.

2.2. The DEMATEL method

The DEMATEL method was developed at the Battelle Geneva Institute to analyze causal relationships between elements in order to understand and solve real-world problems [7]. It is used to construct Influential Network Relation Maps (INRM) to define interactions between dimensions and criteria, based on expert judgement. Additionally, it allows to analyze indirect relationships between criteria [14].

The DEMATEL steps according to [15–18] are as follows:

(1) Establish a direct influence relationship matrix using expert questionnaires on a proportional scale from 0 (no influence) to 4 (maximum influence), in order to obtain data for the influence relationship between any two elements. The value in each cell indicates the relationship level of the element in the row relative to the element in the column. An element cannot directly influence itself.
(2) Calculate the normalized relationship matrix. The mean matrix is calculated and normalized.

(3) Obtain the complete direct/indirect influence matrix or total influence matrix. The sums of the rows indicate the sum of the influence that an element in the row exerts on the other elements, both directly and indirectly. The sums of the columns indicate the sum of the influence that an element in the column receives from the other elements.
(4) Analyze the causal relationships between the elements by setting a threshold value for impact. From the complete direct/indirect influence matrix or total influence matrix, the sum of each row plus the sum of each column is called impact and shows the relational intensity of the element. The higher the impact, the higher the degree influence relationship between the elements. On the other hand, the sum of each row minus the sum of each column is called relationship. If the difference is positive, the element is considered a cause that actively affects other elements. If the difference is negative, the element is considered an effect that is affected by other elements. Cause and effect relationships can be represented in an impact digraph with the elements having a value in the total influence matrix above the threshold.

Other manuscripts in which different scales have been used in DEMATEL can be found in the literature review: in [19] used major contribution (8), considerable contribution (4), some contribution (2), negligible contribution (0), with negative values if necessary; in [20] used a 1–3 scale; in [21–23] used a 0–3 scale; in [24] used a 0–5 scale; [25,26] designed a 11-categories scale, from 0 to 10.

2.3. The new DANP proposal

[3] proposes a new approach in the hybrid integration of DEMATEL and ANP models, considering the weaknesses of the ANP regarding the complexity of the method, the time required for the application and the uncertainty in making judgements, especially at the cluster level [27]. The proposal relates to the way the Supermatrix is calculated, specifically the dependencies between criteria and alternatives in the network. In [3,27] the following steps are defined:

(1) A procedure of structuring the problem by means of a weighted graph in order to model the dependencies between criteria as well as the intensities of those dependencies, and thus define their degree of influence. It therefore structures the problem using part of the DEMATEL algorithm to establish a matrix of influences between elements, equivalent to steps one and two of the method.
(2) Calculate a weighted ANP matrix from the influence matrix obtained in the previous point. It is important to note that, although the proposed method allows the calculation of the unweighted matrix and the cluster matrix, they recommend looking at all elements as a single cluster, thus obtaining the ANP Weighted Supermatrix directly. To assess the influences, they indicate that the Table 1 scale should be used.
(3) Finally, the ANP limit matrix must be calculated to obtain the global influences.

2.4. Clustering

Since most MCDM methods do not make explicit in the literature the concept of clustering, nor which algorithm they propose to use to group the criteria, in order to have a clear understanding of the concept of clustering, it is helpful to start from the concept of "data mining". [28], defines data mining as the use of algorithms and techniques to transform and convert data into something useful by extracting patterns and information from a large amount of data in the process of knowledge discovery. In a first classification, data mining is categorized as supervised and unsupervised learning [29]. Supervised learning considers an objective variable together with a set of input data (predictors, independent) as the basic supply, in order to characterize the output in terms of the predictors. Supervised learning can be subdivided according to the nature of the objective variable, which can be binary (dichotomous), categorical with several unordered categories (polytomous or multinomial), ordered or quantitative (discrete or continuous). In contrast, unsupervised learning does not require prior information, either a non-response data set or a known specific objective variable, to search for clusters, connections, trends or patterns. According to [30] the main data mining techniques are association (association rules, sequential patterns and a priori algorithm), clustering (hierarchical methods, network methods and segmentation methods), classification (decision trees, neural networks and fuzzy sets) and prediction (regression).

Clustering, as indicated by [30–32], is an unsupervised multivariate technique where a set of data samples is divided into categories or a number of clusters, where the particularity is the homogeneity that allows belonging to the same cluster. Clustering algorithms based on unsupervised learning are characterized by not possessing or requiring prior knowledge and treating the input data as a set of random variables [33]. Clustering techniques have been successfully applied in knowledge discovery and data engineering [34], since the first works on k-means, clustering algorithms have been developed, such as kernel k-means, spectral clustering, hierarchical clustering, probabilistic-based clustering, metric clustering, clustering nonumerical data, clustering high dimensional data, clustering graph data, among others [35].

[31] define clustering in N-dimensional Euclidean space R^N is the process of partition a given set of n points into a number, say K, of groups or clusters in such a way that patterns in the same cluster are similar in some sense and patters in different clusters are dissimilar in the same sense. Let the set of n points $\{X_1,X_2,X_3,\ldots,X_n\}$ be represented by the set S and the K clusters be represented by $C_1,C_2,\ldots,C_K$. Then $C_i \neq \varnothing$ for $i = 1, 2, \ldots, K$ and $C_i \cap C_j = \varnothing$ for $i = 1, 2, \ldots, K$, $j = 1, 2, \ldots, K$ and $i \neq j$ and $\bigcup_{i=1}^{K} C_i = S$.

MCDM methods can be classified by criterion dependence into structural dependence, causal dependence and preferential dependence [7], and ANP is classified as a method of the structural dependence type. ANP uses the decomposition (or aggregation) of the elements of a problem within a structure in order to develop a proportion scale of relative priorities between these elements. The measuring instrument that Saaty proposes to use is the pairwise comparison matrix. In this matrix Saaty proposes to cut the scale of measurement into nine. These two things limit on the one hand the number of elements to be compared and on the other hand the proportion of things to be compared. That is why one of the main axioms defined by Saaty requires that only homogeneous elements are compared [36] as defined in the basic foundations of clustering. In situations where the 1 to 9 scale defined by Saaty would be inadequate to cover the spectrum of comparisons needed, i.e., there is no homogeneity, a clustering process is used with a pivot from one cluster to an adjacent cluster, establishing an order of magnitude larger or smaller than the given group, and continuing to use the 1 to 9 scale within each group, thus extending it as far as desired [11]. If all elements cannot be compared to each other directly, it is useful to represent them in groups to avoid large errors of judgement.

The clustering algorithm defined by Saaty determines that clusters are constructed according to the relative size of the priorities of the elements in each cluster, and if any priority differs by an order of magnitude or more, it is moved to the appropriate cluster in order to

Table 1
Classic 5 categories influence scale.

Influence category	Influence Value
No influence (NI)	0
Low influence (LI)	1
Medium influence (MI)	2
Strong influence (SI)	3
Very strong influence (VSI)	4

make the transition from cluster to cluster a well-designed operation [11,37].

In terms of causal dependence, cause and effect relationships are the main distinction of the methods, with DEMATEL being the most representative. However, no research papers have been found with this method in which any clustering technique is used.

3. Methodology and treatments applied

To answer the research questions raised to check whether considering a single cluster with this new proposal, the results are closer or further than considering multiple clusters to the results of the original ANP, the steps shown in Fig. 1 and described below have been followed.

3.1. Searching and selecting ANP articles

To conduct our research, we need cases from the literature solved with ANP that include the unweighted matrix, the cluster matrix and the weighted matrix and/or global priorities. The first phase carried out was a literature review to identify articles on ANP applications. For this purpose, the keyword "ANP" or "analytic network process" was used, performing a search in "Abstract, title and keywords" of journal articles in the Scopus, Web of Science or Google Academic databases published between 2000 and 2021. In addition, this search was filtered by subject areas, such as "Engineering", "Computer Science", "Business, Management and Accounting", "Mathematics", "Energy", among others. Conference papers, master's theses, doctoral dissertations, textbooks and unpublished articles were excluded from the review process. A total of 880 documents were identified.

The next step was to review them and discard those that did not include the matrices and results indicated above. Of the total obtained, the articles that applied the ANP method and included the unweighted, cluster and weighted matrices were identified. Incomplete models that did not include the matrices or had inconsistent data were discarded. Finally, a total of 16 articles were selected [38–53], equivalent to 17 cases of application. We have experienced the problem of missing information in the articles published by the ANP researchers pointed out by [54]. This bad practice prevents the validation of these models and their numerical use by other researchers.

3.2. Calculating five categories scale ANP matrices

Before explaining how the ANP matrices have been transformed to the scale to be used, we want to emphasize that using a measurement scale to assess the influences (such as the one in Table 1), instead of the pairwise comparison matrices and the Saaty scale, implies that the numerical values associated with the levels or categories of the scale are indicating the ratio of influence between these levels, not only their order of influence.

If we had the DMs for each of the cases found in the literature, after

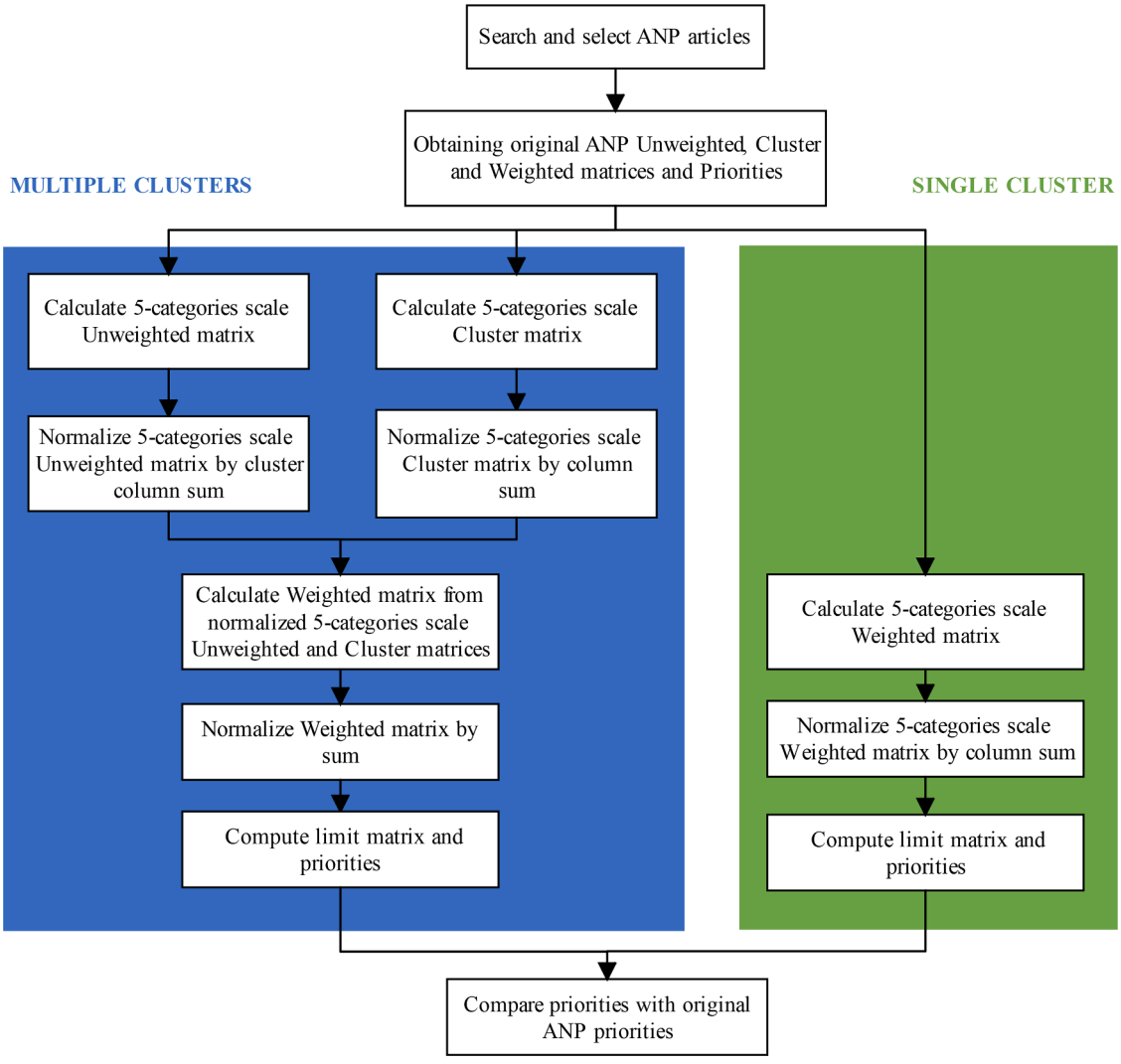

Fig. 1. Methodology and steps performed.

building the three models (classical ANP with several clusters and with pairwise comparison matrices; ANP model with multiple clusters and with measurement scale; ANP model with a single cluster and with a measurement scale), three questionnaires would be given to the DMs to obtain the answers of each model and to be able to calculate the three different weighted matrices with their priorities and compare them. The questions for each model would be similar to the following:

- Original ANP model.
  - Question 1. Identifying relationships. Given a row element, does it influence the column element?
  - Question 2. Assessing elements' influences. Given a column element, and given a pair of row elements of the same cluster, how much more a given member of the pair influence that column element than the other member?
  - Question 3. Assessing clusters' influences. Given a column cluster, and given a pair of row clusters, how much more a given member of the pair influence that column cluster than the other member?
- ANP model with multiple clusters and with measurement scale.
  - Question 4. Identifying relationships and assess elements' influences. Given the elements of a row cluster, how much does each influence a given column element?
  - Question 5. Assessing clusters' influences. Given a column cluster, how much influence does each of the row clusters have?
- ANP model with a single cluster and with measurement scale.
  - Question 6. Identifying the relationships and assessing the influences of the elements. Given a column element, how much is it influenced by the row elements?

Questions 2 and 3 are answered with the Fundamental Saaty's scale [11]. Questions 4, 5 and 6 are answered with the scale of Table 1. But since these DMs are not available, in this step the DMs' response are proposed if they would had been asked how much influence this element/cluster has on this other element/cluster and they would have answered with the scale of Table 1.

To propose the DMs' response, two data must be considered: (1) The influence value of each cell of the published matrices of the original ANP; (2) That this value represents the proportion of influence among a set of items. In the unweighted matrix the influences on each column element are normalized for each of the clusters. In the clusters and weighted matrices, the normalization covers each complete column. The response will be based on the numerical influence values associated with each category of the scale. It should be considered that the numerical values of the scale also reflect their proportion to each other: e.g., SI (3) is three times more influential than LI (1). So, how to convert the influence values of the matrices in each normalization block into influence values 0–4?

This transformation can be formally described as follows: given an influence vector $w = (w_1, w_2, \ldots, w_n)$ where $w_i \in [0, \ldots, 1]$ and $\sum w_i \in \{0, 1\}$, to obtain the vector $v = (v_1, v_2, \ldots, v_n)$ where $v_i \in \{0, 1, 2, 3, 4\}$. This transformation must be a positive monotonic transformation, and if $w_i = 0 \rightarrow v_i = 0$, and ideally, the proportions between them is maintained, $(v_i/v_j \cong w_i/w_j, \forall i.j = 1, \ldots, n)$ and/or together $(v_i / \sum_j v_j \cong w_i, \forall i.j = 1, \ldots, n)$. In the following points, the vector w (1) will be used as an example to illustrate the calculations.

$$w = (0, 0.015, 0.186, 0.05, 0.093, 0.038, 0.105, 0.134, 0.38) \quad (1)$$

Some possible options for this transformation are:

(1) Regarding the maximum value, taking the highest value to transform, $w_{\max}$, the ratios $u_i = w_i/w_{max}$ are calculated. Obviously, $u_i \in [0, 1]$. Since there are 5 categories in the measurement scale, the interval [0,1] is divided into five parts, and the v_i values sought are obtained according to Table 2.

Table 2
0–4 scale equivalent value of influence.

Value Range	Influence
[0, 0.2)	NI-0
[0.2, 0.4)	LI-1
[0.4, 0.6)	MI-2
[0.6, 0.8)	SI-3
[0.8, 1]	VSI-4

Thus, with the vector w (1), the vector $v_1 = (0, 0, 2, 0, 1, 0, 1, 1, 4)$ is obtained.

(2) Regarding the maximum value, considering that the largest value of each block to be transformed, $w_{\max}$, represents the last step in the measurement scale, in this case it would correspond to the influence value 4. The value $u_i = 4 \times (w_i/w_{min})$ converted to an integer is the value v_i sought. This conversion can be done in three possible ways:
   (a) the greatest integer less than or equal to u, denoted floor(u) or $\lfloor u \rfloor$;
   (b) the least integer greater than or equal to u, denoted ceil(u) or $\lceil u \rceil$;
   (c) the nearest integer with tie breaking towards positive infinity, rounding u, denoted $rpi(u) = \lfloor u + 1/2 \rfloor$.

   With the vector w (1), the vectors $v_2 = (0, 0, 1, 0, 0, 0, 1, 1, 4)$, $v_3 = (0, 1, 2, 1, 1, 1, 2, 2, 4)$ y $v_4 = (0, 0, 2, 1, 1, 0, 1, 1, 4)$ would result with each procedure, respectively.

(3) Regarding the minimum value, considering that the smallest non-zero value of each block to be transformed, $w_{\min}$, represents the first non-zero value step in the measurement scale, in this case with the scale of 0–4, it corresponds to the influence value 1. If $w_{\min}$ is the unit value, the ratio w_i/w_{min} converted to an integer is the value v_i sought, with the restriction $w_i/w_{min} > 4 \rightarrow v_i = 4$. This conversion can be carried out with the three procedures indicated above (floor, ceil, round). With the vector w (1), the vectors $v_5 = (0, 1, 4, 3, 4, 2, 4, 4, 4)$, $v_6 = (0, 1, 4, 4, 4, 3, 4, 4, 4)$ y $v_7 = (0, 1, 4, 3, 4, 3, 4, 4, 4)$ are obtained respectively.

(4) Regarding the distribution of values as a whole, for example, an allocation of categories based on quartiles of values can be considered.

In this study, the latter option could be realized given that the resulting influence values of the ANP are known, but it is difficult to imagine the DMs considering the full set of influence values and their distribution, in order to make his assessment of categories. Instead, it seems more plausible to think that it will be easier to take as a reference that which influences more or that which influences less, and to make proportional judgements with respect to that reference. Therefore, option 4 is discarded. In this conversion, either the minimum or the maximum value will be considered as the reference. If the least influential element is assigned a value of 1 (LI), regardless of how the whole number is obtained, it is easy to lose proportionality because ratios greater than 4 are assigned 4 (VSI). It is common to have influence ratios in w much higher than 4. This can be seen in the cases studied in this article and should not be surprising because in the paired comparison matrices the Saaty scale is used, with influence ratios up to 9. Despite this lack of proportionality, using the minimum value as the reference value to obtain the vectors v sought have not been ruled out, for the following reasons: (1) because this guarantees that all the elements that influence the w vectors, continue to influence the v vectors; (2) because based on our experience, it fits with the way many DMs make their judgements: "it does influence, a little, but it influences" and they would assign it the lowest value; (3) because it follows Saaty's

recommendation to look for that unitary element, the smallest among those to be valued, and use it as a pattern to compare the rest with it. In the case of using the minimum, with ceil() the proportionality as a whole would be lost even more. With floor() the value obtained will be "is at least x times", greater than the minimum considered the unity. For this work, the round() function have been chosen, because this method appears to be the most neutral or balanced of the three.

As for considering the most influential element as the reference, looking at the three options in 2), the conversion with floor() is considered too downward, since influences are being compared with respect to the most influential element. With this, only the upward transformation, ceiling, and rounding remains to be considered. All seem reasonable. The upward transformation is acceptable precisely because the comparison is being made with respect to the highest influence, and it allows relatively low influence values to still reach the LI category with a value 1. Although the rounding conversion is more balanced and, in our opinion, more neutral. Thus, three options are left on the table considering the maximum: 1), 2b) and 2c). Of these three options, the first option is considered to be best, not so much for numerical reasons, but conceptually. This is best explained with an example: taking the 4th component of the vector w (1), method 1) gives $0.05/0.38 = 0.1316$, i.e., being 13.16% of the most influential element, it has the value 0 (NI). With methods 2b and 2c), $4 \times (0.05/0.38) = 0.5263$ is obtained, and both methods assign the value 1 (LI). The fundamental difference is that method 1) gives the actual ratio of the influences between w_4 and w_{max}, while methods 2b) and 2c) amplify this influence with the bottom of the measurement scale, which implies assigning the category by taking a part of the value (and information) that not only is the ratio of its influences. In other words, 1) assigns the scale value in proportion to 1, while 2b) and 2c) assign it in proportion to 4.

With all this, four matrices have been obtained with influence values on the 0–4 scale, equivalent to the original ANP matrices. Each one obtained with a different method that will henceforth be used to name the four models: Max Single cluster, Max Multiple clusters, Min Single cluster and Min Multiple clusters. Each of them is a different approximation to the matrix that the DMs would have answered if they followed each of the explained strategies to issue their assessments using the scale in Table 1.

For practical purposes, using the most influential item or the least influential item modifies the procedure used by the DMs to answer the above questions. In the Max models, the DMs must first identify the most influential item (element/cluster) of those they have to evaluate, and assign it the highest level of the scale, the rest will be assigned their level of influence proportionally to this most influential one. In the Min models, the DMs must first identify the non-influential elements, then identify the least influential of the influential elements, and finally assign their level of influence to the remaining items in proportion to the least influential.

3.3. Limitations

We make explicit the following limitations of our work:

- We do not have DM to make judgments for the three questionnaires. Therefore, we cannot compare the results calculated from them. We are calculating through a systematic procedure, what level of influence intensity is appropriate to consider for the influence values of the ANP matrices, and assuming that the responses of the DMs would be those calculated levels. This does not have to be so, because experts are not usually perfectly coherent, proportional or transitive in their answers.
- The number of categories in the scale. The proposal under study [3] utilizes a 5-point scale as presented in Table 1, with the following categories: no influence, low influence, medium influence, strong influence, and very strong influence. Although, as already indicated, other scales with different numbers of categories can be used.
- The numerical values of influence associated with the categories. It is important to note that the proposed numerical values for the scale (0, 1, 2, 3, 4) are not necessarily obligatory, as alternative real numbers may also be assigned. Furthermore, the scale need not necessarily be linear. Our prior experience with ratings in decision-making scenarios has demonstrated that non-linear scales are not uncommon. However, we will adhere to the specific proposal made by the authors of the study, despite the possibility of alternative numerical values or non-linear scales.
- The number of cases studied. With 17 cases, it is evident that the answers to the research questions cannot be generalized to the universe of all possible ANP models.

With these limitations, we will not be able to answer the RQs with the premise "While using a measurement scale to directly measure influence in ANP". Rather, we will approach RQs by "Considering the proposed transformation of influence values of ANP matrices derived from pairwise matrices; considering the proposed scale of five degrees and considering the 17 cases found in the literature". RQ1 being worded in a more measured way: “Which results are more similar to those obtained with the original ANP, considering single cluster or multiple clusters?

3.4. Normalize by sum

Matrices with influences in 5-categories scale, must be normalized to sum 1 in their influence groupings, to do so:

(1) 5-categories Unweighted matrix: Each column values are divided by the sum values of same column cluster.
(2) 5-categories Cluster and 5-categories Weighted matrices: Each column is divided by the sum of the column.

The normalized weighted matrices of the single cluster models are ready to calculate the limit priorities. Calculating the weighted matrices considering multiple clusters is the only step left.

3.5. Calculate weighted matrix of multiple clusters model

By multiplying each value of the normalized 5-categories unweighted matrix by the cluster’s influence value in the normalized 5-categories cluster matrix, the weighted matrix of multiple clusters model is obtained. Finally, this matrix must be normalized by column sum to ensure stochasticity.

3.6. Compute limit matrices and priorities

All priorities, min and max single cluster models and min and max multiple clusters models, are calculate from each weighted matrix.

3.7. Compare of the results with original ANP results

Finally, the four priority results obtained have been compared with the published prioritization obtained by the conventional ANP method. Different indicators have been proposed to measure how similar two vectors are, some based on rank, and others on values. Spearman’s rank correlation coefficient and the mean squared error (MSE) are the indicators that the authors use in their new proposal to compare their results with ANP. In this article, the following indicators have been additionally calculated: Saaty’s Compatibility Index based on the Hadamard product [11] and the Garuti’s Compatibility index [55,56] both proposed in the AHP/ANP context; Cosine similarity as additional measure of similarity between two nonzero vectors; and Kendall correlation coefficient, Tau-b. With these indicators, the aim is to measure to

what extent the priority vectors of the four models (min and Max Single cluster and min and Max Multiple clusters), are similar to the priority vector obtained with ANP. With Saaty's, Garuti's y Cosine's indicators, the priority vectors are compared as a whole; with the Spearman and Kendall indicators, it is verified the extent to which the elements of each vector are in the same order of priority and are therefore in the same ranking. Finally, the MSE is used to evaluate the differences in the values of the priorities obtained.

4. Cases study and results

The size of the cases (number of elements and clusters) is shown in Table 3, as well as the number of questions to be answered using the original ANP and the new method using a single cluster or multiple clusters. This section presents Case 1 [38] of the 17 case studies to which the evaluated proposals has been applied. Only the resulting 0–4 scale matrices are shown. All cases with all matrices are included in the Supplementary material of the article.

4.1. Multiple clusters models

(1) Transform the original Unweighted matrix to 5-categories scale values. From the original ANP Unweighted two 0–4 scale Unweighted matrices were obtained using the procedures previously explained:
   (a) Using max value as the reference. Each value is divided by the maximum cluster column value. For example, in CL3 rows (E6 to E10) on E1 column, influence values are: 0.0678, 0.0502, 0.1443, 0.6199, 0.1178. Dividing by the maximum (0.6199), the values obtained are: 0.1094, 0.0809, 0.2328, 1, 0.19. Converting these values with Table 2, the resulting influence values in 0–4 scale are: 0, 0, 1, 4, 0. The resulting 0–4 scale unweighted matrix using max value as reference is shown in Table 4.
   (b) Using min value as reference. First, each value is divided by minimum cluster column values not equal to 0. For example, in CL3 rows (E6 to E10) on E5 column, influence values are 0.11343, 0.05239, 0.28393, 0.55025, 0. The minimum non-0 value is 0.05239. Dividing by this reference value, the following values are obtained: 2.165, 1, 5.42, 10.503, 0. Secondly, to obtain the values on the 0–4 scale, they are converted in the following way: values greater than 4 are replaced by 4, values 0 are maintained as 0, and the rest are rounded to integer numbers. In our example, the resulting values are: 2,1,4,4,0 The resulting 0–4 scale unweighted matrix using min value as reference is shown in Table 5.

(2) Transform the original Cluster matrix to 0–4 scale values. Applying the two procedures of the previous point, in this case, by columns of the Cluster matrix, the Cluster matrices in the 0–4 scale are obtained. The resulting 0–4 scale Cluster matrices are shown in Tables 6 and 7. For example, the first column CL1 values (0, 0, 0.15762, 0.1603, 0.02636, 0.06955, 0.04297, 0.5432) are divided by 0.5432 or 0.02636 depending on the case, resulting (0, 0, 0.2902, 0.2951, 0.0485, 0.128, 0.0791, 1) and (0, 0, 5.980, 6.081, 1, 2.638, 1.630, 20.607) and then finally (0, 0, 1, 1, 0, 0, 0, 4) and (0, 0, 4, 4, 1, 3, 2, 4).

(3) Normalize the 0–4 scale Unweighted matrix. For example, in the 0–4 Unweighted matrix max model, CL5 rows (E12 to E14) on E1 column, influence values are: 1, 4, 4. Dividing by their sum, 9, the proportional influence of CL5's elements on E1 are: 0.1111, 0.4444, 0.4444. In min model, this positions' values are: 1, 2, 3. Dividing by 6, the proportional influence values are: 0.1667, 0.3333, 0.5.

(4) Normalize the 0–4 scale Cluster matrix. For example, in the 0–4 Cluster matrix max model CL7 column, the influence values are: 0, 0, 4, 4, 1, 1, 0, 0. Dividing by their sum, 10, the proportional influence on CL7 are: 0, 0, 0.4, 0.4, 0.1, 0.1, 0, 0. In the 0–4 Cluster matrix min model CL7 column, the influence values are: 1, 1, 4, 4, 1, 1, 0, 0. Dividing by their sum, 12, the proportional influence on CL7 are: 0.0833, 0.0833, 0.333, 0.333, 0.0833, 0.0833.

(5) Calculate the Weighted matrix from the normalized 0–4 scale Unweighted and Cluster matrices. This matrix is calculated by following these two steps:
   (a) Multiply the unweighted and cluster normalized matrices. Each value of the normalized unweighted matrix from step 3, is multiplied by the influence value between clusters of the normalized cluster matrix from step 4. For example, in the max model, in the non-zero values of the column of the element E3 of cluster CL2: 1, of element E1 is multiplied by 0 (CL1, CL2); (0.5714,0.2857,0.1429) of elements E2, E4 and E5, are multiplied by 0.2222 (CL2, CL2); (0.8,0.2) of E15 and E18, are multiplied by 0.1111 (CL6, CL2); 1 of element E18 is multiplied by 0 (CL7, CL2). This way, the obtained vector of column E3 is: (0, 0.127, 0, 0.0635, 0.0317, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.0889, 0, 0, 0.0222, 0, 0, 0).

Table 3
Cases' number of elements, clusters and questions by model.

Case	Elements	Clusters	No. questions							
			ANP model				Single cluster model	Multiple clusters model		
			Elements' relationships	Elements' influence	Clusters' influence	Total	About elements	About elements	About clusters	Total
01	22	8	484	268	159	911	484	484	54	538
02	19	8	361	181	148	690	361	361	52	413
03	36	6	1296	483	154	1933	1296	1296	46	1342
04	15	3	225	182	9	416	225	225	9	234
05	18	5	324	603	24	951	324	324	18	342
06	13	4	169	39	24	232	169	169	16	185
07	21	4	441	542	13	996	441	441	12	453
08	13	4	169	158	15	342	169	169	13	182
09	18	5	324	147	22	493	324	324	17	341
10	18	6	324	159	42	525	324	324	25	349
11	13	5	169	123	21	313	169	169	17	186
12	18	4	324	109	14	447	324	324	12	336
13	19	4	361	218	11	590	361	361	11	372
14	21	4	441	453	16	910	441	441	13	454
15	13	5	169	60	6	235	169	169	4	173
16	13	4	169	27	6	202	169	169	8	177
17	20	10	400	80	59	539	400	400	26	426

Table 4
Case 01. Max Multiple Clusters Model.0–4 Unweighted Supermatrix.

		CL1	CL2				CL3					CL4	CL5			CL6					CL7	CL8
		E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16	E17	E18	E19	E20	E21
CL1	E1	0	0	4	0	0	0	0	0	0	0	4	4	4	0	4	4	4	4	4	4	4
CL2	E2	0	0	4	4	4	4	4	4	0	0	0	4	4	1	4	4	4	4	3	4	4
	E3	0	1	0	0	2	0	0	0	0	0	0	0	0	0	3	0	0	4	4	4	4
	E4	0	4	2	0	1	4	3	4	4	4	4	4	1	1	2	1	1	4	1	4	4
	E5	0	1	1	0	0	0	0	0	4	0	4	0	1	4	2	1	4	4	0	4	4
CL3	E6	0	0	0	0	1	0	1	2	3	1	1	0	0	1	1	0	1	0	0	0	0
	E7	0	0	0	0	0	0	0	1	3	1	1	1	1	4	3	4	4	4	3	4	1
	E8	1	1	0	0	2	4	4	0	4	4	2	1	2	4	4	4	4	4	4	4	4
	E9	4	4	0	0	4	4	4	4	0	0	4	4	4	0	4	0	0	0	1	4	3
	E10	0	2	0	0	0	4	4	4	0	0	2	0	2	0	0	4	2	0	2	0	2
CL4	E11	4	4	0	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	4
CL5	E12	1	1	0	0	0	2	1	0	0	0	0	0	1	4	0	0	0	0	4	4	1
	E13	4	4	0	0	4	0	0	0	0	4	0	1	0	1	4	4	4	0	4	4	0
	E14	4	4	0	0	4	4	4	0	0	0	4	4	4	0	0	0	0	0	0	4	4
CL6	E15	1	1	4	0	0	1	0	0	0	0	0	4	0	0	0	0	0	0	4	4	0
	E16	0	0	0	0	0	0	4	0	0	4	4	0	0	0	0	0	4	4	4	3	0
	E17	0	0	0	0	4	0	4	0	0	4	4	0	0	0	0	0	0	0	4	3	0
	E18	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	4	1	0
	E19	4	4	0	0	0	4	0	0	0	0	0	3	0	0	4	0	0	0	0	0	4
CL7	E20	4	4	4	0	4	4	4	0	0	0	0	4	4	0	4	0	0	0	0	0	4
CL8	E21	4	0	0	0	0	4	0	0	0	0	0	4	0	0	0	4	0	0	0	0	0

Table 5
Case 01. Min Multiple Clusters Model.0–4 Unweighted Supermatrix.

		CL1	CL2				CL3					CL4	CL5			CL6					CL7	CL8
		E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16	E17	E18	E19	E20	E21
CL1	E1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	1	1	1	1	1	1	1
CL2	E2	0	0	4	1	3	1	1	1	0	0	0	1	3	1	2	4	3	1	3	1	1
	E3	0	1	0	0	1	0	0	0	0	0	0	0	0	0	2	0	0	1	3	1	1
	E4	0	4	2	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	E5	0	1	1	0	0	0	0	0	1	0	1	0	1	3	1	1	2	1	0	1	1
CL3	E6	1	1	0	0	2	0	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1
	E7	1	1	0	0	1	1	0	1	1	2	1	3	2	4	2	4	3	4	4	4	3
	E8	3	4	0	0	4	4	3	0	2	4	2	3	3	4	3	4	3	4	4	4	4
	E9	4	4	0	0	4	4	3	4	0	0	3	4	4	0	3	1	0	0	4	4	4
	E10	2	4	0	0	0	4	3	4	0	0	2	0	3	0	0	4	2	0	4	0	4
CL4	E11	1	1	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
CL5	E12	1	1	0	0	0	2	1	0	0	0	0	0	1	3	0	0	0	0	1	1	1
	E13	2	4	0	0	1	1	0	0	0	1	0	1	0	1	1	1	1	0	1	1	1
	E14	3	3	0	0	1	4	4	0	0	0	1	3	4	0	0	0	0	0	0	1	4
CL6	E15	4	3	3	0	0	4	0	0	0	0	0	4	0	0	0	0	0	0	1	4	4
	E16	1	1	0	0	0	1	1	0	0	1	1	1	0	0	0	0	1	1	1	3	1
	E17	2	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	3	1
	E18	4	1	1	0	0	2	0	0	0	0	0	0	0	0	1	0	0	0	1	1	2
	E19	4	4	0	0	0	4	0	0	0	0	0	4	0	0	4	0	0	0	0	0	4
CL7	E20	1	1	1	0	1	1	1	0	0	0	0	1	1	0	1	0	0	0	0	0	1
CL8	E21	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0

Table 6
Case 01. Max Multiple Clusters Model. 0–4 Cluster Matrix.

	CL1	CL2	CL3	CL4	CL5	CL6	CL7	CL8
CL1	0	0	0	0	0	2	0	3
CL2	0	2	0	1	1	0	0	2
CL3	1	4	4	4	4	4	4	4
CL4	1	2	3	0	2	2	4	4
CL5	0	0	0	0	0	1	1	2
CL6	0	1	0	0	2	0	1	2
CL7	0	0	0	0	0	0	0	2
CL8	4	0	0	0	3	0	0	0

Table 7
Case 01. Min Multiple Clusters Model. 0–4 Cluster Matrix.

	CL1	CL2	CL3	CL4	CL5	CL6	CL7	CL8
CL1	0	1	0	1	2	4	1	2
CL2	0	4	2	3	2	4	1	1
CL3	4	4	4	4	4	4	4	2
CL4	4	4	4	0	4	4	4	2
CL5	1	1	2	2	1	4	1	1
CL6	3	3	2	2	4	3	1	1
CL7	2	1	1	0	1	1	0	1
CL8	4	0	2	0	4	3	0	0

(b) Normalize by column sum. The values of the matrix obtained in the previous point are divided by the sum of its column. This ensures the stochasticity of the weighted matrix. For example, in the max model, the values in column E3 are divided by 0.3333, resulting (0, 0.381, 0, 0.1905, 0.0952, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.2667, 0, 0, 0.0667, 0, 0, 0).

(6) Compute the limit matrix and priorities. The limit matrix and the priorities of the elements are calculated from the weighted matrix. These are the priorities considering several clusters and

using scale of influence measures instead of pairwise comparison matrices.

4.2. Single clusters models

(1) Transform the Original Weighted matrix to 0–4 scale values. According to the aforementioned procedures, as all elements are considered in a single cluster, they are divided by the maximum or minimum of its column. For example, in max model CL1 column, those values are divided by 0.543, resulting (0, 0, 0, 0, 0, 0.0197, 0.0146, 0.0419, 0.1799, 0.0342, 0.2951, 0.0082, 0.0188, 0.0215, 0.0244, 0.0037, 0.0062, 0.0138, 0.08, 0.0791, 1). The resulting 0–4 scale Weighted Max Single cluster model matrix is shown in Table 8. In the min model, CL1 column values are divided by 0.002, resulting (0, 0, 0, 0, 0, 5.5, 4, 11.5, 49, 9.5, 80, 2, 5, 6, 6.5, 1, 1.5, 3.5, 21.5, 21.5, 271.5). The resulting 0–4 scale Weighted Min Single cluster model matrix is shown in Table 9.
(2) Compute the Normalized 0–4 Weighted matrix. Normalizing by column sum, the Weighted matrices are obtained.
(3) Compute limit matrix and priorities. The limit matrices and element priorities are calculated from the weighted matrices. These are the priorities considering a single cluster model and using scale of influence measures instead of pairwise comparison matrices.

4.3. Compare results with ANP

The results obtained for each case have been compared with the original ANP results. As mentioned above, compatibility/similarity indicators, as well as correlation and error indicators have been calculated: Saaty's index, Garuti's index, Cosine similarity, ρ Spearman, τ Kendall and SME (Tables 10–14). The calculation of Saaty's and Garuti's indexes presents problems with null values in the components of the vectors. As in this case, several 0′s were present in the priorities, the null values have been replaced by 0.000001 for the calculation of the Saaty's index and the Garuti's index. In the similarity and correlation indicators, values greater than or equal to 0.9 are highlighted in green. In the case of the Garuti's index, its author indicates that a value in the range 85–89.9% indicates "High compatibility (almost compatible vectors)" [56], so these values have also been highlighted, but in yellow. Indicator statistics have also been calculated for each of the four models (Table 15). The results for each indicator are discussed below:

(1) Saaty compatibility index (Table 10). According to the Saaty's compatibility index, out of the 17 cases studied, only 3–9 cases are compatible with the ANP results according to the model. If the most influential element is used as the reference, there is no obvious conclusion about which is the best approach, to use single or multiple clusters. On the other hand, if the minimum influence element is used, a greater number of results more similar to the original ANP are obtained with the multi-cluster model. If a single cluster model is considered, the Saaty index results are much closer to unity if the minimum influence element is used as the reference, although the number of compatible results is similar. With multiple clusters, it is also much more results more similar to the original ANP are also obtained using the minimum influence element as a reference. These results indicate that Saaty's insistence on using the smallest element to make comparisons and judgements, is corroborated by his indicator.
(2) Garuti's compatibility index (Table 10). The number of cases with compatible results with those obtained with ANP are 2 to 4. If almost compatible values are included, the count is between 5 and 8. With this indicator it is not so clear which strategy is preferable, given that using the model with multiple clusters the number of cases with results more similar to the results with ANP is slightly higher, but looking at the cases individually, in some particular cases the similarity has worsened.
(3) Cosine similarity index (Table 11). According to this indicator, practically all models and cases are similar to the results with ANP (12 to 17 cases). Considering the number of cases with a result greater than 0.9, whether the most influential or the least influential value is used, the number of results similar to the original ones with ANP using multiple clusters is higher. If the single cluster model is chosen, it is slightly greater the number of results similar to the originals with ANP using the most influential element as reference. If the model with multiple clusters is used, it is almost a tie, slightly in favor of using the least influential or the largest element as the reference. The fact that such overwhelming results are obtained for similarity between results, when other indicators of compatibility or correlation are not so clear, might indicate that the Cosine similarity index could not be the most useful indicator for discriminating results.

Table 8
Case 01. Max single cluster model. 0–4 weighted matrix.

		CL1	CL2				CL3					CL4	CL5			CL6					CL7	CL8
		E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16	E17	E18	E19	E20	E21
CL1	E1	0	0	1	0	0	0	0	0	0	0	1	1	2	0	4	4	4	4	4	0	3
CL2	E2	0	0	4	4	2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	E3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	E4	0	2	2	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
	E5	0	0	1	0	0	0	0	0	0	0	2	0	0	1	0	0	0	0	0	0	0
CL3	E6	0	0	0	0	0	0	0	1	2	1	1	0	0	1	1	0	1	0	0	0	0
	E7	0	0	0	0	0	0	0	0	2	1	1	1	1	4	2	2	3	4	2	1	0
	E8	0	1	0	0	2	2	2	0	3	4	2	1	2	4	3	3	3	4	3	1	1
	E9	0	4	0	0	4	2	2	2	0	0	4	3	4	0	3	0	0	0	1	1	1
	E10	0	1	0	0	0	2	2	3	0	0	2	0	2	0	0	2	1	0	1	0	0
CL4	E11	1	4	0	0	4	4	4	4	4	4	0	2	4	4	4	4	4	4	4	4	4
CL5	E12	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
	E13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	1	0	0
	E14	0	0	0	0	0	0	0	0	0	0	2	0	1	0	0	0	0	0	0	0	1
CL6	E15	0	0	3	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	E16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0
	E17	0	0	0	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	E18	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	E19	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
CL7	E20	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	2
CL8	E21	4	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0

Table 9
Case 01. Min single cluster model. 0–4 weighted matrix.

		CL1	CL2				CL3					CL4	CL5			CL6					CL7	CL8
		E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16	E17	E18	E19	E20	E21
CL1	E1	0	0	1	0	0	0	0	0	0	0	1	4	4	0	4	4	4	4	4	4	4
CL2	E2	0	0	4	1	4	4	2	1	0	0	0	4	4	1	3	4	3	1	3	2	4
	E3	0	4	0	0	2	0	0	0	0	0	0	0	0	0	2	0	0	1	3	2	4
	E4	0	4	2	0	2	4	2	1	1	2	1	4	2	2	2	1	1	1	1	2	4
	E5	0	4	1	0	0	0	0	0	1	0	1	0	2	4	1	1	2	1	0	2	4
CL3	E6	4	4	0	0	2	0	4	3	4	2	1	2	3	3	4	3	4	2	2	3	3
	E7	4	4	0	0	1	4	0	2	4	4	1	4	4	4	4	4	4	4	4	4	4
	E8	4	4	0	0	4	4	4	0	4	4	2	4	4	4	4	4	4	4	4	4	4
	E9	4	4	0	0	4	4	4	4	0	0	4	4	4	0	4	3	0	0	4	4	4
	E10	4	4	0	0	0	4	4	4	0	0	2	0	4	0	0	4	4	0	4	0	4
CL4	E11	4	4	0	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	4
CL5	E12	2	1	0	0	0	4	1	0	0	0	0	0	1	3	0	0	0	0	4	4	4
	E13	4	4	0	0	1	4	0	0	0	2	0	1	0	1	4	4	4	0	4	4	4
	E14	4	3	0	0	1	4	4	0	0	0	2	3	4	0	0	0	0	0	0	4	4
CL6	E15	4	3	3	0	0	4	0	0	0	0	0	4	0	0	0	0	0	0	1	4	4
	E16	1	1	0	0	0	1	3	0	0	1	1	1	0	0	0	0	4	3	1	3	1
	E17	2	1	0	0	4	1	3	0	0	1	1	1	0	0	0	0	0	0	1	3	2
	E18	4	1	1	0	0	2	0	0	0	0	0	0	0	0	1	0	0	0	1	1	2
	E19	4	4	0	0	0	4	0	0	0	0	0	4	0	0	4	0	0	0	0	0	4
CL7	E20	4	4	1	0	2	4	2	0	0	0	0	3	4	0	2	0	0	0	0	0	4
CL8	E21	4	0	0	0	0	4	0	0	0	0	0	4	0	0	0	4	0	0	0	0	0

Table 10
Saaty and Garuti Compatibility Indices.

	Saaty Index				Garuti Index			
Case	Max Single cluster	Max Multiple clusters	Min Single cluster	Min Multiple clusters	Max Single cluster	Max Multiple clusters	Min Single cluster	Min Multiple clusters
01	176.663	117.083	1.57	1.48	0.825	0.716	0.674	0.634
02	1.187	1.925	1.236	1.102	0.897	0.735	0.799	0.749
03	1051.398	535.219	1.422	1.339	0.431	0.723	0.688	0.7
04	1602.995	5767.463	1.547	1.076	0.852	0.831	0.586	0.829
05	1.208	1.011	1.064	1.006	0.775	0.923	0.836	0.927
06	1.007	1.003	1.004	1.004	0.954	0.959	0.943	0.953
07	1.132	9917.256	1.176	1.181	0.804	0.465	0.715	0.699
08	1.039	1.012	1.032	1.126	0.919	0.935	0.866	0.8
09	524.676	1.041	1.302	1.069	0.844	0.901	0.699	0.826
10	595.997	700.766	1.425	1.142	0.745	0.88	0.642	0.74
11	5268.329	1.699	1.305	1.032	0.645	0.768	0.581	0.841
12	5987.334	5987.334	1.4	1.038	0.51	0.51	0.879	0.882
13	2521.683	2319.058	1.354	1.115	0.8	0.751	0.713	0.83
14	12,245.533	1.013	1.02	1.004	0.327	0.899	0.883	0.95
15	1.028	1.122	1.008	1.035	0.897	0.811	0.947	0.859
16	1904.932	375.414	1.647	1.134	0.408	0.867	0.608	0.778
17	2931.579	1663.669	1.158	1.092	0.775	0.868	0.803	0.836

Table 11
Cosine Similarity Index.

Case	Max Single cluster	Max Multiple clusters	Min Single cluster	Min Multiple clusters
01	0.9899	0.975	0.9151	0.917
02	0.9925	0.9513	0.9674	0.9453
03	0.6955	0.9502	0.9053	0.9231
04	0.9864	0.9812	0.8473	0.9794
05	0.9669	0.9968	0.9646	0.9967
06	0.999	0.9989	0.9974	0.9979
07	0.9862	0.8966	0.8828	0.9635
08	0.9942	0.9972	0.9889	0.962
09	0.9806	0.9922	0.9002	0.97
10	0.9658	0.9926	0.8335	0.9375
11	0.9599	0.9813	0.8738	0.9858
12	0.9349	0.9349	0.9953	0.9953
13	0.9791	0.9554	0.8822	0.9798
14	0.7871	0.9898	0.9839	0.9983
15	0.992	0.9615	0.9978	0.9862
16	0.5521	0.9956	0.9332	0.9874
17	0.9802	0.9942	0.9683	0.9878

(4) Rho Spearman (Table 12). According to Spearman's correlation, between 7 and 12 cases have a high correlation with the ANP results. It is worth noting that with the Min Single cluster model, the results of case 04 should not be considered different from chance as it has a p-value of 0.23. Taking the value of greatest influence as the reference, it seems to be appropriate to use the single cluster model, not so much because of the number of cases with a very high correlation (12vs. 11) but because the rho values are greater in general. On the other hand, if the least influential value of each block is taken as the reference, higher correlation values are obtained when using the model with multiple clusters. Using a single cluster, higher correlation values are obtained using the most influential element as the reference, while with multiple clusters, the results are very similar when using the most influential or the least influential element as the reference.

(5) Tau-b Kendall (Table 13). Analyzing the Kendall correlation results, the first thing to highlight is the large difference in correlations with Spearman. With the Max Single cluster models, there are only between 1 and 6 cases with a high correlation with the ANP results. It does concur in discarding the results obtained with

Table 12
Spearman Correlation.

Case	Max Single cluster		Max Multiple clusters		Min Single cluster		Min Multiple clusters	
	rho	*p*-value	rho	*p*-value	rho	*p*-value	rho	*p*-value
01	0.9932	3.360E-19	0.7584	6.763E-05	0.8896	6.850E-08	0.874	2.255E-07
02	0.9596	8.502E-11	0.9193	2.679E-08	0.8596	2.397E-06	0.8579	2.648E-06
03	0.8679	7.271E-12	0.9189	2.727E-15	0.6723	7.091E-06	0.7786	2.251E-08
04	0.9536	3.792E-08	0.9262	7.236E-07	0.3286	2.318E-01	0.9464	9.448E-08
05	0.9648	1.069E-10	0.9896	6.473E-15	0.8984	4.119E-07	0.9855	9.429E-14
06	0.9725	2.619E-08	0.9835	1.611E-09	0.989	1.751E-10	0.9835	1.611E-09
07	0.9701	3.812E-13	0.7334	1.551E-04	0.8688	3.241E-07	0.9662	1.203E-12
08	0.967	7.064E-08	0.989	1.751E-10	0.9176	9.906E-06	0.78	1.660E-03
09	0.848	8.798E-06	0.8579	5.305E-06	0.5993	8.580E-03	0.8848	1.083E-06
10	0.9525	1.117E-09	0.9814	6.726E-13	0.5232	2.587E-02	0.8576	5.389E-06
11	0.9269	5.206E-06	0.9505	6.364E-07	0.967	7.064E-08	0.989	1.751E-10
12	0.5455	1.922E-02	0.5455	1.922E-02	0.9897	6.315E-15	0.9917	1.066E-15
13	0.9341	5.049E-09	0.8423	6.068E-06	0.8298	1.108E-05	0.9421	1.722E-09
14	0.6978	4.371E-04	0.9634	2.547E-12	0.9851	5.613E-16	0.9817	3.756E-15
15	0.9491	7.433E-07	0.685	9.777E-03	0.9298	4.182E-06	0.9298	4.182E-06
16	0.8884	4.955E-05	0.9461	1.015E-06	0.8143	7.017E-04	0.989	1.751E-10
17	0.9639	8.742E-12	0.9754	2.848E-13	0.963	1.074E-11	0.945	3.597E-10

Table 13
Kendall Correlation.

Case	Max Single cluster		Max Multiple clusters		Min Single cluster		Min Multiple clusters	
	tau-b	*p*-value	tau-b	*p*-value	tau-b	*p*-value	tau-b	*p*-value
01	0.9594	1.261E-09	0.6095	1.110E-04	0.7429	2.468E-06	0.7524	1.832E-06
02	0.848	3.918E-07	0.8012	1.643E-06	0.6842	4.253E-05	0.6491	1.030E-04
03	0.6828	6.469E-09	0.7815	2.396E-11	0.5079	1.308E-05	0.5873	4.662E-07
04	0.8857	4.178E-06	0.7945	5.085E-05	0.219	2.550E-01	0.8476	1.061E-05
05	0.88	5.004E-07	0.9467	6.412E-08	0.8133	3.394E-06	0.9333	9.780E-08
06	0.8974	1.949E-05	0.9231	1.120E-05	0.9487	6.342E-06	0.9231	1.120E-05
07	0.8762	2.757E-08	0.6136	4.454E-04	0.7333	3.314E-06	0.8762	2.757E-08
08	0.8718	3.345E-05	0.9487	6.342E-06	0.7949	1.552E-04	0.6672	1.930E-03
09	0.6954	6.647E-05	0.7261	2.973E-05	0.4835	5.554E-03	0.7657	1.066E-05
10	0.8431	1.028E-06	0.9085	1.402E-07	0.3464	4.469E-02	0.6993	5.058E-05
11	0.8239	1.571E-04	0.8718	3.345E-05	0.9231	1.120E-05	0.9487	6.342E-06
12	0.4588	2.451E-02	0.4588	2.451E-02	0.9474	4.703E-08	0.9605	3.058E-08
13	0.8343	1.055E-06	0.6727	7.053E-05	0.6842	4.253E-05	0.8129	1.156E-06
14	0.611	6.267E-04	0.8686	6.082E-08	0.9187	6.510E-09	0.9076	1.515E-08
15	0.8645	4.218E-05	0.529	1.221E-02	0.8129	1.178E-04	0.8387	7.098E-05
16	0.7849	3.051E-04	0.8687	4.865E-05	0.7097	7.745E-04	0.9487	6.342E-06
17	0.8953	1.735E-07	0.9214	2.742E-08	0.871	1.279E-07	0.8236	5.332E-07

Table 14
SME.

Case	Max Single cluster	Max Multiple clusters	Min Single cluster	Min Multiple clusters
01	1.859E-04	5.836E-04	8.655E-04	7.980E-04
02	6.057E-05	5.203E-04	2.581E-04	4.184E-04
03	1.567E-03	1.739E-04	2.090E-04	1.715E-04
04	2.851E-04	4.010E-04	2.102E-03	3.144E-04
05	4.107E-04	2.915E-05	3.071E-04	3.007E-05
06	1.572E-05	1.717E-05	4.303E-05	3.192E-05
07	2.938E-04	3.080E-03	1.050E-03	6.380E-04
08	8.661E-05	4.388E-05	1.672E-04	5.481E-04
09	1.542E-04	5.950E-05	7.686E-04	2.255E-04
10	5.575E-04	9.992E-05	1.450E-03	5.831E-04
11	2.061E-03	8.813E-04	2.697E-03	3.799E-04
12	5.435E-03	5.435E-03	1.590E-04	1.562E-04
13	3.344E-04	5.906E-04	1.149E-03	2.299E-04
14	7.176E-03	7.144E-05	1.242E-04	1.400E-05
15	1.298E-04	5.532E-04	3.129E-05	1.975E-04
16	2.059E-02	2.843E-04	3.420E-03	8.320E-04
17	4.490E-04	1.302E-04	4.494E-04	1.737E-04

the Min Single cluster model in Case 4. With these results, according to Kendall's tau-b, whether the maximum or minimum value is used as the reference, the models with multiple clusters have turned out to obtain priorities more correlated with the original ANP. Similarly, more correlated results are obtained by using the minimum value as the reference, both in single and multi-cluster models.

(6) MSE (Table 14). Regarding the MSE, the largest error occurs with the Max Single cluster in Case 16, with a value of 2.059E-2, which is excessive. The rest of the MSE values, in general, have exponentials -4, and some with E-3 and E-5.

Below, the strategies have been compared for each case, and for each indicator. For example, for Case 1, according to Saaty's index, which model, single or multiple clusters, generates results more similar to ANP? And if the minimum influence value is used as the reference? In Case 1, both answers are the multiple cluster model, because Saaty's index is closer to 1. If the single cluster model or the multiple cluster models is used, according to Saaty's index, which reference value, maximum or minimum, generates results more similar to ANP? Using the minimum value as a reference generates results more akin to those obtained with ANP in Case 1 for both models.

Overall, according to the Saaty index, in the max models, the ANP with multiple clusters yields more similar results in 11 cases and with single clusters in 5 cases. In the min models, ANP with multiple clusters produces results more similar in 13 cases, and with single clusters in 3 cases. Tables 16 and 17 display the case counts for all indicators and model combinations.

Table 15
Indicators Statistics.

	Max Single cluster	Max Multiple clusters	Min Single cluster	Min Multiple clusters
Saaty's index				
Mean	2048.1012	1611.3581	1.2747	1.1162
Min	1.007	1.003	1.004	1.004
Max	12,245.533	9917.256	1.647	1.48
Garuti's index				
Mean	0.7299	0.7966	0.7566	0.8137
Min	0.327	0.465	0.581	0.634
Max	0.954	0.959	0.947	0.953
Cosine				
Mean	0.926	0.9732	0.9316	0.9714
Min	0.5521	0.8966	0.8335	0.917
Max	0.999	0.9989	0.9978	0.9983
Rho Spearman				
Mean	0.9032	0.8803	0.825	0.9225
Min	0.5455	0.5455	0.3286	0.7786
Max	0.9932	0.9896	0.9897	0.9917
Tau-b Kendall				
Mean	0.8066	0.7791	0.7142	0.8201
Min	0.4588	0.4588	0.219	0.5873
Max	0.9594	0.9487	0.9487	0.9605
SME				
Mean	2.34E-03	7.62E-04	8.97E-04	3.38E-04
Min	1.60E-05	1.70E-05	3.10E-05	1.40E-05
Max	2.06E-02	5.44E-03	3.42E-03	8.32E-04

Regarding the Single cluster vs Multiple clusters comparison, all indicators except Garuti's index show more cases with results more similar to ANP using multiple clusters. In the Min vs Max comparison, discrepancies between indicators and models are more pronounced. With single clusters, the Saatty, Garuti, and SME indices yield results closer to ANP using the minimum value as a reference. However, with the Spearman, Kendall, and Cosine coefficients, the opposite is true. With multiple clusters, the Cosine indicator is the only one in favor of using the maximum value as a reference in the comparisons, while the Garuti and SME indicators show an even distribution (9–8).

Another aim of this work is to analyze which of the four models had the best and worst results for each of the 17 cases and for the statistics in Table 15:

(1) Cases. For example, in Case 1, which model has obtained the best Saaty's index? Min Multiple clusters, so a vote in favor of this model being the best because it gives the best result according to this indicator. Also in Case 1, which model has obtained the worst Saaty's index? Max Single cluster model is the worst and should not be used, having obtained the worst Saaty's index. Repeating this procedure for all cases and indicators yields the results in Table 18.
(2) Indicator Statistics. A similar process is followed as for the cases, but with the three statistics. According to the Saaty's index, which model has the best and which has the worst average value? Min Multiple clusters and Max Single clusters. Repeating the procedure for the three statistics and all the indicators, the results shown in Table 19 are obtained.

Finally, the votes for best and worst model were subtracted to obtain the results of Table 20.

5. Discussion

First of all, it has to be pointed out that the number of cases studied is small, which makes it difficult to make clear-cut statements based on the results obtained. In this sense, we must ask authors who publish case studies with ANP to include all the ANP matrices in their articles, both to be able to reproduce their results and to be able to use their work in other research studies.

Table 16
Single cluster vs multiple clusters. No Cases.

	Max						Min					
	Saaty	Garuti	Spearman	Kendall	Cosine	SME	Saaty	Garuti	Spearman	Kendall	Cosine	SME
Single cluster	5	10	6	6	7	7	3	12	6	5	3	3
Multiple clusters	11	6	10	10	9	9	13	5	10	12	14	14
Tide	1	1	1	1	1	1	1	0	1	0	0	0

Table 17
Min vs Max. No Cases.

	Single cluster						Multiple clusters					
	Saaty	Garuti	Spearman	Kendall	Cosine	SME	Saaty	Garuti	Spearman	Kendall	Cosine	SME
Min	15	10	4	4	5	15	14	9	10	10	6	9
Max	2	7	13	13	12	2	3	8	6	6	11	8
Tide	0	0	0	0	0	0	0	0	1	1	0	0

Table 18
Best vs Worst Models. Cases.

Best	Total	Saaty	Garuti	Spearman	Kendall	Cosine	SME
Max Single cluster	26	1	4	5	6	5	5
Max Multiple clusters	33	3	7	5	5	7	6
Min Single cluster	8	1	1	2	2	1	1
Min Multiple clusters	36	12	5	5	5	4	5
Worst	**Total**	**Saaty**	**Garuti**	**Spearman**	**Kendall**	**Cosine**	**SME**
Max Single cluster	34	11	6	4	4	4	5
Max Multiple clusters	27	6	6	4	5	2	4
Min Single cluster	37	0	6	7	6	10	8
Min Multiple clusters	12	1	2	3	3	2	1

Table 19
Best vs worst models. Indicator statistics.

Best	Total	Saaty	Garuti	Spearman	Kendall	Cosine	SME
Max Single cluster	2	0	0	1	0	1	0
Max Multiple clusters	3	1	1	0	0	1	0
Min Single cluster	0	0	0	0	0	0	0
Min Multiple clusters	13	2	2	2	3	1	3
Worst	**Total**	**Saaty**	**Garuti**	**Spearman**	**Kendall**	**Cosine**	**SME**
Max Single cluster	9	3	2	0	0	2	2
Max Multiple clusters	2	0	0	1	1	0	0
Min Single cluster	8	0	1	2	3	1	1
Min Multiple clusters	0	0	0	0	0	0	0

Table 20
Best – worst models. Global result.

	Cases	Statistics	Total
Max Single cluster	−8	−7	−15
Max Multiple clusters	6	1	7
Min Single cluster	−29	−8	−37
Min Multiple clusters	24	13	37

When this study was set up, the expectation was that the results were more clearly favorable to using multiple clusters, because it was assumed that proportionality would be more maintained by having two influence values combined instead of just one. But this has not always been the case, with some cases even worsening the results. On the other hand, it needs to be emphasized that the indicators have not given uniform results, so that when their values were examined, one could justify any conclusion by keeping the indicators that were in favor of the assertions made. For this reason, all the indicators calculated have been included and they will be used in the subsequent analyzes, so that the reader can make a critical analysis of them.

Regarding the main research question, RQ1, we can answer after this study that it is not indifferent whether clusters are used or not, because the results between the models with clustering and without, are very different from each other, but we cannot say that one model is always more suitable, because there are cases in which the single cluster model offers a result more similar to the ANP result and in other cases the result of the model with multiple clusters is more similar. Moreover, these cases change depending on the similarity indicator considered.

Concerning RQ2) How much does the result change when considering multiple groups versus a single group? Although there are contradictory results between indicators and cases, based on Table 15, we have prepared Table 21, which shows how much the mean values of the indicators improve (shaded in green) or worsen (shaded in red). For example, the mean value of the Saaty's index goes from 2048.1012 with Max Single cluster models to 1611.3581 with Max Multiple Clusters models, improving by 21.32%. The Min Multiple clusters models improve the average value by 12.43%, from 1.2747 to 1.1162. Also shown in the same table is the variation of the mean values when using the minimum value as a reference. For example, with the use of the minimum value as a reference, the mean value of Kendall's Tau-b has worsened by 11.46% in the Single models, but improved by 5.26% in the Multiple models. Returning to RQ2), according to the results in Table 21, there is a slightly more than 60% improvement in the mean SME when using the multi-cluster models. This improvement is 55% or 61% when using the minimum value as a reference to evaluate the influences.

Table 21
Variation of the mean of the indicators.

	With multiple clusters		With min as reference	
	Max models	Min models	Single cluster models	Multiple cluster models
ΔMean Saaty's index	21.32%	12.43%	99.94%	99.93%
ΔMean Garuti's index	9.14%	7.55%	3.66%	2.15%
ΔMean Cosine	5.10%	4.27%	0.60%	0.18%
ΔMean rho Spearman	2.54%	11.82%	8.66%	4.79%
ΔMean Tau-b Kendall	3.41%	14.83%	11.46%	5.26%
ΔMean SME	67.45%	62.32%	61.68%	55.64%

Regarding RQ3) Does the variation of questions compensate for the variation of the results? Reviewing the number of questions in each model in Table 3, we are of the opinion that wanting to reduce the number of questions is not the determining factor in choosing a single cluster model versus a multiple cluster model. The reason for this is that between a model with multiple clusters and a single cluster/no clusters model, the difference in the number of questions is very small, one question for each cluster of elements that influence each other. Once the number of questions has been reduced by not using comparison matrices, the number of clusters does not add a significant number of questions. Furthermore, from the results in Tables 9, 10 and 21, it does not seem, in general, to be the best decision to model as a single cluster, although there are cases that have better indicators with this option.

Regarding the secondary questions: RQ4) when the DMs are asked to assess the influence with a measurement scale, which should be taken as a reference, the most or the least influential element? and RQ5) does this affect the result depending on how the elements are grouped into one or several clusters? The results also show differences between the two modeling approaches, but there are also conceptual differences that should not be forgotten. DMs find it easier to respond by using one element as a mental reference of influence than by considering a range and having to adjust the rest of the ratings proportionally. On the other hand, in our experience with network models, it is always important to incorporate all relationships in the model, because even a small relationship between two elements allows influence paths to form between other elements that would otherwise remain isolated. Whether the model is more or less connected affects reducibility and other properties of the network, as well as the final priorities. But if the model considers all elements in the same cluster, using the minimum value of influence as the reference for the measurement scale may shorten the influence range and it may be more appropriate to place the elements proportionally distributed with respect to the most influential value.

In view of the results obtained, and although there are indicators for all tastes, the following idea must be highlighted: with a model with multiple clusters, results more similar to ANP are obtained using the least influential element as the reference, or, in other words, if the least influential element is used as a reference, a model with multiple clusters should be built. Whereas, if modelled with a single cluster, the results are more similar if either the most influential or the least influential element is used. It should also be noted that, in this study, the distribution of influences has been as proportional as possible when operating with the values of the ANP matrices. In reality, these values are not known, so the question arises, would the DMs be more proportional by assigning the maximum influence to the most influential and

distributing the rest of the elements in the range of influence, or by assigning 1 to the least influential element and, taking this as a reference, evaluate the proportion of the rest of the elements?

In addition to the results obtained, the use of clusters must also be considered from a more theoretical or conceptual point of view. In our opinion, the fact that Saaty proposes to cluster the elements of the model is almost obligatory when using comparison matrices and the Saaty scale to measure the influences. And it is due to the limitation of 7 to 9 elements to compare in order to maintain the consistency of the judgements in the pairwise comparison matrices. The pairwise comparison matrices also force the grouping of things that are comparable. In return it forces a comparison of the clusters, or grouping levels, to obtain the prioritization of the influence of the elements on each other. But in ANP (and AHP) the concept of clusters, or hierarchy levels, is also used and is useful in the very elaboration of the model and identification of criteria. Thinking about a group of cost criteria, short term, long term, …, helps the decision-maker to think about other criteria or elements that must be considered, to mentally organize the information, to think about his priorities, etc. In this respect, Saaty has always put forward two basic modeling strategies: top-down and down-top. Of course, these sets of criteria do not have to be the definitive ones to be incorporated into the decision problem, e.g., because the elements end up grouped differently, but they are very useful in the process of knowing and learning about the particular decision problem. In summary, thinking in hierarchical clusters/levels helps to approach a complex problem by analyzing it in parts. However, should these clusters/levels be used in the evaluation of influences/priorities of the elements of the model? By not using pairwise matrices, it is true, as proposed by the authors of the new DANP proposal, that it is not necessary to group the elements into clusters, but just as the model must reflect the reality of the problem, the influence measurement system should also consider the nature of the problem under study. For example, in a criteria prioritization model without alternatives, it may make sense to group all the criteria in a cluster and the DMs will be able to assess the influences directly because the influence comes directly from the criterion, not from the group in which the criterion is grouped. But in another model where the elements are for example people, surely the model should incorporate, in addition to the influences between the individual people, the influence of the groups to which these people belong (social groups, research teams, etc.). The fact that the relationships to be evaluated are as similar as possible to reality will make the DMs' assessments of influence more correct and will increase the similarity between the results and reality.

Finally, it is important to point out that in the DANP proposal at the origin of this study, there is an aspect that has been considered closed by its authors, which influences the result, but which is not unique: the measurement scale. The range of the scale directly influences the proportionality of the results. The authors state that DEMATEL uses a scale in numerical terms from 0 to 4, which is not strictly true. As indicated above, other authors have used scales other than the 0–4 scale. A unique scale does not exist, and as a ratio scale, there is no reason why it should numerically finish in 4.

6. Conclusions

In this paper, 17 cases solved with ANP in the literature have been analyzed. An attempt has been made to answer the research questions posed, showing that there are substantial differences in the results depending on whether a single cluster or multiple cluster model is used.

The results also suggest that results more similar to those of ANP can be obtained using the most influential element as a reference in models with a single cluster, but that using the least influential element the results will be more similar in models with multiple clusters.

But being only 17 cases and not having obtained strong results, it is necessary to increase the number of cases in the study to obtain more conclusions. Since real cases with all the necessary data are not available, the study presented here should be continued in future developments with simulations. The use of scales other than the 0–4 scale should also be evaluated in these simulations. Although more closed to ANP results have been obtained using the minimum value as the reference with multiple clusters, the authors' experience with ANP indicates that DMs tend to avoid making judgments at the extremes of the scales, so it should be studied the results obtained using a distribution with respect to the most influential element versus using the least influential element but without pushing the bottom of the scale. In this regard, it would be advisable to simulate decision-makers with different tendencies in their evaluations (one very strict, another laxer, …) to study their influence on the results.

CRediT authorship contribution statement

Erik Schulze-González: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration. **Juan-Pascual Pastor-Ferrando:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration. **Pablo Aragonés-Beltrán:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision.

Declaration of Competing Interest

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Data availability

In the attached file, all 17 cases' matrices are included into a Excel file.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.orp.2023.100275.

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