

## PERFORMANCE EVALUATION OF BURIED UPVC PIPES BY NUMERICAL SIMULATION OF SOIL-PIPE INTERACTION

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### Abstract

Pipelines are the most important and valuable assets in a water distribution network. Design, commissioning and maintenance of the pipes costs a huge amount of money and effort for water supply companies. Understanding the behaviour of buried pipelines is a major concern for engineers. In this paper, soil-pipe interaction has been investigated under various conditions using the numerical simulation of in-situ pipes.

A finite element model has been developed to numerically simulate the interaction of soil with pipes. More than 180 scenarios have been designed to cover the different conditions. Three segments of uPVC pipes with diameters between 160-450 mm were modelled. To cover a range of soils, three different soil types were modelled. The value of Young's modulus was decreased in various scenarios to consider the effects of age on the mechanical properties of pipe material. Four different external loads (corresponding to light to heavy vehicles) were exerted on top of the road pavement. Also, a large external load was applied to study the pipes failure under heavy loads. A variety of internal fluid pressures from 0 to 6 bars was applied inside the pipelines and the ultimate internal pressure which would cause failure in the pipe was determined.

The model domain was discretized for finite element calculations. Finer mesh sizes were selected around the pipe. An appropriate mesh size was determined by mesh sensitivity analysis. The Mohr-Coulomb failure criterion was chosen to simulate the soil behaviour. Also, an elastoplastic stress-strain relationship was considered for the uPVC material with von-Mises failure criterion.

Running the finite element model, the distribution of stresses and strains in the soil and the pipe were computed. In the absence of external load, the values of axial and hoop stresses were compared with the theoretical values. The comparison of the results showed a high correlation between the computed and theoretical values. In both absence and presence of external loads, the values of shear stresses are relatively small and negligible in comparison with axial and hoop stresses. As a result, the minimum and maximum principal stresses are close to the axial and hoop stresses, respectively. This implies that, only measuring the axial and hoop stresses can provide the principal stresses with a reasonable accuracy.

Both internal and external pressures in which the pipe fails are significantly higher than the usual loads on roads and water distribution networks. It justifies the fact that most of the pipe failures occur not due to exceeding the yield stress, but due to fatigue in elastic region. The results of numerical modelling of soil-pipe interaction help engineers to study the pipe performance under different conditions.

### Keywords

Numerical simulation; Soil-pipe interaction; uPVC pipes; von-Mises stress.

## 1 INTRODUCTION

Nowadays, water distribution networks (WDNs) are invaluable parts of our life. Water pipelines are one of most expensive assets among urban infrastructures. Every year, failure in water pipelines costs a considerable amount for water utility companies. A study on 11 failures on main pipelines in United States with 762 – 3048 mm diameter shows a lost production between 6'800 and 10'000'000 m<sup>3</sup> and impact costs between 3.3 and 85.3 \$ Million [1].

Better understanding of the behaviour of pipes under various loads could help the engineers to reduce the consequences of pipe failure. This knowledge may lead to more appropriate design of pipes. Analysing the performance of the pipes under loads could be carried out by physical models [2, 3, 4], analytical models [5, 6], or numerical models [7, 8]. In numerical models, by discretising the domain of the problem, the governing equations are solved for each element. A significant amount of research has been done to investigate the interaction of soil and pipe under various loading conditions using the finite element method [9, 10, 11].

Accuracy of numerical models depends on the mesh size of the model. The finer the mesh, the more accurate the model. In large models, millions of elements are needed to be generated to capture the small deformations of the pipe or soil, which makes these models very time-consuming.

Data-driven techniques have been developed to predict the behaviour of a system using observed data. Artificial Neural Network (ANN) is a widely used method for solving engineering problems. However, ANN suffers from the lack of a transparent relationship between inputs and outputs. Also, due to the black box structure of ANN, the model constants and coefficients are not easy to modify. Genetic Programming (GP) is another evolutionary modelling approach, which inspires from natural selection to fit a mathematical expression to a set of observed data. Although it is very popular in solving engineering problems, the drawbacks are weakness in finding constants and producing appropriate functions [12]. Evolutionary Polynomial Regression (EPR) is one of the machine learning methods which combines the capabilities of genetic programming and polynomial regression to find the coefficients of generated polynomial functions. Comparing the performance of different polynomial expressions, user can find the best relationship to describe the dataset.

In this research, EPR has been used to predict the behaviour of soil-pipe system. Various FE models of soil-pipe system with different conditions were generated in ABAQUS software. By simulating the developed scenarios, a dataset was created representing the interaction of soil and pipe. The dataset was used to train an EPR model in order to find simplified relationships between soil-pipe system properties and stress and strain on pipe wall. The results of this research show that data-driven models are capable of accurately predicting the mechanical behaviour of soil-pipe system at significantly reduced computational time.

## 2 METHODOLOGY

The present research is divided into two stages: the first is a FE analysis of the soil-pipe system, and the second is the use of EPR to capture the mechanical behaviour of PVC pipes.

### 2.1 Finite Element Analysis of soil-pipe system

In real world, there are various factors that affect the mechanical behaviour of PVC pipes in water distribution networks during their life time, e.g. internal hydraulic pressure, external load on the soil surface, pipe's diameter, thickness, and Young's modulus, density, depth, friction angle, and Young's modulus of soil. Internal pressure, external load, pipe's diameter and thickness, and Young's modulus of soil were regarded as key variables impacting stress and strain on pipe walls

in this study, while the other variables were held constant. The values of the parameters used in the model are shown in Table 1.

*Table 1. Values of independent variables in generated scenarios for soil-pipe numerical modelling.*

Internal Pressure (bar)	External Load (ton)	Young Modulus of soils (MPa)	Diameter of pipes (mm)	Thickness of pipes (mm)
0	Normal Sedan	15	160	6.2
2	light goods trucks	40	630	24.1
3	Six axel artic lorries	120	1200	29.4
4	Two axel heavy lorries			
5				

In this study, a range of conditions were employed in which the investigated variables were altered to create 180 scenarios for finite element simulations.

For numerical modelling, ABAQUS was used. ABAQUS is a well-known commercial software for finite element analysis of engineering problems.

Three different diameters of PVC pipe (160 mm, 630 mm, and 1200 mm) were used to create a model of a soil pipe system, which covers a wide range of PVC pipes. The pipe thickness was chosen based on the usage of PN10 PVC pipes in water distribution networks. The pipes are in a soil trench with width and depth of 5 × 5 metres, and a trench length equal to the length of the pipe. The pipe and soil components were chosen as solid homogenous, with cell partitions based on the model geometry for burying pipe at a depth of 1 metre of trench.

In ABAQUS, an eight-node continuum element (C3D8R) was used to better imitate the behaviour of plastic pipe. For modelling the homogeneous and isotropic soil around the pipe, a twenty-node quadratic brick element with reduced integration (C3D20R) was adopted. The optimal mesh size for the pipe and soil was determined through a mesh sensitivity analysis. As seen in Fig. 1, the mesh becomes finer closer to the pipe and coarser further away. In addition, depending on its thickness, the pipe wall was modelled in one, two, or three layers.

Properties of PVC pipes were taken from [13], in which 16 PVC pipes with four different diameters and four different thicknesses were investigated. They used tensile tests on dog-bone samples to produce a standard stress-strain curve for PVC pipes. The stress-strain curve employed in this study is shown in Fig. 2. The yield stress of PVC pipes is 43.67 MPa, which is substantially greater than the usual stresses in water distribution pipelines, as illustrated in this diagram.

On the soil-pipe system, two primary loads were evaluated, as previously stated. The maximum internal pressure in the urban water distribution network is limited at 5 bar. Internal pressures of 0, 2, 3, 4, and 5 bars were applied in the pipes in various conditions. Different external loads were applied on top of the soil trench, taking into account ordinary urban vehicles such as standard sedans, light goods trucks, six axel artic lorries and two axel heavy lorries.

A Mohr-Coulomb model was used to describe the mechanical behaviour of the soil. The Young's modulus of soil was altered in different FE simulation scenarios to study the effects of soil material type on pipes. The Mohr-Coulomb yield criterion is described as [14]:

$$\tau = C - \sigma \tan \varphi \quad (1)$$

where  $\sigma$  and  $\tau$  are the effective normal stress and shear stress;  $C$  is the cohesion of soil; and  $\varphi$  is the effective angle of shearing resistance. The mechanical properties of considered soils are presented in Table 2.

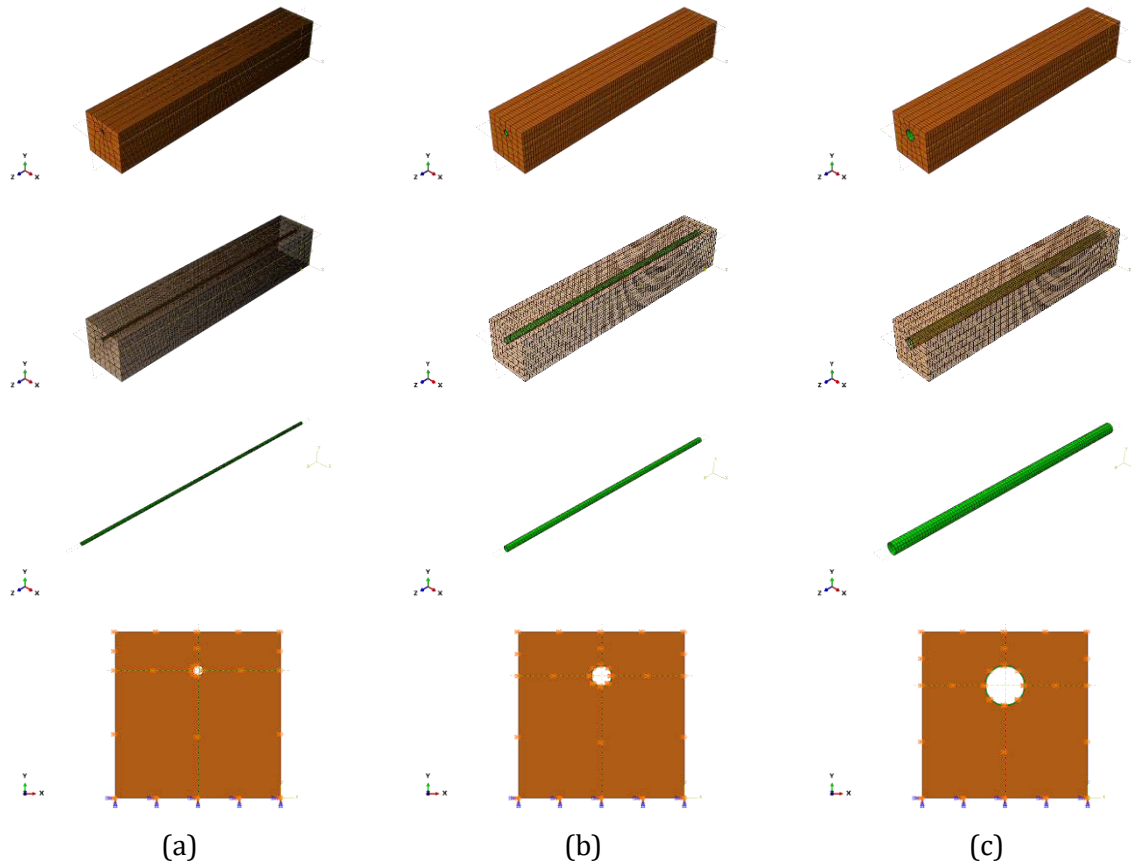


Fig. 1. Soil-pipe system geometry for (a) 160mm; (b) 630mm; and (c) 1200mm; pipes.

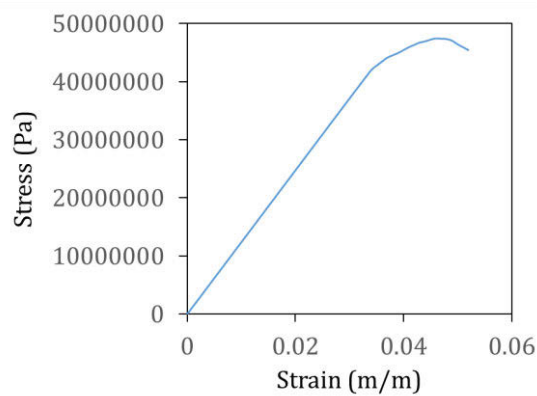


Fig. 2. Stress-strain curve used for characterizing the PVC material [13].

Table 2. Mechanical properties of soil.

Elastic properties	Density (Kg/m <sup>3</sup> ): 1950 Young's modulus (MPa): 15, 40, 120 Poisson ratio: 0.3
Plastic properties	Cohesion (KPa): 15 Angle of shearing resistance (deg): 29 Dilation angle (deg): 2

Contact modelling is essential for accurately simulating the interaction between the soil and the pipe. In the 3D soil pipe interaction problems, contact between soil and underground pipe brings significant non-linearity. The "surface to surface" contact is employed in ABAQUS/Standard version 2021, with the pipe outer surface and the soil interior surface designated as "master" and "slave," respectively. The "penalty" technique approximates a "hard" contact between two surfaces in a non-linear way.

In this study, an adequately long section of underground pipe was modelled. Because the underground pipe can move with the soil, it is appropriate to treat the pipe ends as roller boundary. The displacement/rotation degrees of freedom were fixed ( $U_1=U_2=U_3=UR_1=UR_2=UR_3=0$ ) for trench bottom boundaries; the bottom section of the soil was regarded fixed, and pipe rings and soil were selected as displacement/rotation ( $U_1=U_3=0$ ) for side walls of trench and pipe ends.

All of the scenarios were analysed with the above assumptions and consideration. For the crown, springlines, and invert position in the middle of the pipes, axial stress, hoop stress, von-Mises stress, axial strain, and hoop strain were determined. The mechanical behaviour of the PVC pipes was captured using these data under a variety of internal and exterior loads.

## 2.2 Evolutionary Polynomial Regression (EPR)

EPR is a hybrid approach for fitting an equation to a set of observation data. It has been utilised to identify the link between dependent and independent variables in numerous civil engineering applications [12, 15]. To identify the optimum regression, EPR employs a two-stage technique: 1) developing equations that best match the data, and 2) applying a genetic algorithm (GA) to determine the coefficients and exponents of the equations. EPR's general expression can be shown as:

$$y = \sum_j^m F(X, f(x), a_j) + a_0 \quad (2)$$

where  $y$  is the system's estimated output;  $a_j$  is a constant value;  $F$  is a function built by the process;  $X$  is the matrix of input variables; and  $f$  is a user-defined function. The number of terms in the expression excluding bias  $a_0$  is  $m$ . EPR is able to consider different functions, e.g., polynomial, sinusoid, logarithmic, etc., among which polynomial is the simplest form.

In most of the cases, GA uses least square (LS) of difference between observations and predictions to determine the best coefficients and exponents. Also, coefficient of determination (CoD) is used to select the equations with better correlation.

In this research, many factors affect the stresses and strains on the pipe wall:

$$y = f(W, P, D, t, E_s, E_p, H, \gamma_s, \varphi, \dots) \quad (3)$$

where  $P$  is internal pressure of the pipe;  $W$  is external load on the soil surface;  $D$  and  $t$  are pipe diameter and thickness, respectively;  $E_s$  and  $E_p$  are soil and pipe Young's modulus, respectively;  $H$  is soil depth on the pipe crown;  $\gamma_s$  is density of soil; and  $\varphi$  is angle of shearing resistance of soil. Three main factors mainly affect the stresses and strains in pipes: internal pressure, external load, and soil weight. Nevertheless, using the superposition principle, the above equation could be written as follows:

$$y = f_1(W, D, t, E_s, H, \gamma_s, \varphi) + f_2(P, D, t, E_p) + f_3(H, \gamma_s, D, t) \quad (4)$$

Functions  $f_1$ ,  $f_2$  and  $f_3$  indicate the effect of external load, internal pressure and soil weight, respectively. Here,  $y$  is stress/strain on cardinal points of pipe. To simplify the problem, only  $W$ ,  $P$ ,  $D$ ,  $t$  and  $E_s$  were considered as the variables, while other factors were treated as constant. On the other hand, assuming a polynomial regression, Eq. 3 could be reduced to:

$$y = a_1 W^{m_1} E_s^{m_2} D^{m_3} t^{m_4} + a_2 P^{n_1} D^{n_2} t^{n_3} + a_3 \quad (5)$$

where  $a_1 - a_3$  are coefficients; and  $m_1 - m_3$  and  $n_1 - n_3$  are exponents. In this work, the EPR was used to determine the exponents between -2 and 2 with steps of 0.5. Since all the soil weight factors are constant, it has been shown as a bias ( $a_3$ ).

70% of data was used to train the EPR model and the remaining 30% was used to validate the developed model. Ingesting the data from FE analysis to EPR model, it generates a large number of equations with different structures, including different combinations of independent variables and various CoDs. To select the best equation describing the soil-pipe mechanical behaviour, these criteria were considered:

- 1) Highest amount of CoD,
- 2) Model consisting of the minimum number of terms (parsimony principle),
- 3) Model including the maximum number of independent variables,
- 4) Capability of the model to respond properly in extreme conditions. For example, if  $P$  and  $W$  are being multiplied in a single term of the equation, when each of them is equal to zero, the whole term could be equal to zero, regardless of the amount of the other variable.

Considering the above criteria, the best equations were selected to calculate the axial stress, hoop stress, von-Mises stress, axial strain and hoop strain in crown, springlines and invert of the pipe. The results are presented and discussed in the next section.

### 3 RESULTS

The values of axial stress, hoop stress, von-Mises stress, axial strain and hoop strain were computed by the FE models for each pipe in all scenarios. Fig. 3 presents the distribution of these variables in pipes with 160mm, 630mm and 1200mm diameter. The findings revealed that all of the pipes are long enough so that the boundary conditions do not affect their middle section.

The results demonstrate that for 1200mm pipes, von-Mises stress is high in the springlines, the maximum amount of axial strain is in the springlines, and hoop strain is high in the invert and crown sections, as shown in the Fig. 3. In 630mm pipes, the crown and invert sections have large quantities of axial stress, axial strain, hoop stress, and hoop strain, while the springlines have significant von-Mises stress. In 160mm Pipes, the pipe's shoulder and haunch sections have the

highest hoop stress and hoop strain, whereas the crown and invert sections have the highest von-Mises stress as well as axial stress and axial strain.

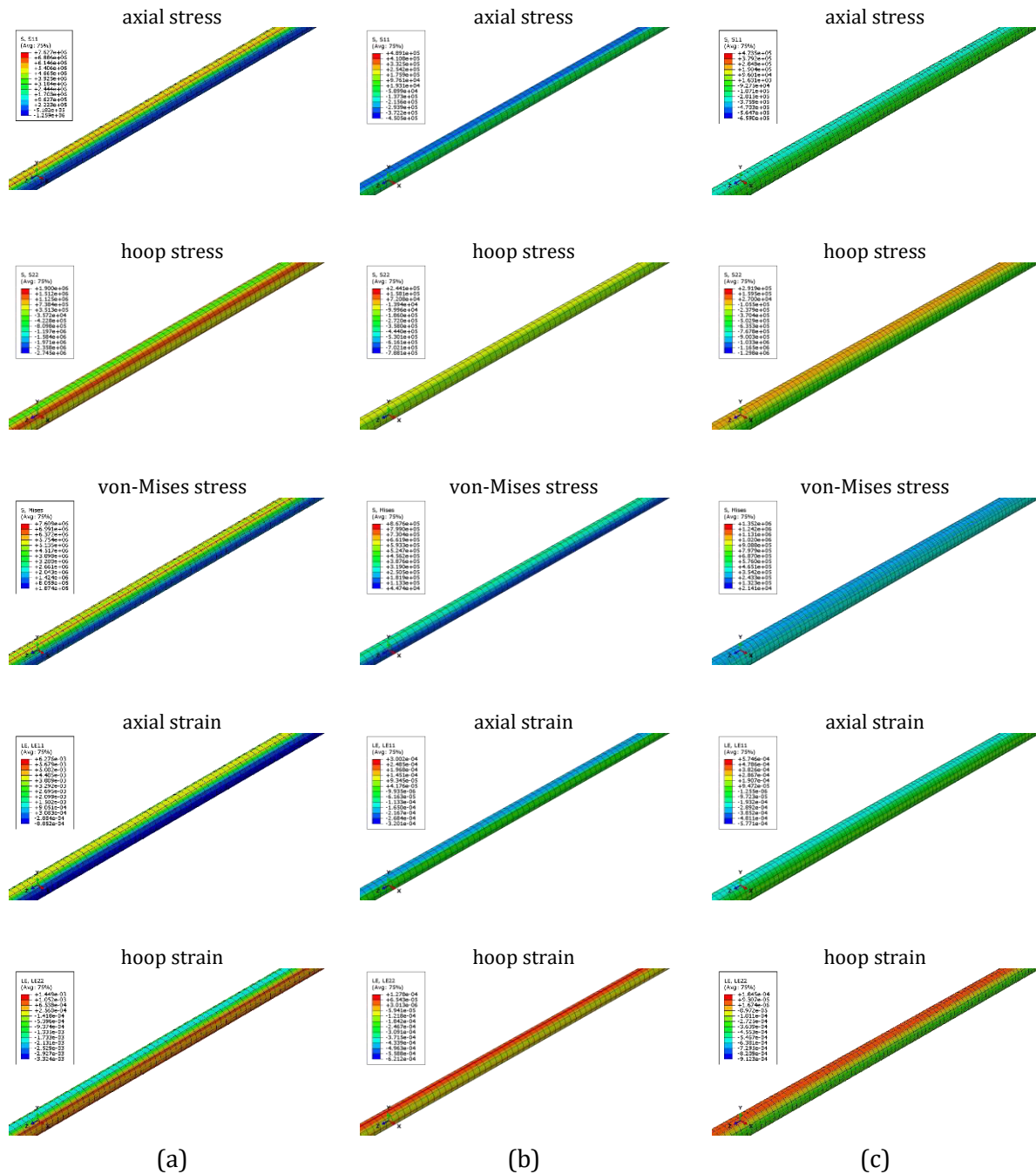


Fig. 3. Distribution of axial stress, hoop stress, von-Mises stress, axial strain and hoop strain for pipes in the loading scenarios with maximum internal pressure for pipes with (a) 160mm; (b) 630mm; and (c) 1200mm diameter.

Using the EPR model, a number of equations were developed for calculating each of the dependent variables. In each case, the best equation was selected considering the criterion mentioned in Methodology section. These equations are presented in Table 3. In most of the cases, the CoD is

higher than 98%, indicating a high level of correlation between prediction model and observation data.

Table 3. The best equations generated by EPR for calculating each dependent variable.

Dependent variable	Position	Equation	CoD (%)
Axial Stress	Crown	$\sigma_a = 6.0968 \times 10^{-12} \frac{W^{0.5} \cdot E_s^2}{t^2} - 1.3090 \times 10^3 \frac{P}{D \cdot t}$	98.64
	Springlines	$\sigma_a = 2.7545 \times 10^8 \frac{W^{0.5}}{D^2 \cdot t^2} - 3.1592 \times 10^4 \frac{P}{D^2} - 6.2909 \times 10^4$	98.49
	Invert	$\sigma_a = 6.2824 \times 10^8 \frac{W}{D^{1.5} \cdot t^2 \cdot E_s^{0.5}} - 2.5167 \times 10^4 \frac{P}{D^2} - 1.1791 \times 10^4$	98.16
Hoop Stress	Crown	$\sigma_h = -1.2204 \times 10^7 \frac{W}{D^2 \cdot t^2} + 1.6963 \times 10^6 \frac{P}{D^{1.5} \cdot t^2} - 2.6147 \times 10^5$	99.47
	Springlines	$\sigma_h = -2.8343 \times 10^7 \frac{W}{D^2 \cdot t^2} + 4.9327 \times 10^4 \frac{P}{D \cdot t^{1.5}} - 4.7687 \times 10^5$	98.95
	Invert	$\sigma_h = -1.9357 \times 10^9 \frac{W}{D^2 \cdot E_s^{0.5}} + 3.3832 \times 10^6 \frac{P}{D^2 \cdot t} - 3.1034 \times 10^5$	98.91
von-Mises Stress	Crown	$\sigma_{Mises} = -5.8982 \times 10^4 t - 0.3W - 0.35 \frac{P}{D^2} + 6.9148 \times 10^2 \frac{P^{1.5}}{D^2} + 1.4096 \times 10^6$	97.88
	Springlines	$\sigma_{Mises} = 6.63819 \times 10^5 \frac{W}{D^2} - 0.00405 \frac{P}{t^2} + 1.0087 \frac{P^{1.5}}{t^2} + 1.46407 \times 10^5$	96.84
	Invert	$\sigma_{Mises} = 1.74507 \times 10^5 \frac{W^{0.5}}{t^2} - 7.1972 \times 10^2 \frac{P^{1.5}}{D^2} + 1.44697 \times 10^5$	98.19
Axial Strain	Crown	$\varepsilon_a = 3.6346 \times 10^{-18} \frac{W^{1.5} \cdot E_s}{t^{1.5}} - 0.00024375 \frac{P}{D^{1.5} \cdot t^{1.5}}$	97.90
	Springlines	$\varepsilon_a = 2.1318 \times 10^{-18} \frac{W^{1.5} \cdot E_s^{1.5}}{D^2} - 1.6404 \times 10^{-6} \frac{P^{1.5}}{D^2 \cdot t}$	90.52
	Invert	$\varepsilon_a = 31.209 \frac{W}{D^2 \cdot t^2 \cdot E_s^{0.5}} - 0.00312 \frac{P}{D^2 \cdot t^{1.5}} + 9.6095 \times 10^{-5}$	99.08
Hoop Strain	Crown	$\varepsilon_h = -9.6493 \times 10^{-3} \frac{W}{D^2 \cdot t^2} + 1.3546 \times 10^{-3} \frac{P}{D^{1.5} \cdot t^2} - 1.9574 \times 10^{-4}$	99.47
	Springlines	$\varepsilon_h = -1.137 \times 10^{-17} \frac{W^2 \cdot E_s}{D^{1.5} \cdot t^{0.5}} + 5.4566 \times 10^{-7} \frac{P^{1.5}}{D^2}$	98.97
	Invert	$\varepsilon_h = -4.2125 \times 10^{-6} \frac{W^{1.5}}{t^2 \cdot E_s^{0.5}} - 5.5012 \times 10^{-4} \frac{P^{0.5}}{D} + 5.5579 \frac{P}{D^2} - 8.7481 \times 10^{-5}$	99.05

As indicated in Table 3, in most of the cases, a two-term equation (with a bias term) is achieved. High CoD values indicate the suitability of Eq. 5 for predicting the dependent variables using internal pressure, external load, pipe diameter, pipe thickness and soil Young's modulus.

Fig. 4 shows the performance of developed prediction models to forecast the axial stress, hoop stress and von-Mises stress in different positions of a pipe, (crown, springlines and invert). The



figure shows the correlation for both training and testing data. It is shown that the model is able to predict the unseen testing data with the same (high) accuracy of training data. In addition, the capability of the model in prediction in crown and invert is higher than that of springlines.

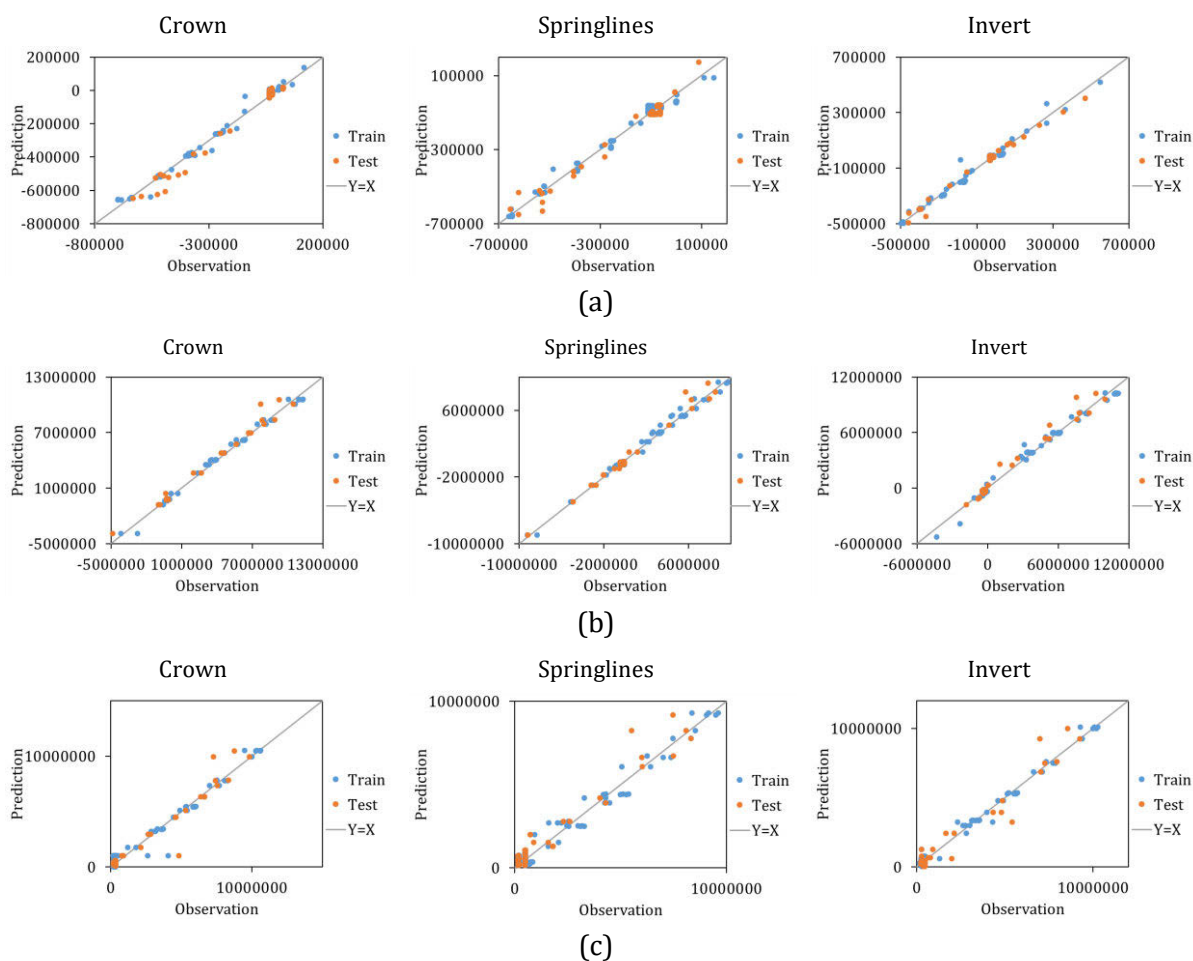


Fig. 4. Performance of prediction model to forecast observation data for (a) axial stress; (b) hoop stress; and (c) von-Mises stress.

Fig. 5 shows the capability of the proposed models to predict the axial and hoop strains in crown, springlines and invert of the pipe. Similar to the stress, the models are better in predicting the strains in crown and invert, than springlines. Also, model proposed for hoop strain has a better correlation than that of axial strain.

Fig. 6 presents the amounts of hoop stress-strain curves for crown, springlines and invert of different pipes. As mentioned earlier, the common internal pressures and external loads on soil-pipe system have been applied in the FE model, therefore, the achieved stress-strain values only cover a part of the entire curve (left) and failure has not occurred in any of the cases.

In this figure, the stress-strain curve presented by [13], which was used as a representative of PVC pipes mechanical behaviour, is shown in black lines. As it shown, the results of EPR model are perfectly fitted on the original stress-strain curve of [13], which indicates the capability of the EPR model to simulate the mechanical behaviour of PVC material using simple variables such as internal and external load and pipe’s diameter and thickness.

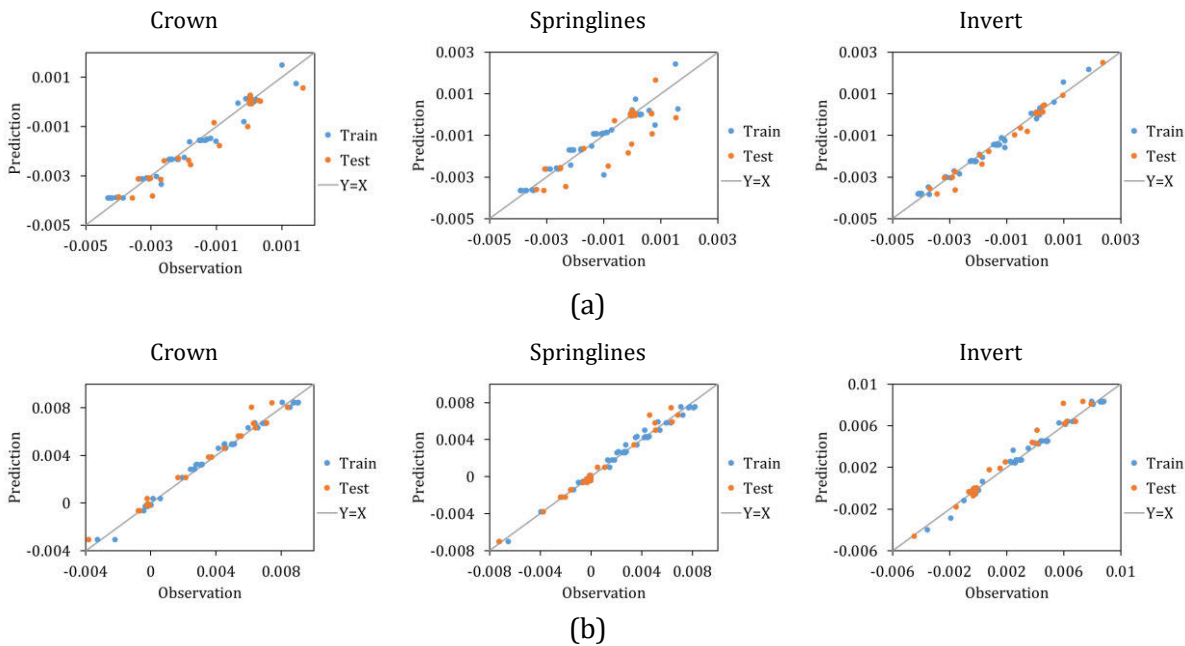


Fig. 5. Performance of prediction model to forecast observation data for (a) axial strain; and (b) hoop strain.

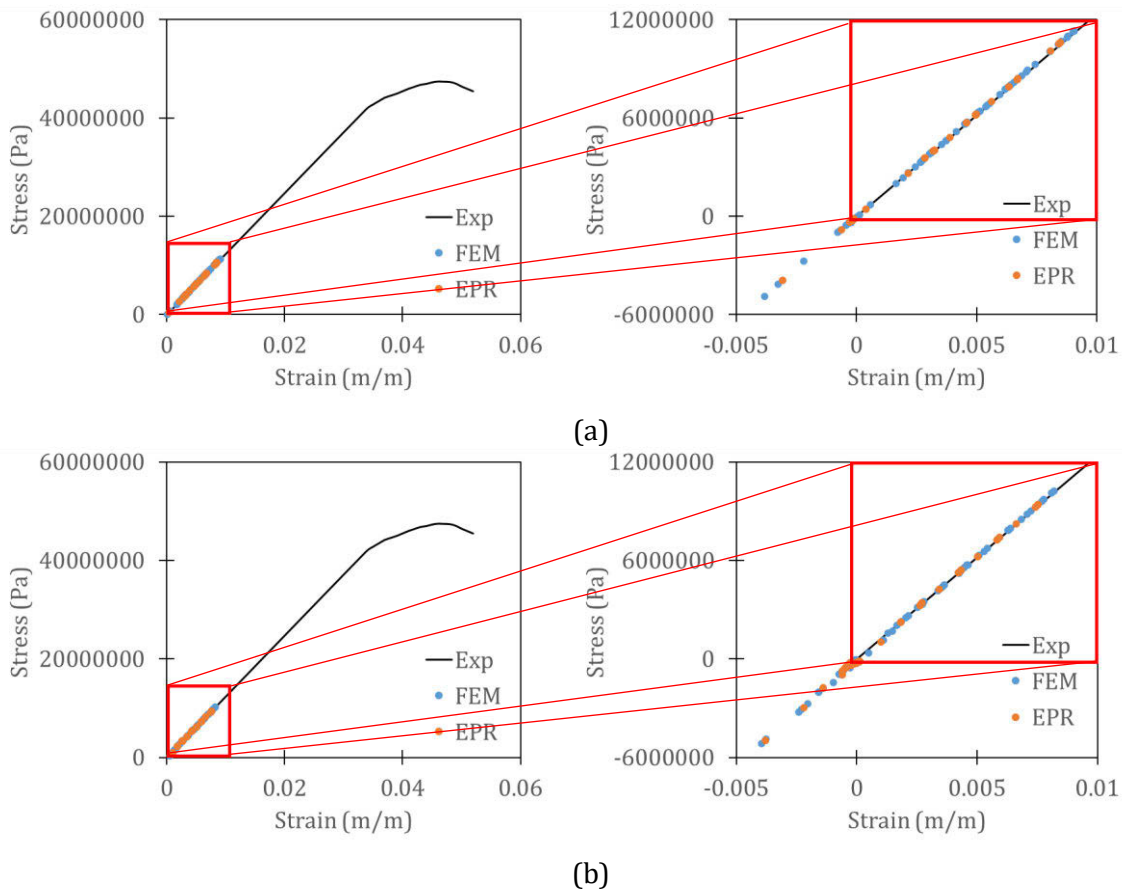
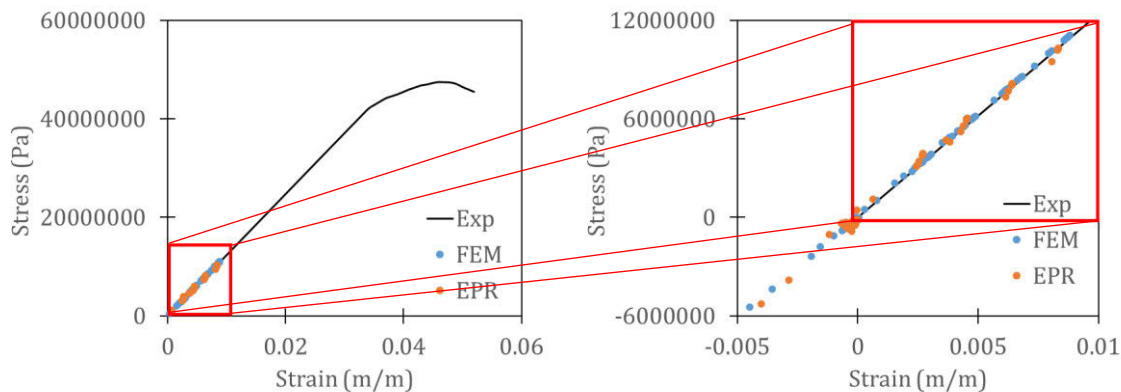


Fig. 6. Hoop stress-strain curves for (a) crown; (b) springlines; and (c) invert; of the pipes.



(c)

Fig. 6 (continue). Hoop stress-strain curves for (a) crown; (b) springlines; and (c) invert; of the pipes.

## 4 DISCUSSION

In this research, PVC pipes with adjusted properties and various diameters and thicknesses were simulated using finite element method with Mohr-Coulomb model for soil. 180 scenarios were constructed with different soil Young's modulus and a variety of loading conditions such as internal pressure and external load. Axial stress, axial strain, von-Mises stress, hoop stress, and hoop strain were collected from three critical positions of the pipe; crown, springlines, and invert. The results of FE modelling were ingested to an EPR model to extract appropriate regression equations for predicting stresses and strains on pipe wall using independent variables.

The constructed relationships are supported by physical principles to consider the main loads on soil-pipe system. In all of the achieved equations, external load and internal pressure are showing opposite trends. This is expected, due to the fact that these two forces act in opposite directions.

The results indicate that EPR is able to find relationships between independent variables and dependent variables with high degree of accuracy. The proposed equations are actually a white-box model to predict the mechanical behaviour of pipes under different combinations of loads. Using these equations is much less time-consuming in comparison with running a finite element model. The equations are developed for pipes from 160mm up to 1200mm, which means they could be used for a wide range of pipe diameters.

It is proposed to consider other factors such as soil depth, soil density and different pipe materials, in the future investigations, in order to develop a more comprehensive model.

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