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CHARACTERIZING THE EFFECTS OF WATER DISTRIBUTION SYSTEM TOPOLOGY MODIFICATIONS ON ITS DYNAMIC BEHAVIOUR THROUGH CONNECTIVITY METRICS

Valentina Marsili[1], Stefano Alvisi[2], Filomena Maietta[3], Caterina Capponi[4], Silvia Meniconi[5], Bruno Brunone[6] and Marco Franchini[7]

[1] Research Assistant, Department of Engineering, University of Ferrara, Ferrara, Italy
[2] Associate Professor, Department of Engineering, University of Ferrara, Ferrara, Italy
[3] PhD Student, Department of Civil and Environmental Engineering, University of Perugia, Perugia, Italy
[4] Research Assistant, Department of Civil and Environmental Engineering, University of Perugia, Perugia, Italy
[5] Associate Professor, Department of Civil and Environmental Engineering, University of Perugia, Perugia, Italy
[6] Professor, Department of Civil and Environmental Engineering, University of Perugia, Perugia, Italy
[7] Professor, Department of Engineering, University of Ferrara, Ferrara, Italy

[1] valentina.marsili@unife.it, [2] stefano.alvisi@unife.it, [3] filomena.maietta@studenti.unipg.it, [4] caterina.capponi@unipg.it, [5] silvia.meniconi@unipg.it, [6] bruno.brunone@unipg.it, [7] marco.franchini@unife.it

Abstract

Water distribution networks (WDNs) are complex combinations of nodes and links and their structure has an impact on their behaviour, considering both quantitative (i.e. related to pipe flows and nodal pressures) and qualitative (i.e. related to water age and quality) aspects. The complexity of WDNs has been the basis of several studies that have resorted to the graph theory to relate connectivity properties to system behaviour (e.g. its reliability and water age/quality), evaluated under the assumption of steady-state conditions. Within this framework, in recent years the tendency toward reducing network interconnection through the closure of isolation valves has emerged, mainly to (i) facilitate its monitoring and management, and (ii) increase flow velocity and reduce water age. However, changes in the topology of a network can affect not only aspects evaluated under the assumption of steady-state conditions, but also its dynamic behaviour. Based on these considerations, the present study investigates whether some metrics derived from graph theory, already applied in the context of networks' steady-state analyses, can also provide useful indications for assessing the effects of changes in the topological structure, which could be consequences of branching operations, on the dynamic response of a network subjected to users' activity. The analyses highlight that connectivity metrics can reflect the pressure dynamic behaviour of the hydraulic systems and support in their macroscopic understanding during design and management operations. Thus, their application can be effectively extended from the steady-state to the dynamic framework.

Keywords
Transient analysis, unsteady flow, water demand, topology, graph theory.

1 INTRODUCTION

Water distribution networks' (WDNs) are normally designed and realized with topological structures, or connectivity structures of pipes and nodes, generally characterized by looped layouts that guarantee high hydraulic and mechanical reliability of the system (Elshaboury et al., 2021; Sirsant et al., 2020). However, these benefits from a reliability point of view imply higher costs of the network (Todini, 2000). In this regard, several methods have been developed and proposed in the scientific literature to identify the optimal topological structure ensuring a balance between reliability and cost (e.g. Choi and Kim, 2019; Farmani et al., 2005). As a result, real WDNs are complex combinations of loops and branches (Walski et al., 2003). This complexity has been the basis for the development of a series of studies that have led to the application to WDN of theories and indicators originally developed in the field of graph theory aimed to manage system hydraulic functioning and water quality (e.g. Abokifa et al., 2019; Giudicianni et al., 2018; Torres et al., 2016; Yazdani & Jeffrey, 2012).

Nowadays, along with the network reliability, other issues are becoming of interest for the water utilities, such as (*i*) the observability and controllability of water networks and (*ii*) the compliance with certain operating conditions (e.g. the values of average and minimum velocity able to ensure sufficient water quality).

Concerning the observability, looped systems have proved to be less controllable and, in recent years, a tendency to reduce the interconnection of networks through, for example, sectorisation techniques (i.e. DMA creation), has emerged. This practice consists of dividing a looped system into portions of networks (i.e. districts) that are connected at a limited number of points where flow meters are placed, while all remaining interconnection points are closed through isolation valves (IVs). The closure of such IVs results in an alteration of the topological/connectivity structure of the network and a reduction of the number of loops. These aspects have been extensively investigated in the literature, also resorting to methodologies based on graph theory (e.g. Diao et al., 2013; Di Nardo et al., 2013; Ferrari et al., 2014; Giustolisi, 2020). It is worth noting that procedures for identifying the optimal districts layout often rely on optimization algorithms that require the resolution of a significant number of connectivity configurations (e.g. Zhang et al., 2017).

With reference to the compliance with certain operating conditions, it is worthy of note that the presence of strongly interconnected network structures typically results in a reduction of average water velocity in pipes with consequent (*i*) ageing of the water before it is delivered to the users and consequent reduction of free residual chlorine or formation of chlorination by-products (Quintiliani, 2017), and (*ii*) non-reaching of the daily self-cleaning velocity that guarantees cleaner pipes (Abraham et al., 2017). To ensure optimal conditions of water velocity, several approaches have been developed in order to identify the optimal interventions on the topological structure of the network based, for example, on the closure of IVs and the reduction of the level of interconnection (Brentan et al., 2021). Again, the optimal set of IVs to be closed is often identified through optimization algorithms evaluating a very large number of solutions.

In summary, on the one hand the closure of the IVs present in the network reduces the level of interconnection and produces an alteration of its topological structure, on the other hand these problems have been so far faced mainly under the assumption of steady-state conditions. However, several studies have shown that, in reality, modern WDNs rarely achieve steady states for more than a small fraction of their operational times and are subjected to continuous pressure transients that can be induced by manoeuvres on main devices in the network, whose status is changed to respond to different conditions of water demands (e.g. changes in the setting of pumps). Recently, there has been evidence that even the operation of regulation valves (Changklom and Stoianov, 2017; Meniconi et al., 2015) and users' activity (Marsili et al., 2021), i.e. the opening and closing of domestic devices, can result in the generation of pressure transients.

Within this framework, several studies have shown that the response of the WDN, to these driving forces depends on its characteristics and, in particular, on the topological structure (i.e. network connectivity) and the geometric and mechanical characteristics of the network, i.e. diameters and materials of the pipes (Ellis, 2008) as well as the presence of elements such as dead-ends (Meniconi et al., 2021). With specific reference to the topological structure (i.e. network graph), it is thus important to observe that modifications of the structure through the closure of IVs can also determine a variation in the response of the system to transients' generation and propagation. From an operational standpoint, the evaluation of the dynamic behaviour of a water network with an assigned topological structure and subjected to a given driving force can be conducted through unsteady flow simulation based on numerical approaches as, for example, the Method of the Characteristics (MOC). However, this approach for complex real WDNs can be extremely expensive from a computational point of view, especially when simulations have to be replicated for a significant number of configurations of the system considered.

In the current study, the ability of some indicators belonging to graph theory – already used in the context of network reliability evaluations – to provide useful indications on the effects of the topological structure variations on the dynamic response of a network is evaluated. To this end, four network connectivity indicators are examined: two basic connectivity metrics and two spectral metrics. These indicators are compared in terms of their ability to represent the dynamic behaviour of a simple WDN, the latter evaluated through a numerical model based on MOC, in which the topological structure is modified by closing an increasing number of IVs for reducing the number of loops. In the following, first, the connectivity indicators are introduced. The methodology adopted is then presented and applied to a simple case study. Results are discussed and finally, some conclusive considerations are provided.

2 MATERIALS AND METHODS

In the following, the connectivity indicators considered in the analysis and the methodology proposed aimed at evaluating the ability of these indicators to reflect useful information on the effects of the structure of the network on its dynamic response, the latter obtained through numerical simulations, are introduced.

2.1 The connectivity metrics

In order to study the relationship between the connectivity properties of networks and their dynamic behaviour, four indicators were considered which belong to graph theory. Among the metrics aimed at investigating the structure and the behaviour of complex networks, the basic connectivity metrics and the spectral measures are the most widespread and well-known. On the one hand, the basic connectivity metrics indicate the degree of connectivity of the vertices (i.e. the *nodes*) and the edges (i.e. the *links*) representing the cohesion of the network and its sensitivity to the removal of nodes and links. Spectral metrics, on the other hand, relate the topology of the network to the connectivity strength and cohesion of the graph by analysing the spectrum of the adjacency matrix, A, of the network, which indicates which vertices are connected (i.e. *adjacent*): the generic element of A $a_{ij} = 1$ if a link connects nodes i and j, otherwise $a_{ij} = 0$. These metrics quantify network properties that depend only on the abstract structure of the graph, independently of its representation (Torres et al., 2016; Yazdani et al., 2010).

More in detail, in this study two basic connectivity metrics and two spectral metrics were considered. Concerning basic connectivity metrics, the first metric considered is the average degree, k, defined as:

$$k = \frac{2m}{n} \tag{1}$$

with m and n the number of links and nodes, respectively. The second basic connectivity metric considered is the meshed-ness coefficient, R_m, defined by the ratio between the total number $(m - n + 1)$ and the maximum theoretical number $(2n - 5)$ of independent loops in the network:

$$R_m = \frac{m - n + 1}{2n - 5} \tag{2}$$

A uniform distribution of the average degree indicates the tendency of the network to be invulnerable to faults, whereas the meshed-ness coefficient can effectively describe the redundancy of the network (Yazdani & Jeffrey, 2012). With reference to spectral metrics, the spectral gap, $\Delta\lambda$, and the algebraic connectivity, λ_2, are considered. Specifically, the spectral gap $\Delta\lambda$ is defined as the difference between the first and second eigenvalue of the adjacency matrix A of the network and it is effective in the quantification of *Good Expansion* (GE) properties of the system. In particular, networks characterised by GE are those whose topological structure presents vertices connected robustly to other nodes, even if the graph is not dense with links (Estrada, 2006). The algebraic connectivity, λ_2, corresponds to the second smallest eigenvalue of the Laplacian matrix, $L = D - A$, of the network (Fiedler, 1973), where D is the diagonal matrix of nodal degrees, the latter indicating the number of links converging at each node. Higher values of λ_2 indicate higher network robustness (and higher reliability) and higher fault tolerance (Yazdani & Jeffrey, 2012).

2.2 The methodology

To evaluate the applicability of the indicators previously introduced to characterise the changes of the dynamic response of a WDN when its topological structure is modified, the approach described below was developed. Consider a generic WDN characterised by n nodes that can be connected with a variable number of links (m) for a total of l loops ($l = m - n + 1$). In Figure 1, a simple WDN is reported as an example and its layout is henceforth referred to as original or *reference configuration* (C1). On the one hand, the four indicators previously introduced can be quantifiable for the considered network. On the other hand, in the face of a generic driving force, such as water demand change at a node, a transient is generated and propagates throughout the network, interacting with its topological structure.

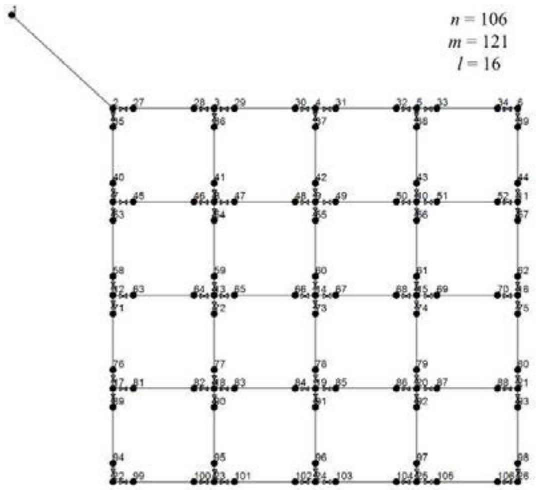

Figure 1 Reference layout of the simple network (i.e. C1).

With reference to the simple network with layout C1, the dynamic response of the system can be obtained through a numerical model and, in the current study, the model introduced in Marsili et al. (2022) and based on MOC is adopted. In greater detail, for the numerical simulation under unsteady flow conditions, it is important to specify that a reservoir located at node 1 fed the network and that all the pipes in the network are characterised by the same diameter (DN125), length (100 m) and wave speed a (500 m/s).

Considering, as an example, an instantaneous closure manoeuvre operated at a generic node, the numerical simulation in a defined time window allows obtaining the value of the interval $\delta p = |p_{max} - p_{min}|$, for each node, representing the range in which pressure oscillates in the considered node, being p_{max} and p_{min} the maximum and minimum values of the pressure. In Figure 2, the interval δp is shown graphically for each node of the network in configuration C1 through a chromatic scale that varies from 0 to 5 m in the face of an instantaneous closure manoeuvre operated, for sake of example, at node 86 highlighted with a red arrow. δp is evaluated considering a simulation time window t_{sim} = 30 s.

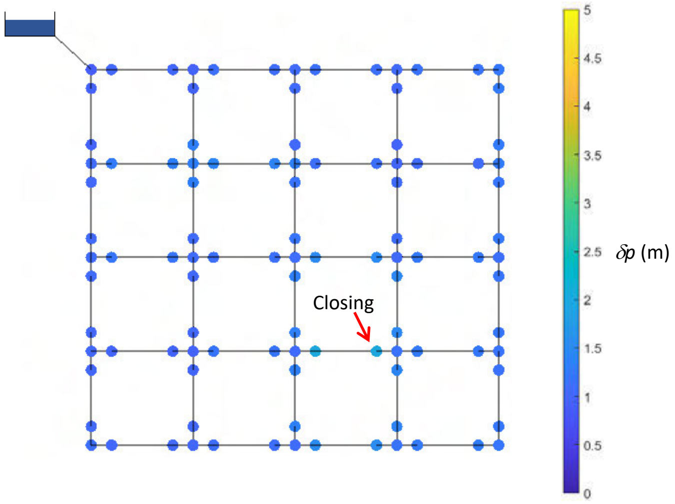

Figure 2 Results of unsteady flow simulation of the network in configuration C1 subjected to an instantaneous closure at node 86 in terms of δp at each node.

More in general, the stress of each node, in terms of the values of δp, can be summarised in the cumulative frequency of δp values. In Figure 3, the cumulative frequency of δp for configuration C1 of the network is shown in blue. Furthermore, the values at the 50th and 90th percentiles of this cumulative distribution – henceforth indicated as δp_{50} and δp_{90}, respectively – can be considered representative of the average and extreme dynamic response of the given network configuration.

If the topological structure, i.e. the connectivity of the system, is modified, for example by closing some IVs to create districts or to improve water quality aspects, this impacts on the connectivity indicators. If *nc* new configurations are considered, *nc* new values for each connectivity indicator is obtained. Moreover, this would lead to different dynamic responses of the system, that can be evaluated in face of the same manoeuvre considered for the *reference configuration* through the numerical model previously introduced. Based on these numerical results, the cumulative frequencies of δp for each new configuration is traceable. The pressure stress variation of each one of the *nc* configuration with respect to the *reference configuration* can be evaluated as $\frac{\delta p_{50}}{\delta p_{50,ref}}$ and $\frac{\delta p_{90}}{\delta p_{90,ref}}$, where $\delta p_{50,ref}$ and $\delta p_{90,ref}$ denote the average and extreme dynamic response of the network in the *reference configuration*.

The ability of the connectivity indicators to provide useful indications on the effects of topological structure variations on the changed dynamic response of a network is evaluated considering their correlation with the corresponding pressure variation indicator obtained through hydraulic simulation. In the following, a numerical example is provided.

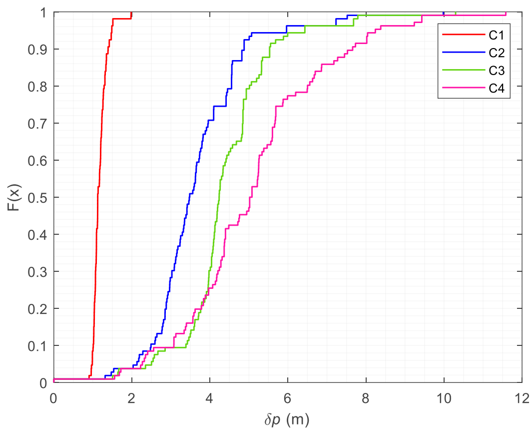

Figure 3 Cumulative frequency of pressure variations δp stressing the network nodes in different configurations (C1-C4).

3 RESULTS AND DISCUSSION

The results of the application of the approach proposed to the simple network considered are hereinafter reported.

Starting from the *reference configuration* C1 (Figure 1), three additional configurations of the network are obtained by closing an increasing number of IVs (12, 15, and 16) and indicated as C2, C3 and C4, for a total of nc = 4 configurations considered, including the reference one. The topological structure of the new configurations is then characterised by the indicators adopted before. The dynamic responses of the simple network in the modified layouts (i.e. C2, C3 and C4) subjected to the same instantaneous manoeuvre at node 86 (which had induced the pressure state in the reference network shown in Figure 2) is obtained through the numerical model and result to be different compared to the response of the network in configuration C1. This is evident in Figure 4, Figure 5 e Figure 6, where the results of the numerical simulations in terms of δp observed in all the nodes are reported when configurations C2, C3 and C4 are considered. Specifically, a greater number of nodes are affected by more significant δp as the number of branches increases and interconnections decrease. In fact, if in the case of the looped configuration (i.e. C1) the nodes are stressed by δp that remains in the order of 1 m (Figure 2), considering a completely branched configuration (i.e. C4), almost all the nodes are stressed by δp of 5 m (Figure 6). Figure 3 compares the pressure dynamic responses of the simple network in the nc configurations in terms of cumulative frequency of δp. As the number of branches in the network increases, the cumulative frequency δp tends to shift to the right. The reciprocal position of the curves - and therefore the values of δp_{50} and δp_{90} - highlights how, geometric and mechanical characteristics being equal, the topological structure influences the dynamic response of the system and, in particular, that the state of pressure stress of the system in terms of δp increases as the number of loops decreases.

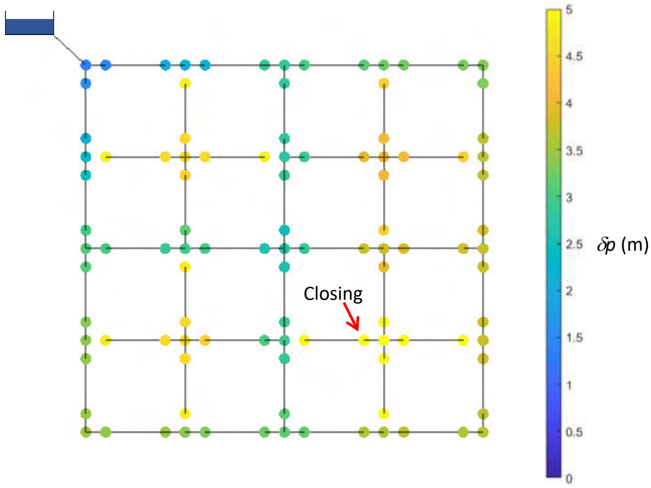

Figure 4 Results of unsteady flow simulation of the network in configuration C2 subjected to an instantaneous closure at node 86 in terms of δp at each node.

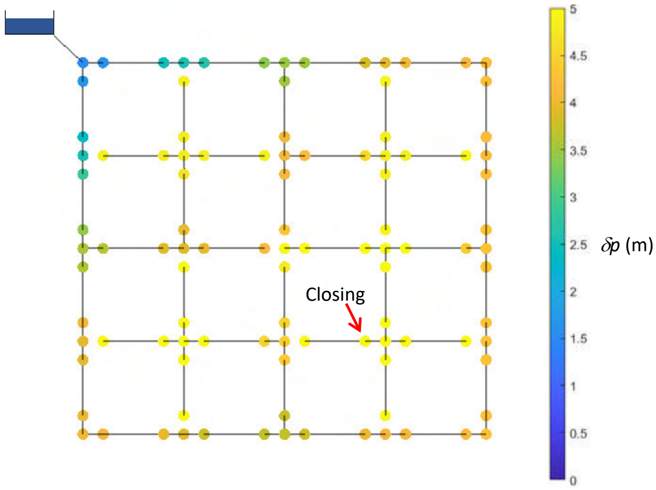

Figure 5 Results of unsteady flow simulation of the network in configuration C3 subjected to an instantaneous closure at node 86 in terms of δp at each node.

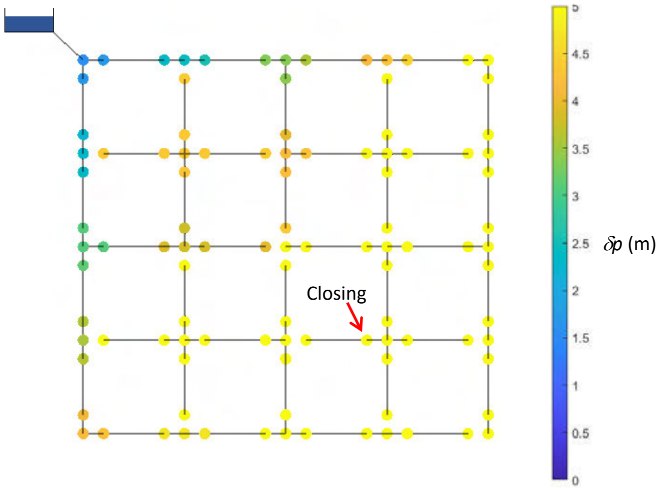

Figure 6 Results of unsteady flow simulation of the network in configuration C4 subjected to an instantaneous closure at node 86 in terms of δp at each node.

The results of the unsteady flow simulations of the nc configurations of the simple network in terms of the dimensionless response of the system at the 50th and 90th percentiles, $\frac{\delta p_{50}}{\delta p_{50,ref}}$ and $\frac{\delta p_{90}}{\delta p_{90,ref}}$, are related to the four connectivity metrics (k, R_m, $\Delta\lambda$ and λ_2) and the result of this evaluation is shown in Figure 7, where the red stars indicate the simple network in its reference configuration whose response in terms of $\frac{\delta p_{50}}{\delta p_{50,ref}}$ and $\frac{\delta p_{90}}{\delta p_{90,ref}}$ is unitary.

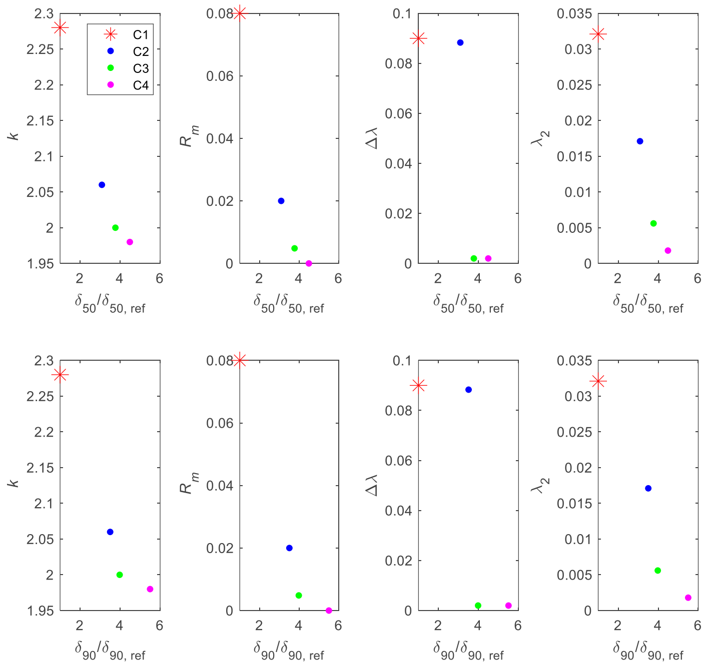

Figure 7 Connectivity metrics (k, R_m, Δλ and λ_2) as a function of the dimensionless dynamic responses of the simple network considered in the nc configurations. The red stars indicate the network in its reference configuration (i.e. C1).

Firstly, it can be observed that as the number of the branches in the network increase (from configuration C1 to C4), k, R_m, $\Delta\lambda$ and λ_2 tend to decrease. Moreover, it is evident that the average degree k, the meshed-ness coefficient R_m and the algebraic connectivity λ_2 are the most effective metrics in reflecting the dynamic behaviour of the system. They indicate, through regular fronts, how as the number of branches increases (so as k, Rm and λ_2 decrease), and therefore as the redundancy of the network decreases (i.e. the network has a lower number of loops), the dynamic response of the network subjected to a single instantaneous manoeuvre is emphasised up to about 4.5 times compared to the average response $\delta p_{50,ref}$ of the reference configuration (i.e. C1) and about 5.5 times compared to the extreme response $\delta p_{90,ref}$. The spectral gap $\Delta\lambda$, on the other hand, is not representative of the dynamic behaviour of the network as its configuration changes and responds with a scattered distribution.

Although it does not emerge in the current case, it is worth stressing the limitation of the metrics k and R_m in distinguishing configurations that are different from the point of view of the adjacency matrix but have the same number of nodes n and links m.

The effectiveness of some connectivity metrics to represent the dynamic behaviour of the network in different configurations could be applied in the evaluation of the impact of a solution of sectioning of the system on the state of stress to which it will be subjected once the modifications have occurred. The use of connectivity metrics could compensate for the execution of a significant number of simulations under unsteady flow conditions, expensive from a computational point of view, since the dynamic response of the network that presents a certain connectivity, and therefore a certain metric, can be easily approximated after the classification of a very small number of exact solutions, perhaps evaluated for "extreme" network configurations, i.e. highly looped or highly branched layouts.

4 CONCLUSIONS

In this study, the effectiveness of some connectivity indicators to represent the effects of the change of the topological structure of a WDN on the resulting dynamic response of the system is evaluated. To this end, four connectivity metrics from graph theory (i.e. average degree k, mesh-ness coefficient R_m, algebraic connectivity λ_2 and spectral gap $\Delta\lambda$) are compared and evaluated in terms of the ability to represent the dynamic behaviour of a WDN. Specifically, this network is subjected to an instantaneous manoeuvre at a node, and its dynamic behaviour is obtained through a numerical model based on MOC, in which the topological structure is modified by closing an increasing number of IVs, thus reducing the number of loops. The results show that k, R_m and λ_2 are the most effective metrics in reflecting macroscopically the dynamic behaviour of the system, while $\Delta\lambda$ is not representative and tends to respond with a scattered distribution. The ability of the metrics considered to represent the dynamic behaviour of a WDN as the configuration changes could be exploited to evaluate the impact of different solutions of sectioning of the system on the stress to which it will be subjected compared to the original configuration. The application of the approach proposed to more complex WDN is the main future objective.

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