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WHY AREN'T SURROGATE RELIABILITY INDICES SO RELIABLE? CAN THEY BE IMPROVED?

Joaquim José de Oliveira Sousa[1], João Muranho[2], Marco Amos Bonora[3], Mario Maiolo[4]

[1]Polytechnic Institute of Coimbra, Coimbra, Portugal
[2]University of Beira Interior, Covilhã, Portugal
[3]Virtual Laboratory of Smart Water Management, NTT DATA Italia S.p.A., Cosenza, Italy
[4]Virtual Laboratory of Smart Water Management, University of Calabria, Cosenza, Italy

[1] *jjoseng@isec.pt*, [2] *jmuranho@ubi.pt*, [3] *marcoamos.bonora@unical.it*, [4] *Mario.maiolo@unical.it*,

Abstract

Water distribution networks are known to be costly infrastructures. A few decades ago, the research efforts concerning water distribution network design were focused on economic aspects and the goal was to obtain the least-cost solutions. Beyond economics, these infrastructures must mostly be reliable since they provide an essential service to society. But reliability assessment is a complex task and involves various aspects: mechanical, hydraulic, water quality, and water safety, among others. This paper focuses on hydraulic reliability.

As hydraulic reliability is computationally hard to measure directly, researchers came up with surrogate measures, like entropy and the resilience index. But these surrogate measures had some flaws and researchers quickly started suggesting new ones trying to avoid those known flaws, like the diameter-sensitive flow entropy or the modified resilience index. But even these new approaches are still not so reliable to be used in the design of water distribution networks.

This paper presents a performance analysis of the resilience index and a modified version as reliability surrogate measures, supported by illustrative examples. A new version of the resilience index is also proposed, introducing additional coefficients in the attempt to overcome some of the flaws of the previous versions. Some results are presented to compare the performance of the new index with those from the previous versions.

Keywords
Water distribution network design, Reliability, Entropy, Resilience index.

1 INTRODUCTION

The increase of water resources demand due to the development of inhabited areas and production processes, and, concomitantly, the progressive reduction in the availability of water resources due to climate change, impose the necessity for optimization and improvement of the existing infrastructure.

To assess the functioning regime performances, the network managers and the scientific community use synthetic indicators. These indicators numerically describe one or more intrinsic characteristics of a distribution network.

Very often water distribution networks (WDNs) designers refer to the concept of Reliability.

Kaufmann *et al.* [1] define reliability as the probability that the system will perform its specified tasks under specified conditions and during a specified time. Cullinane *et al*. [2] and Goulter [3] define WDN reliability as the ability of the system to meet the demands. In several studies, it is defined as the weighted time-averaged value of the ratio of the flow delivered to the flow required by the users [5,6].

Ciaponi *et al*. [7] define reliability as the ability of a WDN to satisfy users in all possible operating conditions. Reliability is assessed using the ratio between the volume of water delivered to users and the one requested in a given period.

Todini [8] considers the concept of reliability as not completely defined and influenced by numerous factors that are difficult to define. The author introduces the concept of resilience as the system's ability to overcome failures. The formulation allows estimating resilience without the need to analyse all types and combinations of possible failures. An increase in resilience leads to an increase in reliability.

Prasad *et al*. [9] consider the network resilience to be representative of reliability. Network resilience is based on the resilience index proposed in [8]. The resilience index has been modified to reward the presence of loops with similar pipe diameters.

Di Nardo *et al*. [10] consider reliability as an indicator difficult to define, due to the uncertainties affecting the WDN operating conditions knowledge. The authors consider the robustness a better metric. Robustness is defined as the ability of the system to maintain a certain level of service in the presence of unfavourable operating conditions.

Muranho *et al*. [11] define reliability as the ability to satisfy the water demand with sufficient pressure, even in the case of critical operating scenarios. The authors present a comparison between surrogate measures of reliability: resilience [8], network resilience [9], and flow entropy [12,13,14]. A new reliability surrogate measure is presented, the WNG Index (WaterNetGen Index). The index represents the ability to satisfy the water demand in the presence of a pipe failure. The authors present a design methodology that identifies the optimal solution with the maximum reliability subjected to a certain budget. The study shows no strong correlations between the analysed indices (resilience, network resilience and flow entropy) and reliability, but entropy maximization shows an improvement in reliability.

This paper presents a new formulation for the resilience index that keeps the simplicity of the original one but, at the same time, takes into account some network characteristics. Many of the commonly used resilience indices are based on an energy balance, considering the surplus of energy to be dissipated in the event of failure as an indicator of robustness. However, these formulations do not take into account the network topology. In some scenarios, this approach tends to overestimate the network resilience schemes, particularly in tree-shaped networks with a good pressure regime.

The new index presented here is based on the formulation of the resilience indices presented in [8] and [15-18]. Three weight coefficients are integrated within the classic formulation. These coefficients take into account the topology of the network, the importance of the nodes and the uniformity of the diameters of the pipes supplying each node.

2 MATERIALS AND METHODS

2.1 Todini resilience index

[8] introduces the concept of resilience as a surrogate measure of robustness. Robustness is a measure of the system's ability to overcome failures. The author defines the resilience index and the failure index as design metrics. The optimal design scheme is achieved by maximizing resilience and limiting the cost. [8] builds the resilience index from an energy balance.

The Todini resilience index, like many other known ones, refers to the "requested" or "design" conditions. These conditions are water demand and piezometric head values that must be reached to ensure proper network functioning. The requested conditions and variables are indicated by an asterisk apex.

The formulation is based on the concept that a network that has a pressure surplus is more robust in case of pipe breaks or anomalous hydraulic events. The resilience index, I_r - equation (1), is structured as the complement to the ratios between the power/energy dissipated in the WDN (P_D*) and the maximum dissipable value (P_{Dmax}) to meet the target/design values.

$$I_r = 1 - \frac{P_D^*}{P_{D\,max}} \quad (1)$$

The total available power (P_{tot} - equation (2)) at the entrance of a WDN is:

$$P_{tot} = \gamma \sum_{k=1}^{r} Q_k H_k + \sum_{j=1}^{p} P_j \quad (2)$$

in which:

- γ water specific weight;
- r number of sources (tanks, reservoirs);
- Q_k source discharge (flow entering the network);
- H_k source piezometric head;
- p number of pumps;
- P_j pump power.

The global minimum output power (P_{Emin} - equation (3)) is the global sum of the power that must be delivered at each demand node to satisfy the design piezometric head and demand:

$$P_{E\,min} = \sum_{i=1}^{n} p_i^* = \gamma \sum_{i=1}^{n} q_i^* h_i^* \quad (3)$$

in which:

- n number of network nodes;
- p_i^* design power of the i^{th} node;
- q_i^* design water demand of the i^{th} node ;
- h_i^* design piezometric head of the i^{th} node.

The maximum dissipable power (P_{Dmax} - equation (4)) is the highest power that can be used without compromising the accomplishment of the design values:

$$P_{D\,max} = P_{tot} - P_{E\,min} = \left(\gamma \sum_{k=1}^{r} Q_k H_k + \sum_{j=1}^{p} P_j\right) - \gamma \sum_{i=1}^{n} q_i^* h_i^* \quad (4)$$

The total amount of actual power delivered to the demand nodes (P_E - equation (5)) is:

$$P_E = \gamma \sum_{i=1}^{n} q_i h_i \tag{5}$$

in which:

- q_i water delivered to the i^{th} node;
- h_i piezometric head of the i^{th} node.

The total amount of power dissipated in the network ($P_D{}^*$ - equation (6)) to satisfy the total demand is:

$$P_D^* = P_{tot} - \gamma \sum_{i=1}^{n} q_i^* h_i \tag{6}$$

The resilience index (I_r - equation (7)) can be written as:

$$I_r = \frac{\sum_{i=1}^{n} q_i^* (h_i - h_i^*)}{\sum_{k=1}^{r} Q_k H_k + \sum_{j=1}^{p} \frac{P_j}{\gamma} - \sum_{i=1}^{n} q_i^* h_i^*} \tag{7}$$

2.2 Di Nardo *et al.* alternative resilience index

Di Nardo *et al.* [15-18] present an alternative formulation for the resilience index, equation (8).

$$I_R = 1 - \frac{P_D}{P_{D\,max}} \tag{8}$$

The index uses the total dissipated power (P_D - equation (9)) instead of the amount of power dissipated in the network. The total dissipated power is obtained as:

$$P_D = \gamma \sum_{j=1}^{m} q_j \Delta h_j \tag{9}$$

in which:

- m total number of pipes in the network;
- Δh_j piezometric head dissipated along the j^{th} pipe;
- q_j flow along the j^{th} pipe.

The total dissipated power (P_D - equation (10)) can be obtained as:

$$P_D = P_{tot} - P_E \tag{10}$$

The resilience index (I_r - equation (11)) is:

$$I_R = 1 - \frac{P_D}{P_{D\,max}} = \frac{\sum_{i=1}^{n}(q_i h_i - q_i^* h_i^*)}{\sum_{k=1}^{r} Q_k H_k + \sum_{j=1}^{p} \frac{P_j}{\gamma} - \sum_{i=1}^{n} q_i^* h_i^*} \tag{11}$$

The use of resilience indices in WDN design rewards networks with an energy surplus that can be dissipated in the event of a failure or an increase in user demand. The limitation of the resilience indices is that their assessment does not take into account the network topology or the necessary connectivity and pipe diameter balance, assumes that nodes without demand do not contribute to the reliability and every node with demand has the same importance. In some cases, a tree-like network topology, obviously not very resilient to failure, with a high enough piezometric head surplus can obtain high resilience values.

2.3 Network resilience index

[9] present an alternative formulation of the Todini resilience index (I_r) which reward the presence of loops in the network, penalizing sudden changes in diameter. To take into account the variability of the diameter, the authors define a uniformity coefficient (C - equation (12)):

$$C_i = \frac{\sum_{j=1}^{npi} D_j}{npi * \max(D_j)} \tag{12}$$

in which:

- C_i uniformity coefficient for the i^{th} node;
- npi number of pipes connected to node i;
- D_j diameter of pipes connected to node i.

The coefficient gets a value $C = 1$ if pipes connected to a node have the same diameter and $C < 1$ if pipes connected to a node have different diameters. The weighted surplus power combines the effect of surplus power and nodal diameter uniformity. The weighted surplus power for node i (X_i - equation (13)) is:

$$X_i = C_i p_i = C_i \gamma q_i (h_i - h_i^*) \tag{13}$$

where p_i is the surplus power of the i^{th} node.

The total weighted surplus power (X - equation (14)) is:

$$X = \sum_{i=1}^{n} X_i = \sum_{i=1}^{n} C_i \gamma q_i (h_i - h_i^*) \tag{14}$$

The network resilience (I_r - equation (15)) is:

$$I_r = 1 - \frac{X}{X_{max}} = \frac{\sum_{i=1}^{n} C_i q_i (h_i - h_i^*)}{\sum_{k=1}^{r} Q_k H_k + \sum_{j=1}^{p} \frac{P_j}{\gamma} - \sum_{i=1}^{n} q_i h_i^*} \tag{15}$$

where X_{max} is the maximum surplus power. It is assumed that the nodal design demand is fully satisfied ($q_i^* = q_i$). This version of the reliability index intends to take into account the pipe diameter balance. However, the uniformity coefficient is computed from all the pipes connected to a node (in and out) and not only from the pipes supplying the node (in). A node that is supplied by one single pipe (like in a tree-like network) may present a good uniformity coefficient and consequently a good resilience index.

3 WEIGHTED RESILIENCE INDEX

This paper presents a set of three coefficients that modify the weight of each WDN junction in the resilience index assessment. The three coefficients represent characteristics of the network which are not commonly taken into account by the classical formulation of the resilience indices.

The *Topological coefficient* takes into account the network topology. This coefficient aims to penalize the junctions that have a low number of connections in the resilience assessment. In order not to add complexity to the calculations using graph theory algorithms, this coefficient is calculated knowing only the number of pipes supplying each node.

The *Importance coefficient* defines a hierarchy of importance of the pipes. The break of main pipes carrying larger volumes of water has a greater negative impact on the WDN hydraulic behaviour.

The *Uniformity coefficient* takes into account the pipes' diameter uniformity. A network for which there are several pipes of similar diameter connected to each junction reacts better in case of the failure of one of them.

3.1 Topological coefficient

The *topological coefficient* reduces the contribution of the junctions for which there is a single entering pipe. It is assumed that junctions with a single entering pipe contribute less to the network resilience.

The *topological coefficient* (K^T - equation (16)) is a multiplicative coefficient that can assume values between 0.5 and 1.5 and can be estimated as:

$$K_j^T = 0.5 + \frac{N_{in}^j - 1}{N_{in}^j} \tag{16}$$

where:

- N_{in}^j: Total number of pipes entering junction j.

This coefficient aims to reduce the flaw in the resilience assessment for tree-like networks. This type of network is notoriously non-resilient (the break of a pipe completely stops the supply of water for all the downstream pipes). The calculation of Todini's resilience index for a tree-like network can lead to misleading results (high resilience) in the presence of a fairly high pressure surplus.

The presence of a coefficient that tends to reduce the importance of the junctions for which there is a low connection redundancy should considerably reduce the resilience of the tree networks, reflecting more the reality.

As defined, this coefficient assigns a reduction value (0.5) to the configurations in which the network junctions are supplied by a single pipe. For configurations in which two pipes are supplying a junction, the coefficient is unitary. The increase in the number of incoming pipes has a progressively smaller effect on the improvement of resilience. Due to its mathematical structure, the coefficient has an upper limit of 1.5 (Figure 1).

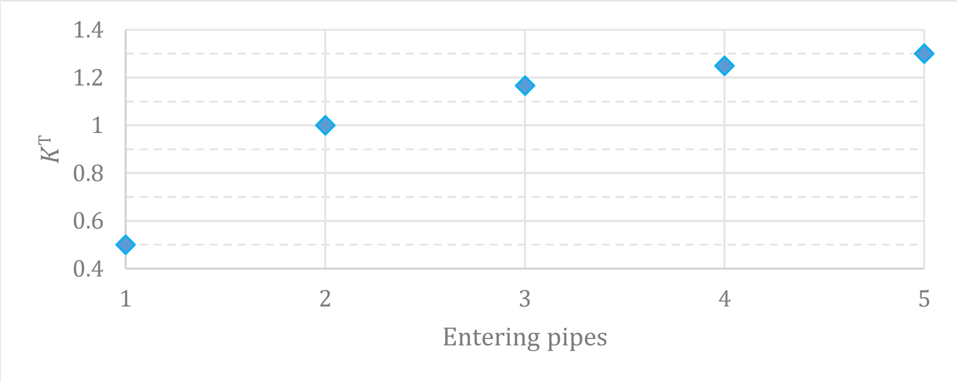

Figure 1. The topological coefficient variation.

3.2 Importance coefficient

The *importance coefficient* is a multiplicative coefficient that assumes values lower than 1. In drinking WDNs, not all junctions have the same importance. This coefficient is based on the assumption that the junctions through which a greater flow passes are more important for the network functioning, and therefore their resilience (or lack of it) is more impactful overall.

The *importance coefficient* (K^I - equation (17)) can be estimated as:

$$K_j^I = \frac{\sum_{i=1}^{N_{in}^j} Q_{in_i}^j}{Q_{in}^{MAX}} \tag{17}$$

where:

- N_{in}^j: Total number of pipes entering junction j;
- $Q_{in_i}^j$: Flow of the i^{th} pipe that enters junction j;

The denominator of the formula refers to the maximum value among all the flows entering the network junctions (Q_{in}^{MAX} - equation (18)):

$$Q_{in}^{MAX} = \max\left(Q_{in}^1, \dots, Q_{in}^j, \dots, Q_{in}^n\right) \tag{18}$$

where:

- Q_{in}^i: Flow entering junction *i*.

The presence of a coefficient that weights more on the junctions through which more flow passes, allows to better take into account the areas near the tanks or main pipelines in the resilience assessment. A junction located in the peripheral area of the network has a marginal impact compared to the water mains near a tank.

3.3 Uniformity coefficient

In a WDN, the connection redundancy does not ensure resilience by itself. The *uniformity coefficient* is based on the assumption that the pipes converging into a junction are effectively redundant, and therefore resilient, the more their diameters are similar.

The *uniformity coefficient* is a multiplicative coefficient that rewards the uniformity of the diameters and penalizes situations in which the diameters of the incoming pipes are very different because they are not very resilient. In general, it will assume values between 0 and 1, but in specific situations it can surpass 1.

The *uniformity coefficient* (K^U - equation (19)) can be assessed as:

$$K_j^U = \frac{\sum_{i=1}^{N_{in}^j} \left(D_{in_i}^j\right)^2}{MIN(N_{in}^j, 2) * \left(MAX(D_{in_i}^j)\right)^2} \tag{19}$$

where:

- N_{in}^j: Total number of pipes entering junction j;
- $D_{in_i}^j$: Diameter of the i^{th} pipe that enters junction j.

The structure of the *uniformity coefficient* is similar to that presented by [19] but here it takes into consideration only the pipes entering the junctions and not the diameter but its square because the pipe section is proportional to the square of the diameter. As it is formulated, the coefficient assumes unitary values when there is one single pipe entering the junction (in this case the resilience index is reduced by the topological coefficient) or when the two pipes entering a junction have the same diameter, and it can present lower or higher values in other situations.

3.4 Weighted resilience indices

The *Topological*, *Importance* and *Uniformity coefficients* are three dimensionless multiplicative coefficients that can be integrated into different formulas for the assessment of resilience indices. The coefficients are calculated for each junction and multiply the numerator of the formula. In general, these coefficients reduce the numerator, but in specific situations may increase it. As they are formulated, a resilience index that integrates these coefficients should be lower than the original one.

Equations (20) and (21) respectively show the suggested new formulations for the resilience index of [8] and [15-18].

$$I_r = \frac{\sum_{i=1}^{n} (K_i^I K_i^T K_i^U)\, q_i^* (h_i - h_i^*)}{\sum_{k=1}^{r} Q_k H_k - \sum_{i=1}^{n} q_i^* h_i^*} \tag{20}$$

$$I_R = \frac{\sum_{i=1}^{n} (K_i^I K_i^T K_i^U)(q_i h_i - q_i^* h_i^*)}{\sum_{k=1}^{r} Q_k H_k - \sum_{i=1}^{n} q_i^* h_i^*} \tag{21}$$

4 CASE STUDIES

The network used in the simulations serves the zone of Villa Rosa. This area is an isolated portion of the network serving the area called Northwest System, in the city of Tampa (Florida, USA). The served area covers approximately 2 square kilometres and is a residential area. The topography of the area is very regular. The junctions of the network have an average elevation of 17.6 m (and are between 16.4 and 19.62 m). The network is supplied by a reservoir which represents the connection of the subnet to the water main of the network. The reservoir has a total head of 54.75 m and supplies 162.95 l/s to the network.

4.1 The three scenario networks

Three WDNs (depicted in Figures 1, 2 and 3) called Normal, Looped and Treelike were used to test the performance of the proposed resilience index and compare it with the original version.

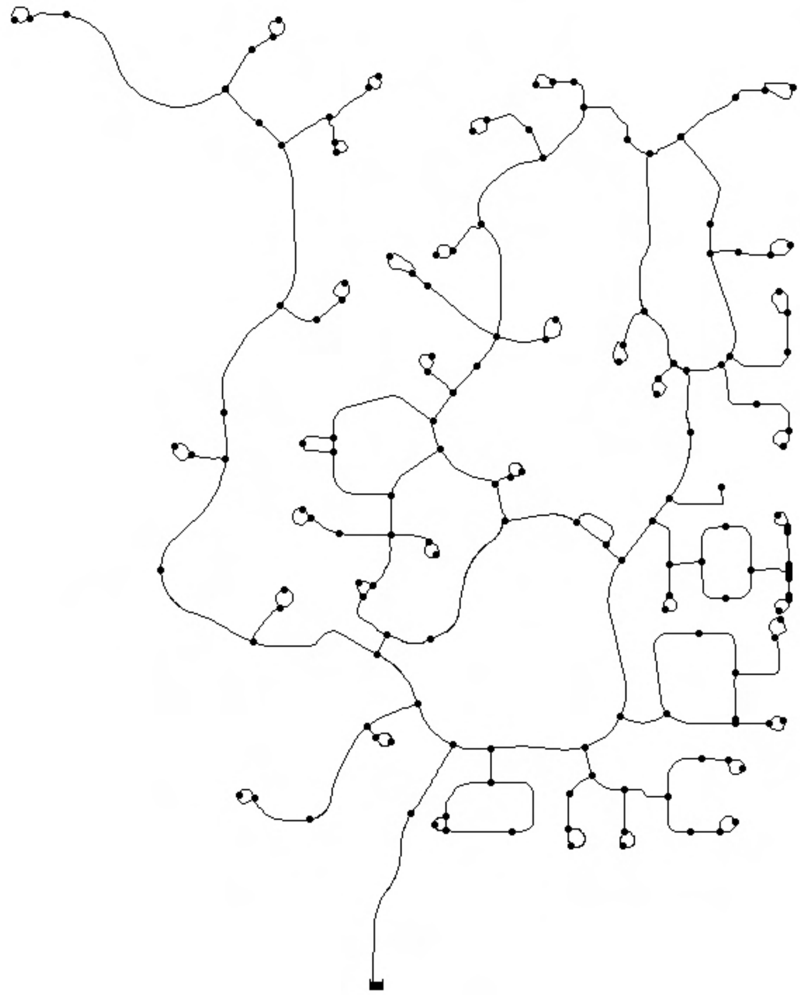

Figure 2. Model of the Villa Rosa neighbourhood WDN in the city of Tampa, Florida - Normal.

The Normal variant is the network that currently supplies Villa Rosa. The other two networks are based on the Normal one but have been modified to test the effect of the weight coefficients on the resilience index.

Table 1. The number of junctions and pipes in the three variants of the Villa Rosa network.

	Normal	Looped	Treelike
Number of Junctions	163	180	163
Number of Pipes	208	254	163

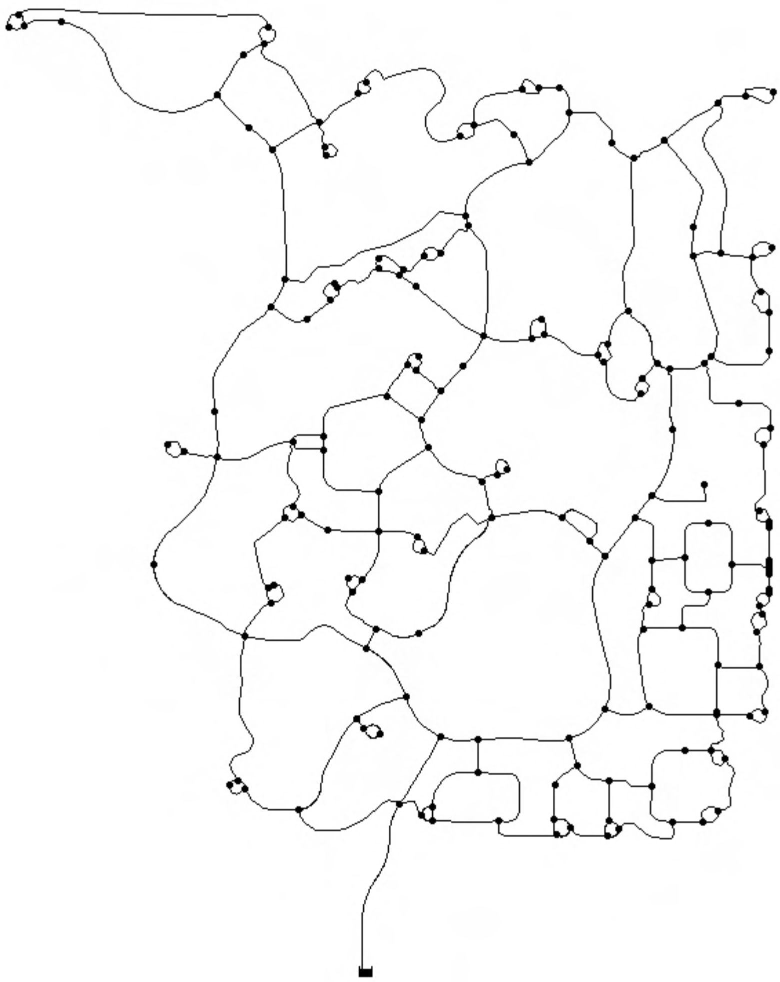

Figure 3. Model of the Looped variant of the Villa Rosa WDN.

To build the Looped network, 17 junctions and 46 pipes were added to the network currently in operation. The pipes added to the network were chosen to maximize the number of loops within the network. The diameter of the added pipes was chosen consistently with the diameters currently present. The 17 junctions added to the network have the function of simplifying the connection of the new pipes and do not modify the distribution or the value of the water demand.

To build the Treelike network, 45 pipes from the network currently in operation were deleted. Pipe loops and some connections between different areas of the network were removed. This operation made it possible to build a completely treelike network (each junction is supplied by a single upstream pipe). Also, in this case, the changes to the network did not change the distribution or the value of the water demand.

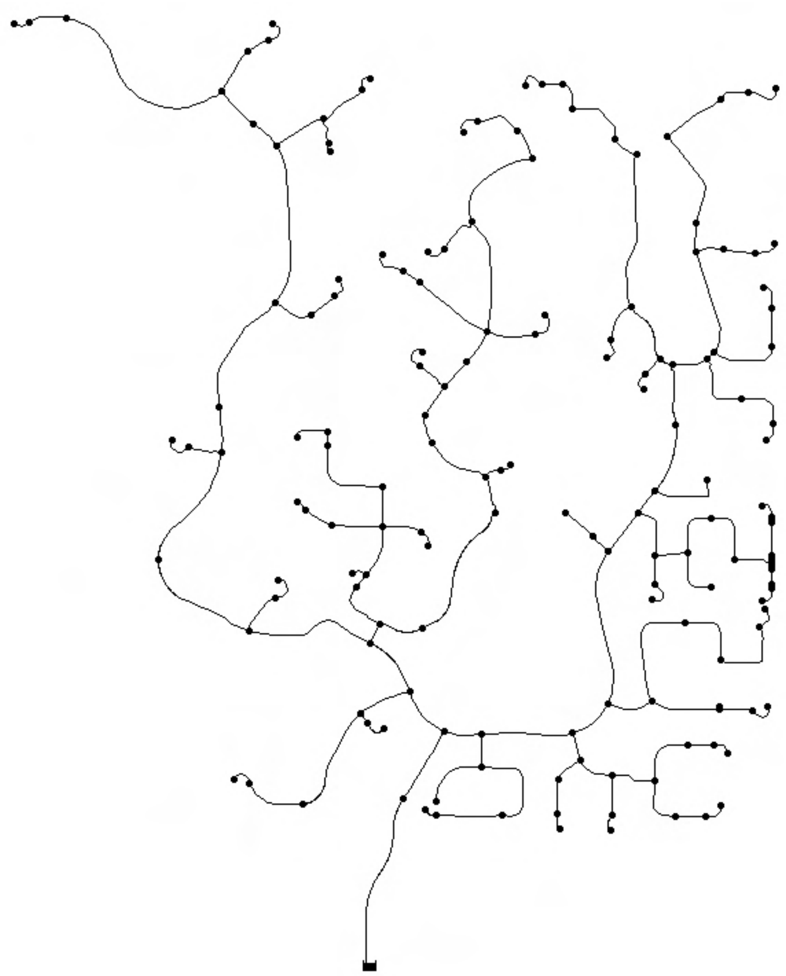

Figure 4. Model of the Treelike variant of the Villa Rosa WDN.

4.2 Resilience indices assessment

The assessment of the resilience indices requires knowledge of the WDN operating regime and a set of information that describes the design or required conditions. Design conditions are used in many of the resilience indices found in the literature [8-9,15-18]. The literature in most cases does not investigate the methodology or the criterion used to assess these required values.

Authors have investigated on several occasions the necessary hypotheses and simplifications useful for estimating the conditions required in different application cases [20-22].

Taking into account the purpose of this work, authors used a simplified approach, defining a single requested pressure value. This simplifying hypothesis does not overly trivialize the results thanks to the homogeneity of the served area. The Villa Rosa neighbourhood is almost entirely residential. The supplied dwellings are small, single-storey houses.

Many of the literature indices use a simplified approach by defining a single required pressure value (*P**) for the entire network [9,15-18]. In the cited bibliography the requested demand value is not defined, assuming that the supply always corresponds to the demand. This hypothesis was also used here. Under this hypothesis, the index defined by di Di Nardo *et al.* and the one defined by Todini are the same.

The resilience indices were assessed for the three networks (Table 2). The Villa Rosa network is characterized by a good pressure regime. The absence of pressure deficits makes it possible to achieve a fairly high resilience index value. The Treelike network has a slightly lower I_r value (-

3.09%) while the Looped network has a slightly higher value (+1.79%). The variations are due to changes in the hydraulic operating regime of the WDN.

Table 2. Value of the resilience indices assessed for the three variants of the Villa Rosa network.

$P^* = 20m$	Treelike	Normal	Looped
Resilience index:	0.784	0.809	0.823
Weighted Resilience index:	0.0239	0.0282	0.0295

The weighted resilience index is characterized by lower resilience values. The multiplicative coefficients, which are usually less than unity, reduce the overall resilience value.

An interesting aspect of this index is its sensitivity to topological changes in the network. As known, tree networks are not resilient. In the case in question, the variant of the Villa Rosa network, modified to be a tree network, sees a reduction in the resilience index of 15.25% (the original resilience index was only reduced by 3.09%). Even the Looped network, characterized by a more redundant structure, undergoes an increase of 4.61% in the weighted resilience index (the original resilience index only increased by 1.79%). Tables 3 to 5 present some statistics of the results obtained for the three networks.

Table 3. Statistics on weight coefficients estimated for the Treelike network. The average value of the coefficients and percentage of the coefficients that are lower, higher and equal to 1.

Treelike				
	Average	<1	=1	>1
K_t	0.500	100.00%	0.00%	0.00%
K_u	1.000	0.00%	100.00%	0.00%
K_i	0.065	99.39%	0.61%	0.00%

Table 4. Statistics on weight coefficients estimated for the Villa Rosa (Normal) network. The average value of the coefficients and percentage of the coefficients that are lower, higher and equal to 1.

Normal				
	Average	<1	=1	>1
K_t	0.638	72.39%	27.61%	0.00%
K_u	0.994	1.84%	98.16%	0.00%
K_i	0.067	99.39%	0.61%	0.00%

Table 5. Statistics on weight coefficients estimated for the Looped network. The average value of the coefficients and percentage of the coefficients that are lower, higher and equal to 1.

Looped				
	Average	<1	=1	>1
K_t	0.702	60.00%	38.89%	1.11%
K_u	0.964	13.33%	85.56%	1.11%
K_i	0.063	99.44%	0.56%	0.00%

As from Tables 3-5, a tree network has the most reductive set of coefficients. The topological coefficient has the greatest reduction effect, while the uniformity coefficient is higher since each junction of the network is supplied by a single pipe. Values greater than 1 of the multiplicative coefficients are very rare and only arise in the parts of the network with greater redundancy.

5 CONCLUSIONS

This paper presents a variant of the resilience index proposed in [8] and in [15-18]. Commonly, the indices aim to assess the resilience of a WDN taking into account an energy balance (pressure surplus) and referring to the required conditions. Such an approach is often not very effective in representing resilience in specific cases (i.e. tree-shaped networks) since it does not take into account the topological characteristics.

The variant proposed here allows taking into account directly some aspects that are ignored or indirectly taken into account by the analysed indices. The use of three coefficients allowed to give greater weight to the redundancy of the connections, especially if using pipes with similar diameters, and to differentiate the impact on the resilience of junctions on water mains and the ones connected to smaller pipes.

Three case studies were analysed. The topological changes made to the Villa Rosa network correspond to a decrease (*Treelike*) and an increase (*Looped*) in resilience.

Using the weighted resilience index, as can be seen from the results, a change in the topology of the analysed network produces a greater variation in the resilience index compared to the classic formulations.

The proposed variant was found to be more effective in representing the various aspects that contribute to the resilience of the network. Although authors are convinced that using these coefficients is a step forward in improving the reliability assessment using the resilience index, there are still some issues to solve and those will be addressed in future works.

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