

WATER DISTRIBUTION NETWORK OPERATION OPTIMIZATION: AN INDUSTRIAL PERSPECTIVE

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Abstract

A significant portion of the operating costs associated with drinking water distribution networks is related to energy usage, which is mostly employed to drive pumps. One strategy to improve energy efficiency and to reduce energy cost is to operate water pumps in an optimal manner that allows a reduction in energy consumption. This produces also environmental benefits, since decreasing energy consumption contributes to the reduction of the associated greenhouse gas emissions, helping utility providers to reduce their carbon footprint and to reach sustainability goals. However, numerical optimization of water distribution network operation is a difficult problem to solve, given the combination of non-linear hydraulic dynamics and the presence of discrete decision variables, corresponding to pumps and valves having on/off or open/closed characteristics. In this work, we address such problem from an industrial perspective, reformulating the mathematical program that is at the core of such operation optimization solutions using complementarity constraints to transform the resulting mixed integer nonlinear program into a nonlinear program having only continuous variables. This allow us to obtain a tractable optimization problem, that could be solved in a short amount of time even for large-scale water networks, making it compatible with industrial implementation and real-time optimization.

Keywords

Water distribution networks, water network operation optimization, mixed integer nonlinear programming, nonlinear programming.

1 INTRODUCTION

Given the increasing awareness of environmental problems, resulting in the call for a substantial reduction of greenhouse gas emission, and considering the increasing energy cost, there is a significant benefit coming from a more efficient use of energy. This is particularly true for drinking water distribution networks, where a significant portion of the operating costs is related to energy usage, and up to 70% of that energy is employed to drive pumps [1], [2], [3]. The adoption of optimization algorithms to schedule pump operation in water distribution networks brings several benefits: it helps reducing energy consumption, thus lowering energy cost and the carbon footprint associated with energy use; it contributes to a better control of water pressure across the distribution network, lowering peak pressure and consequent leakage, and mitigating the risk of pipe burst. Energy and cost optimization of water distribution networks represents an important practical problem [4], and optimal pump operation enables utility providers to reduce non-revenue water, to decrease operating costs, to reduce their carbon footprints, and to reach sustainability goals. Moreover, operation optimization problems can easily be formulated to take into account additional cost components like electricity price, energy production from renewable sources, CO₂ production of the current energy mix, allowing utility providers to develop custom control strategies to target their economic and environmental goals.

It is generally known that the numerical optimization of water distribution networks operation is a difficult problem to solve, since it combines the non-linear and non-convex hydraulic dynamics

of water networks with the presence of discrete decision variables, corresponding to pumps and valves having an on/off or open/closed characteristic [4], [5], [6], [7]. Moreover, the optimization of a medium-sized real-world water distribution network over a one-day horizon, already leads to large-scale, mixed integer nonlinear programs (MINLP) that need to be solved in a relatively short amount of time. The practical challenges range from the computational effort required to numerically solve such a class of optimization problem, to the presence of multiple local minima. An additional difficulty comes from the presence of uncertainties associated with water demand, renewable power generation and volatility in energy prices.

For the described reasons, both in the literature and in industrial applications based on mathematical programming approaches, it is common to rely on heuristics to simplify the problem structure, or to improve convergence time of the optimization. Heuristics-based solutions often require a significant number of ad hoc decisions, which negatively affects the feasibility of the resulting solutions in face of uncertainties and its generality, that limits the possibility to easily extend the solution to different water networks without an extensive engineering effort. In addition to deterministic methods based on mathematical programming, since the 1990s, metaheuristic algorithms have been applied to the problem of water distribution network optimal operation, among which we can cite genetic algorithms [8], [9], ant colony optimization [10], and simulated annealing approaches, see e.g. the review paper [4] and the references therein. However, recent years showed an increasing interest for mathematical programming approaches, which are more suitable for real-time control and can now be applied more easily on industrial products thanks to the growth of the available computational power.

In this work, we present the results of the application of nonlinear programming techniques to the problem of pump operation scheduling for medium-sized water distribution networks. We propose an approach developed for industrial case studies, but we demonstrate the potential showing only results pertaining to open-source network models for confidentiality reasons. We discuss the reasoning that led to the current problem formulation, and we highlight the practical challenges. The proposed approach relies on standardized EPANET [11], [12] hydraulic modelling, without creating alternative surrogate models for optimization. Despite the use of standard EPANET description of water networks, our approach allows to automatically generate analytical gradient information relative to the hydraulic model, and it allows to simultaneously obtain dynamics simulation and optimization results for the operation of drinking water distribution network.

2 MOTIVATIONS AND METHODOLOGY

The problem addressed here is how to optimally operate pumps in water distribution network, managing flow, pressure and storages in the network to minimize operational costs, while complying with operational requirements, i.e. satisfaction of customer demands at all network nodes, and capability to maintain nodes pressure above a minimal threshold under all demand conditions. Here, as a starting point, we assume to have at our disposal a network model defined by its topology and hydraulics parameters according to EPANET standard description.

Operational costs could be mathematically described by combining the different elements contributing to the total costs, among which one could consider pumping energy costs, costs related to water losses and water treatments, costs related to planned or unplanned maintenance and reparation works after failures or damages. The operation optimization is then performed on a day-ahead basis, where we want to schedule pumps operation over a 24 hours time horizon. We assume to have at our disposal reliable demand patterns for all network nodes, thus the problem of demand forecast will not be addressed here. Moreover, in our analysis we are not considering water treatment processes since our target is water distribution. As stated in the introduction, we resort to mathematical programming to tackle the problem of pump operation optimization,

where the main control variables are discrete decision variables associated with pumps, which define if a pump is on or off at every given time instant.

In this work, we consider the full operation optimization problem, which includes binary and continuous decision variables in discrete time, resulting in a mixed integer nonlinear program. We then reformulate such optimization problem as a nonlinear program (NLP) using a relaxation-based approach that resorts on complementarity constraints to transform the original MINLP into a more easily tractable NLP. To do so, the binary decision variables associated with pumps operation are substituted by continuous variables, and appropriate slack variables and constraints are added to the problem formulation. This allows us to obtain an optimization problem that can be solved in a limited amount of time even for medium and large-scale network, and that is suitable for the implementation on industrial applications. More details regarding the mathematical programming formulation and the problem relaxation using complementarity constraints are described in Section 2.2.

2.1 Network model

Here, we present the modelling strategy that we adopt to describe the hydraulics of water networks, which sits at the core of the operation optimization algorithm. Water distribution networks can be described by a directed graph $G = (N, L)$ consisting of vertices, or nodes (N), and arcs, or links (L). Here, following the nomenclature commonly adopted in the literature [7], [13] and by EPANET [12], we classify as nodes all tanks (N_t), reservoirs (N_r), junctions and end points/demand points (N_j), while pumps (L_{pu}) and pipes (L_{pi}) are considered links, thus $N = N_t \cup N_r \cup N_j$ and $L = L_{pu} \cup L_{pi}$.

Table 1. Basic notation

Symbol	Explanation	Value	Unit
q_{lt}	Flowrate at link l at time t		m^3/s
q_{pt}	Flowrate at pump p at time t		m^3/s
d_{nt}	Demand flowrate at node n at time t		m^3/s
h_{nt}	Head at node n at time t		m
\bar{H}_n	Constant head at reservoir n		m
Δh_{lt}	Headloss at pipe l at time t		m
Δh_{pt}	Head increase at pump p at time t		m
ρ	Water density	1000	kg/m^3
g	Gravity acceleration	9.81	m/s^2

To obtain a tractable model that can be used for optimization, we adopt a discrete time model, where the considered time horizon is divided into T equidistant intervals indexed by $t \in \{1, \dots, T\}$. Here, we consider an optimization horizon of one day, divided into 24 periods of one hour each, which gives a discretization interval $\Delta t = 1$ hour. This usually corresponds to the discretization with which demand forecast and electricity prices are provided [6], [7].

For each link $l = (i, j), l \in L$, we denote its flow variable with q_{lt} , which is positive if the flow is directed from i to j , and negative otherwise, with $i, j \in N$ representing network nodes. The flow is always non-negative for pumps and for certain pipes allowing only unidirectional flow via check valves. It is customary for water distribution network problems to measure pressure as the sum of geodetic height and elevation difference $\Delta h = \frac{p}{\rho g}$ due to hydraulic pressure. This goes under the name of nodes head, and here it is denoted by h_{nt} , with $n \in N$. The head increase for pumps is defined by Δh_{pt} , with $p \in L_{pu}$.

The water distribution network is then described by a quasi-stationary, discrete-time, hydraulic model [13], and here we present the equations adopted to model the various network elements. More details regarding hydraulic modelling can be found e.g. in [5], [13].

Reservoirs are considered as unlimited sources of water, where the head is always equal to a known constant value \bar{H}_n with $n \in N_r$, resulting in:

$$h_{nt} - \bar{H}_n = 0, \forall n \in N_r. \quad (1)$$

Tanks are modelled via a discrete-time flow balance equation:

$$h_{nt+1} - h_{nt} - \frac{\Delta t}{A_n} \left(\sum_{l \in L^{in}(n)} q_{lt} - \sum_{l \in L^{out}(n)} q_{lt} \right) = 0, \forall n \in N_t, \quad (2)$$

where A_n represents the cross-section area of the tank, and $L^{in}(n)$ and $L^{out}(n)$ are respectively the set of incoming and outgoing links for node n .

The flow balance in node $n \in N_j$, having a given demand profile d_{nt} , is described by:

$$\sum_{l \in L^{in}(n)} q_{lt} - \sum_{l \in L^{out}(n)} q_{lt} - d_{nt} = 0, \forall n \in N_j. \quad (3)$$

Here, we assume that the demand d_{nt} is positive if the water flow is leaving the network at node n , and it is fixed to zero for nodes having no demands.

Friction losses in pipes can be generically described using the formula expressing the head loss as a function of the flow and of the pipe resistance coefficient a_l :

$$\Delta h_{lt} = a_l q_{lt}^B, \forall l \in L_{pi} \quad (4)$$

where B is the generic flow exponent. Different headloss expressions are available, and Hazen-Williams and Darcy-Weissbach formulas are among the most widely adopted both in industry and literature [5], [7]. In the Hazen-Williams formula, (4) becomes:

$$\Delta h_{lt}(q_{lt}) = \frac{10.67 L_l}{C_l^{1.852} d_l^{4.871}} \text{sign}(q_{lt}) |q_{lt}|^{1.852}, \forall l \in L_{pi}, \quad (5)$$

where L_l and d_l are respectively the pipe length and diameter, and C_l is the Hazen-Williams pipe roughness coefficient. In case of the Darcy-Weissbach formula, (4) becomes:

$$\Delta h_{lt}(q_{lt}) = \frac{8 L_l \lambda_l}{g \pi^2 d_l^5} \text{sign}(q_{lt}) q_{lt}^2, \forall l \in L_{pi}, \quad (6)$$

where $\lambda_l = \lambda_l(q_{lt})$ represents the pipe friction coefficient and depends on the Reynolds number, which nonlinearly depends on the flow, see [5], [6], [14], [15], [16] for more details. Additional minor losses in pipes, that could be caused by turbulence induced by the network layout (e.g. bends, fittings, etc.), are given by:

$$\Delta h_{m_{lt}}(q_{lt}) = \frac{\text{sign}(q_{lt})q_{lt}^2}{2g(\pi(d_l/2)^2)^2}, \forall l \in L_{pi}, \quad (7)$$

and can be added to the main headloss term described by (5) or (6) to obtain the total headloss. Thus, for every pipe, it is possible to write the following head balance:

$$h_{it} - h_{jt} - \Delta h_{lt}(q_{lt}) - \Delta h_{m_{lt}}(q_{lt}) = 0, \forall l = (i, j) \in L_{pi}. \quad (8)$$

The pumps head increase Δh_{pt} for fixed-speed pumps is obtained by the head-flow characteristic diagrams of the pump, given the flow defined by the network current operating point. Here, we assume that the pump characteristic curves are defined by a number of operating points provided by the pump constructor or measured by dedicated “experiments”. We then fit a continuous function of the form:

$$\Delta h_{pt}(q_{pt}) = A_p - B_p q_{pt}^{C_p}, \quad (9)$$

obtaining the constants A_p , B_p and C_p describing the head-flow curve for each fixed-speed pump $p \in L_{pu}$. Then, since in the graph $G = (N, L)$ representing the water network pumps are modelled as links, we obtain the following equation:

$$h_{jt} - h_{it} - \Delta h_{pt}(q_{pt}) = 0, \text{ with } \forall p = (i, j) \in L_{pu}. \quad (10)$$

At present time, we are not considering variable speed pumps in the optimization problem, since we use EPANET hydraulic simulation engine to validate the optimization results, and EPANET does not directly support variable speed pumps.

To conclude, pump efficiency can be provided either as a constant value $\eta_p(q_{pt}) = \eta_p$, or as an efficiency curve describing the relationship between efficiency and flowrate, which we then represent using a continuous linear or quadratic function, resulting respectively in $\eta_p(q_{pt}) = B_\eta q_{pt} + C_\eta$ or $\eta_p(q_{pt}) = A_\eta q_{pt}^2 + B_\eta q_{pt} + C_\eta$. Finally, the electric power [W] consumed by the pump is obtained as:

$$P_{pt} = \frac{\rho g q_{pt} \Delta h_{pt}(q_{pt})}{\eta_p(q_{pt})}. \quad (11)$$

2.2 Optimization

The goal of this work is to schedule daily pump operations to minimize the associated operating costs, and to guarantee the fulfilment of demand requirements. To do so, we start by formulating a mixed integer nonlinear optimization problem, having a linear objective function and nonlinear constraints, with both binary and continuous variables, generically described by:

$$\begin{aligned} & \min_x f(x) \\ & \text{s. t. } x_{lb} \leq x \leq x_{ub} \\ & \quad g_{lb} \leq g(x) \leq g_{ub} \end{aligned} \quad (12)$$

where x represent the vector of optimization variables, x_{lb} and x_{ub} its lower and upper bounds respectively. To reduce the computational complexity of (12), we opt for a linear objective function $f(x) = c^T x$, moving all nonlinearities into the constraints defined by $g(x)$. In (12) we use a generic constraints formulation, which contains both equality and inequality constraints. The cost vector c can be easily tuned to account for different forms of operative costs, from pump energy cost to CO2 and non-revenue water cost terms. Here, for the sake of simplicity, we focus on pumping energy as main term of the objective function.

In addition to the constraints derived from the model equations (1)-(10), further constraints can be included in (12) to take into account minimum pressure requirements at nodes and tank level limits, to impose initial and final conditions (e.g. tank initial and final level):

$$h_{nt} - \underline{H}_{nt_{\min}} \geq 0, \forall n \in N_j, \quad (13)$$

$$h_{n0} - H_{n_{\text{initial}}} = 0, \forall n \in N_t, \quad (14)$$

$$h_{nT} - H_{n_{\text{final}}} = 0, \forall n \in N_t. \quad (15)$$

Finally, (12) can then be rewritten as:

$$\begin{aligned} & \min_x c^T x \\ & \text{s. t. } (1) - (3), (5), (7) - (10), (13) - (15), \forall t \in \{1, \dots, T\} \\ & \quad x_{\text{lb}} \leq x \leq x_{\text{ub}} \end{aligned} \quad (16)$$

Considering the presence of discrete variables, which scale with the number of pumps and the number of considered time samples, and the nonlinear, non-convex nature of the hydraulics constraints, the resulting MINLP is notoriously an NP-hard problem [7], [17], which can easily become intractable for medium and large-scale water networks. Considering that, when a pump is turned off, no head increase constraints should be imposed for said pump, those constraints should be removed at every time instant when the pump is not operating. However, from a mathematical perspective, relaxing a constraint is better than removing it, so an approach often adopted in MINLP is the so-called big-M formulation, where the head constraint is relaxed up to a large value M when the corresponding pump is off. This means that in (16) the set of constraints of the type of (10) is replaced by the following constraints:

$$\begin{aligned} -M(1 - \omega_{pt}) \leq h_{jt} - h_{it} - \Delta h_{pt}(q_{pt}) \leq M(1 - \omega_{pt}), \forall p = (i, j) \in L_{\text{pu}} \\ 0 \leq q_{pt} \leq q_{\max} \omega_{pt} \end{aligned} \quad (17)$$

where ω_{pt} is the pump status binary indicator, and M is a constant whose value express how much the head constraint can be relaxed when the pump is not operating. In the literature there are several approaches that could be used to relax the mixed integer problem into a continuous one. Here we adopt a complementarity formulation see e.g. [18], [19], [20], [21], [22]. Using suitable slack variables to construct the complementarity constraints, (17) becomes:

$$\begin{aligned} -s_{pt}^- \leq h_{jt} - h_{it} - \Delta h_{pt}(q_{pt}) \leq s_{pt}^+ \\ s_{pt}^+ q_{pt} = s_{pt}^- q_{pt} = 0 \\ s_{pt}^+ \geq 0, s_{pt}^- \geq 0, q_{pt} \geq 0, p \in L_{\text{pu}}, t \in \{1, \dots, T\} \end{aligned} \quad (18)$$

This formulation allows us to drop the pump binary indicator, transforming the mixed integer nonlinear (MINLP) optimization problem into a nonlinear (NLP) one, adding a penalty function $p(x)$ to the objective function:

$$\begin{aligned} & \min_x c^T x + p(x) \\ & \text{s. t. } (1) - (3), (5), (7) - (9), (13) - (15), (18), \forall t \in \{1, \dots, T\} \\ & \quad x_{\text{lb}} \leq x \leq x_{\text{ub}} \end{aligned} \quad (19)$$

The mathematical program with complementarity constraints in itself is difficult to solve, since standard regularity assumptions are violated and the resulting feasible region is connected only by one point, i.e. the origin. Several approaches have been proposed in the cited literature to solve such issues, including the use of smoothing functions, of penalization terms, or the use of approaches based on enumeration of branches. Here, we implement a complementarity

formulation based on penalisation, see [22]. This formulation provides practical advantages, but comes with a potential drawback, since a local minimum of the complementarity optimization program with penalty reformulation might not be a local minimum of the original problem, see [18]. However, the penalty formulation allows us to solve the operation optimization problem for medium and large-scale water networks (up to 13000 nodes) in a short amount of time, making it feasible for practical implementation and for real-time optimization. In the next section, we present some results to showcase the performance of such formulation on three different water network models.

3 NETWORKS DESCRIPTION AND RESULTS

The complementarity formulation of the pump operation optimization problem was tested on several water network models. Here, we present results related to open-source academic network models, which constitute a good benchmark to showcase the benefits and the practical applicability of the proposed formulation. We selected three network models which, we believe, are representatives of a small-scale (BWSN), a medium-scale (C-Town), and a large-scale (DWES) water distribution network. These network models have been used extensively in the literature as benchmark for various engineering problems, including water distribution network design and operation optimization, see e.g. [23], [24].

3.1 Water networks description

BWSN is a small-scale network with 126 junctions, 2 tanks, 1 reservoir, 168 pipes and 2 pumps. It includes different demand patterns, and pump pressure-flowrate characteristic curves. We added quadratic efficiency curves, obtained by appropriately rescaling the efficiency-flowrate curves of a different network.

C-Town is a medium-scale network with 388 junctions, 7 tanks, 1 reservoir, 429 pipes and 11 pumps, including demand patterns at nodes, and pump pressure-flowrate and efficiency-flowrate characteristic curves.

DWES is representative of a large-scale network, it has 12523 junctions, 2 tanks, 2 reservoirs, 14822 pipes and 5 pumps, with head-flowrate characteristic curves. Also in this case, we included efficiency curves taken from a different network model, which are rescaled to fit DWES's pumps operating regions.

The networks called BWSN and DWES are respectively Network 1 and Network 2 proposed as benchmark in [25]. These network models were created for the Battle of the Water Sensor Networks initiative, an engineering design challenge aimed at addressing the problem of optimal placement of sensors in water distribution systems. The network model called C-Town is based on a real-world medium-sized water network, and was proposed as benchmark for the Battle of Water Calibration Network initiative [26], a challenge regarding the problem of water network model calibration, i.e. the process of comparing model results with measurements, making the appropriate adjustments so that model results and data provide a correct fit.

3.2 Results

The NLP resulting from the use of the complementarity formulation discussed in Section 2.2 is implemented with CasADi, an open-source tool for nonlinear optimization including a symbolic framework for algorithmic differentiation, used to construct gradients, Jacobians and Hessians, which allows rapid and efficient implementation of NLP [27]. IpOpt [28], [29] is then used as solver, as it can be directly called from within the CasADi environment. IpOpt is an open-source solver that implements an interior point line search filter method to tackle large-scale nonlinear optimization problems.

Here, we report an extract of the results of the pump operation optimization. In Table 2, a comparison of the required computational effort for the three network models is presented, measured in terms of number of iterations needed to reach convergence. For every model, Table 2 also lists its dimension in terms of number of nodes and links, of number of variables and constraints, and of number of complementarity constraints.

Table 2. Results of pump operation optimization based on the complementarity formulation.

Name	N. pumps	N. nodes	N. links	N. variables	N. constr.	N. compl. constr.	N. iter.
DWES	4	12527	14831	970855	970704	184	872
C-Town	11	396	444	30965	30873	506	969
BWSN	2	129	178	11433	11249	92	927

Figure 1 shows the topology of C-Town network, while Figures 2, 3 and 4 depict a comparison between the optimized and the non-optimized pump operation of C-Town, showing the flowrate values of pumps over the considered 24 hours.

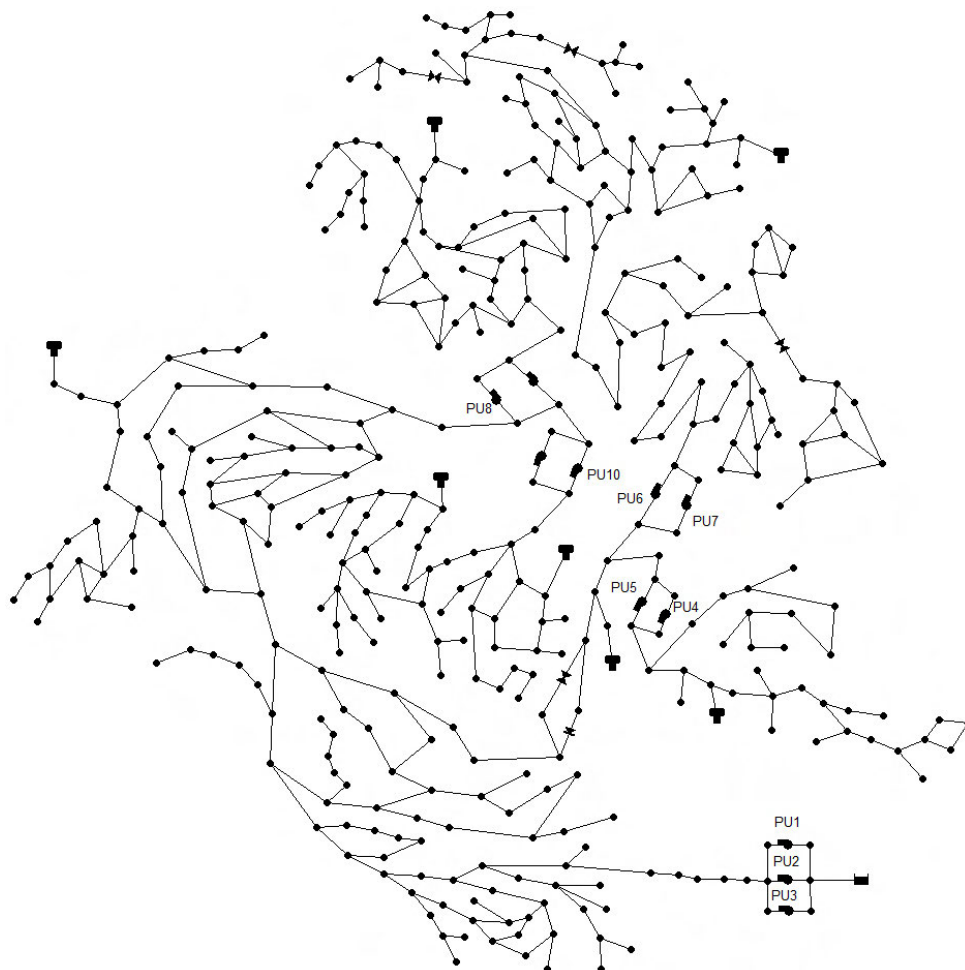


Figure 1. C-Town - Network topology

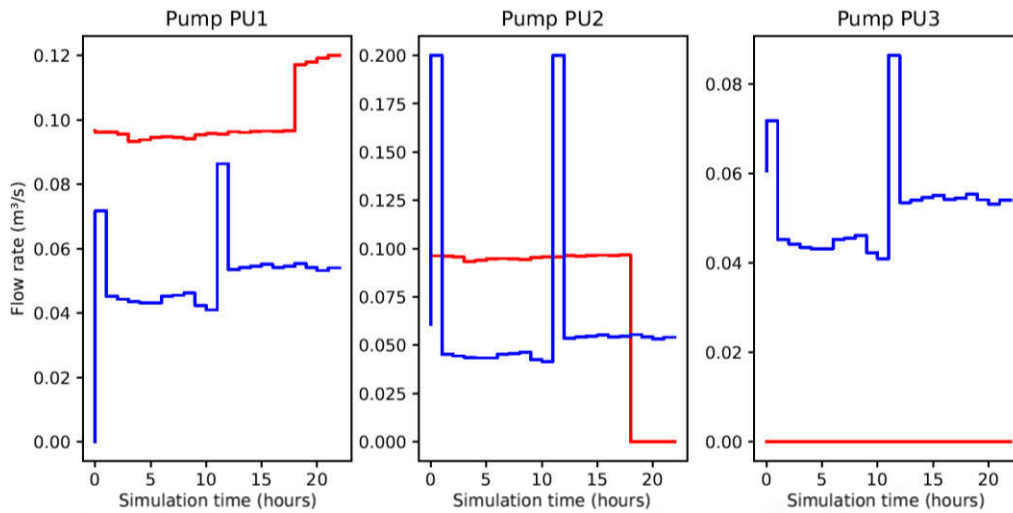


Figure 2. C-Town – Daily schedule for pumps PU1, PU2, PU3. Comparison between pump flowrate with optimized scheduling (blue) and non-optimized operation (red).

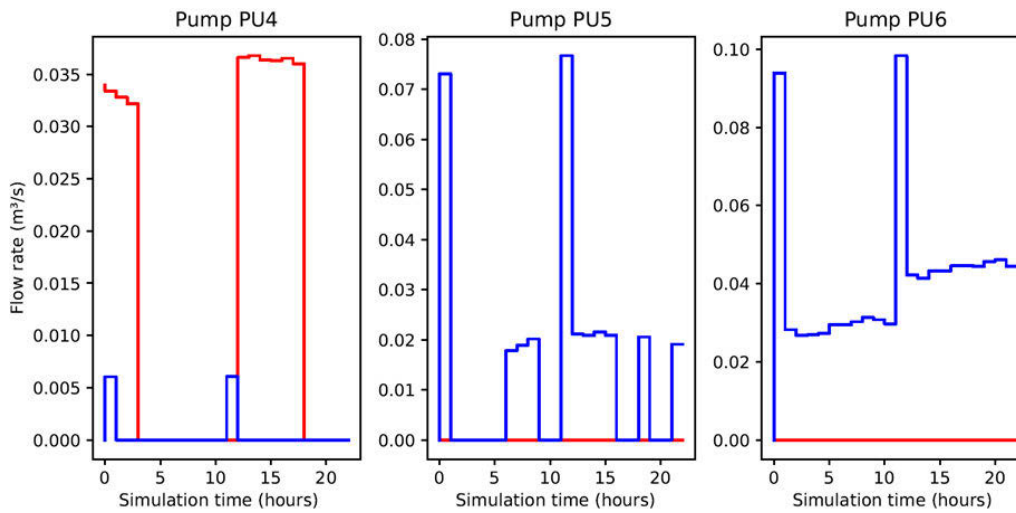


Figure 3. C-Town – Daily schedule for pumps PU4, PU5, PU6. Comparison between pump flowrate with optimized scheduling (blue) and non-optimized operation (red).

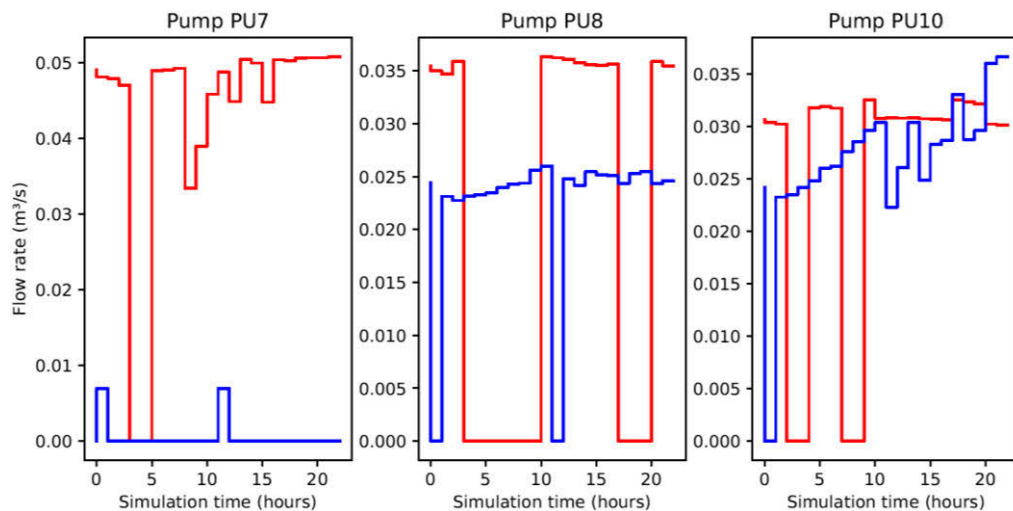


Figure 4. C-Town – Daily schedule for pumps PU7, PU8, PU10. Comparison between pump flowrate with optimized scheduling (blue) and non-optimized operation (red).

4 DISCUSSION

The results presented in Table 2 and in Figures 2-4 provide interesting insights on the benefits and performance of the proposed water distribution network (WDN) operation optimization approach based on a complementarity formulation with penalty terms for the pump constraints, which results in a nonlinear program. This formulation combines the advantages of deterministic optimization approaches based on mathematical programming, with a reduced computational effort that allows its industrial implementation even for large-scale water networks.

From Table 2 it is possible to appreciate that the number of iterations required for the solver IpOpt to converge to a solution scales nicely with the network dimension and with the number of pumps, and the optimization remains tractable even for DWES, the largest water network used here as benchmark. This suggests that the presented approach could be suitable to tackle the operation optimization problem even for large-scale WDN.

The results obtained by applying the complementarity-based optimization to several real-world and open-source water models demonstrated the potential to achieve a reduction of energy consumption and subsequent pumping energy costs that is in line to what described in the relevant literature, e.g. [4] and [7]. The pump flowrate comparison shown in Figures 2-4 exhibits a clear and significant reduction of pump usage when the optimization approach is used, with respect to the non-optimized scenario which is a representative of the current network operation standard. In all the addressed case studies, the use of the presented optimization approach provided energy savings, while guaranteeing satisfaction of water demand and operational requirements. Additional benefits of a reduced pump usage are the increased pump lifetime and less maintenance costs.

5 CONCLUSION

In this work we presented an industrial perspective on the problem of water distribution network operation optimization. A deterministic optimization approach based on mathematical programming with complementarity constraints was proposed, and its application to case studies constructed using open-source WDN models demonstrated the benefits and the performance of the approach. In particular, the presented optimization program achieves a significant reduction of pump usage, lowering energy consumption and the associated costs, while guaranteeing satisfaction of operational constraints. Moreover, the proposed complementarity formulation allowed to move from a mixed integer nonlinear optimization program to a nonlinear one having only continuous variables. This resulted in a tractable problem requiring reduced computational effort, even for large-scale networks. These results demonstrate the potential of the industrial implementation of such optimization approach to medium and large-scale real-world water distribution networks, thus providing a new tool for water utilities, to reduce their energy costs and their carbon footprint.

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