

## DYNAMIC EDGE BETWEENNESS CENTRALITY AND OPTIMAL DESIGN OF WATER DISTRIBUTION NETWORKS

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### Abstract

The multi-objective design of water distribution networks (WDNs) as a nonlinear optimization problem is a challenging task. With two contradicting objectives (e.g., minimizing costs and maximizing resilience), Pareto fronts of optimal solutions can be obtained with, e.g., evolutionary algorithms. However, the main drawback of these algorithms is the high computational effort required to optimize large WDNs. Recently, a highly efficient method based on complex network theory (CNT) was developed, where within seconds, a wide range of Pareto near-optimal solutions can be obtained for the design of WDNs (i.e., determining optimal diameters). The developed method is based on a customized graph measure called demand edge betweenness centrality ( $EBC^Q$ ). This measure is based on the frequency of occurrence of an edge in the shortest path from a source node to a demand node. In addition,  $EBC^Q$  sums up the demands routed through that edge, giving a valid flow estimation for an optimal design. In the graph of a WDN, the edges can have different weights. The weighting function used for  $EBC^Q$  calculations can be 'static' or 'dynamic'. A constant value is utilized for edge weights in the static weighting approach, while a dynamic weighting function implies that edge weights are modified when iterating through all demand nodes. In this context, using dynamic weighting functions for  $EBC^Q$  (i.e., dynamic  $EBC^Q$ ) avoids concentrating  $EBC^Q$  values in just a few edges (shortest-path trees) by considering redundancy in flow paths and better approximation of the hydraulic behavior. However, it is not clear how the parameters of dynamic weighting functions should be defined to achieve the best approximation of the Pareto-optimal front. This work performs a systematic investigation of dynamic weighting functions and gives guidance for optimal parameter selection. The comparative study between the CNT approach (with static and dynamic weights) and evolutionary optimizations on four WDN design problems confirms the capability of the proposed dynamic functions in providing optimal/near-optimal solutions.

### Keywords

Graph theory, Multi-objective evolutionary algorithms, Pareto fronts, Complex network analysis.

## 1 INTRODUCTION

Water distribution networks (WDNs) as pivotal urban infrastructures are comprised of various components interacting in a complex way [1]. Apart from the inherent complexity, major financial investments are required for the construction, operation, and renovation of WDNs. Therefore, various aspects of these infrastructures have come into focus of optimization methods to identify the best solutions ensuring maximum benefits (e.g., resilience, performance) with minimum costs [2]. Evolutionary algorithms (EAs) are the most widely used approaches to solve multi-objective optimization problems in WDNs [3]. However, using EAs for optimal design problems (i.e., selecting optimal pipe diameters), is a major challenge, particularly when considering real-world

large-scale networks [4]. The reason derives from the fact that the design problem is a nonlinear and NP-hard problem [5], which cannot be tackled in a reasonable time frame for large WDNs, as the size of search space grows exponentially [6]. Several attempts have been made to improve the efficiency of EAs by reducing the search space during the optimization procedure [3], [6], [7], [8]. Nevertheless, finding optimal/near-optimal solutions for complex and large WDNs remains a computationally expensive task since multiple runs and numerous function evaluations are still required [9]. Hence, a fast and efficient approach is needed to overcome the limitations of EAs and achieve the best approximation of optimal design solutions. To this end, complex network theory (CNT) approaches are of interest. In CNT, WDNs are converted to mathematical graphs, including sets of nodes (junctions) and edges (pipes), where the relationships between components of complex networks can be described more efficiently. In addition, CNT is applicable to large-scale WDNs (with thousands to millions of pipes) in a relatively short execution time [4].

CNT has been employed in various aspects of analyses of WDNs, such as resilience assessment [10], temporal evolution [11], district metered areas creation [12], water quality analysis [13], or vulnerability assessment [14]. CNT has also gained attention in WDN optimizations. Sitzenfrei et al [15] explored patterns and network characteristics of Pareto-optimal solutions based on different graph measures. They showed that using a centrality metric called edge betweenness centrality (EBC) could drive the layout of low resilient optimal solutions. Giustolisi et al. [16] customized certain centrality metrics according to WDNs characterization to predict hydraulic behavior. One of those customized metrics (i.e., tailored edge betweenness) was later used as a topological measure to narrow down the search space of EAs for pipe re-sizing optimization [3]. Sitzenfrei et al. [17] developed a fast and efficient CNT-based design approach for WDNs, where a range of near-optimal solutions are obtained without conducting hydraulic simulation. The proposed approach is based on a modified EBC measure denoted as demand EBC (EBC<sup>Q</sup>). This measure determines how often a pipe (edge) is part of the shortest path from a reservoir/tank (source node) to a demand node, and it sums up the demands routed through each path. For identifying shortest paths, the edges of a graph are weighted, and often the Euclidean length is used as the edge weighting function. However, weighting functions can be constant (i.e., static) or changed iteratively (i.e., dynamic), which could result in obtaining different design solutions.

It was shown that choosing proper dynamic weighting functions could noticeably improve the quality of design solutions compared to the results obtained with static weighting functions [18]. However, no systematic attempts have been made to discover the potential of using different dynamic weighting functions. To fill this gap, two dynamic weighting functions for WDN design are proposed and their parameters are tailored to reproduce hydraulic behavior of WDNs. The proposed functions are tested in four case studies, including three known benchmark problems and one real-world large network, and the results are compared with those obtained with multi-objective optimization using EAs. The outcome of this paper gives a better insight into applying proper dynamic weights for the optimal design of WDNs, leading to a move toward higher quality design solutions based on CNT.

## 2 MATERIAL AND METHODS

In the first section of methods (2.1), graph measures based on CNT used for designing WDNs are described, and the procedure is explained with a simple illustrative example. In section 2.2 the tool and literature used to find multi-objective optimization results based on EAs are introduced. Also, detailed descriptions of considered case studies are provided in section 2.3.

### 2.1 WDN design using CNT

#### Graph measures used for CNT



In CNT, urban water infrastructures are presented as a mathematical graph  $G$  consisting of sets of vertices/nodes ( $\#N$ ) connected via a set of edges/pipes ( $\#E$ ). In a WDN, demand nodes (sinks) and source nodes (reservoirs/tanks) are represented with  $D$  and  $S$ , respectively, which are subsets of  $N$ . Depending on the analysis objective, various weights can be assigned to edges/nodes. For instance, pipe length (Euclidean distance) can be used as edge weight for WDN design. In this work, the following two graph measures are utilized for CNT-based design [17]:

The first measure is the shortest path length indicated with  $SP$ , which is utilized to describe the shortest distance between two nodes. Distance in this context refers to the sum of positive edge weights in a connecting path between two nodes [19]. As an example, calculating  $SP$  with the weight of pipe length from a source to a demand node results in finding a path between them where the pipe length is minimal.

The second measure is the single-source  $EBC$ , which describes the frequency of occurrence of an edge in the  $SP$  from  $S$  (source node) to every demand node ( $i \in N$ ) [15]. This measure was modified for WDN design denoted as demand  $EBC$  ( $EBC^Q$ ) [17].  $EBC^Q$  finds  $SP$  connecting  $S$  (source node) and every demand node  $i \in D$ , and adds the nodal demands  $Q_i$  ( $Q_i > 0$ ) to the  $EBC^Q$  values of all edges along the  $SP_{S,i}$ . For instance,  $EBC^Q$  of an edge  $e$  is formulated regarding Equation (1):

$$EBC^Q(e) = \sum_{S,i \in D} SP_{S,i}(e) \cdot Q_i \quad (1)$$

One can realize that  $EBC^Q$  value of each edge (pipe) can be used for estimating water flow in that pipe. On the other hand, the  $EBC^Q$  value itself depends on the weights chosen for  $SP$ . As mentioned, pipe length is often employed for weighting edges in the design procedure. However, the value of edge weights can be constant or modified with a function in the course of the  $EBC^Q$  calculation, denoted as static and dynamic weights, respectively [17]. Dynamic weights use alternative paths for routing the demand, which are not necessarily the shortest path but could be for example the second shortest path. Therefore, dynamic weights avoid concentrating  $EBC^Q$  values in just a few pipes (shortest-path tree) by considering redundant flow paths. This idea is based on the concept of energy balance in WDNs. One method to define dynamic weights could be increasing pipe length in a path (shortest path) to a maximum value (i.e., a threshold) per iteration with a function approximating increased friction losses in that path due to an increased flow. This paper gives guidance for selecting the proper parameters of such functions and thresholds used for defining dynamic weights. Detailed information regarding suggested dynamic weighting functions is described in the following section. Note that hereinafter, 'dynamic  $EBC^Q$ ' refers to  $EBC^Q$  calculation with dynamic weighting functions.

### Design procedure using static and dynamic $EBC^Q$

The  $EBC^Q$  value of an edge gives an estimation of the volumetric flow rate in that edge (pipe), which can be employed for pipe sizing of WDNs. Thus, according to the continuity equation, pipe diameter  $DN_e$  required for the edge  $e$  with  $EBC^Q(e)$  is estimated as follows [17]:

$$DN_e = \left\lceil \sqrt{\frac{4}{\pi} \cdot \frac{EBC^Q(e)}{V_{design}}} \right\rceil \in DN_{available} \quad (2)$$

$V_{design}$  in Equation (2) is the assumed velocity used for pipe sizing. In order to design a pipe based on the CNT approach, the corresponding  $EBC^Q$  of the pipe is first calculated, and then the required diameter is estimated by assigning a value between 0.5 to 2.5 m/s to  $V_{design}$ . Therefore, various ranges of solutions can be found by varying the design velocity. For instance, with 0.01 m/s steps, 201 different design options can be created. Note that no hydraulic stimulation is involved during the course of pipe sizing. It is only after the design that a hydraulic solver (Epanet2) is used to

examine solutions to ensure required constraints (e.g., required minimum pressure) are met. An illustrative simple WDN is shown in Figure 1 to explain the CNT design method with static and dynamic EBC<sup>Q</sup>.

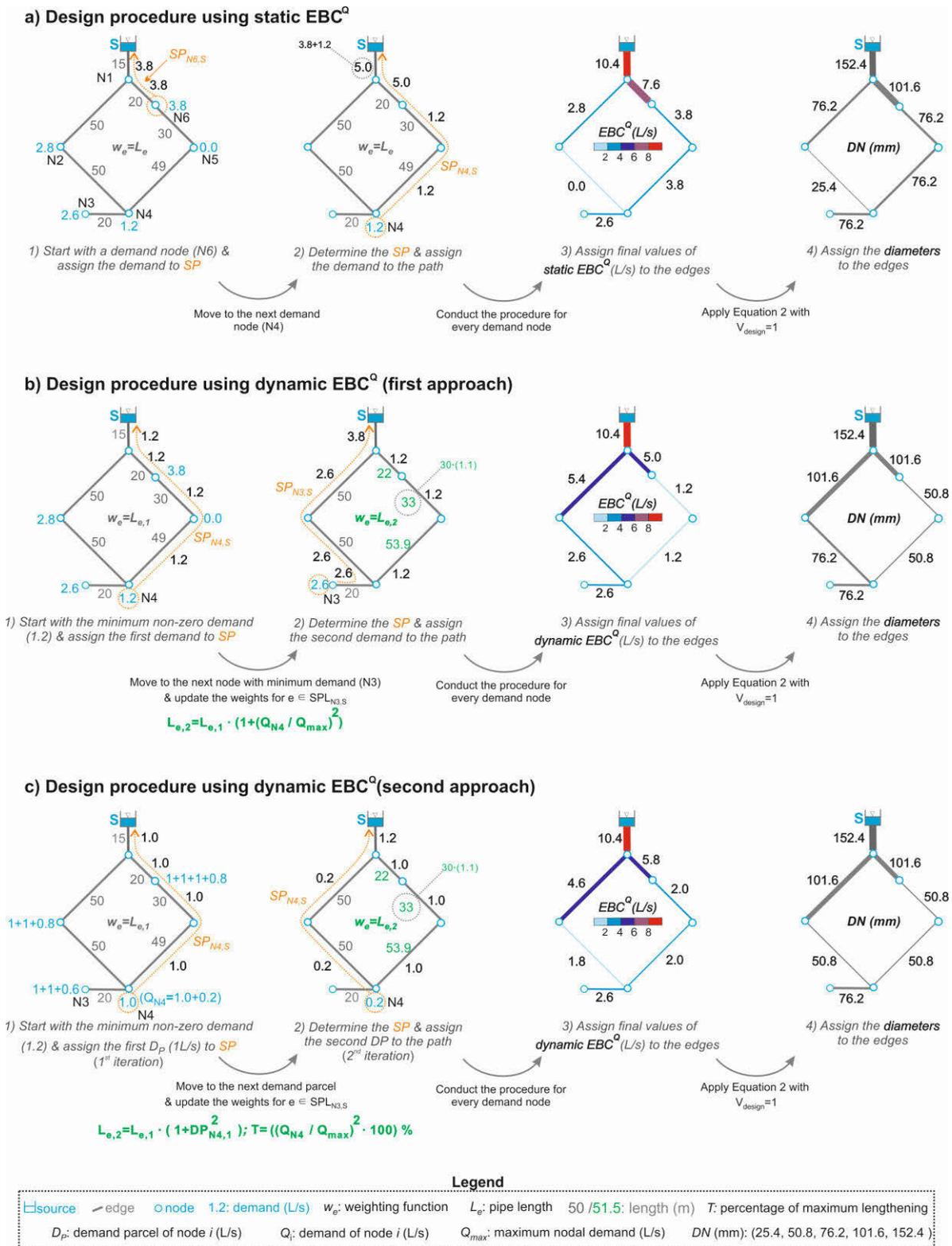


Figure 1: CNT design procedure using static and dynamic EBC<sup>Q</sup>

Before initiating the procedure, the WDN Epanet2 file needs to be converted to a graph object, including the network's information (e.g., nodal demands, pipe length). In the next step, pipe lengths are assigned to the graph edges as weights. In order to design pipe diameters, we need to distribute the nodal demands through the edges (pipes). This process is conducted with the  $EBC^Q$  measure, which can be either based on using static (static  $EBC^Q$ ) or dynamic weights (dynamic  $EBC^Q$ ). Required steps for the static  $EBC^Q$  design are described in Figure 1a. In static  $EBC^Q$  the procedure can be initiated with any node. In Figure 1a, the design process starts for example with N6, and then its corresponding nodal demand (3.8L/s) is routed to the path between N6 and S with the shortest path length. We conduct this process for all the demand nodes resulting in  $EBC^Q$  value of each edge using static weights. As shown in the third step of Figure 1a,  $EBC^Q$  values are concentrated in the shortest path (i.e., right branch), implying that the existing redundant capacity in the left branch is overlooked since it is a slightly longer route. However, based on the energy balance concept in the looped networks, alternative paths' capacity should also be considered to balance friction losses along different pipes. This issue can be addressed in CNT design by utilizing dynamic  $EBC^Q$ . Dynamic  $EBC^Q$  design is conducted based on two approaches shown in Figures 1b and 1c. In the first dynamic  $EBC^Q$  approach (see Figure 1b), we initiate the design process with the node that has the minimum non-zero demand (N4) and route its corresponding nodal demand (1.2L/s) through the edges located in the right-hand branch (the orange-colored path where  $SP$  is minimal). Moving to the next larger demand node (N3), the edge weights of the orange-colored path of the first iteration are increased (updated) using a dynamic function, resulting in routing the demand of N3 to the left-hand branch (i.e., the second shortest path before weights updating). The proposed dynamic function updates the weights (pipe length) in  $j+1^{th}$  iteration using the following equation:

$$L_{e,j+1} = L_{e,j} \cdot (1 + (Q_{i,j}/Q_{max})^2) \quad (3)$$

Where,  $L_{e,j}$  is the length of each edge  $e$  in the shortest path between demand node  $i$  and source node  $S$  in  $j^{th}$  iteration (m),  $Q_{i,j}$  is the demand of node  $i$  in  $j^{th}$  iteration  $j$  (L/s), and  $Q_{max}$  is the maximum nodal demand (L/s). Note that  $j$  in this equation is between 1 and the total number of demand nodes.

It is worth mentioning that the idea of using this dynamic weighting function is derived from the quadratic relationship between water flow and friction losses in the Darcy-Weisbach equation. By conducting this procedure for all the demand nodes and updating weights in each iteration,  $EBC^Q$  values of edges are calculated. Comparing the obtained values for static and dynamic  $EBC^Q$  in Figures 1a and 1b indicates that the potential capacity of the left-hand branch is further utilized using the dynamic approach, better approximating the energy balance in loops.

The second method for determining dynamic  $EBC^Q$  divides the demand of each node into smaller parcels ( $D_p$ ) to reproduce flow division in a looped network (see Figure 1c). For instance, two demand parcels ( $D_p = D_{p,1} + D_{p,2}$ ) with the values of 1 L/s and 0.2 L/s are considered for N4 with a total nodal demand of 1.2 L/s. After routing  $D_p$  of each nodal demand to its corresponding  $SP$  in the  $j^{th}$  iteration, the related edge weights are increased by the following equation in  $j+1^{th}$  iteration as follows:

$$L_{e,j+1} = L_{e,j} \cdot (1 + D_{p,i,j}^2) \quad (4)$$

Where,  $D_{p,i,j}$  is the demand parcel of node  $i$  in  $j^{th}$  iteration (L/s). Note that  $j$  in this equation is between 1 and the total number of demand parcels.

The proposed function in Equation (4) could excessively increase the weight of a path, especially when a network has a few large demands. In order to avoid this issue, we limit the maximum lengthening percentage in each iteration. Therefore, the corresponding threshold ( $T$ ) is calculated as  $T = ((Q_i/Q_{max})^2 \cdot 100)\%$ , where ( $Q_i$ ) is nodal demand and ( $Q_{max}$ ) is maximum demand.

The capacity of neglected paths in the static approach is activated with the proposed dynamic  $EBC^Q$ , resulting in creating loops in the course of design procedures. This issue leads to assigning more capacity to the left-hand branch of the WDN in Figures 1b and c, compared to those obtained in Figure 1a. In this work, CNT design with static and dynamic  $EBC^Q$  approaches are systematically tested on four design problems to select the appropriate method and give guidance for future applications. The selected approaches are then compared with the results found by multi-objective optimization using EAs.

## 2.2 Multi-objective optimization based EAs

The best outcome of the CNT approach is compared to those derived from multi-objective optimization based on EAs to validate the results and compare the computational efficiency of procedures. For benchmark case studies, the best-known solutions obtained based on five state-of-the-art EAs in literature are used, referred to as best-known Pareto fronts (BPF) [20]. For the real case, we solve the optimization problem using the state-of-the-art tool called GALAXY [9]. GALAXY was proposed based on the framework of multi-objective EAs to deal with the multi-objective and combinatorial design of WDNs [9]. Detailed information regarding the methodology and algorithms used for this tool was described in [9].

In both BPF and GALAXY, the multi-objective design is conducted using two conflicting objectives, i.e., maximizing resilience and minimizing total costs. The resilience is formulated according to a measure proposed by Prasad and Park [21], taking the surplus nodal head, as well as pipes' uniformity, into account. Besides, total costs are calculated based on unit pipe expenses (as a function of diameter) and the corresponding pipe length.

Parameters of optimization problems (e.g., diameter classes, pressure constraints) are selected according to the characteristics of case studies, which are described in the next section.

## 2.3 Case studies

Four WDNs, including three benchmark problems and one real network, are considered the case studies. The first case (Figure 2a) is known as the two-loop network (TLN) [22], which is a hypothetical WDN consisting of six demand nodes, a single reservoir, and eight pipes with an equal length of 1000 m. The second case (Figure 2b) is a subdivision of the WDN in Blacksburg town located in Virginia [23]. This network comprises 33 demand nodes, a reservoir, and 32 pipes, 12 of which have fixed diameters. The third case (Figure 2c) is BakRyun (BAK) network in South Korea which has 35 demand nodes, a single reservoir, and 58 pipes [24]. The fourth case is a real-world large network located in Austria, comprising 3,558 nodes, one reservoir, and 4,021 pipes with a total length of 211 km (the layout of this case study is anonymized, however, the real hydraulics are preserved).

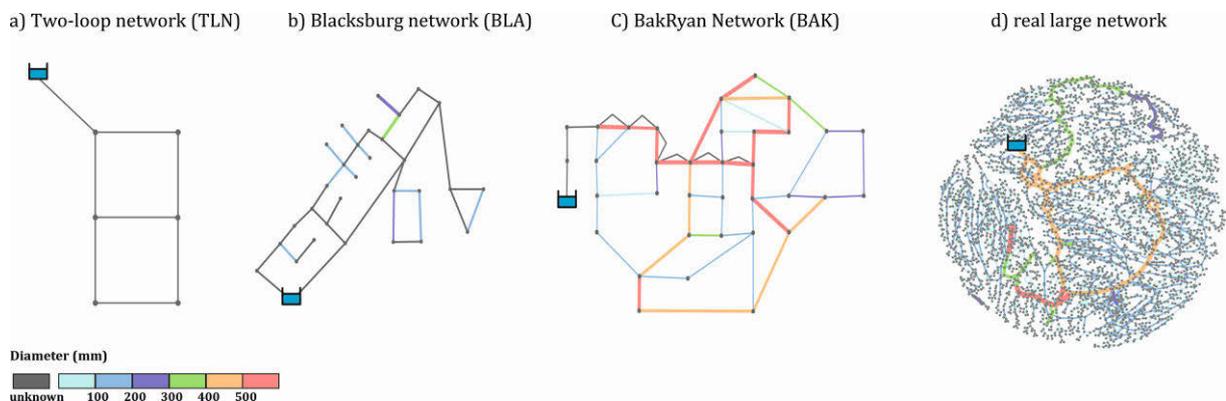


Figure 2: Layouts of considered case studies

BAK has been presented as an ‘extended design (rehabilitation) problem’ with the objective of finding the best possible diameters for the parallel and new pipes shown in Figure 2c. While for TLN, BLA, and real networks, the aim is to determine the possible diameters of all pipes, which is considered a ‘design problem’. The minimum pressure constraint is considered 15 m for BAK and 30 m for other WDNs. In addition to minimum pressure, the maximum pressure of each demand node in the BLA network is limited to a particular value presented in [23]. For benchmark problems, diameter options and their costs are defined according to the values suggested in the literature [20]. For the real network, 15 diameter classes from 76.2 to 914.4 mm with the unit costs ranging from 8 to 1,200 \$/m are considered. The optimization of this network is solved using GALAXY, with 500,000 and 100,000 generations and population size of 100. Regarding the size of the search space, the problems can be categorized into three groups: small (TLN and BAK), Intermediate (BLA), and very large (real WDN). As an example, a complete enumeration of  $15^{4,021}$  ( $DN^{#E}$ ) different design solutions are needed to optimize the real network using EAs, making it a very large design problem.

### 3 RESULTS AND DISCUSSION

#### 3.1 Comparing the results of static and dynamic EBC<sup>Q</sup> approaches

The obtained CNT results using static and dynamic EBC<sup>Q</sup> are indicated in Figure 3. Dynamic EBC<sup>Q</sup>(1) and EBC<sup>Q</sup>(2) in this figure refer to the results found based on the first and second dynamic methods, respectively.

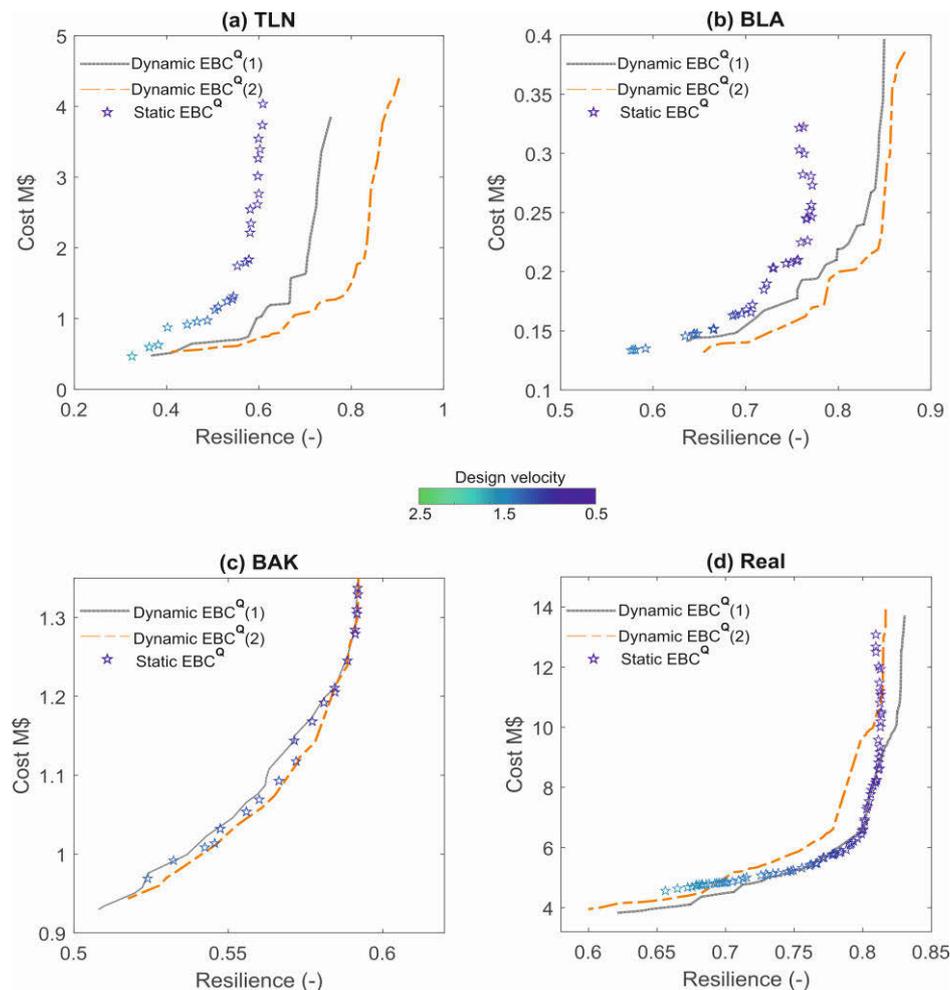


Figure 3: Comparing the results of static and dynamic EBC<sup>Q</sup> approaches for different case studies

At the end of the CNT procedure, we examined all the obtained solutions using Epanet2 to exclude those which do not meet the pressure constraints. For instance, the pressure constraint for BLA cannot be fulfilled for the obtained solutions based on static  $EBC^Q$  with the design velocity larger than 1.7 m/s.

A lower number of unique design solutions (i.e., design options with different values of resilience and cost) is obtained for benchmark cases based on static  $EBC^Q$  compared to those found for the large WDN. For instance, varying the design velocity in the range of 0.5-2.5 m/s with 0.01 m/s steps using the static approach results in 20 and 113 unique design options for BAK and real WDNs, respectively (see Figures 3c and 3d). This issue can be clarified by simple structures and less redundancy associated with benchmark networks in comparison with the real WDN with more than 4,000 pipes.

Figures 3a and 3b indicate that applying proposed dynamic approaches to TLN and BLA networks yields a better trade-off between the resilience and costs compared to the static approach. TLN is a looped network consisting of equal pipe lengths. Therefore, applying the static approach to such networks results in concentrating water flow ( $EBC^Q$ ) in particular pipes and assigning zero value to the  $EBC^Q$  of others. Similar behavior can be seen in the BLA network, where the reservoir is connected to two pipes and  $EBC^Q$  values are mostly assigned to the shorter pipe. In contrast, dynamic  $EBC^Q$  can activate the overlooked paths and distribute nodal demands uniformly, deriving cheaper design options with higher resilience values.

For the BAK network as an extended design problem, only the diameters of six parallel and three new pipes need to be determined (see Figure 2c). In this case, the resilience and costs of the design solutions are mainly influenced by those three new pipes that are connected to the reservoir. Using either the static or dynamic approach for BAK results in the same value of  $EBC^Q$  for each of those pipes. Therefore, dynamic  $EBC^Q(2)$  can slightly improve the quality of obtained solutions compared with static  $EBC^Q$ , which is mainly due to the less capacity assigned to the parallel pipes.

Figure 3d shows that in contrast with the benchmark problems, the outcome of using dynamic  $EBC^Q(2)$  for the real WDN cannot outperform those found by  $EBC^Q(1)$ . The real WDN is composed of thousands of demand nodes and edges, implying that a great number of alternative paths exist for routing a demand to the source in every iteration. Using dynamic  $EBC^Q(2)$  for designing this network increases edge weights excessively as demand division is conducted during the procedure (see Figure 1d). Consequently, the excessive increase of edge weights leads to incorrect identification of the shortest path, hence a sub-optimal route.

In this study, dynamic  $EBC^Q(2)$  and  $EBC^Q(1)$  are cherry-picked for the small-to-intermediate and large design problems, respectively.

### 3.2 Comparing dynamic $EBC^Q$ results with optimal design solutions

Figure 4 illustrates a comparison between the best dynamic  $EBC^Q$  results for each case study with their optimal solutions obtained based on multi-objective optimization (OPT) using EAs. As shown in Figure 4a, near-optimal solutions can be found by applying dynamic  $EBC^Q(2)$  to TLN. For the BLA network,  $EBC^Q(2)$  results cover part of the optimal Pareto front with the resilience values ranging from 0.63 to 0.73. However,  $EBC^Q(2)$  solutions with a Resilience > 0.75 cannot compete with OPT solutions. This is because redundant capacity plays a more crucial role in high resilient design solutions and the  $EBC^Q(2)$  results with high resilience value can only partly benefit from that potential capacity. The reason lies in the fact that no hydraulic information is involved in the CNT design, and the procedure is solely performed based on the spatial distribution of water flow using  $EBC^Q$ .

Figure 4c indicates that a couple of optimal/near-optimal solutions are obtained using dynamic  $EBC^Q(2)$  for BAK WDN, highlighting the potential of the CNT design approach for extended

design/rehabilitation problems. For the real network, optimal solutions are obtained with GALAXY, using two different generation numbers. Figure 4d shows that promising solutions can be found by applying dynamic EBC<sup>Q</sup>(1) to the real network. According to the figure, EBC<sup>Q</sup>(1) outcomes outperform those found with GALAXY with a generation number of 100,000 in the resilience range of 0.62 to 0.79. In addition, dynamic EBC<sup>Q</sup>(1) covers the knee bend area in the Pareto front of these optimal solutions, which is of interest to operators and decision-makers. Dynamic EBC<sup>Q</sup>(1) results can also outperform OPT results with 50 million function evaluations ( $g = 500,000$ ) in a specific resilience range from 0.62 to 0.72.

The advantages of using dynamic EBC<sup>Q</sup> over EAs approaches for large-scale networks become more noticeable by comparing their computational efficiency. Multi-objective design optimization of the real network with 100,000 and 500,000 generations requires 8 and 35 weeks of execution time, respectively, while the design procedure based on dynamic EBC<sup>Q</sup> is performed only in 47 seconds.

The high efficiency of CNT design approach with dynamic EBC<sup>Q</sup> in terms of computational time makes it possible to find a range of optimal/near-optimal solutions for large networks within seconds/minutes, enabling a broad range of applications. In addition, CNT can be used in combination with EAs to reduce the search space and improve the computational efficiency of optimizations for large networks.

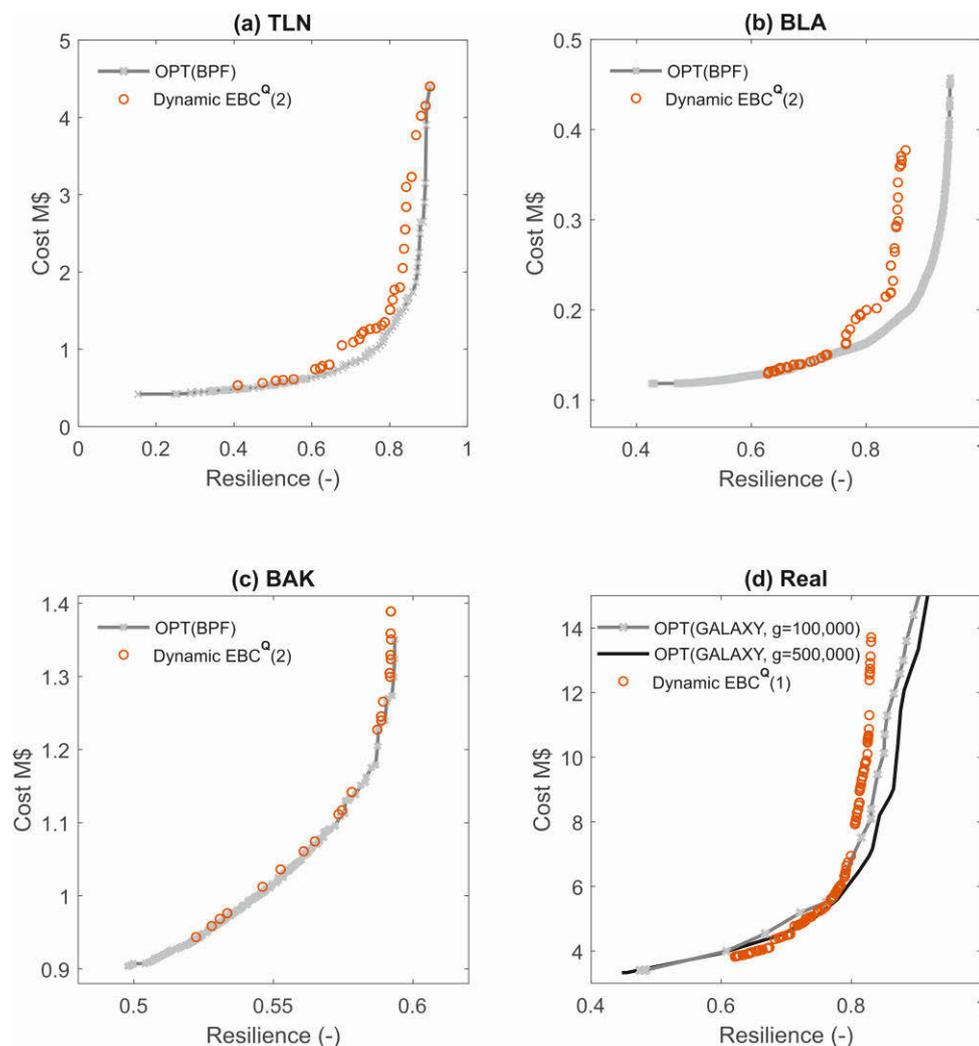


Figure 4: Comparing dynamic EBC<sup>Q</sup> results with optimal design solutions (OPT) according to the best-known Pareto front (BPF) and GALAXY with  $g$  generation number.

## 4 SUMMARY AND CONCLUSIONS

This paper explores the potential of using dynamic weighting functions for a complex network theory-based (CNT) design approach. For this purpose, two dynamic weighting functions are proposed and integrated into a graph measure known as  $EBC^Q$  which is used for WDNs design. This measure provides a valid estimation of water flow by determining the frequency of occurrence of an edge in the shortest path from a source node (reservoir) to a demand node and summing up the demands routed through an edge (pipe). In order to determine the shortest path, pipe length is used as a weighting function. Using a constant value as a weighting function in the course of  $EBC^Q$  calculation (static  $EBC^Q$ ) results in centralizing  $EBC^Q$  values in a few pipes (shortest-path tree). In comparison, changing (modifying) edge weights when calculating  $EBC^Q$  (dynamic  $EBC^Q$ ) employ the potential redundancy of alternative paths neglected with static  $EBC^Q$ , which could result in higher resilient and cheaper cost design solutions.

The first dynamic  $EBC^Q$  proposed in this work increases edge weights according to the corresponding demand node  $i$  ( $Q_i$ ) and maximum demand ( $Q_{max}$ ) in a WDN with a quadratic function. The second dynamic  $EBC^Q$  splits  $Q_i$  into smaller parcels ( $D_{pi}$ ) and increases the weights during the design process based on  $(D_{pi})^2$ . The results of applying dynamic  $EBC^Q$  approaches to three designs and one extended design problem confirm the ability of proposed weights in enhancing the quality of solutions, especially for complex design problems. For instance, the results of applying dynamic  $EBC^Q$  to a real large WDN with 4,021 pipes can partly outperform those obtained with evolutionary algorithms with 50 million function evaluations. Further, in terms of execution time, dynamic  $EBC^Q$  needs 47 seconds to find optimal/near-optimal design options for this network, whereas evolutionary algorithms require 35 weeks.

Regarding the obtained results in this work, it is recommended to utilize the second dynamic method for small-to-intermediate size networks and the first one for large/very large-scale WDNs. In future work, the focus will be on the design of multi-source WDNs with CNT approach using dynamic  $EBC^Q$ .

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