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MULTI-OBJECTIVE OPTIMIZATION OF WATER DISTRIBUTION SYSTEM UNDER UNCERTAINTY USING ROBUST OPTIMIZATION

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Abstract

Uncertainty is inevitable while trying to tackle any real-world problem, Water distribution system design problem is not an exception for that. Among the many uncertainties involved in the design problem, demand uncertainty is the most important. The current study aims to provide efficient designs that can handle the predicted variations in demand without compromising the resilience of the network system. Most previous studies explored stochastic solutions to handle the uncertainty that consumed immense computational power. Innovations in developing non-probabilistic techniques like robust optimization paved the way to reduce computational time and handle uncertainty efficiently. A new methodology is proposed in this study to obtain efficient designs for the multi-objective design problem of WDS under uncertainty. The proposed methodology uses a combination of robust optimization approaches to handle the uncertainty and a multi-objective cuckoo search algorithm. The proposed methodology is applied to a common benchmark water distribution system problem, and the designs are compared with the nominal designs obtained when demand is assumed as certain. Furthermore, the effect of considering different uncertainty sets is discussed.

Keywords
Water Distribution System, Optimization Under Uncertainty, Multi-objective Optimization, Robust Optimization, Self Adaptive Multi-Objective Cuckoo Search Algorithm (SAMOCSA).

Optimal design and management of water distribution systems (WDS) is an extensively explored research area in the water resources field. The main objective of the optimal design of WDS is to minimize the design cost (i.e. Pump capacity, Tank size, Pipe diameter, etc.) such that the system satisfies the hydraulic and water quality constraints. Over the past four decades, multiple variants of this problem have been explored using many optimization approaches. The research explored linear programming, non-linear programming, as well as dynamic programming approaches(Alperovits and Shamir 1977; Avi Ostfeld and Shamir 1996.; Kessler and Shamir 1989). Later with the development of evolutionary meta-heuristic algorithms, these techniques have also been explored to solve the problem (Ostfeld et al. 2008.; Savic and Walters 1997; Vasan and Simonovic, 2010.; Wu et al. 2005). Even though these approaches could provide the least cost design alternative that could satisfy the required constranits, the obtained designs could not meet the expected reliability. Walski (2001) emphasized incorporating additional objectives to the least-cost design problem like reliability, capacity, and resistance to uncertainty. With the development of multi and many-objective optimization algorithms and Pareto-front representation of designs, the research moved towards incorporating multiple conflicting objectives into the problem definition. Over the last two decades, optimal multi-objective WDS design has been extensively explored with studies considering two objectives: minimization of cost and resilience/robustness/ reliability (Pankaj et al. 2020; Perelman et al. 2008; Prasad and Park 2004; Wang et al. 2014), three objectives(Farmani et al. 2006; Wu et al. 2013) and also six objectives (Fu et al. 2013). Most of these studies considered the design problem to be deterministic and ignored the uncertainties associated with various design variables. In most real cases, this assumption is not true. Babayan et al. 2005 stated that almost all the design parameters associated with the WDS design problem have some uncertainty. Even though the multi-objective optimization approach considering maximization of resilience/ reliability provided some protection against uncertianiy, they lacked quantitative realization of the protection level achieved. This motivated the resercheres to solve stochastic formulation of the WDS design problem.

Lansey and his team were the pioneers in developing a methodology to solve the stochastic least-cost design problem. They used chance constraints to address the stochastic nature of the problem. They used GRG-2 methodology to obtain the stochastic optimal designs with various levels of protection for the two-loop network (Lansey et al. 1989). Later, Sumer and Lansey (2005) proposed a stochastic optimization model using First Order Second Moment (FOSM) uncertainty analysis. Other stochastic approaches have been explored using various probabilistic analysis methods to solve the stochastic least-cost design of the WDS problem (Jung et al. 2012, 2014; Seifollahi-Aghmiuni et al. 2013). Table -1 summarizes few of the studies that used probabilistic approaches to solve the WDS design problem.

Although the probabilistic approach successfully handled the uncertainty, the considerable computational time and the uncertainty in the assumption of probability density function (PDF) hindered its practical application. The researchers moved toward applying non-probabilistic approaches to handle the uncertainty to overcome these disadvantages. The robust optimization (RO) is a non-probabilistic approach that has been getting attention in the recent past to handle the uncertainty. This technique was successfully applied to WDS least-cost design by (Perelman et al. 2013a, b) under single loading conditions and (Schwartz et al. 2016) under multiple loading conditions. This approach has also been used to handle water quality constraints (Pankaj et al. 2022). Non-probabilistic approach application in the design of WDS is limited to the least cost design problem. There have not been any studies that used these methods to solve a multi-objective problem considering cost minimisation and resilience maximisation as objectives.

Table 1 List of few works in the area so WDS design and analysis under uncertainty

Uncertain parameters	PDF assumed	Uncertainty handling techniques	Optimization techniques	References	Type
q,H, RC	Normal	FORM	GRG2	(Xu and Goulter 1999)	Hydraulic analysis
q,H, RC	Normal	MCS	GRG2	(Lansey et al. 1989)	SO Design
q,RC	Normal	MCS	SFLA	(Seifollahi-Aghmiuni et al. 2013)	Hydraulic analysis
q,RC	Normal	FOSM & MCS	--	(Hwang et al. 2018)	Hydraulic analysis
q	Gaussian	LHS	GA	(Babayan et al. 2004)	SO Design
q,RC	Normal	FORM	GA	(Tolson et al. 2004)	SO Design

q	Gaussian	LHS	RNSGA-II	(Kapelan et al. 2005a)	MO Design
q	--	Robust optimization	cross entropy	(Perelman et al. 2013a, b)	SO Design
q- Demand; RC - roughness coefficient; H- Pressure head; LHS-Latin Hypercube Sampling; MCS - Monte Carlo Simulations; GA- Genetic Algorithm; FORM -First order reliability method; SO -single objective; MO- multiobjective design; SFLA -Shuffled frog leap algorithm					

The current study proposes a new robust multi-objective design optimization formulation that simultaneously minimizes construction cost and maximizes network resilience considering consumer demands as uncertain. In this study, the uncertain problem is reformulated using a robust counterpart for both objectives. A 'self-adaptive multi-objective cuckoo search algorithm' (Pankaj et al. 2020) and 'fmincon' optimization algorithms have been used to solve the problem. The proposed methodology is demonstrated on the Hanoi water distribution system.

Robust optimization is a recent non-probabilistic optimization under uncertainty (OUU) technique. The main advantage of this technique is that the uncertain problem is reformulated into a tractable form, and the solution obtained is feasible for all possible realizations of the uncertain parameter within the specified uncertainty set. This method is gaining popularity in engineering applications due to its fast computational ability and ability to handle uncertain parameters that do not follow any standard probability distribution.

The main problem with optimal water distribution system design under uncertainty is computational time, and consumer demands do not follow any probability distribution. This robust optimization technique can solve both problems. The robust approach to solving the multi-objective design optimization of WDS involves converting the uncertain problem into a deterministic form using a robust counterpart approach and a self-adaptive multi-objective cuckoo search algorithm. This methodology is applied to a Hanoi WDS.**Robust optimization**

The aim of robust optimization is to obtain a unique solution for the optimization problem whose feasibility is independent of the uncertainty in the data. The general robust optimization formulation is as follows,

$$\begin{gathered} min\, f(x) \\ s.t\; g_i(x, \alpha_i) \geq 0\; \forall \alpha_i \in U_i, i \in [1, m] \end{gathered} \tag{1}$$

Here x is decision vectors, $f(.)$ is the objective function and $g_i(.)$ is the i[th] constraint function. α_i is the uncertain parameter and U_i is the uncertainty set corresponding to the i[th] constraint function.

In general, the problem in equation (1) is intractable, leading to infinite constraints (all possible realizations within the uncertainty sets), we can write this in-tractable constaint as shown in equation (2), if we can obtain the minimum value of g(x,a) for all values of a in the uncertainty set U and check its value to the inequality, it implies that all the values of g(x,a) within the set of U satisfy the inequality. By using this we can covert the infine constraints into one single tractable cosntriant.

$$s.t \min_{\alpha_i \in U_i, i \in [1,m]} g_i(x, \alpha_i) \geq 0 \tag{2}$$

This reformulation is called the robust counterpart of the problem. A detailed description of robust optimization and robust counterpart formulation and its applications is provided by Bertisimas et al (2011). The reformulation approach varies with the type of uncertainty set. For the current study, we use ellipsoidal uncertainty sets as they are less conservative compared to box uncertainty sets and can also incorporate the correlation between the uncertain variables (Baron et al. 2011).

2.2 Ellipsoidal uncertainty set [(Ben-Tal and Nemirovski 1998)]:

Let us assume that for every ith constraint, α_i, can vary within the interval $[\hat{\alpha}_i - \delta, \hat{\alpha}_{ij} + \delta]$, where $\hat{\alpha}$ is the nominal value of α and δ is the maximum deviation from the nominal value.

For any uncertain coefficient α with the nominal value $\hat{\alpha}$ and covariance matrix Σ, the ellipsoidal uncertainty can be defined using Mahalanobis distance in the form:

$$U(\Omega) = \{\alpha | (\alpha - \hat{\alpha})^T \Sigma^{-1} (\alpha - \hat{\alpha}) \leq \Omega^2\} \quad (3)$$

'Ω' is a value controlling the size of the ellipsoidal uncertainty set, which is also referred to as the protection level.

Let us consider a simple function as the constraint (2), $g(x, \alpha) = \alpha^T x$ and the uncertainty set be ellipsoidal uncertainty set described in equation (3), then the constraint in the equation (2) can be written as

$$\min_{\alpha_i \in U(\Omega)_i, i \in [1,m]} \alpha_i^T x \geq 0 \quad (4)$$

Then the solution to this minimization problem can be easily attained by Karush Kuhn tucker conditions, the minimum value of $\alpha^T x$ can be attained when $\alpha = \hat{\alpha} - \frac{\Omega}{\sqrt{(x^T \Sigma x)}} \Sigma x$

By substituting this value, we can re-write the constraint as

$$\min_{\alpha_i \in U(\Omega)_i, i \in [1,m]} \alpha_i^T x \geq 0 \Rightarrow \hat{\alpha}_i x - \Omega \sqrt{x^T \Sigma x} \geq 0 \quad (5)$$

2.3 Cost vs resilience optimal design of WDS problem formulation

The optimal design of WDS is an np-hard problem containing complex non-linear equations in energy constraint and discrete search space. Initial efforts were made considering this as single objective as explained in the introduction section, and then the research moved towards multi-objective optimation, considering maximization of reliability or resilance as the second objective. Although there is no exact maximization way to realize resilience, many authors suggest a few surrogate measures to indicate the system's resilience. The most popular surrogate measure is the resilience index. In the current study we incorporate this resilience index as the second objective. The mathematical representation of the problem is expressed as follows:

$$Minimize \sum_{i=1}^{np} U_C(D(i)) * L(i) \quad (6)$$

$$Maximize\ RI = \frac{\sum_{i=1}^{nn} q_i (h_i - h_i^{min})}{(\sum_{s=1}^{nr} Q_s H_s + \sum_{b=1}^{npu} \frac{P_b}{\gamma}) - \sum_{i=1}^{nn} q_i h_i^{min}} \quad (7)$$

$$subject\ to: \ A_{21} Q - q = 0 \quad (8)$$

$$A_{11}Q + A_{12}\text{h=0} \tag{9}$$

$$h \geq h^{min} \tag{10}$$

$$D_i \in \{D_C\} \tag{11}$$

Here U_c- Unit cost per length of pipe corresponding to the diameter, D – set of design diameters, $\{D_C\}$ is the set of commercially available diameters, L – length of pipe, np – number of pipes, q_i- demand of node 'I', h- pressure head at node, $h^{\min}$ – minimum pressure required, nr – number of reservoirs, Q_s- flow from reservoir 's', H_s – pressure head of reservoir 's', npu – number of pumping units, P_b – energy of pump 'b', γ – efficiency of the pump, nn – number of nodes in the network, A_{21}=A_{12}^T is the connectivity matrix of the network based on topology, A_{11}- nonlinear elements representing the frictional resistance of the pipe, Q- Flow values in each pipe.

The equation (9) represents the energy constraint where the $A_{11}Q$ is the non-linear head loss term that can be expressed as :

$$\Delta h = \Delta h(Q) = R_c Q^{a1}\} \tag{12}$$

Where $R_c(resistance\ coefficent) = {}^{a_3 L}/_{f_c^{a1} \times D^{a2}}$, f_c- pipe friction coefficient, $a1$ = 1.852, a2 = 4.87 and $a3$ is the Hazen-Willams coefficient

2.4 Robust Counterpart formulation considering demand(q) as uncertain

Among all the uncertain parameters affecting WDS design, demand is the most important parameter (Babayan et al. 2005). For the current study, demand is assumed to be uncertain. To explicitly formulate the constraint with demand uncertainty, we use the linearization method proposed in (Perelman et al. 2013) to replace the head loss function. Among the two linearization methods they proposed, for the current study we incorporate the linearisation within a range [Q1, Q2] that under estimates the head within the range. For the case study we used Q1 = mean demand -2*standard deviation and Q2 = mean demand +2*standard deviation.

$$\Delta h = (\frac{\Delta h(Q_2) - \Delta h(Q_1)}{Q_2 - Q_1})\ Q + \frac{\Delta h(Q_1)Q_2 - \Delta h(Q_2)Q_1}{Q_2 - Q_1};\ L_1 Q + L_0 \tag{13}$$

$$\begin{bmatrix} A_{12} & L_1 \\ 0 & A_{21} \end{bmatrix} \begin{bmatrix} h \\ Q \end{bmatrix} = G \begin{bmatrix} h \\ Q \end{bmatrix} = \begin{bmatrix} -L_0 + h_o \\ q \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} h \\ Q \end{bmatrix} = K \begin{bmatrix} L_0^* \\ q \end{bmatrix} = \begin{bmatrix} K_{11} & K_{21} \\ K_{12} & K_{22} \end{bmatrix} \begin{bmatrix} L_o^* \\ q \end{bmatrix} \tag{14}$$

Where $G^{-1} = K = \begin{bmatrix} K_{11} & K_{21} \\ K_{12} & K_{22} \end{bmatrix}$ is the inverse of the matrix $\begin{bmatrix} A_{12} & L_1 \\ 0 & A_{21} \end{bmatrix}$, K_{11} is of the size $[nn \times np]$, K_{12} is of size $[nn \times nn]$ and $L_o^* = -L_o + h_o$, where h_o is a given vector of fixed known heads.

From the equation (15), the nodal heads' h' can be computed as

$$h = K_{11} L_0^* + K_{12} q \tag{15}$$

Using this formulation, we can rewrite the optimization problem as

$$Minimize \sum_{i=1}^{np} U_C(D(i)) * L(i) \tag{16}$$

$$Maximize\ RI = \frac{\sum_{i=1}^{nn} \tilde{q}_i((K_{11}L_0^* + K_{12}\tilde{q})_i - h_i^{min})}{(\sum_{s=1}^{nr} Q_s H_s + \sum_{b=1}^{npu} \frac{P_b}{\gamma}) - \sum_{i=1}^{nn} \tilde{q}_i h_i^{min}}; \tag{47}$$

$$K_{11}L_0^* + K_{12}\tilde{q} \geq h^{min} \tag{18}$$

$$\tilde{q} \in U \tag{19}$$

Now consider the equation (18,19), the equation contains demand (q) as an uncertian parameter, the robust optimization formulation for this is assuming the demand varies in the uncertainty set $U(\Gamma) = \{q_{ij} | (\tilde{q}_{ij} - \hat{q}_{ij})^T \Sigma^{-1} (\tilde{q}_{ij} - \hat{q}_{ij}) \leq \Gamma^2\}$ and $P: \Sigma = P.P^T$. Then as explained in the ellipsoidal robust optimization formulation (equation 3-6), we can write the formulation as follows.

$$\min_{\tilde{q} \in U} K_{11}L_0^* + K_{12}\tilde{q} \geq h^{min} \Rightarrow K_{11,i}L_0^* + \tilde{q}^T K_{12,i}^T - \Gamma \| P^T K_{12,i}^T \| \geq h^{\min} \tag{20}$$

Robust optimization formulation for resilience index, for simplifying the problem, lets assume that the network consists of only one source and no pumps. Then the simplified resilience index equation can be written as:

$$Maximize\ RI = \frac{\tilde{q}^T K_{12}\tilde{q} + \tilde{q}^T K_{11} L_o^* - \tilde{q}^T h^{min}}{(\Sigma q) H_s - \tilde{q}^T h^{min}}; K_{11}L_0^* + K_{12}\tilde{q} \geq h^{min}; \tilde{q} \in U \tag{21}$$

The robust optimization formulation for the problem in equation 21 is,

$$\max \tau$$

$$subject\ to: \min_{\tilde{q} \in U(\Gamma)} \frac{\tilde{q}^T K_{12}\tilde{q} + \tilde{q}^T K_{11} L_o^* - \tilde{q}^T h^{min}}{(\Sigma q) H_s - \tilde{q}^T h^{min}} \geq \tau;\ K_{11}L_0^* + K_{12}\tilde{q} \geq h^{min} \tag{5}$$

The resilience index formulation is still nonlinear with a form similar to quadratic over linear, but all the elements in the matrix K_{12} are negative (Perelman et al, 2013). The denominator is always positive as energy at the source ($(\Sigma q)H_s$) is always greater than energy reached at the nodes ($\tilde{q}^T h^{min}$), this problem will never be of the form quadratic over linear with the positive definite quadratic matrix.

In order to solve the optimization problem in equation 22, an inbuilt nonlinear optimization algorithm in MATLAB named "fmincon" is used.

The overall robust multiobjective formulation used in this study is as follows

Objective function (1)	$$\text{Minimize} \sum_{i=1}^{np} U_C(D(i)) * L(i)$$ $$\text{Subject to: } K_{11,i}L_0^* + \tilde{q}^T K_{12,i}^T - \Gamma \| P^T K_{12,i}^T \| \geq h^{\min}$$	(63)
Objective function (2)	$$\max \tau$$ $$subject\ to:\ \min_{\tilde{q} \in U(\Gamma)} \frac{\tilde{q}^T K_{12}\tilde{q} + \tilde{q}^T K_{11} L_o^* - \tilde{q}^T h^{min}}{(\Sigma \tilde{q}) H_s - \tilde{q}^T h^{min}} \geq \tau;$$ $$K_{11}L_0^* + K_{12}\tilde{q} \geq h^{min}$$	(24)

The proposed method is applied to a standard benchmark problems Hanoi WDS proposed by (Fujiwara and Khang 1990)

3.1 Multiobjective optimization method:

Self-adaptive multiobjective cuckoo search algorithm (SAMOCSA) combined with fmincon nonlinear optimization model is used to solve the robust multiobjective WDS design optimization problem. SAMOCSA is an improved version of the multiobjective cuckoo search that adapts the algorithm's exploration and exploitation governing parameters at every iteration. This algorithm has been tested on the two-loop network, Hanoi network and Pamapur network (Indian network) for deterministic multiobjective design problem of WDS. The complete details of the algorithm and its efficiency can be obtained from (Pankaj et al. 2020).

3.2 Hanoi WDS case study

Hanoi WDS is a medium gravity-based WDS proposed by Fujiwara and Khang 1990. The network consists of 32 demand nodes and 34 pipes connected to a single source with a head of 100m. The minimum pressure head required at every node is 30m. The network needs to be designed with 6 different sized pipes. The unit cost corresponding to the available diameter are shown in Table -1.

The full data for this example can be found
https://emps.exeter.ac.uk/engineering/research/cws/resources/benchmarks/design-resiliance-pareto-fronts/medium-problems/

Table 2 Diameter options and associated unit costs for Hanoi WDS

Diameter (in.)	Unit Cost ($/m)	Diameter (in.)	Unit Cost ($/m)	Diameter (in.)	Unit Cost ($/m)
12.0	45.73	20.0	98.39	30.0	180.75
16.0	70.40	24.0	129.33	40.0	278.28

To model uncertainty in demands, the WDS nodes were partitioned into three demand regions: region 1—nodes 1:15, region 2—16:24, and region 3—25:32 (Figure 1). Demands in region 2 were assumed to be certain and in regions 1 and 3 as uncertain with a standard deviation of 12% from the mean demand of each region [i.e., 80 and 50 (m3/h)], respectively. Two different protection levels are studied $\boldsymbol{\Omega} = [\mathbf{1}, \mathbf{2}]$. Furthermore, the correlation between the nodes within the region and the correlation between the regions are also altered. The intraregional correlation values are set to be $\boldsymbol{\rho} = \mathbf{0.8}$, and the interregional correlation varies between positive, no-correlation and negative correlation $\boldsymbol{\rho} = [\mathbf{0.6}, \mathbf{0}, -\mathbf{0.6}]$. SAMOCSA algorithm is used to solve the outer design problem, and the "fmincon" algorithm is used to solve the nonlinear inner optimization problem for minimization of the resilience index within the demand search space.

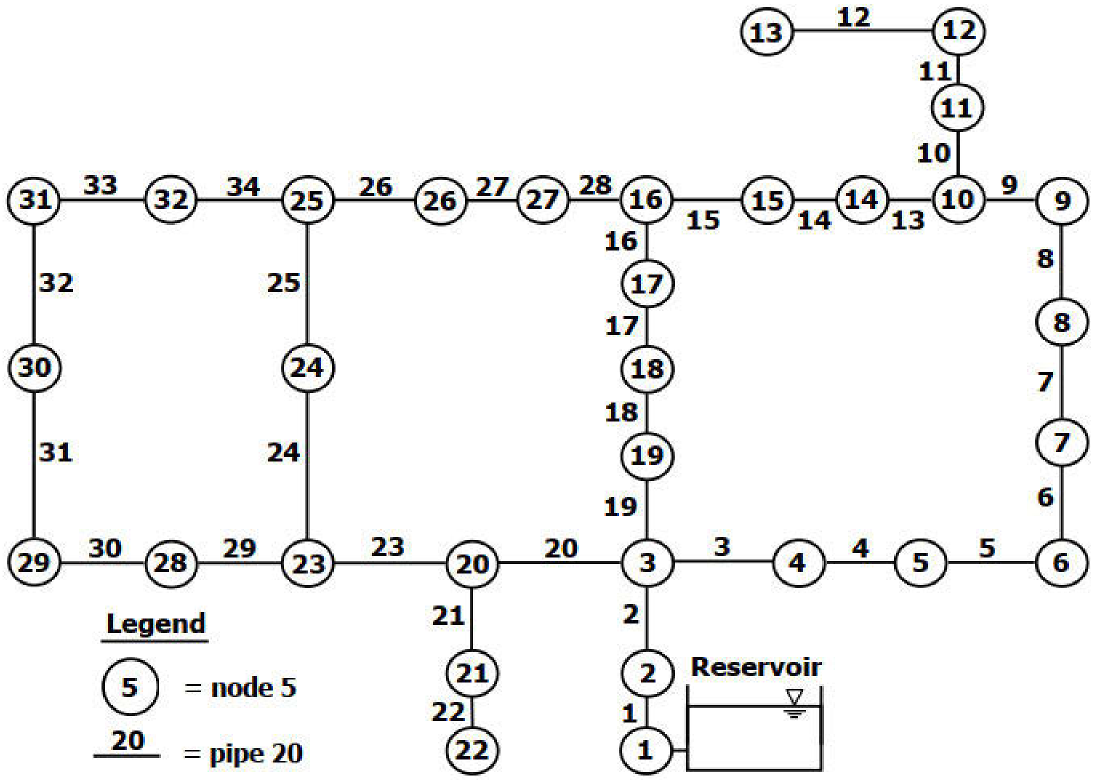
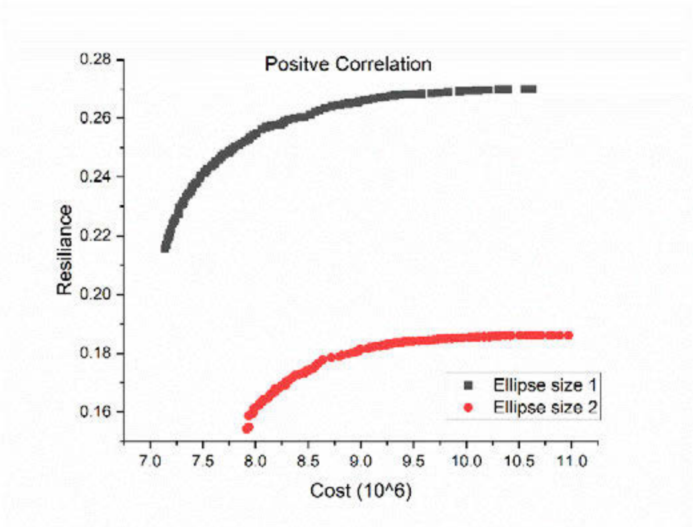


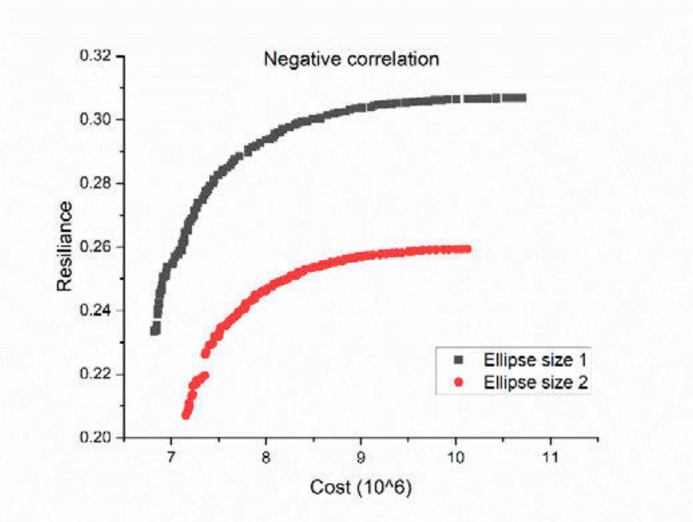
Figure 1 Graph of the topology of Hanoi WDS (Fujiwara & Khang 1990)

The graphs are shown in fig(2) compare the results obtained for different uncertainty sets (Ellipsoidal-1, Ellipsoidal -2) for positive, negative and no correlation. We can infer that uncertainty changes the cost to resilience trade-off from the graphs. As the ellipsoidal set size increases ($\Omega = 1\ to\ 2$), the cost vs resilience trade-off gets worst. The worst case is obtained when the demands are positively correlated among the three cases. We need to provide a higher cost design, even for a small resilient design.

(a)



(b)



(c)

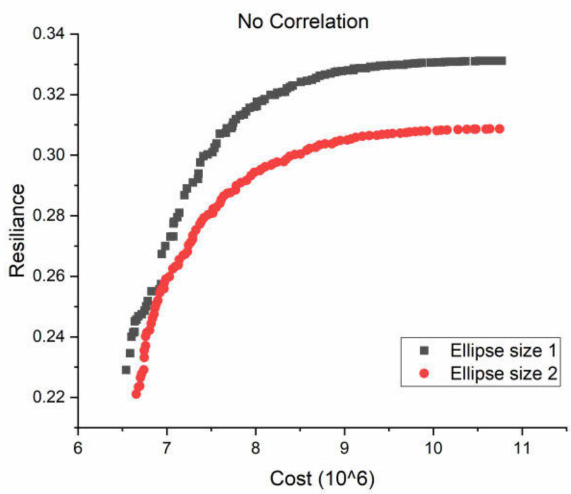


Figure 2 Cost vs Resilience Pareto Fronts for two different sizes of uncertainty sets [Ω = 1,2] when the demands are considered (a) Positively Correlated (b) Negatively Correlated (c) no correlation

This work proposes using a robust counterpart approach to handle the demand uncertainty in WDS. The results show significant promise of this new approach in terms of ease of computation and model formulation. Once the problem is converted to tractable deterministic robust counterparts, the methodology for solving the problem is similar to solving a conventional multi-objective design problem. Even though the formulation is similar to max-min approach, the ellipsoidal uncertainty set makes it less conservative. As the size of the uncertainty set to increase, the trade-off also increases; even for little resilience, we need to incorporate high-cost designs. The worst-case scenario is attained when the demands are assumed to be positively correlated. The results of the ellipsoidal set with $\Omega = 1$ is a less conservative and robust solution. The further scope of this work is trying to incorporate other objectives like (minimizing the leakages or carbon emissions)

5 ACKNOWLEDGEMENT

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