

ITERATIVELY TUNING THE REGULARISATION PARAMETER IN AN INVERSE METHOD FOR LOCALISING LEAKS IN WATER DISTRIBUTION NETWORKS

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Abstract

Novel methods for detecting and localising leaks in water distribution networks are being increasingly investigated as water utilities face unprecedented financial and environmental challenges in reducing water losses. A promising method includes the solution of the regularised inverse problem that minimises the difference between simulated and measured data in addition to the regularisation term. However, the results of leak localisation are sensitive to the choice of the regularisation parameter. In this paper, we propose and investigate a method for iterative tuning of the regularisation parameter to improve the leak localisation performance in case of multiple time steps measurements. The numerical investigation utilises a benchmarking network model, and different metrics are applied to evaluate the results of the leak localisation performance, such as the performance and the distance metric.

Keywords

Inverse Problem, Leakage localisation, Water Distribution Networks.

1 INTRODUCTION

Water distribution networks are used to satisfy water demand at all planned locations with an adequate pressure head, deliver the necessary volume of water during firefighting, minimise water loss and supply disruptions, and preserve water quality in the distribution pipes. However, 126 billion cubic metres of water are wasted each year globally, resulting in a loss of 39 billion US dollars [2]. Moreover, water leaks from pipes also contribute to contamination and health problems as pollutants are involved in the water and flow back to the pipe through the breaks due to negative pressure [3].

Detecting and localising leaks in the water distribution networks is challenging and usually consists of three steps. First, an anomaly event is detected in the system, and then the localisation methods are applied to reduce the research area. In the end, pinpoint techniques are used to precisely locate the leaks.

Leak detection technologies are used to signal a burst event in the water supply system without providing any specific information about the location of the burst event in the system. In the last 20–25 years, it has been increasingly common to use transient analysis approaches for leak detection because advances in metering technology have made it possible to gather near real-time data from pressure and flow sensors. For example, Liggett and Chen [4] developed an inverse transient analysis that models the pressure responses using least squares regression between measured and computed pressure responses. Leaks can be detected when there are deviations in the pressure response of the system. Numerous transient analysis-based approaches are described in the literature [5]–[8]. However, due to the significant effect of system uncertainties, these approaches are limited to a single node or grouped pipelines [9]. More recently, data-driven approaches based on the pressure sensor and flow metre readings have been developed to

identify a leak as a divergence from normal observations [10]–[12]. Jung and Lansley [13] use a nonlinear Kalman filter in conjunction with a hydraulic model to address the issue that data-driven approaches cannot distinguish between real demand changes and leakage, allowing for a known change in the system's operating conditions accommodated by the model.

Leak localisation is a process of narrowing the search region for leaks to make pinpointing approaches more effective [9]. Detecting and localising leaks using pinpointing techniques is practical and exact [9]. For example, using leak noise correlators, at least one sensor is placed in contact with the pipe on both sides of the suspected leak to capture the sound data. The sound data is then analysed mathematically to determine the specific location of the leak on the pipe, which is accomplished by correlating the noises that reach both sensors and computing the time difference between each sensor's leak site. The trace gas technique includes pumping a non-toxic, lightweight combination of hydrogen and nitrogen gas into the pipe where the leak is thought to be occurring. The gas will then escape from the leak location in the pipe and be detected by the gas-sensitive sensors installed in the pipe [14]. Khulief, et al., [15] employed hydrophones to conduct sound measurements within the pipeline. The leak signal can be readily detected when the hydrophone passes over the leak location. While specific leak pinpointing methods are the most precise technology for leak localisation, they are also time-consuming, labour-intensive, and costly, particularly when a broad region has to be searched [9].

Numerous leak localisation approaches rely on hydraulic models, pressure sensor measurements and customer demand information. The leak localisation strategy proposed by Pudar and Liggett [16] is based on addressing an inverse problem by minimising residuals between simulated and measured data. Although the inverse problem may be used to identify multiple leaks that occur simultaneously, the inverse problem is often under-determined in water supply networks since the network's sensor count is far smaller than the number of probable leak locations. Pudar and Liggett [16] minimised the l_2 -norm of the leak parameters to solve the under-determined problem. Sanz, et al., [17] addressed the under-determined problem by using a grouped node technique to minimise the number of unknowns. The leak location is determined as part of the demand calibration process. Demand components representing each node's state in a zone with grouped nodes are computed during the calibration process [18]. The six leakage detection indicators calculate the variance in the present and historical demand components. If the sum of scores for the grouped nodes exceeds the global threshold, this indicates the existence of a leak and its approximate location [17]. The capability to identify leaks is dependent on the relationship between the leak flow and the consumption of demand components. Small leaks that occur in zones with high consumption components are not detected because the small differences generated by leaks are not observable [17]. Most recently, Chew, et al., [19] first calibrated the net demand profile, rectified the observed offsets in the sensors and calibrated the physical parameters such as roughness coefficients and valve setting, followed by the localisation of anomaly events.

Casillas, et al., [20] and Perez, et al., [21] proposed the sensitivity matrix method that compares the difference of the pressure measurement when a leak occurs at a single location to the sensitivity of pressure measurements to a leak flow from every node. However, the Sensitivity Matrix Method is limited to detecting the existence of a single leak since it compares the measurement sensitivity of a leak at one node to the residual [21]. Moreover, although the Sensitivity Matrix Method only generates a single leak candidate, the distance between the leak candidate and the true leak node is sensitive to the uncertainty of the measurements, model accuracy, and assumed leak flow while generating the leak sensitivity matrix [20].

To overcome the constraints of sensitivity-based techniques and solve the under-determined inverse problem. Blocher, et al., [1] formulated and solved a regularised inverse problem for leak localisation by minimising the least squares of residuals between simulated and measured data. By including a regularisation term, this technique replaces the ill-posed original problem with a

well-posed and stable neighbouring problem, enabling the localisation of a leakage hotspot area. However, the results of leak localisation are sensitive to the leak location because the single value of the regularisation parameter cannot guarantee good performance for every leak scenario.

Moreover, Romero-Ben, et al., [22] applied both hydraulic model-based and data-based methods to detect and localise leaks. The hydraulic model-based method relies on the accuracy of the model, demand pattern and sensor information, and the full data-based method is based on graph interpolation and candidate selection criteria.

This article extends the method developed by Blocher, et al., [1] by investigating a method for iterative tuning of the regularisation parameter to improve the leak localisation performance assuming exact model and data. The paper has 4 sections. Section 2 describes the methodology of implementing the iterative tuning approach. Section 3 tests the proposed method for the Net25 network and compares it to the original regularised inverse method. Finally, section 4 concludes the changes made by the iterative tuning approach and future works.

2 METHODOLOGY

2.1 Problem Formulation

In this article, the proposed leak localisation method aims to improve the leak localisation performance by iteratively tuning the regularisation parameter and solving the regularised inverse problem. The leaks in the network are assumed to be detected, and the effects of uncertainties in the hydraulic model, sensor measurements, and customer demand information on the leak localisation results are assumed to be neglected.

The network is modelled as a directed graph with n_n junctions (demand nodes), n_p pipes (links), and n_0 source nodes (reservoirs/tanks). The link-junction incidence matrix is $A_{12} \in \mathbb{R}^{n_p \times n_n}$, which shows the connectivity of demand nodes to the pipes, and the link-source node incidence matrix $A_{10} \in \mathbb{R}^{n_p \times n_0}$ shows the connectivity of source nodes to the pipes. The entries of the incidence matrix can be defined by the relationship between a node $i \in \{1, \dots, n_n + n_0\}$ and a pipe $j \in \{1, \dots, n_p\}$. If there is no relationship between the node i and the pipe j , the entry is 0; if the pipe j leaves node i , the entry is -1; if the pipe j enters the node i , the entry is +1.

The hydraulic model is modelled based on the conservation laws of energy and mass, and the hydraulic equations are shown below:

$$A_{12}h + A_{10}h_0 + \phi(q) = 0 \quad (1)$$

$$A_{12}^T q - d - d_L(c, h) = 0 \quad (2)$$

where $h \in \mathbb{R}^{n_n}$ is the hydraulic head at demand nodes, $q \in \mathbb{R}^{n_p}$ is the pipe flow, $h_0 \in \mathbb{R}^{n_0}$ is the hydraulic head at source nodes, $\phi(q) \in \mathbb{R}^{n_p}$ is the head loss across the pipe and $d_L(c, h) \in \mathbb{R}^{n_n}$ is the leakage at each node. In the present work, the leakage is modelled as pressure-dependent flow, and the leak flow at each node i is defined by a modified orifice equation [23]:

$$d_{L,i}(h) = c_i(h_i - z_i)^{0.5} + c_{var,i}(h_i - z_i)^{1.5} \quad (3)$$

where h_i is the hydraulic head, z_i is the elevation, c_i and $c_{var,i}$ are the unknown coefficients depending on the orifice area and the slope of pressure head to orifice area, respectively. The modified orifice equation indicated in equation (3) has two terms: one for the original flow and another for the enlarged area flow. The first term dominates when the pressure head is small or the pressure head slope to the orifice area is small, whereas the second term dominates when the pressure head is big or the pressure head slope to the orifice area is large. In this article, the $c_{var} = 0$ for all nodes.

The Darcy-Weisbach (DW) or Hazen-Williams (HW) equations are typically used to determine the head losses across the pipe that are caused by friction. However, one of the challenges in using Darcy-Weisbach or Hazen-Williams equations to model head loss is that the fractional exponent in Hazen-Williams equations or explicit approximations for Darcy-Weisbach equations may have a singularity at zero flow [24]. Therefore, a quadratic approximation approach proposed by Pecci, et al., [25] is used to calculate the head loss across the pipe j :

$$\phi(q_j) = q_j(a_j|q_j| + b_j) \quad (4)$$

where a_j and b_j are the unknown coefficients and computed in [25] Moreover, the hydraulic heads and flows are computed by solving equations (1) and (2) as outlined in [26]

The leak localisation problem in this article is similar to the one formulated by Blocher, et al., [1] as a regularised inverse problem and solved by minimising the difference between simulated and measured hydraulic states and the regularisation term. However, the regularisation parameter is not a fixed value, and it will update for each iteration.

$$\begin{aligned} \min_{h,q,c} \quad & w_h \sum_{m \in M} (h_m - \bar{h}_m)^2 + w_q \sum_{o \in O} (q_o - \bar{q}_o)^2 + \rho \|c\|_2^2 \\ \text{subject to} \quad & A_{12}h + A_{10}h_0 + \phi(q) = 0 \\ & A_{12}^T q - d - d_L(c, h) = 0 \\ & h \geq z \\ & 0 \leq c \leq c_{max} \end{aligned} \quad (5)$$

Where M is a set of pressure sensor locations, O is a set of flow meter locations, ρ is the regularisation parameter, W_h and W_q are the weights assigned to the pressure and pipe residuals, respectively, to guarantee that residuals are scaled equally.

Hydraulic head h , flow q , and leak coefficients c are the variables in the optimisation problem. The objective is to minimize the weighted sum of squared residuals and the weighted regularisation term. Equality constraints are based on hydraulic mass conservation and energy conservation laws. The third constraint specifies that the hydraulic head must be larger than the elevation head in order to maintain a non-negative pressure head. The fourth constraint specifies the range of leak coefficients.

The leak localisation results can be evaluated by the performance metric [1] and distance metric [20]. The former metric considers both the presence of the true leak node in the leak candidate set and the distance between the true leak node and the leak candidate set. Once the problem (5) is solved, the normalised attribute $u_i = \frac{c_i}{\max(c)}$ as an indicator of the leakage, the large value corresponds to a higher possibility of leakage. The performance metric is defined as the sum of the reward component and the penalty component:

$$\begin{aligned} \beta &= \gamma - \lambda \\ \gamma &= \sum_{i \in K} \frac{u_i}{|K|} \\ \lambda &= \frac{r^T u}{\sum_{i=1}^{n_n} r_i} \end{aligned} \quad (6)$$

where K is an index set containing the locations of true leak nodes, $|K|$ is the total number of true leak nodes, r_i is the shortest distance between node i and the closest true leak node, γ is the reward component, and λ is the penalty component. In the distance metric, the node i with the highest normalised attribute u_i is selected as the main leak candidate, and the distance between

the main leak candidate and the true leak location D is computed as the measure for the leak localisation results.

2.2 Iterative tuning approach

The proposed approach iteratively solves the problem (5) with the updated regularisation parameter and search area to improve the leak localisation performance. The algorithm terminates when the relative change in the search area is smaller than a given threshold.

First, we simulate single leaks at each network node to determine the optimal regularisation parameter for each location. This is done by solving problem (5) with a set of regularisation parameters and selecting the parameter that results in the highest beta or the shortest distance, depending on the metric used. The regularisation parameter that has the highest frequency of occurrence across the entire network is chosen as the initial regularisation parameter.

When a leak is detected, we use the initial regularisation parameter to solve the regularised inverse problem (5). We consider network nodes for which the optimization procedure has assigned leak coefficients that exceed the threshold. The regularisation parameter is then updated to the value with the highest frequency of occurrence among nodes with leak coefficients above the threshold. Then, using the updated regularisation parameter, we solve a regularised inverse problem and obtain a new set of nodes with leak coefficients greater than the threshold. The iterative procedure is repeated until the relative change in the search area is smaller than 0.1.

3 CASE STUDY

Net25 is a publicly available water distribution network consisting of 22 demand nodes, 37 pipes, and 3 reservoirs. The network topology is depicted in figure (1), where nodes 23, 24, and 25 represent reservoir locations and nodes 1, 8, 13, and 21 represent sensor locations. Pipe properties are given in [27] and nodal characteristics are provided in [28]. To evaluate the Iterative Tuned Regularised Inverse Method (ITRIM), 22 single leak scenarios are simulated, and ITRIM is compared to the Regularised Inverse Method (RIM). Both methods are solved in the case of the different number of time steps measurements, including 1 time step measurements obtained at 12 pm, 6 and 12 time steps measurements obtained over 24 hours in 4 hours and 2 hours intervals, respectively. Leak flow is simulated using equation (3) with the parameters $c_i = 0.6 \times 10^{-3} [m^{2.5}s^{-1}]$ and $c_{var,i} = 0 [m^{1.5}s^{-1}]$. Hydraulic equations (1) and (2) are solved to generate the hydraulic head and flow measurements. The following section implements the performance and distance metrics to compute the optimal regularisation parameter for each location and evaluate the leak localisation results.

3.1 Performance Metric

For each assumed single leak event in the Net25, a set of regularisation parameters ranging from [0, 0.1, 1, 10, 100, 1000, 10000] is implemented to solve the problem (5) with multiple time steps measurements. The optimal regularisation parameter for each location under multiple time steps measurements is the one results in the highest β . The leak localisation results are then computed by the procedure described in section 2.2. Finally, the leak localisation results from ITRIM are compared to those from RIM with the regularisation parameter=1000. Figure (2) shows the performance profiles of both ITRIM and RIM with 1 time step, 6 time steps and 12 time steps measurements, the dotted lines represent the RIM and the solid line represent the ITRIM. The performance profile illustrates the percentage of leak scenarios with performance greater than or equal to a threshold value:

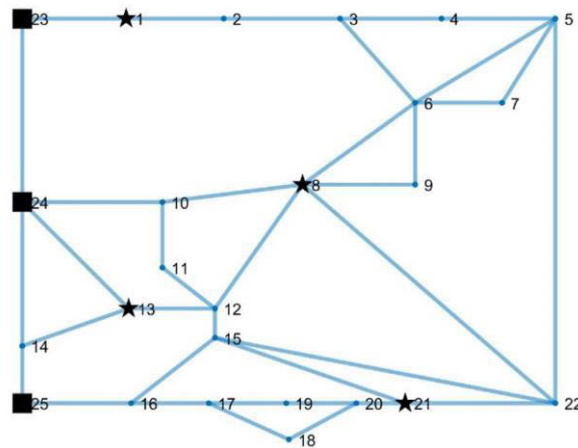


Figure 1. Layout of water distribution network: Net25.

$$P(\tau) = 100 \frac{|\{v \in V: \beta_v \geq \tau\}|}{|V|} \quad (7)$$

where V is a set of leak scenarios, β_v is the performance for leak scenario $v \in V$ and β_v is the threshold value. In Figure (2), the performance profiles obtained by RIM with the different number of time step measurements are similar to one another. However, a comparison with the profiles obtained by ITRIM indicates clearly that the iterative tuning approach improves the leak localisation performance. For example, in the case of one time step measurements obtained at 12 pm, 68% of ITRIM leak localisation results yield a performance $\beta \geq 0.8$, and only 34% achieve the same performance if RIM is used. Additionally, if 6 or 12 time steps measurements are used, around 40% of ITRIM leak localisation results yield a performance $\beta = 1$, which means the leak localisation results correctly identify the true leak as the only main leak candidate. Moreover, the average performance for all leak scenarios increases from 0.73 to 0.89 when ITRIM is used.

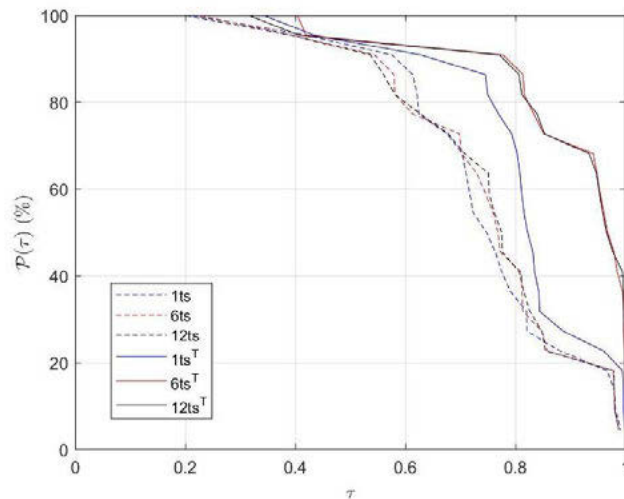


Figure 2. Performance profiles of the Regularised Inverse Method (RIM) and the Iterative Tuned Regularised Inverse Method (ITRIM) with the different number of time step measurements. Dotted lines with legend 1ts, 6ts and 12ts represent the performance profiles of RIM, whereas solid lines with legends 1ts^T, 6ts^T and 12ts^T represent the performance profiles of ITRIM.

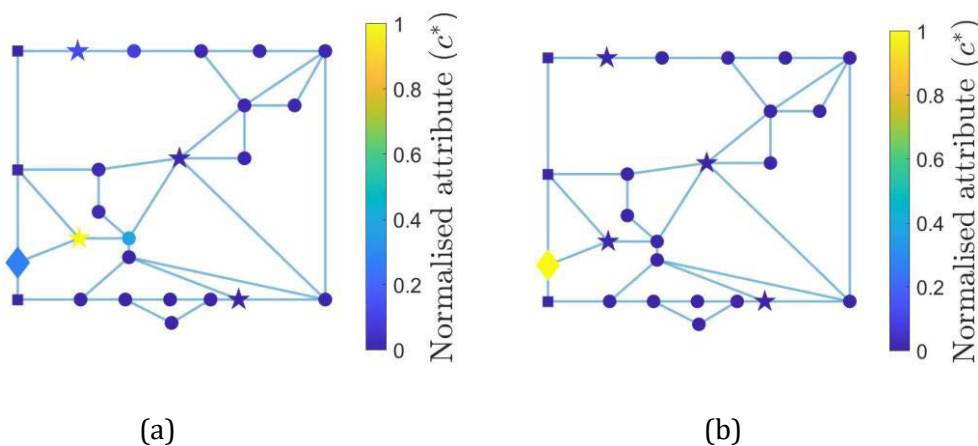


Figure 3. Leak localisation results for (a) Regularised Inverse method and (b) Iterative tuned Regularised method when a leak occurs at node 14 (diamond shape).

The largest positive performance change between RIM and ITRIM occurs when a leak is simulated at node 14. Figure (3) shows the leak localisation results of both RIM and ITRIM, and RIM assigns the largest normalised attribute to the neighbour of the true leak node and a small value of the normalised attribute to the true leak node, which results in a lower performance $\beta = 0.22$. However, ITRIM yields the true leak node as the only leak candidate, significantly improving leak localisation performance to $\beta = 1$.

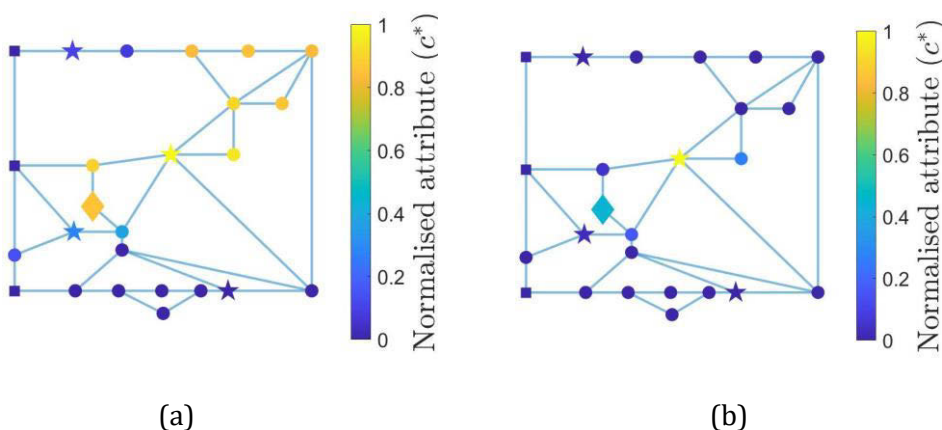


Figure 4. Leak localisation results for (a) Regularised Inverse method and (b) Iterative tuned Regularised method when a leak occurs at node 11 (diamond shape).

The largest negative performance change between RIM and ITRIM is achieved when a leak occurs at node 11. As shown in Figure (4), RIM assigns the high normalised attribute to a set of leak candidates, including the true leak node. In contrast, ITRIM only identifies one main leak candidate but is far from the true leak node. As a result, it leads to the reduction of performance β from 0.61 to 0.41.

ITRIM yields a small set of leak candidates and identifies the true leak node as the main leak candidate in most cases. On the other hand, RIM identifies the true leak node as one of the main leak candidates, which results in a large set of leak candidates.

3.2 Distance Metric

Similar to the performance metric outlined in the previous section, the distance metric is implemented in this section to compute the optimal regularisation parameter for each location in the network that yields the shortest distance between the main leak candidate and the true leak

node. The results are shown in table (1) for ITRIM and RIM with the different number of time step measurements. The distance between the main leak candidate and the true leak node for RIM does not change when multiple time steps measurements are used. However, in most leak scenarios, the distance is decreased when ITRIM is implemented. For instance, in a single-time step measurement, 10 out of 22 leak scenarios result in a shorter distance when ITRIM is utilised, and the average distance decreased from 1024 m to 535 m. In addition, if 12 time steps measurements are used, ITRIM correctly identifies the true leak node as the main leak candidate with a distance of zero in 17 leak scenarios, and the average distance for all leak scenarios is 273 m, which is reduced by more than 75% compared to RIM.

Table 1. Distance between the main leak candidate to the true leak node for both RIM and ITRIM with different number of time step measurements

Leak ID	1TS Distance [m]		6TS Distance [m]		12TS Distance [m]	
	RIM	ITRIM	RIM	ITRIM	RIM	ITRIM
1	0	0	0	0	0	0
2	1930	0	1930	0	1930	0
3	2017	0	2017	0	2017	0
4	2343	326	2343	0	2343	0
5	1858	1858	1858	0	1858	0
6	743	743	743	0	743	0
7	1358	500	1358	500	1358	500
8	0	0	0	0	0	0
9	443	443	443	300	443	0
10	249	249	249	0	249	0
11	791	791	791	791	791	791
12	762	0	762	0	762	0
13	0	0	0	0	0	0
14	1014	1014	1014	0	1014	0
15	832	0	832	0	832	0
16	1746	914	1746	0	1746	0
17	1823	1736	1823	1736	1823	0
18	1412	1412	1412	701	1412	2147
19	1575	864	1575	2407	1575	0
20	711	0	711	0	711	0
21	0	0	0	0	0	0
22	931	931	2252	2578	2252	2578
Average Distance [m]	1024	535	1084	409	1084	273

4 CONCLUSION

This article proposed the method to localise the leaks in the water distribution networks by iteratively solving the regularised inverse problem with the updated regularisation parameter. It determines the optimal regularisation parameter for each location in the network by solving the problem (5) with a set of regularisation parameters. The parameter that yields the highest beta or shortest distance is the optimal regularisation parameter at this location, depending on the choice of metric.

The proposed method has been evaluated with both the performance and the distance metric using a benchmark network Net25 and compared to the standard Regularised Inverse Method in the case of the different number of time step measurements. When the performance metric is applied with multiple time steps measurements, the average leak localisation performance rises from 0.73 to 0.89, representing a 21.3% increase. In the scenario where the distance metric and 12 time steps measurements are used, the average distance between the main leak candidate and the true leak node is reduced by 75%, going from 1084 metres to 273 metres.

In the present work, the effects of uncertainties in the hydraulic model, sensor measurements, and customer demand information on leak localisation performance are not taken into consideration. However, future work can explore the effects of uncertainties on the proposed leak localisation methods. Moreover, the network Net25 only consists of 22 demand nodes, which is relatively small compared to the real operational networks. Therefore, it also recommends evaluating the performance of the proposed leak localisation method in a larger operational network with near-real-time data.

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