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OPTIMAL DESIGN-FOR-CONTROL OF WATER DISTRIBUTION NETWORKS VIA CONVEX RELAXATION

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Abstract

This paper considers joint design-for-control problems in water distribution networks (WDNs), where locations and operational settings of control actuators are simultaneously optimized. We study two classes of optimal design-for-control problems, with the objectives of controlling pressure and managing drinking-water quality. First, we formulate the problem of optimal placement and operation of valves in water networks with the objective of minimizing average zone pressure, while satisfying minimum service requirements. The resulting mixed-integer non-linear optimization problem includes binary variables representing the unknown valve locations, and continuous variables modelling the valves' operational settings. In addition, water utilities aim to maintain optimal target chlorine concentrations, sufficient to prevent microbial contamination, without affecting water taste and odour, or causing growth of disinfectant by-products. We consider the problem of optimal placement and operation of chlorine booster stations, which reapply disinfectant at selected locations within WDNs. The objective is to minimize deviations from target chlorine concentrations, while satisfying lower and upper bounds on the levels of chlorine residuals. The problem formulation includes discretized linear PDEs modelling advective transport of chlorine concentrations along network pipes. Moreover, binary variables model the placement of chlorine boosters, while continuous variables include the boosters' operational settings.

Computing an exact solution for the considered mixed-integer optimization problems can be computationally impractical when large water network models are considered. We investigate scalable heuristic methods to enable the solution of optimal design-for-control problems in large WDNs. As a first step, we solve a convex relaxation of the considered mixed-integer optimization problem. Then, starting from the relaxed solution, we implement randomization and local search to generate candidate design configurations. Each configuration is evaluated by implementing continuous optimization methods to optimize the actuators' control settings and compute feasible solutions for the mixed-integer optimization problem. Moreover, the solution of the convex relaxation yields a lower bound to the optimal value of the original problem, resulting in worst-case estimates on the level of sub-optimality of the computed solutions

We evaluate the considered heuristics to solve problems of optimal placement and operation of valves and chlorine boosters in water networks. As case study, we utilize an operational water network from the UK, with varying sizes and levels of connectivity and complexity. The convex heuristics are shown to generate good-quality feasible solutions in all problem instances with bounds on the optimality gap comparable to the level of uncertainty inherent in hydraulic and water quality models. Future work should investigate the formulation and solution of multiobjective optimization problems for the optimization of pressure and water quality, to evaluate the trade-offs between these two objectives. Moreover, the formulation and solution of robust optimization problems for the design of water networks under uncertainty is the subject of future work.

Keywords
Pressure management, water quality, optimization.

1 BACKGROUND AND MOTIVATION

Water distribution networks (WDNs) are critical infrastructure, facing unprecedented challenges due to increasing water demand, climate change, and more stringent economic and environmental constraints. The rising demand for intelligent water systems requires analytical and technological innovations to support design and automatic control of such complex networks. In particular, the COVID-19 pandemic has highlighted the need for near real-time monitoring and control of water quality to protect public health and provide assurance to customers [1]. The reduction in human resources caused by the pandemic is increasing the demand for automation and reliability of WDNs. The efficient management of WDNs requires the satisfaction of multiple objectives, including optimal control of pressure and water quality. Pressure control allows network operators to minimize leakage [2] and reduce probability of pipe failures [3]. Furthermore, disinfectant residuals in WDNs are critical control variables to preserve water quality and eliminate the risks of pathogen contamination, with chlorine being a commonly used water disinfectant – see for example [4], [5]. Pressure control schemes are implemented by installing and operating pressure reducing valves (PRVs), which control pressure at their downstream node. Optimal pressure control problems in WDNs aim at minimizing average zone pressure (AZP) in WDNs, while satisfying regulatory requirements on minimum pressure at demand nodes [6]. At the same time, water utilities aim to maintain optimal target chlorine concentrations, sufficient to prevent microbial contamination, without affecting water taste and odour, or causing growth of disinfectant by-products [7]–[9]. Moreover, optimized chlorine dosage should avoid spatial and temporal variations, which are perceived as water quality problems by customers. This is achieved by the optimal placement and operation of chlorine booster stations, which re-apply disinfectant at selected locations within WDNs to maintain target chlorine concentrations [10]–[12].

We consider two classes of optimal design-for-control problems, with the objectives of controlling pressure and managing drinking-water quality. First, we formulate the problem of optimal placement and operation of valves in water networks with the objective of minimizing average zone pressure, while satisfying minimum service requirements. Then, we consider the problem of optimal placement and operation of chlorine booster stations, which reapply disinfectant at selected locations within WDNs. In this case, the objective is to minimize deviations from target chlorine concentrations, while satisfying bounds on the levels of chlorine residuals. These problems result in mixed integer optimization problems, combining continuous variables modelling the operational settings of the control actuators, with discrete variables representing their locations.

As shown by the numerical experience reported in [13], [14], off-the-shelf global optimization solvers fail to solve these mixed integer optimization problems, even for small-scale water network models. In [14], the authors proposed new heuristics based on convex optimization to generate good quality feasible solutions when large WDN models are considered. The developed convex heuristic relies on randomization to generate candidate locations for valves and chlorine boosters. Moreover, the heuristic generates lower bounds to the optimal value of the original problem, resulting in estimates on the optimality gap of the computed solutions. This allows a worst-case estimate on the level of sub-optimality of the computed solutions. In this manuscript, we investigate the implementation of the method proposed in [14], using as case study an operational water network from the UK. We evaluate the optimal operation of valves and chlorine boosters over extended time simulations, and compare network pressures, flows, and chlorine concentrations computed by the optimization process, with those obtained by the widely used hydraulic and water quality simulation tool EPANET [15].

2 PROBLEM FORMULATION

In this study, we consider two design-for-control problems in water networks: optimal valve placement and operation, and optimal booster placement and operation. Firstly, we formulate the problem of computing optimal locations of pressure control valves, and their operational settings, while satisfying hydraulic constraints, i.e. mass and energy conservation laws. The objective is to minimize Average Zone Pressure (AZP), while enforcing lower and upper bounds on hydraulic variables. Binary variables are used to model the placement of valves on network links. In our formulation, we consider unidirectional pressure control valves, assuming that the direction of operation of the valves is kept constant for the whole time period. These binary variables are subject to physical and economical constraints, limiting the number of valves considered for installation, and ensuring that only one valve is installed on each link. A detailed description of the problem formulation can be found in [16] .

In comparison, the problem for optimal placement and operation of chlorine boosters is formulated as a mixed integer quadratic program, where decision variables include locations and operational settings of chlorine boosters, and chlorine concentrations at water sources – see as examples [10], [11], [17], [18]. As done in [11], we aim to minimize the deviation from target chlorine concentrations at demand nodes, weighted by nodal demands - we refer to this as Average Target Deviation (ATD). We also enforce lower and upper bounds on water quality variables. In particular, these include maximum allowed chlorine concentrations at network nodes. We include binary variables to model the placement of boosters, with the total number of boosters considered for installation being modelled as optimization constraint. The operational settings of chlorine boosters are defined as the chlorine concentrations at the node where the booster is installed. The transport of chlorine residuals through network pipes is governed by a linear advection reaction partial differential equation (PDE) [19], which we approximate within optimization constraints using an upwind implicit discretization scheme [20]. A complete description of the problem formulation can be found in [21].

3 SOLUTION METHOD

Both design-for-control problems for optimal placement and operation of valves and boosters can be represented as:

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & Ax + g(x) = 0 \\ & Bx + Dz \leq b \\ & z \in \{0,1\}^m. \end{array} \tag{1}$$

The objective $f(x)$ is a (possibly non-linear) convex function representing either AZP or ATD. In the case of optimal valve placement, $g(x)$ models the frictional energy losses across network pipes as non-linear function of the flow rates. In contrast, in the case of optimal booster placement, the discretized PDEs result in $g(x) = Tx + c$, for opportunely define matrix T and vector c. Problem (1) belongs to the class of mixed integer non-linear programming (MINLP). As shown in [13], off-the-shelf MINLP solvers can fail to compute feasible solutions for (1), even when modest size water network models are considered. In [14], the authors have proposed convex heuristics to compute feasible solutions for mixed-integer programs like (1), together with bounds on the level of sub-optimality. For a detailed description of the methods, the reader is referred to [14]. Here, we provide a summary of the proposed convex heuristics.

Step 1. Solve a convex relaxation of (1), where non-convex constraints (if any) are substituted by their polyhedral relaxations:

$$\begin{aligned} \text{minimize} \;\; & f(x) \\ \text{subject to} \;\; & Ax + Rx \le r \\ & Bx + Dz \le b \\ & z \in [0,1]^m, \end{aligned} \tag{2}$$

for opportunely defined matrix R and vector r – see [16]. The convex problem in (2) can be efficiently solved by convex optimization solvers [22], including GUROBI [23] when $f(x)$ is either linear or quadratic. By solving Problem (2), we obtain a lower bound to the optimal value of Problem (1), denoted by f^* , as well as a vector of fractional values $z^* \in \mathbb{R}^m$.

Step 2. Implement the randomized rounding heuristic described in [14], where the fractional values in z^* are used to defined a discrete probability distribution over the set $\{1,\ldots,m\}$, to generate vectors of binary variables $\hat{z} \in \{0,1\}^m$. For each $\hat{z}$, solve the following optimization problem using a non-linear programming solver like IPOPT [24] to compute a locally optimal solution:

$$\begin{aligned} \text{minimize} \;\; & f(x) \\ \text{subject to} \;\; & Ax + g(x) = 0 \\ & Bx \le b - D\hat{z}\,. \end{aligned} \tag{3}$$

Solving Problem (3), we obtain a feasible vector of continuous variables $\hat{x}$, and the corresponding objective function value $f(\hat{x})$. Let $(z^{\text{best}}, x^{\text{best}})$ be the feasible solution corresponding to the lowest objective function value computed by the randomized rounding heuristic. The value $f^{\text{best}} = f(x^{\text{best}})$ is an upper bound to the optimal value of Problem (1). A worst-case estimate on the level of sub-optimality of the solution $(z^{\text{best}}, x^{\text{best}})$ is given by:

$$\text{Gap} = \frac{f^{\text{best}} - f^*}{f^{\text{best}}}. \tag{4}$$

4 CASE STUDY

We evaluate the performance of the considered methods using BWFLnet, the hydraulic model of a water distribution network from the UK – the EPANET model of BWFLnet is given in [25]. BWFLnet has two inlets and it includes 2281 links and 2221 nodes. The network layout is presented in Figure 1. We consider 24 different time steps, one for each hour of the day.

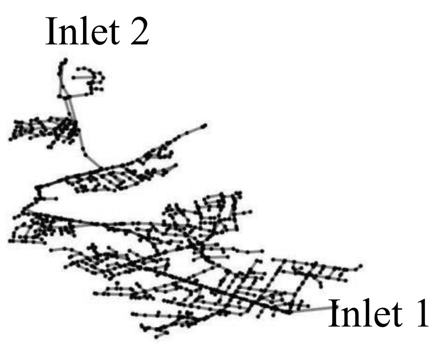

Figure 1. Layout of BWFLnet

4.1 Optimal valve placement and operation in BWFLnet

We formulate the problem of computing optimal locations and operational settings for 3 pressure control valves to be placed in BWFLnet. The convex heuristic method described in Section 3 is implemented to compute a feasible solution with bounds on its level of sub-optimality. The optimization process terminates after 174 seconds, returning a feasible solution corresponding to an AZP value of 33.90 m , and an estimated optimality Gap of 12%. The locations of the three pressure control valves are shown in Figure 2.

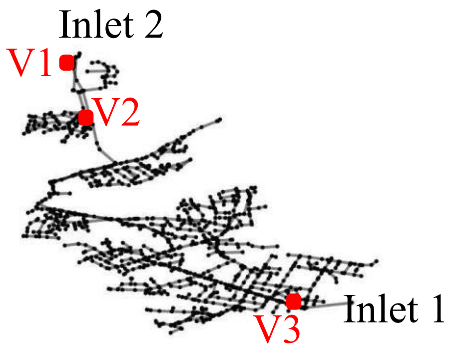

Figure 2. Optimised locations of pressure control valves computed solving Problem (1).

In order to validate the results, we implement the optimised control settings for the three pressure control valves in EPANET. The simulated AZP is very close to the value obtained solving Problem (1), and it is equal to 33.89 m. The largest difference between optimised and simulated pressures is equal to 0.02 m. The accuracy of the predictions made by the optimization model is also illustrated by Figure 3, which reports the temporal average of the errors in nodal pressures:

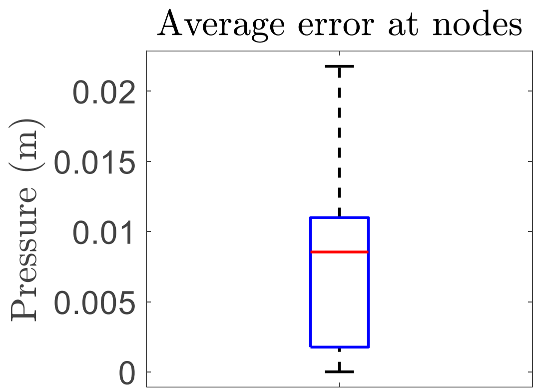

Figure 3. Temporal average of the differences in nodal pressures between EPANET simulations and optimization results.

4.2 Optimal booster placement and operation in BWFLnet

We formulate the problem of optimal placement and operation of 3 chlorine boosters in BWFLnet. We assume a target chlorine concentration of 1 mg/l. The hydraulic head and flows have been fixed based on the solution computed in 4.1, where 3 pressure control valves are optimally operated to minimize AZP. In this case, Problem (1) results in a mixed integer quadratic program (MIQP). We have implemented the state-of-the-art MIP solvers GUROBI and CPLEX but they both failed to compute a feasible solution after two hours of computation. In comparison, our convex heuristic has resulted in a feasible solution with an ATD value of 27.35, and an optimality Gap of 24%. The computational time required by the convex heuristic to converge was 212 seconds. The optimised locations of chlorine boosters are shown in Figure 4.

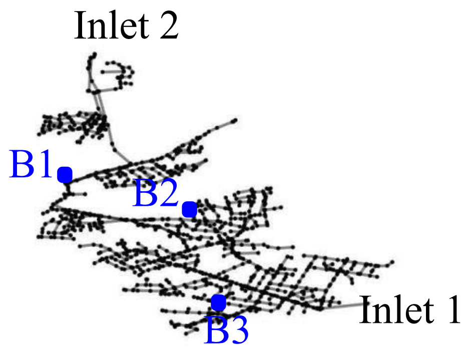

Figure 4. Optimised locations of chlorine boosters computed solving Problem (1).

As done in Section 4.1, we validate our results comparing the optimised chlorine concentrations with those obtained by simulating the boosters' operation in EPANET. As shown in Figure 5, the temporal average differences between simulated and optimised concentrations are smaller than 0.15 mg/l in 99% of nodes. This is comparable to the level of uncertainty inherent in modelling of chlorine residuals in operational water networks.

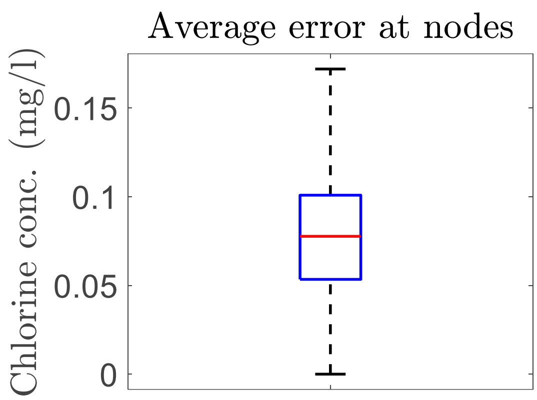

Figure 5. Temporal average of the differences in chlorine concentrations between EPANET simulations and optimization results.

Finally, Figure 6 reports the distribution of simulated chlorine concentrations in BWFLnet at 20:00, where three boosters are optimally operated to minimize ATD. Observe that the vast majority of nodes experience chlorine concentrations close to the target value of 1 mg/l. The few nodes with lower chlorine concentrations have either zero or low demand. Hence, they do not have a significant impact on ATD, whose weights are nodal demands.

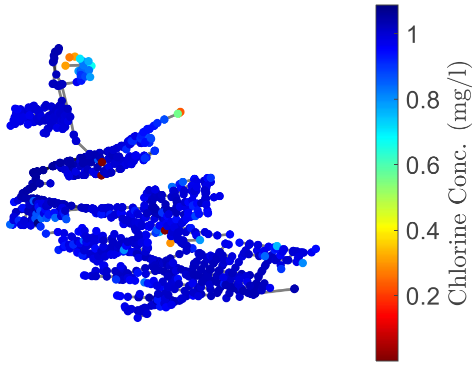

Figure 6. Chlorine concentrations at 20:00 in BWFLnet, with optimized boosters.

5 CONCLUSIONS

The convex heuristics are shown to generate good-quality feasible solutions in for the considered problem instances with bounds on the optimality gap comparable to the level of uncertainty inherent in hydraulic and water quality models. Future work should investigate the formulation and solution of multiobjective optimization problems for the optimization of pressure and water quality, to evaluate the trade-offs between these two objectives. Moreover, the formulation and solution of robust optimization problems for the design of water networks under uncertainty is the subject of future work.

6 REFERENCES

[1] R. S. Quilliam, M. Weidmann, V. Moresco, H. Purshouse, Z. O'Hara, and D. M. Oliver, "COVID-19: The environmental implications of shedding SARS-CoV-2 in human faeces," Environ. Int., vol. 140, no. April, p. 105790, 2020, doi: 10.1016/j.envint.2020.105790.

[2] B. Ulanicki, P. Bounds, J. P. Rance, and L. Reynolds, "Open and closed loop pressure control for leakage reduction," Urban Water, vol. 2, no. 2, pp. 105–114, 2000, doi: 10.1016/S1462-0758(00)00048-0.

[3] A. Lambert and J. Thornton, "The relationships between pressure and bursts – a state-of-the-art update," Water21 - Magazine of the International Water Association, no. 21, pp. 37–38, 2011.

[4] M. M. Polycarpou, J. G. Uber, Z. Wang, F. Shang, and M. Brdys, "Feedback control of water quality," IEEE Control Syst. Mag., vol. 22, no. 3, pp. 68–87, 2002, doi: 10.1109/MCS.2002.1004013.

[5] T. Sakomoto, M. Lutaaya, and E. Abraham, "Managing water quality in intermittent supply systems: The Case of Mukono Town, Uganda," Water (Switzerland), vol. 12, no. 3, 2020, doi: 10.3390/w12030806.

[6] R. Wright, E. Abraham, P. Parpas, and I. Stoianov, "Control of water distribution networks with dynamic DMA topology using strictly feasible sequential convex programming," Water Resour. Res., vol. 51, no. 12, pp. 9925–9941, 2015, doi: 10.1002/2015WR017466.

[7] B. T. Carrico and P. C. Singer, "Impact of booster chlorination on THM production: A simulated analysis," J. Environ. Eng., vol. 135, no. 10, pp. 928–935, 2009, doi: 10.1061/40792(173)126.

[8] T. Walski, "Raising the Bar on Disinfectant Residuals," World Water Mag. Water Environ. Fed., vol. 42, no. 4, pp. 34–35, 2019.

[9] S. Avvedimento, S. Todeschini, C. Giudicianni, A. Di Nardo, T. Walski, and E. Creaco, "Modulating Nodal Outflows to Guarantee Sufficient Disinfectant Residuals in Water Distribution Networks," J. Water Resour. Plan. Manag., vol. 146, no. 8, p. 04020066, 2020, doi: 10.1061/(asce)wr.1943-5452.0001254.

[10] M. Propato and G. Uber, James, "Linear Least-Squares Formulation for Operation of Booster Disinfection Systems," J. Water Resour. Plan. Manag., vol. 130, no. 1, pp. 53–62, 2004, doi: 10.1061/(ASCE)0733-9496(2004)130:1(53).

[11] M. Propato and J. G. Uber, “Booster System Design Using Mixed-Integer Quadratic Programming,” J. Water Resour. Plan. Manag., vol. 130, no. 4, pp. 348–352, 2004, doi: 10.1061/(ASCE)0733-9496(2004)130:4(348).

[12] D. Kang and K. Lansey, “Real-Time optimal valve operation and booster disinfection for water quality in water distribution systems,” J. Water Resour. Plan. Manag., vol. 136, no. 4, pp. 463–473, 2010, doi: 10.1061/(ASCE)WR.1943-5452.0000056.

[13] F. Pecci, I. Stoianov, and A. Ostfeld, “Relax-Tighten-Round Algorithm for Optimal Placement and Control of Valves and Chlorine Boosters in Water Networks,” Eur. J. Oper. Res., vol. 295, no. 2, pp. 690–698, 2021, doi: 10.1016/j.ejor.2021.03.004.

[14] F. Pecci, I. Stoianov, and A. Ostfeld, “Convex Heuristics for Optimal Placement and Operation of Valves and Chlorine Boosters in Water Networks,” J. Water Resour. Plan. Manag., vol. 148, no. 2, pp. 1–14, 2022, doi: 10.1061/(ASCE)WR.1943-5452.0001509.

[15] L. A. Rossman, “EPANET 2,” U.S. Environmental Protection Agency, Cincinnati, 2000. doi: 10.1177/0306312708089715.

[16] F. Pecci, E. Abraham, and I. Stoianov, “Global optimality bounds for the placement of control valves in water supply networks,” Optim. Eng., vol. 20, no. 2, pp. 457–495, 2019, doi: 10.1007/s11081-018-9412-7.

[17] M. E. Tryby, D. L. Boccelli, J. G. Uber, and L. A. Rossman, “Facility location model for booster disinfection of water supply networks,” J. Water Resour. Plan. Manag., vol. 128, no. 5, pp. 322–333, 2002, doi: 10.1061/(ASCE)0733-9496(2002)128:5(322).

[18] K. Lansey, F. Pasha, S. Pool, W. Elshorbagy, and J. Uber, “Locating satellite booster disinfectant stations,” J. Water Resour. Plan. Manag., vol. 133, no. 4, pp. 372–376, 2007, doi: 10.1061/(ASCE)0733-9496(2007)133:4(372).

[19] L. A. Rossman and P. F. Boulos, “Numerical methods for modeling water quality in distribution systems: A comparison,” J. Water Resour. Plan. Manag., vol. 122, no. 2, pp. 137–146, 1996, doi: 10.1061/(ASCE)0733-9496(1996)122:2(137).

[20] M. R. Islam, M. H. Chaudhry, and R. M. Clark, “Inverse modeling of chlorine concentration in pipe networks under dynamic condition,” J. Environ. Eng., vol. 125, no. 3, pp. 296–298, 1999, doi: 10.1061/(ASCE)0733-9372(1999)125:3(296).

[21] F. Pecci, I. Stoianov, and A. Ostfeld, “Optimal Design-for-Control of Chlorine Booster Systems in Water Networks via Convex Optimization,” in Proceedings of the European Control Conference (ECC) 2022, 2022.

[22] S. Boyd and L. Vandenberghe, Convex optimization. Cambridge University Press, 2004.

[23] Gurobi Optimization, “Gurobi Optimizer 9.1 Reference Manual.” 2021, [Online]. Available: https://www.gurobi.com/documentation/9.1/refman/index.html.

[24] A. Waechter and L. T. Biegler, “On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming,” Math. Program., vol. 106, no. 1, pp. 25–57, 2006, doi: 10.1007/s10107-004-0559-y.

[25] F. Pecci, I. Stoianov, and A. Ostfeld, “Case Studies for Optimal Placement and Control of Valves and Chlorine Boosters in Water Networks.” Mendeley Data, 2021, doi: 10.17632/ws9pwxkbb2.4.

[26] B. J. Eck and M. Mevissen, “Non-Linear Optimization with Quadratic Pipe Friction,” 2012.