A STOCHASTIC GOAL-BASED ECONOMIC ORDER QUANTITY MODEL FOR DAIRY PRODUCTS

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Abstract:

Resilient efforts are still sought to optimize the inventory of dairy products under demand uncertainty. Several industries operate under uncertainties that severely affect company performance. In this paper, a multi-objective goal programming model is proposed to optimize the economic order quantity of dairy products under stochastic demand. Adopting a Markov chain approach, the model initially defines the demand transition matrix, inventory cost matrix, objective function, priorities, and goal constraints of dairy products. The model seeks to minimize the deviation variables from the targeted economic order quantity and total inventory costs of dairy products over a finite period planning horizon. The sum of deviations is minimized so that the actual economic order quantity and inventory costs meet the targeted levels of dairy products. Using the simplex method, an analytical solution is obtained and a numerical example is presented for illustration; demonstrating the overachievement or underachievement of the targeted economic order quantity and inventory costs of dairy products. Results indicate how priority-based goal programming solution is sensitive to the highest priority to be achieved. The proposed model can be effective where the relevant economic order quantity and inventory costs can be prioritized if necessary.

Keywords: dairy; goal; inventory; optimization; stochastic.

1. Introduction

The food we eat encompasses the efforts of farmers, traders, food manufacturers, wholesalers, and several other people involved in the food chain. In less middle-income countries, there is a radicle transformation in response to social–economic and demographic changes. The problems associated with the development of a sustainable dairy sector are numerous. The problems/constraints range from inconsistent and insufficient supply of raw materials, poor quality of animal feed supplies, high losses during transportation from farms to factories obsolete processing and ancillary equipment, poor and inconsistent quality of processed dairy products, and sub-optimal use of processing facilities and equipment. Similar eminent problems of the dairy sector also comprise of lack of qualified food technologists, lack of proper hygiene and sanitation practices, inappropriate materials, weak or non-existent market development, lack of technical support and absence of proper management of the dairy processing facility once it is commercialized. However, proper inventory management of dairy products is crucial as it plays an important role in human nutrition in developing countries. This is particularly useful where diets of poor people frequently lack diversity and consumption of animal-source foods may be limited. Better strategies for dairy product inventory management can add the much-needed diversity to plant-based diets in order to foster child growth.

2. Related Literature

Inventory management in the dairy sector (Shafice et al., 2022) can be viewed as a three-level sustainable dairy supply chain under uncertain conditions. The authors propose a multi-objective model to minimize the costs and environmental impacts; and maximize the social impacts of a multi-period and multi-product chain composed of suppliers, producers and retailers and this is applied to a dairy company. In a related development, production planning in the dairy sector was examined where authors developed a mathematical model for the production planning decision of a dairy plant that determines the optimal product mix (Jena and Ray, 2020) The model maximizes the expected profits of a dairy under disruption risk and demand uncertainty in the Indian context. A closely related study of the stochastic model of dairy supply chain valorization was examined by taking into account the multipurpose and batch characteristics of dairies (Ebrahin et al., 2018). The study employs the time resource distribution over the processing nodes and products. Results reflect improved supply chain profits in case of using byproducts instead of discarding them to increase supply chain costs. Shmueli (2012) illustrated how consumer product sector faces volatility in demand on a high scale and level of complexity; thereby posing challenges in the area of inventory management. The project goal was to arrive at an inventory planning policy for perishable classes of goods: yogurt and fresh milk. The inventory policy attempts to balance the costs of understocking versus the cost of overstocking these

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goods. Related research by Roychowdhury (2009) examined the optimal policy for a stochastic inventory model of deteriorating items under time-dependent selling price. The rate of deterioration of the items The author formulated a profit-maximization model and solved the optimum order quantity where numerical examples provided how sensitive the solution was to the system parameter values, lead time distribution and the selling price The profit/market demands/milk vendor's satisfaction trade off problem in the dairy supply chain (Vaklieeva-Pacheva et al., 2007) provide profound insights to the problem. The authors proposed a Pareto frontier control plot to support the planning managers for quick plant profit estimation in case of priorities changing in the dairy supply chain. In a related insight, a replenishment inventory model (Broekmeulen and Van Donselarr, 2006) examines estimated aging and retrieval behavior on products in inventory management. The model assumed the age of inventories which required very simple calculations; but had profound insights especially in terms of randomness of demand. In a related project, the use of a dairy supply chain model to study how market demand for dairy products and how changes in milk composition at the farm level impact on manufacturing performance at the New Zealand dairy industry was studied. (Montes de Oca et al., 2005). The model was developed to mimic the standard supply chain management functions. Within the context of milk products, the effects of constant defective rates and capacity constraints for a chocolate milk manufacturer (Tabuccano and Nagarur, 1997) were studied. In this study, production planning is complex by the stochastic nature of demand for the manufactured products. An analysis of the production planning via dynamic programming approach showed that the company's production order variable was convex. Running the model through first order partial derivatives of the expected total cost function yielded the optimal solution for the production planning problem. Miller (1994) tested the hypothesis that inventory behavior for American cheese, butter and nonfat dry milk was consistent with dynamic cost minimization by dairy manufacturers. In this study, results indicate that firms chose their dairy product inventories so as to minimize quadratic output and inventory lamp costs subject to autoregressive cost shocks. There appears to be a relatively rapid adjustment of commercial dairy product stocks to desired levels. An EOQ model for perishable products under uncertainty (Patriarcha et al., 2020) considers potential backlogging and lost sales where imperfect product quality and deterioration is modeled as a time-dependent variable. In a numerical example, cost-based sensitivity analysis is provided to understand the role of main parameters. Kizito (2012) developed an inventory model with stochastic demand to examine the ordering decisions of milk powder product in supermarkets given a periodic review inventory system under stochastic demand. The decision of hen to order is made using dynamic programming over a finite period planning horizon; demonstrating the existence of an optimal state-dependent ordering policy and total inventory costs. The EOQ problem for a multi-size multi-period single item (Kizito et al., 2013) was presented using milk powder product as a case study. Markov decision process methodologies were suggested where states of the chain represent possible states of demand for different sizes of milk powder product. The decisions of ordering versus not ordering were determined using

dynamic programming and results showed the existence of an optimal state-dependent EOQ for each size of milk powder packet. Extension of inventory using milk powder product examines the joint location inventory replenishment problem at a supermarket chain (Kizito et al., 2017). Using the same supermarket at designated locations, an optimal ordering policy for milk powder product is determined so that the long run inventory costs are minimized for a given state of demand; where states of a markov chain represent possible states of demand for milk powder product. Further research within the context of dairy inventory suggests a supply model design for a dairy company (Orges et al., 2018) in order to allow continuous improvement of supplies necessary for daily production for achieving quality of finished product and satisfaction. Classification of inputs was collected through the ABC method and choice of the inventory model to use resulted into use of the EOQ model; thereby decreasing of orders per month and reducing supply costs. The EOQ model for both ameliorating and deteriorating items by Biswaranjan (2020) explores demand as a cubic function of time where shortages are allowed that are fully backlogged. The model is illustrated with a help of a numerical example to illustrate the different aspects of the model in a practical setting.

The primary contribution of this paper to the literature on EOQ inventory management in the dairy industry are as follows:

- It illustrates how the demand transition matrix and total inventory cost matrix can be computed for a chosen set of dairy products in supermarkets
- The computation procedure for expected demand, total inventory costs and economic order quantity of dairy products
- A stochastic goal programming formulation is presented that allows the decision maker to set priorities for replenishment decisions for optimal inventory costs given different states of demand

3. Model Formulation

A case of a supermarket whose dairy products with stochastic demand was considered. The demand for products during each period over a finite fixed planning horizon is described as either favorable (state F) or unfavorable (state U). The state of a Markov chain represent possible states of demand for dairy products with notation shown in Table 1.

3.1. Computation of Model parameters

Average on hand inventory/Inventory matrix

$$M = \frac{[R+S]}{2} \qquad (1)$$

Demand Transition Probability/Matrix

Considering the customer matrix

$$N(p,q) = \begin{bmatrix} N_{FF}(p,q) & N_{FU}(p,q) \\ N_{UF}(p,q) & N_{UU}(p,q) \end{bmatrix}$$

Table 1: Key Notation used.

i,j	States of demand	d^-_k	Underachievement of kth goal
F	Favorable demand	L	Economic order quantity
U	Unfavorable demand	$X_{ij}(p,q)$	Quantity of product p to be replenished in quarter q
Q	Demand transition matrix	N	Customer matrix
p	Dairy product	c_r	Unit replenishment cost
q	Quarter of the year	c_h	Unit holding cost
FF,FU,UF,UU	State transitions	c_s	Unit shortage cost
Z	Value of objective function	D	Demand matrix
P_k	Pre-emptive priority of kth goal	M	Inventory matrix
d^+_k	Overachievement of kth goal	C	Total inventory cost matrix
R	Beginning inventory	S	Ending inventory

the demand transition probability as demand changes from state i to state j for i,j ϵ (F,U} can be calculated as

$$Q_{ij}(p,q) = \frac{N_{ij}(p,q)}{N_{iF}(p,q) + N_{iU}(P,q)} \tag{3}$$

which yields the demand transition matrix.

$$Q(p,q) = \begin{bmatrix} Q_{FF}(p,q) & Q_{FU}(p,q) \\ Q_{UF}(p,q) & Q_{UU}(p,q) \end{bmatrix} \tag{4}$$

Demand /Matrix, inventory matrix and total inventory cost matrix

Demand matrix

$$D(p,q) = \begin{bmatrix} D_{FF}(p,q) & D_{FU}(p,q) \\ D_{UF}(p,q) & D_{UU}(p,q) \end{bmatrix} \tag{5}$$

Inventory matrix

$$M(p,q) = \begin{bmatrix} M_{FF}(p,q) & M_{FU}(p,q) \\ M_{UF}(p,q) & M_{UU}(p,q) \end{bmatrix} \tag{6}$$

Total inventory cost matrix

When demand outweighs the amount in inventory,

$$C(p,q) = (c_r + c_s + c_h)[D(p,q) - M(p,q)] \tag{7}$$

Similarly, when demand is less than amount in inventory,

$$C(p,q) = c_h[M(p,q) - D(p,q)] \tag{8}$$

As demand changes from state i to state j for i.j ϵ (F,U),

$$C(p,q) = \begin{bmatrix} C_{FF}(p,q) & C_{FU}(p,q) \\ C_{UF}(p,q) & C_{UU}(p,q) \end{bmatrix} \tag{9}$$

where C(p,q) represents the total inventory cost matrix.

3.2. Expected demand, total inventory costs and economic order quantity

Expected demand

Favorable demand:

$$E[D_F(p,q)] = Q_{FF}(p,q)D_{FF}(p,q) + Q_{FU}(p,q)D_{FU}(p,q) \tag{10}$$

Unfavorable demand

$$E[D_U(p,q)] = Q_{UF}(p,q)D_{UF}(p,q) + Q_{UU}(p,q)D_{UU}(p,q) \tag{11}$$

Expected inventory

Favorable demand:

$$E[M_F(p,q)] = Q_{FF}(p,q)M_{FF}(p,q) + Q_{FU}(p,q)M_{FU}(p,q) \tag{12}$$

Unfavorable demand:

$$E[M_U(p,q)] = Q_{UF}(p,q)M_{UF}(p,q) + Q_{UU}(p,q)M_{UU}(p,q) \tag{13}$$

Expected total inventory costs

Favorable demand:

$$E\{C_F(p,q)] = Q_{FF}(p,q)C_{FF}(p,q) + Q_{FU}(p,q)C_{FU}(p,q) \tag{14}$$

Unfavorable demand:

$$E[C_U(p,q)] = Q_{UF}(p,q)C_{UF}(p,q) + Q_{UU}(p,q)C_{UU}(p,q) \tag{15}$$

Expected economic order quantity

Favorable demand

$$E[L_F(p,q)] = E[D_F(p,q) - M_F(p,q)]$$

$$if \; D_F(p,q) > M_F(p,q) \tag{16}$$
$$E[L_F(p,q) = 0 \; otherwise \tag{16.1}$$

Unfavorable demand

$$E[L_U(p,q)] = E[D_U(p,q) - M_U(p,q)]$$

$$if \; D_U(p,q) > M_U(p,q) \tag{17}$$
$$E[L_U(p,q) = 0 \; otherwise \tag{17.1}$$

3.3. Stochastic goal programming formulation

We now formulate the stochastic goal programming model by setting priorities, defining the objective function and goal constraints

Set priorities

P_1: Replenish a batch of $E[L_F(p,q)]$ packets when demand is favorable

P_2: Replenish a batch of $E[L_U(p,q)]$ packets when demand is unfavorable

P_3: Total inventory cost must not exceed E[C_F(p,q)] when demand is favorable

P_4: Total inventory cost must not exceed E[C_U(p,q)] when demand is unfavorable

Objective function

Minimize $Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_q^4 P_k\,(p,q)(d_k^+ + d_k^-)$ (18)

Goal constraints

P_1: Economic order quantity E[L_F(p,q)] – favorable demand

$X_{FF}(p,q) + X_{FU}(p,q) - d_1^+ + d_1^- = E[L_F(p,q)]$ (18.1)

P_2: Economic order quantity E[L_U(p,q)] – unfavorable demand

$X_{UF}(p,q) + X_{UU}(p,q) - d_2^+ + d_2^- = E[L_U(p,q)]$ (18.2)

P_3:Total inventory cost – favorable demand

$X_{FF}(p,q)\,C_{FF}(p,q) + X_{FU}(p,q)\,C_{FU}(p,q) + d_3^- - d_3^+ =$
$= E[C_F\,(p,q)]$ (18.3)

P_4:Total inventory cost – unfavorable demand

$X_{UF}(p,q)C_{UF}(p,q) + X_{UU}(p,q)C_{UU}\,(p,q) - d_4^- - d_4^+ =$
$= \mathrm{E}[C_U(p,q)]$ (18.4)

3.4. Stochastic goal programming model for economic order quantity

Minimize $Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_q^4 P_k\,(p,q)(d_k^+ + d_k^-)$ (19)

Subject to:

$X_{FF}(p,q) + X_{FU}(p,q) - d_1^+ + d_1^- = E[L_F(p,q)]$ (19.1)

$X_{UF}(p,q) + X_{UU}(p,q) - d_2^+ + d_2^- = E[L_U(p,q)]$ (19.2)

$X_{FF}(p,q)\,C_{FF}(p,q) + X_{FU}(p,q)\,C_{FU}(p,q) + d_3^- - d_3^+ =$
$= E[C_F\,(p,q)]$ (19.3)

$X_{UF}(p,q)\,C_{UF}(p,q) + X_{UU}(p,q)\,C_{UU}(p,q) + d_4^- - d_4^+ =$
$= E[C_F\,(p,q)]$ (19.4)

$X_{FF}(p,q), X_{FU}(p,q), X_{UF}(p,q), X_{UU}(p,q)$ ≥0 (19.5)

$C_{FF}(p,q)\,, C_{FU}\,(p,q), C_{UF}(p,q)\,.\,C_{UU}\,(p,q)$ ≥ 0 (19.6)

$d_1^-\,,\,d_1^+\,,d_1^-$ - $d_1^+\,,d_3^-$ - $d_3^+\,,\,d_4^-\,,\,d_4^+$ ≥ 0 (19.7)

4. A case study

In this section, a real case application from Shoprite supermarket in Uganda was used to demonstrate the applicability of the proposed model. The supermarket orders and sells dairy products for customers. The numerical illustration contains real data for the first quarter of the year as shown in Table 2. Data classification was made by state of demand, analyzed and used in the proposed model. Considering packets of cheese (product A) for the given week, demand is favorable (state F) if $N_{ij} > 25$; otherwise demand is unfavorable (state U) if $N_{ij} \leq 25$ as shown in Table 2

Table 2: Data classification by state of demand for packets of cheese (product A).

Month	Week	Customers (N)	Demand (D)	On hand Inventory (M)	State of Demand (i)
	1	15	308	5263	U
	2	24	2891	7337	F
1	3	24	1757	7081	U
	4	35	6619	5654	F
	1	8	231	3525	U
	2	17	2046	6243	U
2	3	15	1617	5922	U
	4	45	4443	5951	F
	1	14	559	3765	U
	2	37	3686	4736	F
3	3	28	1537	4480	F
	4	14	5626	5746	F

4.1. State transitions and on hand inventory

For a particular state transition, given the beginning and ending inventory, the average on hand inventory (in packets) was calculated as presented in Table 3.

Table 3: Average on hand inventory for packets of cheese (product A).

State Transition (i,j)	Beginning Inventory (B)	Ending Inventory (E)	Average on hand Inventory V = (B+E)/2
FF	4980	5746	5363
FU	7081	3765	5423
UF	7337	4738	6037.5
UU	6243	5922	6082.5

4.2. Demand transition probabilities

Data classification by state transition was done as illustrated in Table 4 and then used to calculate the demand transition probabilities of product A.

Table 4: Data classification by state transition for packets of cheese (product A).

Month	State transition	Number of customers $N_{ij}(A,1)$	Demand $D_{ij}(A,1)$
1	FF	0	0
	FU	53	4648
	UF	106	11575
	UU	0	0
2	FF	0	0
	FU	0	0
	UF	60	6060
	UU	57	5940
3	FF	137	12386
	FU	0	0
	UF	51	4245
	UU	0	0

From Table 4, the totals for customers and demand as transitions are made from state to state are as follows:

Customers:

$$N_{FF}(A,1) = 137 \qquad N_{FU}(A,1) = 53$$

$$N_{UF}(A,1) = 106 + 60 + 51 = 217 \quad N_{UU}(A,1) = 57$$

Demand:

$$D_{FF}(A,1) = 12386 \qquad D_{FU}(A,1) = 4648$$

$$D_{UF}(A,1) = 11575 + 6060 + 4245 = 21880$$

$$D_{UU}(A,1) = 5940$$

From equation (3) in section 3.1, the demand transition probabilities are:

$$Q_{FF}(A,1) = \frac{N_{FF}(A,1)}{N_{FF}(A,1) + N_{FU}(A,1)} = \frac{137}{137+53} = 0.7211$$

$$Q_{FU}(A,1) = \frac{N_{FU}(A,1)}{N_{FF}(A,1) + N_{FU}(A,1)} = \frac{53}{137+53} = 0.2789$$

$$Q_{UF}(A,1) = \frac{N_{UF}(A,1)}{N_{UF}(A,1) + N_{UU}(A,1)} = \frac{217}{217+57} = 0.7920$$

$$Q_{UU}(A,1) = \frac{N_{UU}(A,1)}{N_{UF}(A,1) + N_{UU}(A,1)} = \frac{57}{217+57} = 0.2080$$

Hence the demand transition matrix from equation (4) is

$$Q(A,1) = \begin{bmatrix} 0.7211 & 0.2789 \\ 0.7920 & 0.2080 \end{bmatrix}$$

4.3. Demand matrix, inventory matrix and total inventory cost matrix

The demand matrix, inventory matrix and total inventory cost matrix were obtained as follows:

Front equation (5), the demand matrix becomes

$$D(A,1) = \begin{bmatrix} 12386 & 4640 \\ 21880 & 5940 \end{bmatrix}$$

Front equation (6), the inventory matrix becomes

$$M(A,1) = \begin{bmatrix} 5363 & 54238 \\ 6037.5 & 6082.5 \end{bmatrix}$$

Total inventory cost matrix

The total inventory cost matrix can be computed for cheese (product A). Considering equations (7), (8) and (9)

Unit replenishment cost $c_r(A)$ = \$31.5

Unit holding cost $c_h(A)$ = \$0.63

Unit shortage cost $c_s(A)$ = \$3.465

$$C_{FF}(A,1) = [c_r(A) + c_h(A) + c_s(A)][D_{FF}(A,1) - M_{FF}(A,1)]$$

$$C_{FF}(A,1) =]31.5 + 0.63 + 3.465][12386 - 5363] = 249983.685$$

$$C_{FU}(A,1) = c_h(A)[M_{FU}(A,1) - D_{FU}(A,1)]$$

$$C_{FU}(A,1) = (0.63)(5423 - 4648) = 488.25$$

$$C_{UF}(A,1) = [c_r(A) + c_h(A) + c_s(A)[D_{UF}(A,1) - M_{UF}(A,1)]$$

$$C_{UF}(A,1) = (31.5 + 0.63 + 3.465)(21880 - 6037.5) = 563913.7875$$

$$C_{UU}(A,1) = c_h(A)(M_{UU}(A,1) - D_{UU}(A,1))$$

$$C_{UU}(A,1) = (0.63)(6082.5 - 5940) = 89.775$$

From equation (9), $C(A,1) = \begin{bmatrix} 249983.685 & 488.25 \\ 563913.7875 & 89.775 \end{bmatrix}$

4.4. Expected demand, inventory, inventory costs and economic order quantity

The demand transition matrix developed in section 4.2 helps us to determine the expected demand, inventory, total inventory costs and economic order quantity considering both favorable and unfavorable demand.

Expected Demand

Considering equation (10) - Favorable demand (state F)

$$E[D_F(A,1) = Q_{FF}(A,1) * D_{FF}(A,1) + Q_{FU}(A,1) * D_{FU}(A,1)$$

$$E[D_F(A,1) = (0.7211 * 12386) + (0.2789 * 4648)$$

$$E[D_F(A,1 = 11227.8718$$

Considering equation (11) - Unfavorable demand (state U),

$$E[D_U(A,1) = Q_{UF}(A,1) * D_{UF}(A,1) + Q_{UU}(A,1) * D_{UU}(A,1)$$

$$E[D_U(A,1) = (0.7920 * 21880) + (0.3080 * 5940)$$

$$E[D_U(A,1) = 18564.48$$

Expected Inventory

Considering equation (12) - Favorable demand (state F),

$$E[M_F(A,1) = Q_{FF}(A,1) * M_{FF}(A,1) + Q_{FU}(A,1) * M_{FU}(A,1)$$

$$E[M_F(A,1)] = (0.7211 * 5363) + (0.2789 * 5423)$$

$$E[M_F(A,1) = 5379.734$$

Considering equation (13) - Unfavorable demand (state U),

$$E[M_U(A,1) = Q_{UF}(A,1) * M_{UF}(A,1) + Q_{UU}(A,1) * M_{UU}(A,1)$$

$$E[M_U(A,1)] = (0.7920*6037.5) + (0.2080*6082.5)$$

$$E[M_U(A,1) = 6046.86$$

Expected Total Inventory costs

The expected total inventory costs are now computed for the product considering both favorable and unfavorable demand using equations (14) and (15)

Favorable demand (state F)

$$E[C_F(A,1) = Q_{FF}(A,1) * C_{FF}(A,1) + Q_{FU}(A,1) * C_{FU}(A,1)$$

$$E[C_F(A,1)] = (0.7211*229983.685) + (0.2789 *488.25)$$

$$E[C_F(A,1) = 180399.4082 \ USD$$

Unfavorable demand (state U)

$$E[C_U(A,1)] = Q_{UF}(A,1) * C_{UF}(A,1) + Q_{UU}(A,1) * C_{UU}(A,1)$$

$$E[C_U(A,1) = (0.7920 * 563913.7875) + (0.2080 * 89.775)$$

$$E[C_U(A,1) = 446638.3929 \ USD$$

Expected economic order quantity

Using equations (16) and (17), the expected economic order quantity considering both favorable and unfavorable demand was computed whose results are as follows:

Favorable demand (state F)

$$E[L_F(A,1) = E[D_F(A,1)] - E[M_F(A,1)$$

$$if \ E[D_F(A,1)] > E[M_F(A,1)]$$

$$E[L_F(A,1) = 0 \quad otherwise$$

$$E[L_F(A,1) = 11227.8718 - 5379.734 = 5846.1378$$

Unfavorable demand (state U)

$$E[L_U(A,1) = E[D_U(A,1)] - E[M_U(A,1)$$

$$if \ E[D_U(A,1)] > E[M_U(A,1)]$$

$$E[L_U(A,1) = 0 \quad otherwise$$

$$E[L_U(A,1) = 18564.86 - 8046.86 = 12517.62$$

4.5. Stochastic goal programming model

The stochastic goal programming model for the livestock product was formulated by setting priorities, defining the objective function and formulating the goal constraints

Priorities

P_1: Replenish an order quantity of 5848.1378 units when demand is initially favorable

P_2: Replenish an order quantity of 12517.62 units when demand is initially unfavorable

P_3: Total inventory costs must not exceed $180399.4082 when demand is favorable

P_4: Total inventory costs must not exceed $446638.3929 when demand is unfavorable

Objective Function

$$Minimize \ Z = \sum_{k=1}^{4} [P_k(A,1)d_k^+ + P_k(A,1)d_k^-]$$

Goal constraints

Economic order quantity

$$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 5848.1378 \quad favorable \ demand$$

$$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 12517.62 \quad unfavorable \ demand$$

Total inventory costs

$$249983.685 X_{FF}(A,1) + 488.25 X_{FU}(A,1) - d_3^+ =$$
$$= 180399.4082 \qquad favorable \ demand$$

$$563913.7875 X_{uf}(A,1) 89.77 X_{uU}(A,1) - d_4^+ =$$
$$= 446638.3929 \qquad favorable \ demand$$

Non-negativity

$$X_{FF}(A,1), X_{FU}(A,1, X_{UF}(A,1), X_{UU}(A,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

where d_1^-, d_2^- = slack variables

d_3^+, d_4^+ = surplus variables

$X_{FF}(A,1)$ = Economic order quantity of product A when initially favorable demand remains favorable

$X_{FU}(A,1)$ = Economic order quantity of product A when initially favorable demand becomes unfavorable

X_{UF}(A,1) = Economic order quantity of product A when initially unfavorable demand becomes favorable

X_{UU}(A,1) = Economic order quantity of product A when initially unfavorable demand remains unfavorable

5. Results and Discussion

The stochastic goal programming model for dairy products was solved using MATLAB. Data input was done in MATLAB TM; and using the linprog solver, an optimal solution was obtained whose values are presented in Table 5.

The results highlight the optimal values of the economic order quantity of cheese (product A) in the first quarter of the year when demand changes from one state to another. The results were analyzed and discussed based on the priorities set and the optimal values achieved as shown in Table 5. The results can establish the overachievement or underachievement of the economic order quantity priorities needed for inventory planning. An explanation of this case incorporates markov chains by considering changes from one state to another. Based on the results, when demand is initially favorable or unfavorable, additional packets of cheese (product A) cannot be replenished. The amount in stock is sufficient to cater for the prevailing demand since the model predicts zero units (=0) of the economic order quantity for the product in the first quarter of the year. The model also predicts the economic order quantity for the product of 2.3729 units and 1040840 units when initially favorable demand becomes unfavorable; and unfavorable demand remains unfavorable.

As observed in Table 6, priorities 1, 2 and 3 can be fully achieved. However, an underachievement of 5478.7 units is realized in the first quarter of the year when demand is initially favorable (state F).

Priority 4 is partially achieved since the actual stochastic solution is slightly higher than the expected goal value of the targeted total inventory costs in the first quarter. When demand is initially unfavorable (state U) an over achievement of 677.130 units is realized.

6. Conclusion

A stochastic goal programming model that optimizes the economic order quantity of dairy products with stochastic demand was presented in this paper. The model determines the quantity of cheese(in packets) to be replenished during the first quarter of the year when demand changes from state i to state j for i,j ∈ {F,U}; establishing the overachievement or underachievement of the economic order quantity priorities desired for aggregate planning. The model was solved with the help of MATLAB software and the results indicate the optimal economic order quantity as demand changes from one state to another. Further research is however needed to extend the proposed model in order to handle demand and price uncertainty for dairy products. In addition, weighted goal programming can be adopted to improve the computational aspects when handling pre-emptive priorities of the product in question.

6.1. The batch EOQ model versus the stochastic goal-based approach

The batch economic order quantity (EOQ) model is a fundamental concept for minimizing storage costs and maintain efficient inventory levels to match inventory with demand. However, high costs of frequent ordering and high risks of production interruptions due to low product inventory is a major challenge. The method is also limited by the assumption of one product business; not allowing for combining several different products at the same time. However, the proposed stochastic goal-based approach to optimize the economic order quantity gains greater insight into supply chain operations while simultaneously reducing costs. A company can more efficiently identify and prioritize objectives and track progress towards economic order quantity goals. The computational efficiency through the goal-based approach can allow a company to stay within an efficient linear programming computational environment; by considering several products in a business setting.

Acknowledgements

Table 5: Optimal solution from MATLAB.

Variable	X_{FF}(A,1)	X_{FU}(A,1)	X_{UF}(A,1)	X_{UU}(A,1)	d_1^-	d_2^-	d_3^+	d_4^+
Values	0	364.4816	0	125.18	5478.7	0	0	677.150

Table 6: Expected values of stochastic solution considering over and under achievement.

Goals/priorities	Expected value from Goal	Value of stochastic solution	Deviation	Over achievement	Under Achievement
1	5848.1378	5848.1816	0.0438		5478.7
2	12517.62	125.18	0.38		0
3	180399.4082	180399.3912	0.017	0	
4	446638.3929	446673.45	35.0571	6771.3	

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