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# OPTIMAL OPERATION IN SECTORIZED NETWORKS WITH INTERMITTENT WATER DISTRIBUTION

**Rui Gabriel Souza[1], Bruno Brentan[2], Gustavo Meirelles[3] and Daniel Bezerra[4]**

[1, 2, 3, 4]Department of Hydraulic Engineering and Water Resources, Federal University of Minas Gerais, Belo Horizonte, Minas Gerais, Brazil
[1]Deptartment of Civil Engineering, Polytechnical Institute of Pontifical Catholic University of Minas Gerais, Belo Horizonte, Minas Gerais, Brazil

[1] *rui.g182@gmail.com;* [2] *brentan@ehr.ufmg.br;* [3] *gustavo.meirelles@ehr.ufmg.br;*
[4] *danielbezerrab@gmail.com*

## Abstract

The deterioration of water distribution infrastructure over the years, associated with an increase in demand and seasonal droughts can lead to an intermittent operation of the system, as the consumers cannot be supplied with a minimum pressure. The lack of investments to rehabilitate the system can be limited, and in this scenario, the intermittent water supply become a normal operation. This scenario difficulties even more the recovery of the continuous supply, as the expenses for water production increase and the revenue decrease. Thus, strategies to achieve an optimal operation in these conditions are fundamental to overcome this critical period in the best way possible. Two main objectives have to be set: minimize the operational costs, that can be described by the energy consumption in pumping stations and the volume of water lost in leakages, and, maximize water demand supplied to the consumers. Avoiding the supply during night periods, when the pressure remains high, naturally reduces the leakage volume lost. However, as the system is not operated 24h per day, it is expected a different pattern of consumption, with higher peaks during the supply period. This can lead to a significant increase in the power required by the pumps, as its head have to be higher to overcome the increased headlosses. Thus, this paper proposes an optimal operation of water distribution networks based on the scheduling of supply to different sectors of the network. This strategy aims to control the increase of the headlosses, as only part of the consumers will be supplied during a period of the day. Thus, the main pipes will not be overloaded and the power required for the pumping stations will remain low. The proposed procedure first divides the network into sub-systems using a k-means algorithm. Then, with the number of sub-systems defined, an optimal scheduling of their supply will be done. Each sub-system can have different time periods of supply, as bigger sub-system will require a higher water volume to be supplied. In addition, the pumps will be select to optimize the operation, and for each period, their rotational speed will be optimized to minimize the operational costs. The same number of sub-systems will be considered for the number of pumps in the pumping station, so adequate pumps can be selected to supply each sector. PSO algorithm will be used to optimize the operation.

**Keywords**
Water Distribution Network, Intermittent Operation, Sectorization, Optimization.

## 1 INTRODUCTION

Water Distribution Networks (WDNs) are designed to operate continuously to guarantee comfort and security for the consumers. However, the population growth, the deterioration of the infrastructure and periodically droughts lead to a situation where the system is not capable to attend the demand. When this condition is established, the Intermittent Water Supply (IWS) is commonly used to mitigate the problem. In this case, the consumers will only have water supplied during a few hours of the day, and currently, one-third of people are affected by this problem [1].

Despite the attenuation of the water supply problem, [2] highlight that this approach can lead to other issues, such as pipe bursts, water contamination and increase in energy consumption.

The IWS can be the result of a pressure deficit caused by increased headlosses or a diminished water source compared to the consumption. Leakages are a significant issue in these situations, as it contributes both for the pressure and water deficits. As the WDNs becomes larger, its operation, especially in IWS conditions, becomes more complex, as the adjustment of several hydraulic components are added to the decision-making process [3]. Thus, as described by [4], the partition of the network into District Metered Areas (DMAs) can simplify its operation, as each DMA, with less complexity, can be individually studied. As a result, [5] highlight the improvements in leakage control, identification of pipe bursts, water quality and security. In addition, the DMAs can be useful for the IWS, as it allows an equitable water supply for each DMA separately [6].

However, in a WDN supplied by a pump station, the alteration of the operation pattern from continuous to intermittent can significantly increase the energy consumption, as the pumps will not operate in their Best Efficiency Point (BEP). [7] shows the importance of the correct selection of pumps according to the systems characteristics to achieve energy and hydraulic efficiency. Thus, new pumps can be selected to avoid this issue. However, the change in operation pattern will also change the consumption pattern. [8] found that IWS leads to a reduction in consumption per capita, while [9] describes the behaviour to store water in emergency situations and discard unused "old" water when the supply cycle restarts, increasing the consumption. The number of operating hours, the average pressure and the social conditions of the consumers are aspects to consider when evaluating this issue and then, select an appropriate pump to the case. [2] proposes the full supply of water demand in shorter times. The authors found that this can be achieved, but with a high operational cost, since the headlosses are significantly increased. Posteriorly, [10] shown that the rehabilitation of main pipes could solve this issue.

As mentioned above, leakages are a major problem for IWS. Even if the total volume lost is reduced due to the lower number of hours operating, especially during the night period, when the supply cycle starts, pumps need to operate with higher head to try attend the demand, and leakage at some points can increase. Once again, the use of DMAs could be well used in this case, using smaller pumps at the entrance of each sector, or a pressure reducing valve, in case the DMA is located at a lower elevation.

This paper proposes the optimal operation of WDN under IWS conditions. First, DMAs are created using a Social Network Community Detection algorithm. Each sector will be individually supplied during a certain time, with the goal to attend the total water volume demanded in a shorter period. The duration and the period of the day will be defined trough an optimization process, as bigger system can require more time to attend the demand, and the operation of pump stations with higher power can be avoid during periods with higher energy tariffs. The demand pattern will be adjusted accordingly to each case. In addition, the scheduling (rotational speed at each operating hour) and selection (head and flow at BEP) of the pumps located at the entrance of each sector will be also done in this optimization process, where the Particle Swarm Optimization (PSO) will be used. The proposed methodology is applied into the OBCL-1 network [11].

## 2 METHODOLOGY

The proposed methodology to optimize the IWS operation is based on the creation of sectors (DMAs) and the operation of each one during different periods along the day. The operational optimization of each sector is based on the minimization of the cost function compound by two parts: i) energy cost; and ii) water leakage cost. The first is based on the energy consumption of the pump station to supply the water volume required by the customers of each sector at a minimal pressure level. The second aims to improve the pressure of the operated sector to values closer to the minimum required in order to reduce the leakage volume during the operation period of the sector. Figure 1 summarizes the process to achieve an optimized operation and the following sections describe each process presented.

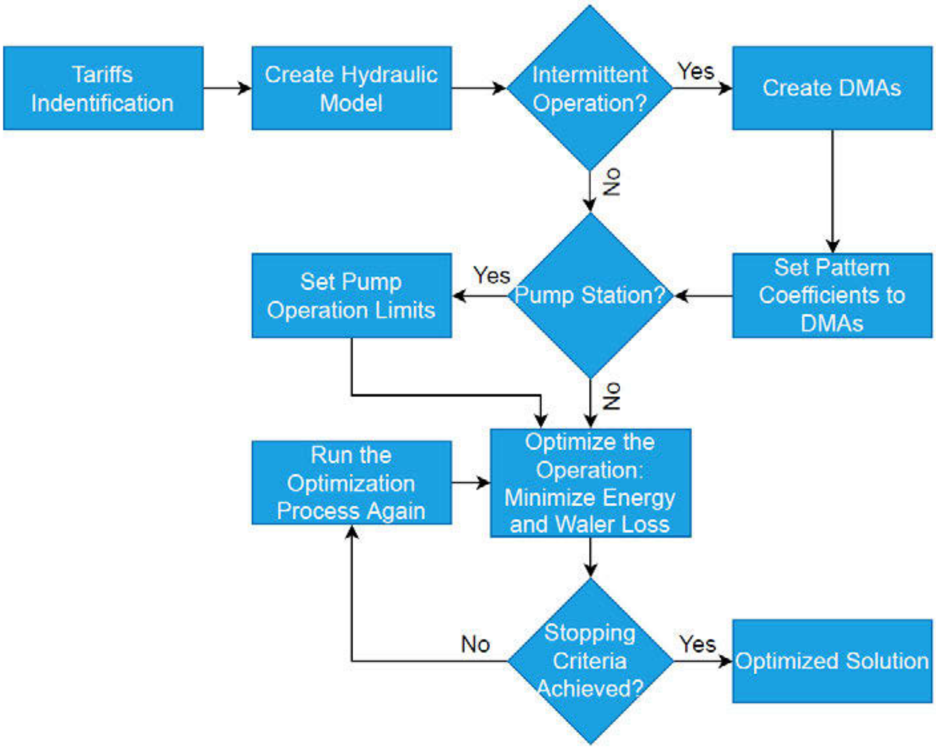


*Figure 1. Flowchart of the proposed optimization for intermittent sectorization operation*

## 2.1. Case Study

The benchmark network OBCL-1, firstly presented by [11] and further studied by [2] for an optimized intermittent operation, is used as a case study. As presented in Figure 2, it has 269 nodes and 294 pipes, with a daily demand of 14,270 $m^3$. The supply is made by a pump station composed by four pumps. Each pump will be used to supply a specific DMA to try to operate as close as possible to its best efficiency point. Finally, the leakage $Q_L$ is modelled trough emitters, using Equation 1, with the coefficients $\alpha$ and $\beta$ set for each node of the network model as 0.03 and 0.5 respectively, the same adopted by [2] to be able to compare the results and visualize the improvements. Then, the daily leakage volume $(DL)$ can be calculated multiplying the Equation 1 by each time step to be simulated $(\Delta t)$ until reaching the total operating time $(t)$ each day from the first node up to the $n^{\text{th}}$ node of the system with emitter as described in Equation 2.

$$Q_L = \alpha \cdot h^{\beta} \tag{1}$$

$$DL = \sum_{j=1}^{n} \sum_{i=1}^{t} Q_{L_{n,i}} \cdot \Delta t \tag{2}$$

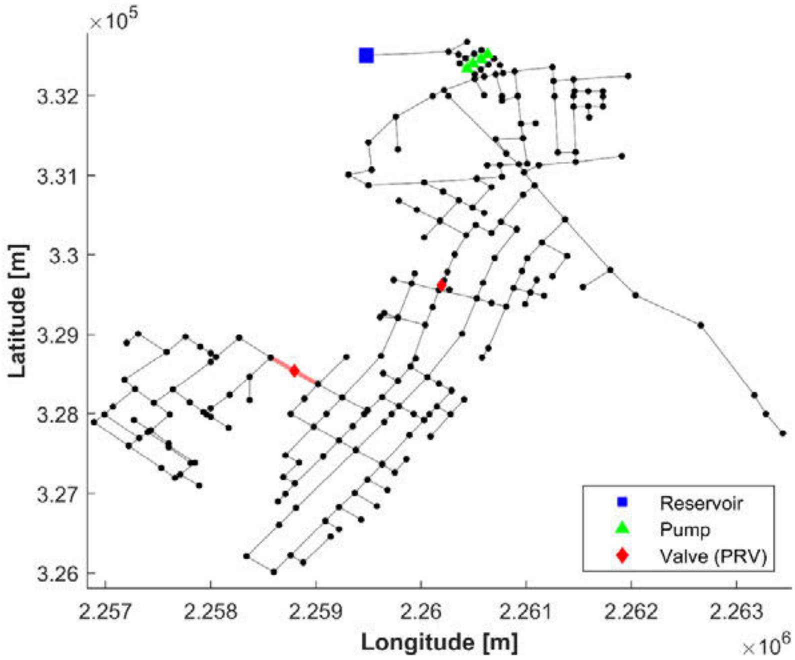


*Figure 2. OBCL-1 network*

For the economic analysis, the energy and power tariffs, and the cost of water production considered are presented in Table 1. As there is a significant difference in the water production costs in Brazil, three different scenarios were evaluated to verify its impact on the operation. The peak hours (PH) considered are between 17 h and 20 h and the other periods are nonpeak hours (NPH).

*Table 1. Energy and water tariffs*

| | |
|---|---|
| Energy tariff - nonpeak [R$/kWh]* | 0.3567 |
| Energy tariff - peak [R$/kWh]* | 0.5342 |
| Power tariff - nonpeak [R$/kW]* | 13.950 |
| Power tariff - peak [R$/kW]* | 43.850 |
| Water production cost – High [R$/m$^3$]** | 7.820 |
| Water production cost – Low [R$/m$^3$]** | 0.300 |
| Water production cost – Average [R$/m$^3$]** | 3.570 |

Source: [12]* and [13]**.

## 2.2. District metered area (DMA) creation

In order to generate the DMAs for intermittent operation, this work applies the methodology presented by [14]. This methodology is based on data mining technique applied to the water distribution features for clustering the nodes of the hydraulic model. As data mining algorithm, k-means [15] is used. K-means is a similarity-based technique for clustering non-labelled data. The similarity of samples is measured as an Euclidian distance $d_{i,j}$ between a normalized input vector, $x$, and the clusters centers $c$ (Equation 3).

$$d_{i,j} = \sqrt{(x_i - c_j)^2} \tag{3}$$

The clustering process starts randomly selecting points in the feature space, according to the number of pre-defined clusters. In this work, the number of clusters (i.e., the number of DMAs) is defined as four (4), considering the topology of the water distribution system. After defining the clusters centers, k-means algorithm calculates the distance among all samples and centers (Equation 4). Each sample is attributed to a cluster according to that distance. Then, the new center position is recalculated as the average value of all samples belonging to a defined cluster *j*.

$$c_j = \frac{\sum x_i}{N_j} \; \forall \, x_i \, \in j \tag{4}$$

where $N_j$ is the number of samples belonging to the cluster $j$. After recalculate the clusters centers, the similarity matrix is recalculated and the samples are re-clustered. This process finishes when the changes on clustering center is lower than a defined limit.

In this work, following [14], the input feature vector is built based on geographic coordinates of nodes, base demand and elevation. Geographic coordinates are responsible to give information about the closeness of nodes, regarding those nodes belonging to a DMA should be interconnected by pipes and control elements. Base demand and elevation can bring to data mining analysis the hydraulic similarity of the nodes, resulting in a more controllable DMA. This because more similar hydraulic nodes require similar controls for improving the hydraulic efficiency of the system.

After the clustering process, the DMAs are almost defined. Nevertheless, a post-processing algorithm is required, since k-means is not able to catch connectivity features of the system. Eventually, non-connected nodes can belong to the same cluster, that is hydraulic impossible. In this sense, the post-processing algorithm evaluate the connectivity of all nodes and rearrange each cluster according to the connectivity of the system.

Finally, boundary pipes should be identified. Those pipes connect two DMAs and they are responsible for isolation and control of each DMA. Usually, flow meters and control valves are installed at open boundary pipes, while isolation valves are installed at closed boundary pipes.

### 2.3. Pattern Distribution

The water consumption ($D_t$), according to Equation 5, is the product of the base demand ($D_m$) set for each node of the hydraulic model – that represents the average daily consumption of customers connected to the system – by the dimensionless pattern coefficients ($q_t$), that adjust the base demand for a pattern consumption for a specific hour ($t$) of the day.

$$D_t = D_m q_t \tag{5}$$

As proposed by [2] and assumed in this work, the daily water consumption in CWS is equal to IWS, being necessary to modify only the pattern coefficients ($q_t$) according to the number of operation hours of each sector, which results in higher demand flows. So, the difference in the operational costs is the result exclusively of the improvements in the management of the system, since the total water volume supplied in CWS and IWS is the same. The pattern for each sector and time operation are presented in Figure 3.

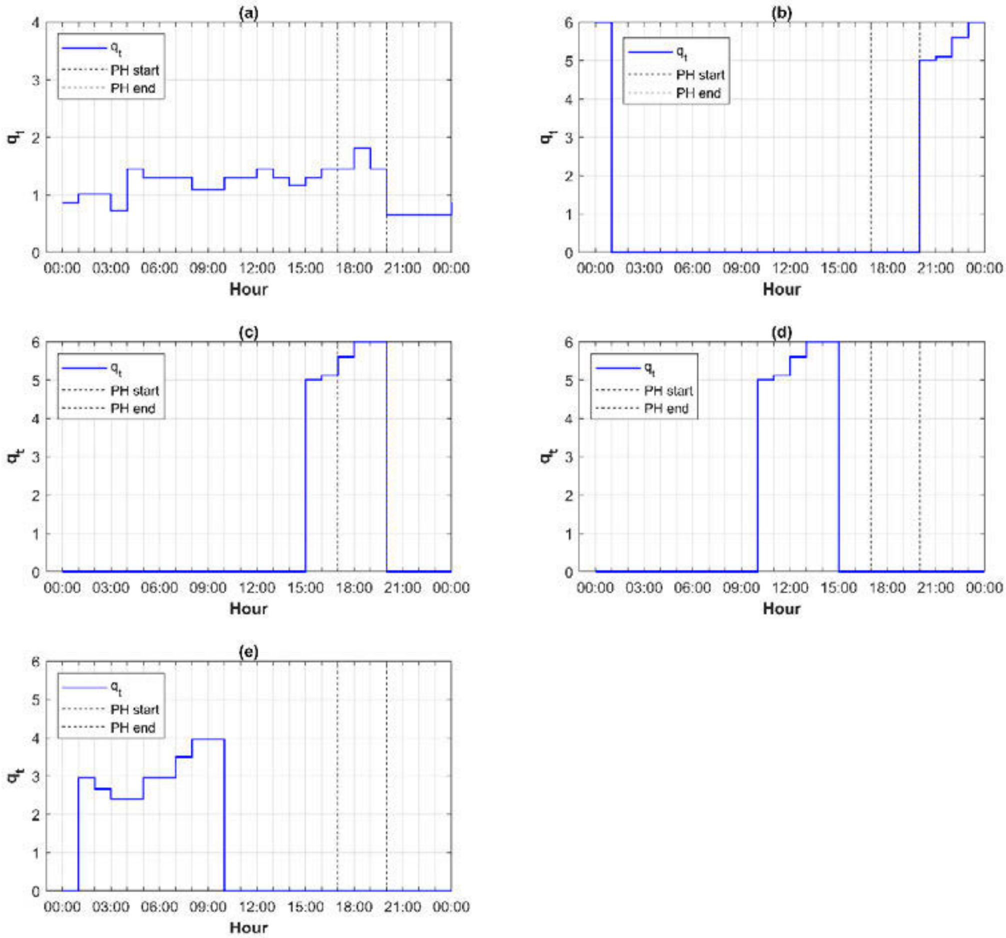


*Figure 3. Demand pattern ($q_t$) for OBCL-1 Network: (a) continuous operation;(b) sector 1 operation; (c) sector 2 operation; (d) sector 3 operation; (e) sector 4 operation*

## 2.4. Optimal Operation

The optimization of the system consists of selecting an optimized pump ($Q_{\text{BEP}}$ and $H_{\text{BEP}}$) and the respective relative rotational speed for every hour of operation ($N_t$) for each sector. Hence, the number of variables to be optimized at each sector are $t$+2, where $t$ is total time of operation. Then, the variables are evaluated by the objective function ($OF$) that represent the operational cost to be minimized, evaluated by the costs with energy and power consumption for each pump and the leakage volume of the systems, as defined by Equation 6.

$$OF = \sum_{p=1}^{Np} \sum_{i=1}^{t} [\frac{\gamma \cdot Q_i \cdot H_i}{1000 \cdot \eta_i} \cdot te_i] + \sum_{p=1}^{Np} P_{max} \cdot tp_i + \sum_{i=1}^{t} [Q_{L,i} \cdot \Delta t \cdot tw] + Pen \tag{6}$$

where:
$OF$ [R$] – objective function to be minimized, describing the operational costs of a day;
$Np$ [dimensionless] – number of pumps operating in the network;
$t$ [h] – time simulation;
$\gamma$ [N/m$^3$] – specific weight of water;
$Q_i$ [m$^3$/s] – pump flow at time *i*;
$H_i$ [m] – pump head at time *i*;
$\eta_i$ [dimensionless] – pump efficiency at time *i*;
$te_i$ [R$/kWh] – energy tariff at time *i*;

$P_{max}$ [kW] – maximum power;
$tp_i$ [R$/kW] – power tariff at time *i*;
$Q_{L,i}$ [m$^3$/h] – flow leakage at time *i*;
$\Delta t$ [h] – time simulation step;
$tw$ [R$/kW] – water production tariff;
$Pen$ [R$] – penalty function.

The energy tariffs and water production costs are obtained from the Energy Company of Minas Gerais – Brasil (CEMIG) [12] and Brazilian Sanitation Information System (SNIS) [13], both presented in the section 2.1. The penalty factor ($Pen$), as presented in Equation 6, is added to consider the restriction problem, as the optimization method is an unconstrained method. The constraint considered is the minimum operational pressure, $p_{min}$, in demand nodes, set as 10 m to respect the minimal pressure established by the Brazilian standards [16]. Thus, the penalty added to the objective function is calculated with Equation 7, where $p$ is the pressure in a demand node and $\beta$ is the penalty coefficient set as $10^8$.

$$if(p < p_{min}) \rightarrow Pen = \beta.|p_{min} - p| \quad (7)$$

The variables are evaluated to achieve an optimized solution by Particle Swarm Optimization (PSO) proposed by Eberhart and Kennedy in 1995, an algorithm based on the collective response of flocks of birds [17]. The initial possible solution ($X$) is randomly initialized and the next solution is defined by the experience of each particle (or birds) searching the space for the best solution with a velocity ($V$) and the collective experience of all the particle (flocks of birds) by three components, namely: i) inertia coefficient ($\omega$); ii) cognitive coefficient ($c_1$); and iii) social coefficient ($c_2$). Equations 8 and 9 describe mathematically the process to achieve an optimized solution.

$$V_i^{t+1} = \omega.V_i^t + c_1.rand_1.\frac{(Xp_i - X_i^t)}{\Delta t} + c_2.rand_2.\frac{(Xg - X_i^t)}{\Delta t} \quad (8)$$

$$X_i^{t+1} = X_i^t + V_i^{t+1}.\Delta t \quad (9)$$

The search for an optimized solution continues until a criterion is met, assumed as 1,000 iterations or a relative change in the objective function in 20 consecutive iterations below $10^{-10}$. The default values defined in the MATLAB® software for the search components are adopt, where inertia coefficient ($\omega$) is 1.1 and the cognitive ($c_1$) and social ($c_2$) coefficients are 1.49.

## 3 RESULTS

The benchmark network OBCL-1, firstly presented by [11] and further studied by [2] for an optimized intermittent operation, is used as a case study. Figure 4 presents the four DMAs created using the proposed methodology. To define the number of hours that each DMA would be supplied, the total demand of each of them was evaluated. As the DMA 4 comprises almost 40 % of the total demand, it was defined a longer period of supply to avoid excessive flows in the pipes, and consequently higher headlosses, which would significantly increase the power required for the pump. Thus, DMA 4 is supplied during 9 h of the day, while the remaining three are supplied in equal periods of 5 h, as there is no significant difference in their demand. Figure 4 shows the demand pattern adopted for each DMA, considering that the same water volume will be consumed.

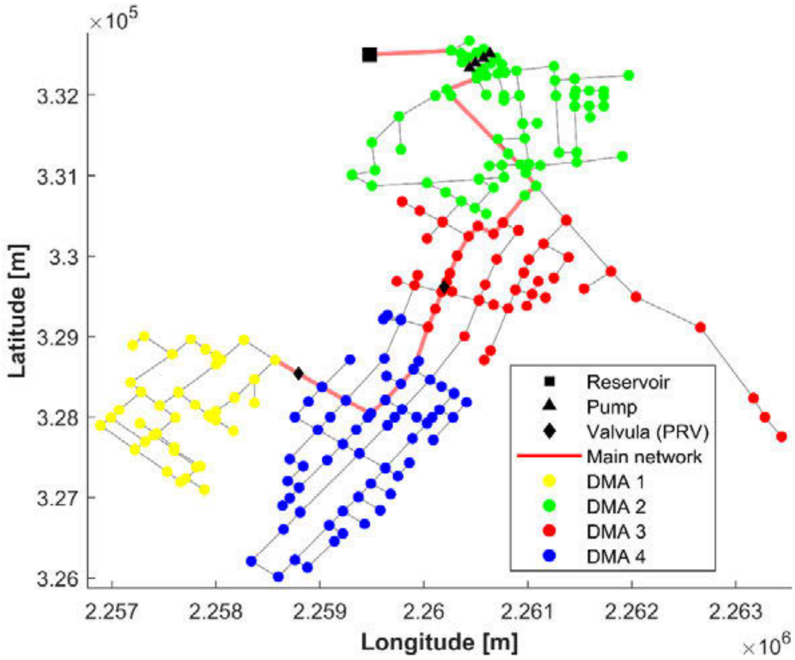


*Figure 4. DMAs created for the intermittent operation*

Tables 2, 3 and 4 shows the results for the different scenarios of water production costs. As expected, the energy consumption increased in all scenarios, as the pumps required more power to attend the same demand in a shorter period. When the water production cost is high, this increase in energy costs is even more significant, as economically, it is more relevant to minimize leakages than to avoid the operation during peak hours. On the other hand, the supply schedule proposed for each DMA significantly reduced the leakages, as some nodes that would have high pressures, and therefore, high leakage flow, could be isolated during a certain period. From the economic point of view, it can be seen that the energy tariffs and the water production costs play an important role, as for the scenario with low water production cost the continuous operation is 75.7 % better, and for the high and average scenarios, the intermittent operation is 27.6 % and 44.0 % better respectively. These results indicate that systems with a lower relation between energy tariffs and water productions costs will greatly benefit from the intermittent operation.

*Table 2. Results for the optimized operation for the low water production cost*

| Description | Scenario 1: *tw* = 0.30 R$/m³ | | | | |
|---|---|---|---|---|---|
| | **24 h*** | **DMA 1** | **DMA 2** | **DMA 3** | **DMA 4** |
| Sector Energy Consumption [kWh] | - | 688 | 59 | 377 | 1,468 |
| Daily Energy Consumption [kWh] | 581 | 2,592 | | | |
| Sector Leakage ($m^3$) | - | 252 | 240 | 272 | 952 |
| Daily Leakage ($m^3$) | 5,900 | 1,717 | | | |
| Daily Leakage (%) | 29.3 | 10.7 | | | |
| Sector Energy Cost (R$) | - | 2,195 | 720 | 1,206 | 2,917 |
| Sector Leakage Cost (R$) | - | 76 | 72 | 82 | 286 |
| Daily Energy Cost (R$) | 2,500 | 7,039 | | | |
| Daily Leakage Cost (R$) | 1,800 | 515 | | | |
| Daily Operation Cost (R$) | 4,300 | 7,554 | | | |
| Economic Efficiency (%) | - | -75.7 | | | |

Source: [2]*.

*Table 3. Results for the optimized operation for the high-water production cost*

| Description | Scenario 2: *tw* = 7.82 R$/m³ | | | | |
|---|---|---|---|---|---|
| | **24 h*** | **DMA 1** | **DMA 2** | **DMA 3** | **DMA 4** |
| Sector Energy Consumption [kWh] | - | 688 | 59 | 377 | 1,494 |
| Daily Energy Consumption [kWh] | 608 | 2,618 | | | |
| Sector Leakage ($m^3$) | - | 252 | 240 | 271 | 940 |
| Daily Leakage ($m^3$) | 5,800 | 1,704 | | | |
| Daily Leakage (%) | 28.9 | 10.7 | | | |
| Sector Energy Cost (R$) | - | 2,197 | 721 | 1,206 | 7,354 |
| Sector Leakage Cost (R$) | - | 1,970 | 1,878 | 2,122 | 2,956 |
| Daily Energy Cost (R$) | 2,500 | 11,479 | | | |
| Daily Leakage Cost (R$) | 45,600 | 8,926 | | | |
| Daily Operation Cost (R$) | 48,100 | 20,406 | | | |
| Economic Efficiency (%) | - | 57.6 | | | |

Source: [2]*.

*Table 4. Results for the optimized operation for the average water production cost*

| Description | Scenario 3: $tw$ = 3.57 R$/m$^3$ | | | | |
|---|---|---|---|---|---|
| | **24h*** | **DMA 1** | **DMA 2** | **DMA 3** | **DMA 4** |
| Sector Energy Consumption [kWh] | - | 687 | 59 | 377 | 1,482 |
| Daily Energy Consumption [kWh] | 556 | 2,605 | | | |
| Sector Leakage (m$^3$) | - | 252 | 240 | 271 | 940 |
| Daily Leakage (m$^3$) | 5,800 | 1,704 | | | |
| Daily Leakage (%) | 29 | 10.7 | | | |
| Sector Energy Cost (R$) | - | 2,196 | 721 | 1,206 | 2,932 |
| Sector Leakage Cost (R$) | - | 901 | 858 | 970 | 3,373 |
| Daily Energy Cost (R$) | 2,700 | 7,054 | | | |
| Daily Leakage Cost (R$) | 20,800 | 6,101 | | | |
| Daily Operation Cost (R$) | 23,500 | 13,156 | | | |
| Economic Efficiency (%) | - | 44.0 | | | |

Source: [2]*.

## 4 CONCLUSIONS

This paper presented a methodology to operate a WDN under an intermittent cycle, trying to supply the same volume for the consumer but in a shorter period. For this, DMAs were created and the supply period of each of them was defined according to their total demand, with the bigger DMA being supplied for a longer period to avoid excessive headlosses. Then an optimization was made to select a specific pump to supply each DMA, so its operation is as close as possible form its best efficiency point, reducing the energy consumption. The results showed that the intermittent operation significantly increase the energy consumption to attend the demand in a shorter period, but also reduces significantly the water losses, as high pressures points can be isolated during some periods. Thus, it is expected that systems with a low relation between energy tariffs and water production costs are the ones that could achieve a greater economic benefit from the intermittent operation.

## 5 ACKNOWLEDGMENTS

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