2nd International Joint Conference on Water Distribution Systems Analysis & Computing and Control in the Water Industry
Valencia (Spain), 18-22 July 2022
doi: https://doi.org/10.4995/WDSA-CCWI2022.2022.14054

COMPARISON OF SECOND AND THIRD ORDER ALGORITHMS FOR STEADY STATE RESOLUTION OF WATER DISTRIBUTION NETWORKS

Enrico Creaco[1], Armando Di Nardo[2], Michele Iervolino[3] and Giovanni Francesco Santonastaso[4]

[1]Dipartimento di Ingegneria Civile e Architettura, Università degli Studi di Pavia, Via Ferrata 3, 27100 Pavia.
[2,3,4] Dipartimento di Ingegneria, Università degli Studi della Campania Luigi Vanvitelli, via Roma 29, 81031 Aversa (CE)

[1] *creaco@unipv.it*, [2] *armando.dinardo@unicampania.it*, [3] *michele.iervolino@unicampania.it*, [4] *GiovanniFrancesco.Santonastaso@unicampania.it*

Abstract

This paper presents the comparison of second and third order algorithms for steady state resolution of water distribution networks (WDNs). The algorithms are obtained by using the direct outflow/pressure relationship and linearizing the global equations using the Newton Raphson method. The increase in the order of convergence from quadratic to cubic is obtained by refining system matrices at half Newton Raphson step. Two variants are considered for the third order algorithm, differing in the evaluation of the matrix expressing the derivative of the outflow/pressure relationship at WDN nodes: the derivative is evaluated analytically and numerically for the first and second versions, respectively. Specifically, the numerical evaluation is obtained by using outflow and head values that are available at the half Newton Raphson step. The results of applications to five case studies of increasing complexity point out that the third order algorithm converges in a smaller number of iterations than the second order algorithm. The third order algorithm with numerical evaluation of the derivative of the outflow/pressure relationship gives significant benefits in terms of convergence performance when the service pressure range for passing from no outflow to full outflow conditions at WDN nodes is small. All the algorithms developed in this work will be considered for implementation inside the SWANP version 4.0 software.

Keywords
Water distribution networks (WDNs), Pressure-driven modelling, Resolution algorithm; High-order convergence, Matrix numerical approximation.

1 INTRODUCTION

Simulation models are traditionally used by water utility operators to replicate the nonlinear behaviour of water distribution networks (WDNs), in both off-line and real-time applications. Off-line applications concern the use of WDN models calibrated based on historical data collected from the field for specific managerial objectives, such as contingency planning, network optimization, and strategy planning [1]. Thanks to the increasing adoption of smart sensors and smart water metering, the real-time modelling of WDNs has recently started to catch on [2, 3], with the main aim to proactively simulate WDN behaviour in emergency and other situations not encountered during the calibration period. Between unsteady flow modelling and extended period simulation, i.e., WDN resolution in a sequence of steady states, the latter seems to offer better applicability in the context of real time modelling, considering the trade-off between consistency of results and computational burden, as long as it is applied with sufficiently long temporal steps to ensure dampening of hydraulic transients in the WDN [4].

The real-time modelling and management of WDNs requires use of fast, stable, and robust solvers, which must be used for both simulation and optimization purposes. While the convergence of

WDN resolution algorithms is very stable in the demand driven modelling, i.e., in the case of no dependence of nodal outflow on service pressure, the implementation of the pressure driven modelling is well known to create difficulties for convergence. To tackle this issue, four main approaches have been used in the scientific literature. The first approach lies in transforming the pressure driven resolution into an iteration of demand driven resolutions, in which nodal outflows at the generic iteration are calculated based on the head values obtained at the previous iteration [5-9]. Among these authors, Alvisi et al. [7] proposed updating nodal outflows across the algorithm iterations and implemented a relaxation procedure, later refined by Ciaponi and Creaco [9], on nodal outflows to facilitate convergence. The second approach consists of modifying the WDN resolution algorithms, by substituting the preferred pressure-driven equation into the mass conservation equations at WDN nodes. Based on this concept, Giustolisi et al. [10], Wu et al. [11], Siew and Tanymboh [12] and Elhay et al. [13] used different methods to improve the convergence behaviour of pressure driven modelling. Giustolisi et al. [10] used a heuristics-based relaxation to correct both pipe water discharges and nodal heads. Siew and Tanyimboh [12] proposed a heuristic algorithm based on backtracking and line search to correct only nodal heads. Elhay et al. [13] proposed a mathematically well-posed damping scheme based on Goldstein's algorithm, to be applied on both nodal heads and pipe water discharges. In the third approach, e.g., [14-16], the inverse outflow-pressure relationship, namely expressing the pressure as a function of the outflow, is used to eliminate the problem of oscillations. The fourth, and last, approach was recently proposed by Creaco et al. [17] and consists of using high order algorithms in the direct outflow-pressure relationship, namely expressing the outflow as a function of the pressure. Starting from the traditional second order algorithms, the high order algorithms, such as the third order ones, are obtained by refining the evaluation of system matrices at the generic iteration of WDN resolution.

The present paper is the follow-up of the paper of Creaco et al. [17] and aims to present some additional results on the comparison of second and third order WDN resolution algorithms based on the direct outflow-pressure relationship, as well as to provide insights on the treatment of system matrices expressing the outflow/pressure relationship, to improve the algorithm convergence performance.

2 ALGORITHMS FOR WDN RESOLUTION

2.1 Pressure driven modelling

For a WDN with p pipes and n nodes, including n_1 nodes with unknown head (demanding nodes) and n_0 nodes with known head (source or tanks), the steady state modelling of WDNs includes the following system of p energy balance equation and n_1 mass balance equations, written in the compact vector form:

$$\begin{pmatrix} \mathbf{A_{11}} & \mathbf{A_{12}} \\ \mathbf{A_{21}} & \mathbf{A_{22}} \end{pmatrix} \begin{pmatrix} \mathbf{Q} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} -\mathbf{A_{10}H_0} \\ \mathbf{0} \end{pmatrix}, \tag{1}$$

in which $\mathbf{Q}$ $(p,1)$ and $\mathbf{H}$ $(n_1,1)$ are the unknown vectors, i.e., the vectors of water discharges at pipe and heads at unknown head nodes, respectively. $\mathbf{H_0}$ $(n_0,1)$ is the vector of heads at known head nodes. $\mathbf{A_{10}}$ (p, n_0) and $\mathbf{A_{12}}$ (p, n_1) are matrices obtaining by extracting the n_0 and n_1 columns associated with the known and unknown head nodes, respectively, from the topological incidence matrix A (p, n). This matrix is constructed in such a way that the generic i-th row helps identifying the upstream and downstream end node, according to the arbitrarily defined positive direction in the generic i-th pipe. In the i-th row, the element $A(i.j)$ associated with the j-th node is equal to −1 or 1 if the j-th node is the upstream or downstream node of the i-th pipe, respectively. Otherwise, $A(i.j) = 0$ if the j-th node does not belong to the i-th pipe. Finally, $\mathbf{A_{21}}$ (p, n_1), $\mathbf{A_{11}}$ (p, p) and $\mathbf{A_{22}}$ (n_1, n_1) are the transpose matrix of $\mathbf{A_{12}}$, a diagonal matrix expressing the resistance of the WDN pipes and a diagonal matrix expressing the ratio of outflow to head for the unknown head nodes, respectively. The various pressure driven relationships for the evaluation of user consumption and leakage can be easily considered for the construction of $\mathbf{A_{22}}$. As an example, for the Wagner et al. [18] pressure driven formulation, the expression of the element A_{22} (i, i) associated with the generic i-th node with unknown head is:

$$A_{22}(i,i) = \begin{cases} 0 & H \leq H_{min} \\ \left(\frac{H - H_{min}}{H_{des} - H_{min}}\right)^{\alpha} \frac{d}{H} & H_{min} \leq H \leq H_{des} \\ \frac{d}{H} & H \geq H_{des} \end{cases} \quad (2)$$

In which d and H are the demand and head of the generic i-th node, respectively. H_{min} = z + h_{min} and H_{des} = z + h_{des}, in which h_{min} and h_{des} are the minimum head for having a positive outflow and the desired head for full demand satisfaction, respectively.

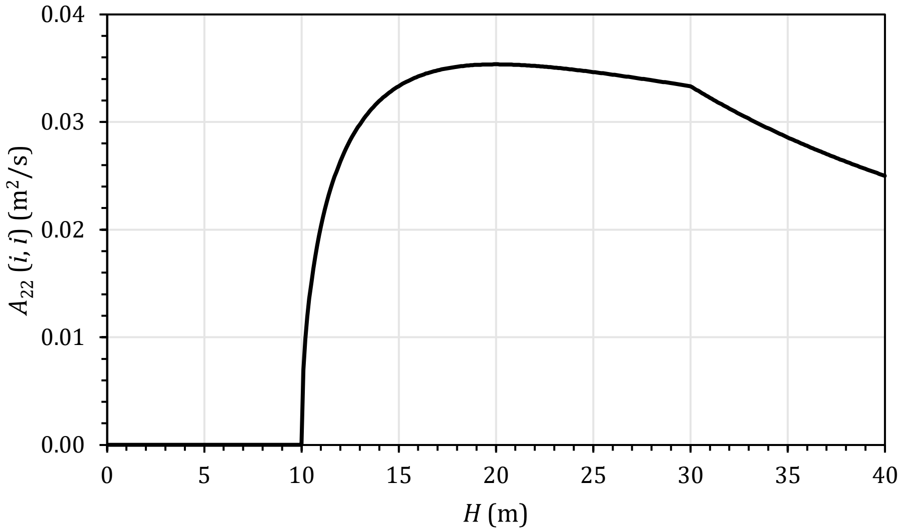

Figure 1. Pattern of the generic diagonal element of $\mathbf{A_{22}}$ *as a function of H for* H_{min}=10 m, H_{des}=30 m *and* α=0.5.

2.2 Second order Newton Raphson Method

The system of equations (1) can be solved iteratively by applying the Newton Raphson method, as explained by Todini and Rossman [19]. If $\mathbf{H^k}$ and $\mathbf{Q^k}$ are the vectors $\mathbf{H}$ and $\mathbf{Q}$, respectively, at the generic k-th iteration, the vectors $\mathbf{H^{k+1}}$ and $\mathbf{Q^{k+1}}$ at the new iteration can be obtained by solving the two following vector equations (3) and (4), respectively:

$$(\mathbf{A_{21}D_{11}^{-1}A_{12}} - \mathbf{D_{22}})\mathbf{H^{k+1}} = \{\mathbf{A_{21}D_{11}^{-1}}[(\mathbf{D_{11}} - \mathbf{A_{11}})\mathbf{Q^k} - \mathbf{A_{10}H_0}] + \mathbf{A_{22}H^k} - \mathbf{D_{22}H^k}\} \quad (3)$$

$$\mathbf{D_{11}Q^{k+1}} = \mathbf{D_{11}Q^k} - (\mathbf{A_{11}Q^k} + \mathbf{A_{12}H^{k+1}} + \mathbf{A_{10}H_0}) \quad (4)$$

in which $\mathbf{D_{11}}$ and $\mathbf{D_{22}}$ are diagonal matrices that can be calculated analytically as $\mathbf{D_{11}}$=d($\mathbf{A_{11}Q}$)/d$\mathbf{Q}$ and $\mathbf{D_{22}}$=d($\mathbf{A_{22}H}$)/d$\mathbf{H}$. As an example, for the Wagner et al. [18] pressure driven formulation, the analytical expression of the element D_{22} (i, i) (Figure 2) associated with the generic i-th node with unknown head is:

$$D_{22}(i,i) = \begin{cases} 0 & H \leq H_{min} \\ \dfrac{\alpha}{(H_{des} - H_{min})^{\alpha}} (H - H_{min})^{\alpha-1}\ d & H_{min} \leq H \leq H_{des} \\ 0 & H \geq H_{des} \end{cases} \tag{5}$$

All matrices $\mathbf{A_{11}}$, $\mathbf{A_{22}}$, $\mathbf{D_{11}}$ and $\mathbf{D_{22}}$ are evaluated based on the values of **Q** and **H** at iteration *k*.

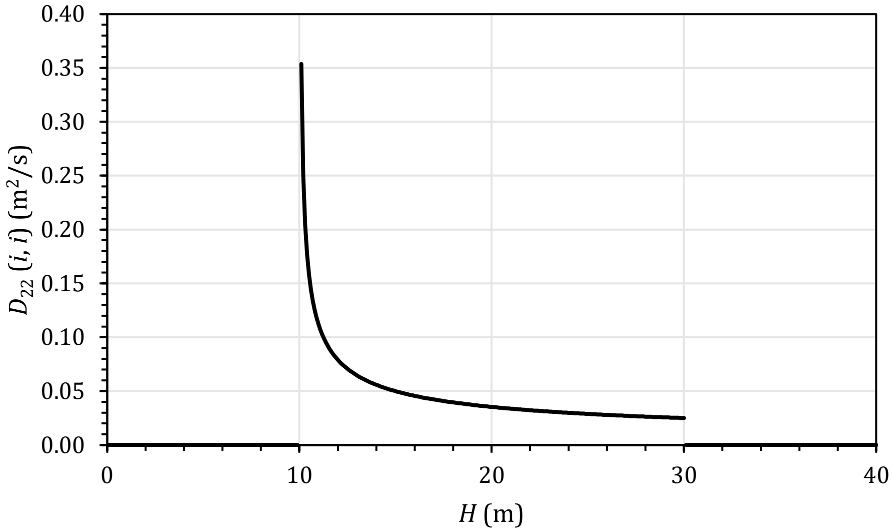

Figure 2. Pattern of the generic diagonal element of $\mathbf{D_{22}}$ *as a function of H for* H_{min}=10 m, H_{des}=30 m *and* α=0.5.

From equation (3), the new vector $\mathbf{H^{k+1}}$ of heads at demanding nodes can be obtained through the solution of the linear system of n_1 equations. From equation (4), the new vector $\mathbf{Q^{k+1}}$ of pipe water discharges can be obtained through the solution of *p* independent linear equations.

To speed up convergence, the second order Newton Raphson method can be dampened by applying the following underrelaxation to the heads:

$$\mathbf{H^{k+1}} = \mathbf{H^{k}} + \lambda^{k}(\mathbf{H^{k+1}} - \mathbf{H^{k}}) \tag{6}$$

in which λ^k is a number between 0 and 1, to be evaluated at each iteration as explained by Creaco et al. [17]. Therefore, the second order dampened Newton-Raphson method is applied by first solving the vector equation (3), then applying underrelaxation (6) and finally solving the vector equation (4).

2.3 Third order Newton Raphson Method

As explained by Creaco et al. [17], the increase in the order of convergence is obtained by refining the evaluation of matrices $\mathbf{D_{11}}$ and $\mathbf{D_{22}}$ at the generic iteration at half Newton Raphson step. To accomplish this, the second order Newton Raphson is initially applied to obtain first estimates for the vectors **H** and **Q** of nodal heads and pipe water discharges respectively. These first estimates are indicated as $\mathbf{H^{k+1,ie}}$ and $\mathbf{Q^{k+1,ie}}$, respectively. Then, the nodal head and pipe water discharge vectors at half Newton Raphson step are derived as:

$$\mathbf{H^{k+1/2}} = \frac{\mathbf{H^{k}} + \mathbf{H^{k+1,ie}}}{\mathbf{2}} \tag{7}$$

$$\mathbf{Q^{k+1/2}} = \frac{\mathbf{Q^{k}} + \mathbf{Q^{k+1,ie}}}{\mathbf{2}} \tag{8}$$

After evaluation of the refined matrices $\mathbf{D_{11}}$ and $\mathbf{D_{22}}$ based on the vectors $\mathbf{H^{k+1/2}}$ and $\mathbf{Q^{k+1/2}}$ at half Newton Raphson step, the sequence of vector equations (3), (6) and (4) is repeated to obtain the vectors $\mathbf{H^{k+1}}$ and $\mathbf{Q^{k+1}}$ at the new iteration. As Creaco et al. [17] proved, the increase in the order of convergence from quadratic to cubic yields significant benefits in terms of convergence performance under pressure driven modelling conditions.

2.4 Numerical approximation of $\mathbf{D_{22}}$

The presence of derivative discontinuities in the outflow/pressure relationship, which makes matrix $\mathbf{D_{22}}$ discontinuous, is known to slow down the convergence of WDN resolution algorithms under pressure driven modelling conditions. As an example, this happens in the Wagner et al. [18] formulation for $H=H_{min}$ and $H=H_{des}$ (see equation 5 and Figure 2). While Creaco et al. [17] proposed the regularization of matrix $\mathbf{D_{22}}$ for the third order Newton Raphson algorithm, this work proposes its numerical evaluation to obtain its smoothening. Remembering that $\mathbf{D_{22}}$=d($\mathbf{A_{22}H}$)/d$\mathbf{H}$, this can be done by calculating the generic element of $\mathbf{D_{22}}$ in the second step of the third order Newton Raphson algorithm as:

$$D_{\mathbf{22}}(i,i) = \frac{A_{22}^{+}H^{+} - A_{22}^{-}H^{-}}{H^{+} - H^{-}} \tag{9}$$

In which H^+ and H^- can be set equal to $H^{k+1,ie}$ and H^k, respectively. Furthermore, A_{22}^{+} and A_{22}^{-} are the elements A_{22} (i,i) evaluated at H^+ and H^-, respectively. In the first step of the third order Newton Raphson algorithm, except for the first iteration in which equation (5) is applied, $\mathbf{D_{22}}$ is set equal to its value in the second step of the previous iteration.

3 APPLICATIONS

Five case studies of increasing complexity (Figure 3) were considered in this paper to show the comparison of the second order algorithm (SO), third order algorithm with analytically calculated matrix $\mathbf{D_{22}}$ (third order variant 1 TO1) and third order algorithm with numerically calculated matrix $\mathbf{D_{22}}$ (third order variant 2 TO2). The first case study is the branched WDN of Gupta and Bhave [20] with n_0=1, n_1=4 and p=4. The second case study is the 2-looped WDN of Deuerlein et al. [14] with n_0=1, n_1=4 and p=6. The third case study is the 3-looped WDN of Hanoi [21] with n_0=1, n_1=31 and p=34. The fourth case study is the 49 looped WDN of Modena [22] with n_0=4, n_1=268 and p=317. Finally, the fifth case study is the 11-looped WDN of Balerma [23] with n_0=4, n_1=443 and p=454. The data concerning the features of WDN nodes and pipes can be found in the referenced works or in [16].

The algorithms SO, TO1 and TO2 of the present work were tested against the five case studies considering various values of h_{min} and h_{des}. Specifically, the values h_{min} = 0 m and h_{des} = 20 m were considered for the first and second case studies. For the three remaining case studies, four pairs of h_{min} and h_{des} were analyzed, namely h_{min} = 10 m - h_{des} = 40 m, h_{min} = 10 m - h_{des} = 30 m, h_{min} = 10 m - h_{des} = 20 m, h_{min} = 10 m - h_{des} = 10.1 m, to create increasingly challenging pressure driven conditions. In fact, the closer h_{min} and h_{des}, the smaller the service pressure variation required for increasing the generic nodal outflow from 0 to the desired demand d.

While the three algorithms analysed always converged to the same solution, the number of iterations required for convergence varied a lot. In case studies 1, 2, 4 and 5, the simple algorithm

SO proved capable of converging in less than 10 iterations in the case of sufficiently large service pressure range [h_{min}, h_{des}], i.e., h_{des} -h_{min}≥10 m. Problems of convergence were always observed in case study 3, in which large head oscillations were noticed during the iterations, and in all case studies when h_{des} -h_{min}=0.1 m. The use of the third order algorithm yielded benefits in terms of convergence performance, in comparison with SO. However, TO1 still needed more than 30 iterations to converge in case studies 4 and 5 for h_{des} -h_{min}=0.1 m. This problem was totally fixed in TO2. Except for case studies 4 and 5 in the case of h_{des} -h_{min}=0.1 m, featuring a number of iterations equal to 14 and 13, respectively, TO2 always converged in a number of iterations lower than or equal to 6.

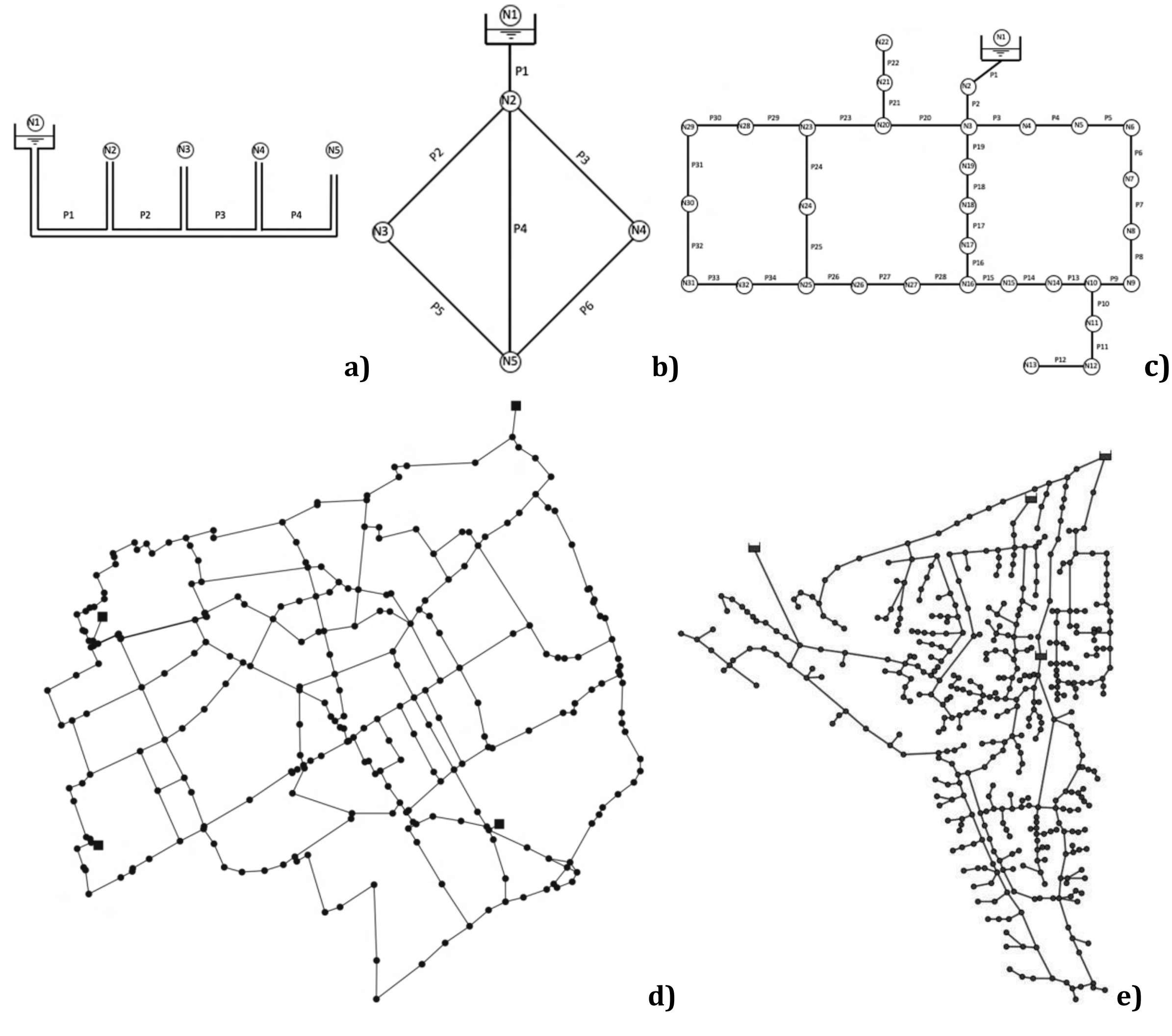

Figure 3. WDNs of case studies a) 1, b) 2, c) 3, d) 4 and e) 5.

Table 1. Convergence performance of second order (SO) and third order (TO1 and TO2) Newton Raphson algorithms for WDN resolution in the five case studies.

Network	h_{min} (m)	h_{des} (m)	SO Iterations	TO1 Iterations	TO2 Iterations
1	0	20	4	3	3
2	0	20	6	5	5
3a	10	40	56	5	4
3b	10	30	56	12	5
3c	10	20	56	15	5
3d	10	10.1	103	9	6
4a	10	40	4	4	4
4b	10	30	4	4	4
4c	10	20	5	4	4
4d	10	10.1	42	38	14
5a	10	40	5	5	4
5b	10	30	5	5	4
5c	10	20	7	5	4
5d	10	10.1	60	34	13

4 CONCLUSIONS

In this work, a comparison of second and third order algorithms for steady state resolution of WDNs was carried out. These algorithms were obtained by using the direct outflow/pressure relationship and linearizing the global equations using the Newton Raphson method. The increase in convergence order from quadratic to cubic was obtained by refining system matrices at half Newton Raphson step. The numerical approximation of the matrix expressing the derivative of the outflow/pressure relationship was proposed as a novel aspect of the present work. Globally, the results of the applications to five case studies of increasing complexity pointed out that:

- All the algorithms analysed converge to the same solution.
- The convergence of the second order algorithm is observed to slow down in case studies where nodal heads tend to oscillate and when the service pressure range for passing from no outflow to outflow is small.
- The third order algorithm features better convergence performance, especially when the matrix expressing the derivative of the outflow/pressure relationship is numerically approximated.

The algorithms developed in the present work are being considered for implementation inside the SWANP version 4.0 software [24], which enables tackling various kinds of modeling, design, and managerial problems for WDNs

5 REFERENCES

[1] Walski, T., D. Chase, D. Savic, W. Grayman, S. Beckwith, and E. Koelle. Advanced water distribution modelling and management, Waterbury, CT: Haestad, 2003.

[2] Machell, J., S. R. Mounce, and J. B. Boxall, "Online modelling of water distribution systems: A UK case study, drinking water." Eng. Sci., vol. 3, no. 1, 2010, pp. 21–27.

[3] Okeya, I., Z. Kapelan, C. Hutton, and D. Naga, "Online modelling of water distribution system using data assimilation." Procedia Eng., vol. 70, no. 1, 2014, pp. 1261–1270. https://doi.org/10.1016/j.proeng.2014.02.139.

[4] Creaco, E., G. Pezzinga, and D. Savic, "On the choice of the demand and hydraulic modeling approach to WDN real-time simulation." Water Resour. Res., vol. 53, no. 7, 2017, pp. 6159–6177. https://doi.org/10.1002/2016WR020104.

[5] Bhave, P. R., "Node flow analysis distribution systems." Transp. Eng. J., vol. 107, no. 4, 1981, pp. 457–467. https://doi.org/10.1061/TPEJAN.0000938.

[6] Tabesh, M. Implications of the pressure dependency of outflows on data management, mathematical modelling and reliability assessment of water distribution systems. Ph.D. thesis, Dept. of Civil Engineering, Univ. of Liverpool, 1998.

[7] Alvisi, S., and M. Franchini "Near-optimal rehabilitation scheduling of water distribution systems based on a multi-objective genetic algorithm." Civ. Eng. Environ. Syst., vol. 23, no. 3, 2006, pp. 143–160. https://doi.org/10.1080/10286600600789300.

[8] Jun, L., and Y. Guoping "Iterative methodology of pressure dependent demand based on EPANET for pressure-deficient water distribution analysis." J. Water Resour. Plann. Manage., vol. 139, no. 1, 2013, pp. 34–44. https://doi.org/10.1061/(ASCE)WR.1943-5452.0000227.

[9] Ciaponi, C., and E. Creaco "Comparison of pressure-driven formulations for WDN simulation." Water, vol. 10, no. 4, 2018: 523. https://doi.org/10.3390/w10040523.

[10] Giustolisi, O., D. Savic, and Z. Kapelan "Pressure-driven demand and leakage simulation for water distribution networks." J. Hydraul. Eng., vol. 134, no. 5, 2008, pp. 626–635. https://doi.org/10.1061/(ASCE)0733-9429(2008)134:5(626).

[11] Wu, Z., R. Wang, T. Walski, S. Yang, D. Bowdler, and C. Baggett "Extended global-gradient algorithm for pressure-dependent water distribution analysis." J. Water Resour. Plann. Manage., vol. 135, no. 1, 2009, pp. 13–22. https://doi.org/10.1061/(ASCE)0733-9496(2009)135:1(13).

[12] Siew, C., and T. Tanyimboh. "Pressure-dependent EPANET extension." Water Resour. Manage., vol 26, no. 6, 2012, pp. 1477–1498. https://doi.org/10.1007/s11269-011-9968-x.

[13] Elhay, S., O. Piller, J. Deuerlein, and A. Simpson. "A robust, rapidly convergent method that solves the water distribution equations for pressure-dependent models." J. Water Resour. Plann. Manage., vol. 142, no. 2, 2016: 04015047. https://doi.org/10.1061/(ASCE)WR.1943-5452.0000578.

[14] Deuerlein, J. W., O. Piller, S. Elhay, and A. R. Simpson. "Contentbased active-set method for the pressure-dependent model of water distribution systems." J. Water Resour. Plann. Manage., vol. 145, no. 1, 2019: 04018082. https://doi.org/10.1061/(ASCE)WR.1943-5452.0001003.

[15] Water Infrastructure Division. EPANET 2.2 users manual. U.S. Environmental Protection Agency Cincinnati, OH 45268. Accessed May 3, 2020. https://epanet22.readthedocs.io/en/latest/index.html.

[16] Todini, E., S. Santopietro, R. Gargano, and L. A. Rossman. "Pressure flow-based algorithms for pressure-driven analysis of water distribution networks." J. Water Resour. Plann. Manage., vol. 147, no. 8, 2021: 04021048. https://doi.org/10.1061/(ASCE)WR.1943-5452.0001401.

[17] Creaco, E., A. Di Nardo, M. Iervolino, and G. Santonastaso. "High-Order Global Algorithm for the Pressure-Driven Modeling of Water Distribution Networks" J. Water Resour. Plann. Manage., vol. 148, no. 3, 2022: 04021109. https://ascelibrary.org/doi/10.1061/%28ASCE%29WR.1943-5452.0001524.

[18] Wagner, J., U. Shamir, and D. Marks. "Water distribution reliability: Simulation methods." J. Water Resour. Plann. Manage., vol. 114, no. 3, 1988, pp. 276–294. https://doi.org/10.1061/(ASCE)0733-9496(1988)114:3(276).

[19] Todini, E., and L. A. Rossman. "Unified framework for deriving simultaneous equation algorithms for water distribution networks." J. Hydraul. Eng., vol. 139, no. 5, 2013, pp. 511–526. https://doi.org/10.1061/(ASCE)HY.1943-7900.0000703.

[20] Gupta, R., and P. R. Bhave. “Comparison of methods for predicting deficient-network performance.” J. Water Resour. Plann. Manage., vol. 122, no. 3, 1996, pp. 214–217. https://doi.org/10.1061/(ASCE)0733-9496(1996)122:3(214).

[21] Fujiwara, O., and D. B. Khang. “A two-phase decomposition method for optimal design of looped water distribution networks.” Water Resour. Res., vol. 26, no. 4, 1990, pp. 539–549. https://doi.org/10.1029/WR026i004p00539.

[22] Artina, S., A. Bolognesi, C. Bragalli, C. D’Ambrosio, and A. Marchi. “Comparison among best solutions of the optimal design of water distribution networks obtained with different algorithms.” In: Vol. 3 of Proc., CCWI2011 Urban Water Management: Challenges and Opportunities, 2011, pp. 985–990. Exeter, UK: Centre for Water Systems.

[23] Reca, J., and J. Martinez. “Genetic algorithms for the design of looped irrigation water distribution networks.” Water Resour. Res., vol. 42, no. 5, 2006, pp. 1–9. https://doi.org/10.1029/2005WR004383.

[24] Di Nardo, A., M. Di Natale, A. Di Mauro, E. Martínez Díaz, J. A. Blázquez Garcia, G. F. Santonastaso, and F. P. Tuccinardi. “An advanced software to design automatically permanent partitioning of a water distribution network.” Urban Water J., vol. 17, no. 3, 2020, pp. 259–265. https://doi.org/10.1080/1573062X.2020.1760322.