MODELLING AND OPTIMIZATION OF RESIDENTIAL ELECTRICITY LOAD UNDER STOCHASTIC DEMAND

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Abstract:

The paper considers a modelling framework for a set of households in residential areas; using electricity as a form of energy for domestic consumption. Considering the demand and availability of units for electricity consumption, optimal decisions for electricity load allocation are paramount to sustain energy management. We formulate this problem as a stochastic decision-making process model where electricity demand is characterized by Markovian demand. The loading and operational framework is governed by the demand and supply phenomena; where shortage costs are realized when demand exceeds supply. Empirical data for electricity consumption was collected from fifty households in two residential areas within the suburbs of Kampala in Uganda. Data collection was made at hourly intervals over a period of four months. The major problem focussed on determining an optimal electricity loading decision in order to minimize consumption costs as demand changes from one state to another. Considering a multi-period planning horizon, an optimal decision was determined for loading or not loading additional electricity units using Markov decision process approach. The model was tested; whose results demonstrated the existence of an optimal state-dependent decision and consumption costs considering the case study used in this study. The proposed model can be cost-effective for managers in the electricity industry. Improved efficiency and utilization of resources for the distribution systems of electricity to residential areas was realised; with subsequent enhanced reliability of service to essential customers of the energy market.

Keywords: demand; electricity load; modelling; optimization; stochastic.

1. Introduction

Residential electricity load optimization had become one of the key issues in solving energy crisis problems in the past few years. In several nations worldwide, residential buildings constituted a large percentage of energy consumption. In the European Union for instance, the residential sector accounted for 26.1% of the total energy consumption in 2018; and this catered for space, water heating, and electric end users such as lighting or appliances. It was observed that in residential areas, variations of social, economic and technical characteristics among consumer groups influenced electricity consumption. This was based on the timing, location, peak and distribution of electric power. Considering the social-economic aspects and technical equipment used in Uganda, residential electricity load had a significant influence considering the type of dwelling and location. Indirect influences on electricity load were also attributed to the number of occupants (Graveia, 2015), the number of bedrooms (Chesser et al., 2019), the dwelling area (Larsen & Nesbakken, 2004), the floor area (Bafer & Rylatt, 2008), incomes (Yohanis et al., 2008) and the household ownership of physical appliances.

Electricity consumption in residential areas was also affected by several social-economic factors worldwide. In some households, it was noted how much electricity appliances consumed so as to enable households to acquire knowledge about the expenditure patterns associated with such appliances. It was therefore prudent to devise appropriate methods for understanding electricity consumption based on household appliances. The degree of how such appliances were used in residential areas was of paramount importance.

In practical situations, the usage of an appliance as well as the related operational cost calculations considered estimates of the daily hours run by appliances; that determined the wattage of the product, daily consumption, annual energy consumption and the annual cost to run the appliance. The estimated run time of appliances on a daily basis was made through a rough estimate by keeping a log. Through a rough estimate, the household predicted the usage rates of an appliance on a daily basis and such a household determined the usage hourly rate. As the number of power appliances consumed varied considerably depending on the setting, realistic estimates of current in residential areas were obtained considering the current and voltage used by the household appliance. Determining the daily consumption, annual consumption and annual cost to run the appliance were critical factors that influenced electricity loading decisions and consumption patterns of households in residential areas.

1.1. Residential electricity load background

The goal of meeting people's energy needs became a crucial research topic of global concern in recent years. In modern society today, however, the use of electric and electronic devices had increased tremendously; and

this contributed to energy consumption for smartphones, televisions, appliances and various related devices. It had been widely believed that load demand for electricity did not vary significantly among households regardless of socialeconomic circumstances; considering inhabitants of a family or an apartment building. However, residential electricity loading lacked predictability guidelines for modelling purposes since a solid understanding of the residential load profile and its prevailing state was needed. Despite this challenge, however; residential electricity load profiles had a big role in capacity planning; by improving the efficiency in system operations, electricity grids, generation investment energy market, electricity tariffs, price structures, incentives, customer satisfaction and other economic considerations.

1.2. Research objectives

i) To develop and optimize the residential electricity load model under stochastic demand

ii) To optimize electricity consumption costs under stochastic demand

iii) To test the residential electricity load model

1.3. Research questions

i) What is the optimal residential electricity loading decision under stochastic demand?

ii) What are the electricity consumption costs under stochastic demand?

1.4. Methodology

1.4.1. The survey instrument

Questionnaires were developed and pre-tested for the field survey; then administered to the residential areas and a sample of households within the residential areas. The first questionnaire established the demand transitions, electricity demand and electricity units available under the decision of loading additional units versus not loading additional units of electricity

1.4.2. Research Participant

Field participants were trained to administer the questionnaire in this study and collected relevant data from households of a given residence.

1.4.3. Study Population/sample

The study was conducted on an accessible population; that comprised two residential areas and one hundred (100) households

1.4.4. Data Analysis

To reduce the data collected to usable dimensions, the raw data collected was edited, processed and analysed; so that the data generated was organised and interpreted. An electronic database was created from the database generated. Data was analysed from frequencies generated and presented to show the relationship

between state transitions, the number of households, demand, available electricity units and the respective electricity loading decisions.

1.5. Residential load management under stochastic demand

The theoretical foundation of the study (Mcloughlin & Duffy, 2012); emphasized the great challenge encountered while considering uncertainty in residential load demands. Considerations to load uncertainty of each residential customer were modelled by utilizing the simplex method with fuzzy numbers. Using the price vector as the input of the optimization problem, the input for electricity consumers was used to optimize this fuzzified demand. The effectiveness of the proposed stochastic load management scheme was validated by solving a two-demand problem. Each demand expected a minimum level of power contribution which was defined by a fuzzy constraint. In our approach, the residential electricity load under stochastic demand considered the nature of demand using a two-state Markov chain. The states of residential electricity demand represented possible states of demand. The optimal electricity loading decision and associated consumption costs were determined using the Markov decision process methodology over a designated finite period planning horizon.

The paper was organized as follows: After reviewing the literature in §2, the model was formulated in §3; where consideration was given to the process of estimating model parameters. The model was solved in §4, and a case study was presented in §5. The study showed the practical application of the proposed model; where analysis/discussion of the results and limitations of the study were presented. Lastly, conclusions followed in §6; with prospects for future research.

2. Related literature

In a recent study, an electrical system framework that measured the accessibility of electrical power (Khorsandl & Cao, 2016) was considered. The author examined stochastic residential load management using fuzzy-based optimization approaches. A novel stochastic optimization framework to model the day-ahead load profile of a residential energy hub (Askeland et al., 2020) was suggested using an incentive-based DR program. That was done through a distributed approach; where the load profile became smoother by considering the related aggregator's desirable load profile limits. Related previous work on residential load considered three steps where an independent system operation ISO day-ahead RTP to a residential load aggregator (RLA) was considered (Nezhad et al., 2020).The RLA predicted individual household loads (step 1), and aggregated the loads that minimized the costs (step 2). In the second layer, the RLA announced incentives to homes, and more energy management systems (EMS) controlled the loads and maximized the reward in real time (step 3). A very recent study of the load combination of power sales companies was based on various power values (Wang et al., 2019) where demand response data was extracted by load characteristics index and power consumption index. The method proposed reduced the power purchase cost

and increased the revenue of the power company. A systematic literature review however pointed to a diversity of modelling techniques and associated algorithms on short-term load forecasting (Liu et al., 2022). The authors concluded that it was desirable to have a unified data set, together with a set of benchmarks and well-defined metrics for a clear comparison of all the modelling techniques and the corresponding algorithms. A related approach that used a stochastic bottom-up model for generating electrical loads for residential buildings (Rodrigues et al., 2023) in Canada was presented. The model investigated the impact of different household characteristics, appliance stock and energy behaviour on the timing and magnitude of non-HVAC energy loads at multiple houses and yielded significant results. The stochastic perturbation method and the transformed random variable method (Mohamed et al., 2023); where energy-demand analysis was performed for the representative single house in Poland produced important results. The expanded polystyrene thermal conductivity and external temperature were considered uncertain. The expected value and central moments of the energy consumption were determined using the stochastic perturbation method while the transformed random variable method obtained the explicit form of the energy consumption probability density function. However, the highly resolved electricity consumption data of Austria, German and UK households (Anvari et al., 2022) and the proposed applicable data-driven load model made critical awareness to model developers. The average demand profiles were disentangled from the demand fluctuations based on time series data. A stochastic model was then introduced to capture the intermittent demand fluctuations. A related study assigned pre-generated electricity and heat demand curves to georeferenced residential buildings in Germany (Büttner et al., 2022). That provided a large variety in residential load profiles which spatially corresponded to official social–demographic data. Results were validated on different aggregation values. The forecasting performance of models based on functional data analysis (Shah et al., 2022) gave important insights. The demand time series was first treated for the extreme values. The filtered series was then divided into deterministic and stochastic components. The additive modelling technique was used to model the deterministic component; whereas the functional autoregressive was used to forecast the stochastic component.

The literature cited showed important insights by current scholars that were crucial in studying the residential electricity load problem. However, the optimality of electricity loading decisions with associated consumption costs was not fully considered under demand uncertainty. The Markov decision process model provided a powerful framework for optimizing electricity loading decisions and electricity consumption costs under demand uncertainty considering several households in residential areas.

The major contributions of this paper to residential electricity load under stochastic demand highlighted the following:

i) The state-transition matrices that characterized the demand and consumption cost were computed under the prevailing electricity loading decisions

ii) The computation procedure calculated the expected consumption costs and accumulated consumption costs for the electricity loading decisions

iii) The Markov decision process formulation allowed the decision maker to load or not load extra units of electricity under different states of demand

3. Model formulation

A discrete-time finite horizon MDP model was developed with decision epochs $t\epsilon T = \{1, 2, \ldots \ldots E\}$. At each decision epoch t, the decision maker (ie, electricity regulator) observed the electricity demand states by conducting some observatory tests concerning the electricity demand levels when the available electricity exceeded demand, loading additional units was stopped and the decision process was terminated. Otherwise, the decision maker decided (based on residential electricity demand) on the optimal loading decision that had to be taken. The decision continued till the loading exercise ended for each action the decision maker took. There emerged therefore an immediate reward representing the total electricity consumption costs based on the decision taken.

Our goal was to solve the trade-off problem between loading additional electricity units with the associated consumption costs versus not loading additional electricity units. A formal definition of the core components of our MDP model followed.

3.1. States

The demand state i was composed of two-state variables: Favourable state (state F) and unfavourable state (state U). The favourable state was defined by the presence of customer N^S, with demand D^S observed by the decision maker at each decision epoch t within residential area r; where $S\epsilon\{0,1\}$, r $\epsilon\{1,2\}$, t=1,2,…………T.

3.2. Actions

We denoted the action space by A= $\{a_0, a_1, \ldots \ldots a_k\}$ where $a_i = 0$ represented not loading and $a_i = 1$ represented loading additional units. We assumed that if $a_i = 0$ was chosen, additional electricity units were not loaded when customers in residential areas were fully supplied; while additional units needed to be loaded whenever electricity demand exceeded available electricity.

3.3. Transition probabilities

When the decision maker chose action $s_t \epsilon$ S at decision epoch t when demand was in state s_t, the demand state moved to s_{t+1} at t+1 with probability $P_t (S_t / S_{t+1}, a_t)$. We assumed that

$$P_t (S_{t+1} / S_t, a_t) = P_t^\lambda(c_t, a_t) \times = P_t^\alpha(\alpha_{t+1} P_t^\alpha (\alpha_{t+1}, \alpha_t) \qquad (1)$$

where

$P_t^\lambda(c_t, a_t)$ and $P_t^\alpha (\alpha_{t+1} / \alpha_t, a_t)$

were the transition probabilities for the favourable demand state and the unfavourable demand state respectively. This assumption was consistent with our proposition that favourable demand and unfavourable demand did not depend on each other; but depended on the decision maker's action only. More specifically, we assumed that

$$P_t^{\lambda}(c_{t+1}/c_t,a_1) > P_t^{\lambda}(c_{t+1}/c_t, a_2) > \ldots\ldots > P_t^{\lambda}(c_{t+1}/c_t, a_k) \quad (2)$$

where c_{t+1} represented a favourable state.

$$P_t^{\lambda}(\lambda_{t+1}/\lambda_t,a_1) < P_t^{\lambda}(\lambda_{t+1}/\lambda_t, a_t) < \ldots\ldots < P_t^{\lambda}(\lambda_{t+1}/\lambda_t, a_k) \quad (3)$$

where λ_{t+1} represented unfavourable state than λ_t

3.4. Reward Functions

Our model included a reward function $\lambda_t(s_t, a_t)$ that reflected the utility/disutility of the decision maker as realized demand state s_t with action a_t which was taken at decision epoch t. This was defined as

$$\alpha_t(s_t/a_t) = \sum_{i=1}^{N} \alpha_{t,i}(c_i, a_t) + \alpha_t(\alpha_t, a_t) \quad (4)$$

where $\alpha_{t,i}(c_i, a_t)$ represented the immediate reward forfavorable demand state c_i and $\alpha_t(\alpha_t, a_i)$ was the immediate reward for unfavorable demand state α_t

Hence the corresponding reward functions were assumed to follow the following inequality for all t.

$$\alpha_t(c_i, a_t) < \alpha_t(c_i', a_t), \; \alpha_t(\alpha, a_t) < \alpha_t(\alpha', a_t) \quad (5)$$

3.5. Value function

The goal of our MDP model was to find the optimal strategy for loading electricity units. A rule was therefore sought for taking action at each state that would minimize the expected total consumption costs of electricity over the planning period. This could be achieved by solving Bellmann's recursive equations for all $s_t \in S$ and t=1,2,………T.

$$V_t(s_t) = min_{a_t \in A}\{\alpha_t(s_t, a_t) + \sum_{s_{t+1} \in S} P_t(s_t+1/s_t,a_t) \, v_{t+1}(s_{t+1})/s_t, a_t)v_{t+1}(s_{t+1})\} \quad (6)$$

where $v_t(s_t)$ represented the minimum expected total reward at the decision epoch t when demand was in state s_t with the boundary condition

$$V_{\alpha+1}(s) = \alpha_{T+1}(s)$$

3.6. Formulating the Finite-period Dynamic Programming Problem

Since demand was considered as favourable state (state F) or unfavourable state (state U), the problem considered an optimal electricity loading decision; and this was modelled as a dynamic programming problem over a finite period planning horizon. We denoted $g_n(i, r)$ as the expected total consumption costs accumulated by residential area r during the periods $n,n+1,…...,N$ given

that the state of the system at the beginning of period n was $i \in \{F, U\}$. The recursive equation relating g_n and g_{n+1} became

$$g_N(i, r) = min_s[e_i^S(r) + Q_{iF}^S(r) \, g_{n+1}(F,r) + Q_{iU}^S(r) \, g_{n+1}(U,r)] \quad (7)$$

The following condition was sufficient

$$g_{N+1}(F,r) = g_{N+1}(U,r) = 0 \quad (8)$$

The consumption costs $C_{ij}^S(r) + g_{n+1}(j)$ resulting from reaching state $j \in \{F,U\}$ at the start of period n+1 from state $i \in \{F,U\}$ at the start of period n occurred with probability $Q_{ij}^S(r)$.

Clearly,

$$e^S(r) = [Q^S(r)] [C^S(r)]^T \quad S\epsilon[0, 1] \quad r = [1,2] \quad (9)$$

The corrresponding dynamic programming recursive equations were thus obtained

$$g_N(i,r) = min_S[e_i^S(r) + Q_{iF}^S(r) \, g_{n+1}(F,r) + Q_{iU}^S(r) \, g_{n+1}(U,r)] \quad (10)$$

$$g_N(i,r) = min_S [e_i^S(r)] \quad (11)$$

Electricity demand in excess of supply yielded the consumption cost matrix

$$C^S(r) = (c_t + cs) [D^S(r) - A^S(r)] \quad (12)$$

Otherwise

$$C^S(r) = c_o[A^S(r) - D^S(r)] \quad (13)$$

when supply exceeded demand

Clearly,

$$C_{ij}^S(r) = \begin{cases} (c_l + c_s + c_o)[D_{ij}^S(r) - A_{ij}^S(r)] \text{ if } D_{ij}^S(r) > A_{ij}^S(r) \\ c_o[A_{ij}^S(r) - D_{ij}^S(r)] \text{ if } D_{ij}^S(r) \le A_{ij}^S(r) \end{cases} \quad (14)$$

for: i,j $\epsilon\{F,U\}$, r = \{1,2\} , Sϵ\{1,0\}

The justification for expressions (13) and (14) was that $D_{ij}^S(r) - A_{ij}^S(r)$ units had to be loaded to meet excess demand. Otherwise loading was cancelled when demand was less than or equal to supply. The following conditions were therefore sufficient to execute the model.

1. S=1 when $c_l > 0$ otherwise S=0 when $c_l = 0$
2. $c_s > 0$ when shortages were allowed otherwise c_s=0 when shortages were not allowed.

4. Optimization

The electricity loading decision /consumption costs were optimized for periods 1 and 2 in residential area r.

4.1. Optimization - period 1

Considering favourable (state F) demand, the optimal loading decision was determined as

$$S = \begin{cases} S = 1 \text{ if } e_F^1(r) < e_F^0(r) \\ 0 \text{ if } e_F^1(r) \geq e_F^0(r) \end{cases} \qquad (15)$$

with expected consumption costs

$$g_1(F,r) = \begin{cases} e_F^1(r) \text{ if } S = 1 \\ e_F^0(r) \text{ if } S = 0 \end{cases} \qquad (16)$$

When demand was unfavourable (i.e. in state U), the optimal loading decision was determined as

$$S = \begin{cases} 1 \text{ if } e_U^1(r) < e_U^0(r) \\ 0 \text{ if } e_U^1(r) \geq e_U^0(r) \end{cases} \qquad (17)$$

with expected consumption costs

$$g_1(U,r) = \begin{cases} e_U^1(r) \text{ if } S = 1 \\ e_U^0(r) \text{ if } S = 0 \end{cases} \qquad (18)$$

4.2. Optimization - period 2

Using (equations (10),(11) and recalling that $a_i^S(r)$ denoted the already accumulated consumption costs at the end of period 1 as a result of decisions made during that period,

$$a_i^S(r) = e_i^S(r) + Q_{iF}^S(r) \min[e_F^1(r), e_F^0(r)] +$$
$$+ Q_{iU}^S(r) \min[e_U^1(r), e_U^0(r)] \qquad (19)$$

$$a_i^S(r) = e_i^S(r) + Q_{iF}^S(r) g_2(F,r) + Q_{iU}^S(r) g_2(U,r) \qquad (20)$$

Therefore, for favourable demand (ie. in state F), the optimal loading decision during period 2 was determined as

$$S = \begin{cases} 1 \text{ if } a_F^1(r) < a_F^0(r) \\ 0 \text{ if } a_F^1(r) \geq a_F^0(r) \end{cases} \qquad (21)$$

while the associated accumulated consumption costs were

$$g_2(F,r) = \begin{cases} a_F^1(r) \text{ if } S = 1 \\ a_F^0(r) \text{ if } S = 0 \end{cases} \qquad (22)$$

Similarly, when demand was unfavorable(ie.in state U), the optimal loading decision during period 2 was determined as

$$S = \begin{cases} 1 \text{ if } a_U^1(r) < a_U^0(r) \\ 0 \text{ if } a_U^1(r) \geq a_U^0(r) \end{cases} \qquad (23)$$

In this case, the associated accumulated consumption costs were

$$g_2(U,r) = \begin{cases} a_U^1(r) \text{ if } S = 1 \\ a_U^0(r) \text{ if } S = 0 \end{cases} \qquad (24)$$

5. A case study of Uganda Electricity Distribution Company (UEDC)

The use of the model developed was presented using a case study of Uganda Electricity Distribution Company in Uganda; that experienced random electricity demand in residential areas. The Electricity Distribution Company (UEDC) sought elimination of excess electricity supply under unfavourable demand (state U) or avoiding shortages when demand was favourable (state F) and hence, UEDC sought an optimal electricity loading decision and consumption costs considering a planning horizon of two week

5.1. Data Collection

A number of households, demand and electricity units available (in kwh) were observed and recorded from two residential areas. The states of demand under electricity loading decisions were considered over ten weeks for favourable demand (state F) and unfavourable demand (state U). The data was captured in Tables 1 – 3.

Table 1: Households Versus State-Transitions for Electricity Loading Decisions.

Residential area (r)	States of demand	Load additional units (S=1)		Do not load units (S=0)	
		F	U	F	U
1	F	91	71	82	30
	U	63	13	55	25
2	F	45	59	64	40
	U	59	13	45	11

Table 2: Demand (in kwh) Versus State-Transitions in Residential Areas for Electricity Loading Decisions Load additional units.

Residential area (r)	States of demand	Load additional units (S=1)		Do not load units (S=0)	
		F	U	F	U
1	F	78	38	62	39
	U	60	65	39	40
2	F	100	30	36	39
	U	30	70	69	60

Table 3: Available Electricity (in kwh) Versus State-Transitions in Residential Areas for Electricity Loading Decisions.

Residential area (r)	States of demand	Load additional units (S=1)		Do not load units (S=0)	
		F	U	F	U
1	F	95	80	34	45
	U	54	75	47	55
2	F	47	40	81	79
	U	36	56	38	72

For either loading decision taken, unit loading cost (c_l)=1.20 USD per kwh, unit operational cost(c_o)=0.80 USD per week and unit shortage cost (c_s)=0.32 USD per week.

5.2. Computation of Model Parameters

We illustrated how the demand transition matrices and consumption cost matrices were determined from empirical data. For example, considering matrix $Q^1(1)$ for residential area 1 and electricity loading decision 1 and referring to (1),

$$Q^1_{FF}(1) = \frac{N^1_{FF}(1)}{N^1_{FF}(1) + N^1_{FU}(1)} = \frac{91}{91+71} = 0.5617$$

$$Q^1_{FF}(1) = \frac{N^1_{FF}(1)}{N^1_{FF}(1) + N^1_{FU}(1)} = \frac{71}{91+71} = 0.4383$$

$$Q^1_{UF}(1) = \frac{N^1_{UF}(1)}{N^1_{UF}(1) + N^1_{UU}(1)} = \frac{53}{63+13} = 0.8289$$

$$Q^1_{UU}(1) = \frac{N^1_{UU}(1)}{N^1_{UF}(1) + N^1_{UU}(1)} = \frac{13}{63+13} = 0.1711$$

$$\text{Hence}, Q1(1) = \begin{pmatrix} Q^1_{FF} & Q^1_{FU} \\ Q^1_{UF} & Q^1_{UU} \end{pmatrix} = \begin{pmatrix} 0.5617 & 0.4383 \\ 0.8289 & 0.1711 \end{pmatrix}$$

We Note that $\sum_{i \epsilon F, U} Q^1_{iF}(1) + Q^1_{iU}(1) = 1$ and $Q_{iF}(1) \leq 0$ for all $i \epsilon \{F, U\}$

Considering matrix $C^1(1)$ for residential area 1 given electricity loading decision 1; from equations (12),(13) and (14).

$c^1_{FF}(1) = (95 - 78)(0.80) = 18.6$

$c^1_{FU}(1) = (80 - 38)(0.80) = 33.6$

$c^1_{UF}(1) = (60 - 54)(12+0.80+ 0.32) = 13.9$

$c^1_{UU}(1) = (75 - 65)(0.80) = 8.0$

$$\text{Hence}, C1(1) = \begin{pmatrix} c^1_{FF} & c^1_{FU} \\ c^1_{UF} & c^1_{UU} \end{pmatrix} = \begin{pmatrix} 18 & 33.6 \\ 13.9 & 8.0 \end{pmatrix}$$

Using a similar approach, the remaining matrices were calculated

Using (19) and (20) the expected consumption costs (in USD) and accumulated consumption costs(in USD) for the two residential areas were computed under favourable demand (state F) and unfavourable demand (state U); whose results were presented in Table 4

5.3. Analysis of Results

Week 1

Residential area 1

Considering residential area 1, when demand was favourable and noting that 22.37 < 48.80, S=1 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 22.37 USD for the case of favourable demand. Since 4.78<12.89, S=0 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 4.78 USD for the case of unfavourable demand.

Residential area 2

Considering residential area 2, since 33.84<57.71, then S=0 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 33.84 USD for the case of favourable demand. Since 9.80<52.73, S=1 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 9.80 USD for the case of unfavourable demand.

Week 2

Residential area 1

Considering residential area 1, since 37.03<66.46, S=1 was chosen as an optimal electricity loading decision for week 2 with associated accumulated consumption costs of 37.03 USD for the case of favourable demand. Since 21.65<32.25, S=0 was chosen as an optimal electricity loading decision for week 2 with associated accumulated consumption costs of 21.65 USD for the case of unfavourable demand.

Residential area 2

Considering residential area 2, since 57.45<78.05, S=0 was chosen as an optimal electricity loading decision for week 2 with associated accumulated consumption costs of 57.45 USD for the case of favourable demand. Since

Table 4: Expected and Accumulated Consumption Costs (in USD) for Residential Areas.

| Residential Area (r) | State of demand (i) | Expected consumption costs $e^S(r)$ | | Accumulated consumption Costs $a^S(r)$ | |
		Load additional units (S=1)	Do not load units (S=0)	Load additional units (S=1)	Do not load units (S=0)
1	F	22.37 12.80	48.80	37.03 32.25	66.46 21.65
	U		4.78		
2	F	57.71	33.84	78.05	57.45
	U	9.80	52.73	35.30	79.75

Table 5: Households Versus State-Transitions for Electricity Loading Decisions.

Residential area (r)	States of Demand (F/U)	Load Electricity units (S=1)		Do not load Electricity units (S=0)	
		F	U	F	U
1	F	70	30	80	20
	U	40	60	25	75
2	F	90	10	55	45
	U	35	65	15	85

Table 6: Demand (in kwh) Versus State-Transitions for Electricity Loading Decisions in Residential Areas.

Residential area (r)	States of Demand (F/U)	Load Electricity units (S=1)		Do not load Electricity units (S=0)	
		F	U	F	U
1	F	80	60	10	30
	U	30	60	30	40
2	F	30	20	70	60
	U	20	40	20	60

Table 7: Available Electricity (in kwh) Versus State-Transitions for Electricity Loading Decisions in Residential Areas.

Residential area (r)	States of Demand (F/U)	Load Electricity units (S=1)		Do not load Electricity units (S=0)	
		F	U	F	U
1	F	70	60	20	20
	U	45	70	40	50
2	F	40	10	60	70
	U	40	30	30	50

35.30<79.75, S=1 was chosen as an optimal electricity loading decision for week 2 with associated accumulated consumption costs of 35.30 USD for the case of unfavourable demand.

5.4. Model Validation

In this section, we considered out of sample data in two residential areas in order to demonstrate the predictive ability of the proposed model. A sample of 100 customers was considered in each residential area and considerations on demand and available electricity units were captured in t Table 5 and Table 6 below:

For either loading decision taken, unit loading cost (c_l) = 1.20 USD per kwh, unit operational cost (c_o) = 0.80 USD per week and unit shortage cost (c_s) = 0.32 USD per week.

5.5. Computation of Model Parameters

5.5.1. Demand Transition Matrices

Residential area 1

$$Q^1(1)=\begin{pmatrix} 0.700 & 0.300 \\ 0.400 & 0.600 \end{pmatrix}$$

$$Q^0(1)=\begin{pmatrix} 0.800 & 0.200 \\ 0.250 & 0.750 \end{pmatrix}$$

Residential area 2

$$Q^1(2)=\begin{pmatrix} 0.900 & 0.100 \\ 0.350 & 0.650 \end{pmatrix}$$

$$Q^0(2)=\begin{pmatrix} 0.550 & 0.450 \\ 0.150 & 0.850 \end{pmatrix}$$

5.5.2. Consumption Cost Matrices

$$C^1(1)=\begin{pmatrix} 6.4 & 1.28 \\ 23.2 & 3.2 \end{pmatrix}$$

$$C^1(2)=\begin{pmatrix} 9.28 & 3.2 \\ 3.2 & 23.2 \end{pmatrix}$$

$$C^0(1)=\begin{pmatrix} 6.94 & 3.2 \\ 3.2 & 3.2 \end{pmatrix}$$

$$C^0(2)=\begin{pmatrix} 1.60 & 12.8 \\ 69.6 & 6.4 \end{pmatrix}$$

5.5.3. Expected consumption costs

Residential area 1

$e^1_F(1) = (0.700)(6.4) + (0.300)(1.28) = 8.32$

$e^0_F(1) = (0.800)(6.94) + (0.200)(3.2) = 6.172$

$e^1_U(1) = (0.400)(23.2) + (0.600)(3.2) = 11.200$

$e^0_U(1) = (0.250)(3.2) + (0750)(3.2) = 3.20$

Residential area 2

$e^1_F(2) = (0.900)(9.28) + (0.100)(3.2) = 8.672$

$e^0_F(2) = (0.55)(1.6) + (0.45)(8.32) = 6.640$

$e^1_U(2) = (035)(3.2) + (0.65)(23.2) = 16.200$

$e^0_U(2) = (0.15)(69,6) + (0.850\backslash)(6.4) = 15.880$

Table 8: Expected and accumulated consumption costs (in USD) for residential areas.

Residential Area (r)	State of demand (i)	Expected consumption costs $e^S(r)$		Accumulated consumption Costs $a^S(r)$	
		Load additional units (S=1)	Do not load units (S=0)	Load additional units (S=1)	Do not load units (S=0)
1	F	8.320	6.170	13.360	11.750
	U	11.200	3.20	15.589	7.143
2	F	8.672	9.200	18.065	18.556
	U	16.200	15.880	29.557	30.679

5.5.4. Accumulated consumption costs

Residential area 1

$a^1_F(1) = 13.60 + (0.700)(6.172) + (0.300)(3.200) = 13.600$

$a^0_F(1) = 6.172 + (0.800)(6.172) + (0.200)(0.540) = 11.750$

$a^1_U(1) = 11.200 + (0.400)(6.172) + (0.600)(3.20) = 15.589$

$a^0_U(1) = 3.200 + (0.250)(6.172) + (0750)(3.2) = 7.143$

Residential area 2

$a^1_F(2) = 8.672 + (0.900)8.672 + (0.100)15.880 = 18.065$

$a^0_F(2) = 6.64 + (0.55)(8.672) + (0.45)(15.880) = 18.556$

$a^1_U(2) = 16.20 + (0.35)(8.672) + (0.65)(15.880) = 29.557$

$a^0_U(2) = 15.880 + (0.150)(8.672) + (0850)(15.880) = 30.679$

5.6. Analysis of results

Week 1

Residential area 1

Considering residential area 1, when demand was favourable and noting that 6.17 < 8.32, S=0 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 6.17 USD for the case of favourable demand. Since 3.20<11.20, S=0 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 3.20 USD for the case of unfavourable demand.

Residential area 2

When demand was favourable (state F) and noting that 8.672<9.200, S=1 was chosen as an optimal electricity loading decision with associated expected consumption costs of 8.672 USD When demand was unfavourable (state U), S=0 was chosen as an optimal electricity loading decision with associated expected consumption costs of 8.67 USD for the case of unfavourable demand (state U).

Week 2

Residential area 1

When demand was favourable(state F) and noting that 11.750<13.60, S=0 was the optimal electricity loading decision with associated accumulated consumption costs

of 11.750 USD. Similarly, since 7.143<15.589, S=0 was the optimal electricity loading decision with accumulated consumption costs of 7.143 USD for the case of unfavourable demand. (state U).

Residential area 2

When demand was favourable (state F) and noting that 18.065<21.116, S=1 was the optimal electricity loading decision with associated accumulated consumption costs of 18.065 USD for the case of favourable demand (state F). Since 29.557<30.679, then S=1 was the optimal electricity loading decision with associated accumulated consumption costs of 29.5557 USD for the case of unfavourable demand (state U)

5.7. Discussion of Results

Considering the case study of Uganda Electricity Distribution Company (UEDC) presented, optimality of electricity loading decisions and consumption costs over a finite period planning horizon yielded important results for discussion. Results indicated optimal state-dependent electricity loading decisions and consumption costs were-dependent and consistent at every stage of the decision problem. This was attributed to the stationary demand transition probabilities considered at the decision epochs.

When demand was initially favourable (state F), additional electricity units were needed for weeks 1 and 2 of residential area 1. However, when demand was initially unfavourable (state U), additional electricity units were not required for weeks 1 and 2 of residential area 2.

6. Conclusions

A Markov decision process model that optimized electricity loading decisions and consumption costs with stochastic demand was presented in this paper. Using dynamic programming, an optimal electricity loading decision was determined for residential areas over a multi-period planning horizon. Therefore, as an optimization strategy for electricity loading decisions and consumption costs in residential areas, computational efforts of using the Markov decision process model showed promising results.

6.1. Model Implications on the electricity industry

The proposed model has interesting implications in practical terms as a decision making tool for sustaining electricity regulation strategies in industry. Considering the case study results, demand uncertainty affected electricity regulatory policies; which was a driving force for comparative analysis of electricity consumption. Although Markov decision processes for optimizing electricity loading options was fundamental for practical purposes, stationarity of demand transition probabilities raised a number of salient issues to consider: for example changing demand patterns for electricity consumption among users, unpredictable power outages during the demand cycle, price fluctuations of electricity supply etc, left a lot to be examined; especially in the Ugandan context. It was also noted that the study was done using a smaller number of residential areas; considering the two areas captured in the case study and the model validation section. Future studies must aim at increasing the number of residential areas in order to establish a realistic representative sample. This can also improve the model's predictive ability as a decision making tool. In effect, the electricity regulatory authorities for energy distribution can be in position to gain a competitive advantage over energy providers for domestic consumption.

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