

ENERGY EQUATIONS TO ANALYZE PRESSURIZED WATER TRANSPORT SYSTEMS

Roberto del Teso¹, Elena Gómez², Elvira Estruch-Juan³ and Enrique Cabrera⁴

^{1,2,3,4} ITA. Department of Hydraulic Engineering and Environment. Universitat Politècnica de València. Valencia, Valencia, (Spain)

¹ rodete@ita.upv.es, ² elgosel@ita.upv.es, ³ maesjua1@ita.upv.es, ⁴ecabrera@ita.upv.es

Abstract

Pressurized water systems are high energy demanders. They must deliver the demanded volume of water at the minimum service pressure. There is currently a marked change in production and energy tariffs, which is causing prices for water services to rise. In addition to the economic aspect, the energy demand of water systems has an environmental implication linked to gas emissions. With increasing demands, the climate change situation and the shift in energy tariffs, it is necessary to improve the efficiency of pressurized water transport systems. The lower the kWh required to supply, the more energy efficient they will be.

The energy analysis of the systems will allow us to know the current situation of the systems, and to know if it is necessary to undertake improvement measures. There are different processes to know the energy status of the networks. The simplest one is to carry out a diagnosis, with little data, which will give an initial idea of the energy status of the networks. This diagnosis can be carry through by applying Bernoulli's equation. Applied from the supply points to the most unfavorable point of the system.

If it is desired to know more precisely the energetic status of the networks, the use of the integral energy equation seems more reasonable. This equation makes it possible to audit the system and to know in detail the energy use for a given control volume. It allows the energy supplied to be broken down into: useful energy, structural energy losses (linked to topography) and operational energy losses (friction losses, pumping losses, leakage losses and excess energy). In order to be able to apply this method, it is mandatory to have the mathematical model of the system.

This paper discusses the advantages and disadvantages of carrying out the energy analysis of a system using the Bernoulli equation (diagnosis) or the energy integral equation (audit), and when it is convenient to apply one or the other. On one hand, the Bernoulli equation makes it possible to estimate the energy level of the network with very little data, without knowing exactly in which processes the energy introduced into the system is invested. The audit based on the integral energy equation, on the other hand, requires precise data collection and mathematical modelling, but it will provide a detailed understanding of the energy breakdown of the network.

Depending on the objective of the energy analysis, it seems reasonable to apply one process or another. For a first estimation and as a start of the energy analysis, it will be sufficient to carry out a diagnosis as quickly as possible, which will allow to know if it is necessary to continue with a more in-depth research of the system status such as an energy audit.

Keywords

Energy audit, water-energy nexus, energy equation, water distribution systems, energy efficiency.

1 INTRODUCTION

Energy consumption in water transport is large. In Europe, pumps are responsible for 10 % of the electrical energy [1], while in California, pressurised water transport accounts for 6 % of the energy demand [2]. In Spain, energy consumption due to irrigation accounts for 3% of the

country's total electrical energy [3], while in Israel pumping accounts for more than 8% of the total electricity consumed in the country [4]. Climate change calls for increased efficiencies, both water and energy.

In order to improve the efficiency of pressurised water systems, it is necessary to analyse their energy status and find out the type of energy losses in order to propose improvements and changes to optimise their efficiency. Water transport systems can be differentiated into simple and complex systems. Simple systems are made up of a pipe and a pump that transfers water between end nodes. Complex systems are networks with multiple delivery points, where the operating conditions are imposed by the most unfavourable or critical point [5].

In complex systems, by guaranteeing the necessary energy at the critical node, the rest of the nodes are given more energy than necessary. The sum of all these excesses, the greater the more irregular the terrain, is the topographic energy [6]. From an energetic perspective, topographic energy is the big difference between simple and complex systems. This energy, as far as possible, should be minimised at the design stage [7], and if the system is already operating, it should be managed as efficiently as possible to minimise its losses [8]. Since these losses do not depend on the system's operating mode, they are called structural losses, which do not exist in simple systems. In these systems, it is sufficient to minimise operational losses (related to the pumping station, those linked to leaks and pipe friction).

The energy analysis in simple and complex systems can be carried out in different ways, depending on the type of system to be analysed and the depth of the analysis, it will be more convenient to choose one method or another. The energy analysis to be carried out will depend on the quantity and quality of the installation's data. A first diagnosis allows detecting whether there is a need for a more specific analysis detailing the energy consumed by the system. This diagnosis can be carried out with little data. If the result of the diagnosis indicates that there is considerable room for improvement, it will be necessary to carry out a network audit, for which it is essential to have the corresponding mathematical model, and therefore much more precise data.

Two main equations can be distinguished for the energy analysis of pressurised water transport. On the one hand, the Bernoulli equation, on the other hand, the energy integral equation.

Contrary to what happens in thermal fluid mechanics, which is always governed by the integral energy equation, the resolution of hydraulic energy problems, in which the thermal term is neglected, is often based on Bernoulli's equation. This equation does not allow heat balances to be carried out, so when these effects are negligible, the fluid is incompressible and certain conditions are given that are typical of the transport of water under pressure, its use is accepted. In this case, both equations are valid, both the general energy equation and the Bernoulli equation, although with their nuances (such as the one-dimensionality of the Bernoulli equation as opposed to the spatiality of the integral energy equation). This work indicates which equation should be applied depending on the type of water transport system and the depth of the study, highlighting similarities and differences between the two equations.

2 BERNOULLI EQUATION

The well-known Bernoulli equation, expressed in energy per unit weight (m), is usually formulated as follows:

$$\frac{p_1}{\gamma} + \frac{1}{2g} V_1^2 + z_1 = \frac{p_2}{\gamma} + \frac{1}{2g} V_2^2 + z_2 = C \quad (1)$$

With p the pressure, γ the specific weight of the fluid (9810 N/m³), g the acceleration of gravity, v the velocity and z the elevation.

Most hydraulic flows are assumed to be one-dimensional, which means that Bernoulli's equation is often used to solve this type of problem. It is an expression that relates the different ways of storing energy in a fluid, with the exception of thermal energy, which is irrelevant in most cases. It poses an energy balance between two points on the same streamline, distinguishing three different summands that allow each type of energy to be recognised: energy in the form of pressure, kinetic energy and potential energy.

For the use of Bernoulli's equation in the energy analysis of pressurised water transport, it must be applied to a pipe, associating its axis to the reference streamline. Furthermore, it must be generalised to include every existing energy input or loss. As an input, the shaft work contributed by a pump, h_p , must be added (if there is shaft work subtracted by a turbine, this would also be included). As energy losses, the friction between the water and the walls of the pipe is included (Darcy Weisbach equation). In this way we arrive at the generalised Bernoulli equation [9], equal to:

$$\frac{p_1}{\gamma} + \frac{1}{2g} V_1^2 + z_1 + h_p = \frac{p_2}{\gamma} + \frac{1}{2g} V_2^2 + z_2 + f \frac{L}{g} \frac{V^2}{2g} \quad (2)$$

where f is the friction factor (dimensionless) and D is the pipe diameter.

2.1 Recommendations of Bernoulli's equation

The Bernoulli equation is suitable for diagnosing the energy efficiency of simple systems because, when applied between two points on the same power line, it is perfectly suited to the condition of simple systems. In complex systems, with well identified end nodes, it provides important but incomplete information.

Bernoulli's equation is well suited to diagnose the energy efficiency of simple systems. From the results of Bernoulli's equation and knowing that the units of the energy intensity indicator (energy per unit volume) coincide with those of the pressure. A quick energy diagnosis can be established, where the energy intensity can also be expressed in metres of water column ($p = \gamma H$). Their ratio $1 \text{ m} = 0.002725 \text{ kWh/m}^3$. Under these conditions, the energy intensity required by the system, I_{ee} , can be estimated:

$$I_{ee} = 0.002725 H_e \text{ kWh/m}^3 \quad (3)$$

H_e is the estimated head needed to get from the supply source to the critical node at the required pressure. This H_e would be obtained by applying the Bernoulli equation:

$$H_e = \frac{1}{\eta_{pe}\eta_{le}} [(z_c - z_s) + (z_s - z_l)\eta_{pe} + h_{fe} + \frac{p_o}{\gamma}] m \quad (4)$$

Where η_{le} is the estimated water yield, η_{pe} the pumping efficiency. The remaining terms in equation 1 are, 0.002725, a unit change factor (metres to kWh/m³), z_c , the most energy demanding, or critical, node elevation. On the other hand, z_s is that of the supply source, and z_l is the minimum level. The remaining variables are h_{fe} (head loss from the source to the critical node), p_o the service pressure and γ the specific weight of the water (9810 N/m³).

Once the estimated energy intensity for the installation is known, it can be compared with the actual energy intensity value (total energy consumed divided by volume of water billed), in which case pump inefficiencies and system leakage must be included.

In previous work [5] [10], energy efficiency has been analysed in both simple and complex systems using the Bernoulli equation. For this purpose, I_{er} , the real energy intensity (energy consumed divided by the final measured volume), is compared with I_{ee} , the intensity estimated from the application of the Bernoulli equation, a comparison that, in order to be carried out correctly, requires disaggregating the energy supplied into natural and shaft energy.

2.2 Limitations of diagnosing systems with the Bernoulli equation

As it is a static equation, it provides information for a specific instant. Therefore, its accuracy depends on the representativeness of the selected instant (there are no problems if the flow is stationary) and the critical node adopted. The same happens if the head losses between the source and the critical node are estimated with a peak flow rate that does not correspond to the average state of the system.

Bernoulli requires a careful selection of the end nodes, usually the supply source and the most energy demanding point (or critical point). This result extrapolates to the rest of the nodes [10]. However, by ignoring the topography (elevation) of the rest of the nodes, it does not quantify the individual pressure exceedances, and the information obtained is incomplete. To obtain it, the Bernoulli equation has to be applied repeatedly between the source and each node, losing the appeal of simplicity. This is information that the integral energy equation, which does require the coordinates of the nodes, provides [6].

Another case that conditions the application of the Bernoulli equation is networks with several supply sources. In these types of complex systems, as there are two or more energy sources, the selection of the points between which to apply the Bernoulli equation is no longer immediate. One of them is the critical one (the one that requires the most energy) and the other the energy source. However, if there are different energy sources, one must be chosen and, at the same time, the energy contribution of the whole must be weighted [10].

Finally, in networks with backflows, the energy balance of the nodes of the system is broken, a balance that is maintained within the subsystems corresponding to each backflow. This makes it necessary to apply Bernoulli's equation to each subsystem, where the nodes do maintain an energy balance.

The integral energy equation, the energy balance, extended in space and time, is not conditioned by these factors that make energy analysis using Bernoulli difficult.

3 ENERGY INTEGRAL EQUATION

The energy integral equation is more general than Bernoulli's equation, but also more demanding, as it requires more data. In its initial approach, the energy integral equation is the result of applying Reynolds' Drag Theorem to the first law of thermodynamics [9]. The total energy per unit mass is the sum of internal energy (u), kinetic energy ($v^2/2$) and gravitational energy (gz). Thus, in its most general form, the equation results:

$$\begin{aligned}
 \frac{dQ}{dt} + \frac{dW_{shaft}}{dt} &= \frac{\partial}{\partial t} \int \int \int_{VC} \left(\frac{v^2}{2} + u + gz \right) \rho dV \\
 &+ \int \int_{SC} \left(\frac{v^2}{2} + u + gz + \frac{P}{\rho} \right) \rho (\vec{v} \cdot d\vec{A})
 \end{aligned} \tag{3}$$

It validates after disaggregating the total work into flow work (between the system and the external medium through the control surface, SC) and shaft work, $(dW_{shaft})/dt$. The remaining variables are, the heat exchange between the external medium and the control system, SC, v , the velocity, u , the internal energy per unit mass, z , the geometric coordinate, the fluid density, dV , the volume differential of the control volume (VC), p , pressure and $\vec{v} \cdot d\vec{A}$ the flow differential across the SC. This general formulation, with few limitations, is widely used in Thermal Fluid Mechanics and very rarely in Mechanical (or Hydraulic) Fluid Mechanics, which usually resorts to the much simpler Bernoulli equation.

The three terms of this equation have a clear physical meaning. The first is the shaft work, which can be contributed (with pumps), subtracted (with turbines) or simply be zero. It is the hydraulic useful work (given to the turbine or delivered by the pump), as the energy changes of the fluid (the system) are evaluated. The inefficiencies (turbomachines and engine/generator) are added afterwards. The first integral of the second member represents the energy change inside the VC, which will be non-zero if there are reservoirs inside the VC (they gain or lose energy depending on whether they are filled or emptied). Without them, the fluid being incompressible, it is zero. The second integral, the flow term, is the balance of powers between the outgoing flow (uses and leaks) and the incoming flow in the VC. Both powers include three summands, the kinetic (negligible in network analysis), the piezometric ($gz + p/\rho$) and the internal energy, whose variation is the power dissipated by friction.

As a consequence of its initial approaches, Bernoulli's equation formulates an energy balance between two points on the same streamline, while the energy equation formulates it for a control volume [9]. With hardly any limitations, the energy integral equation is valid for any regime, static or dynamic, compressible or incompressible, with or without heat transfer. Thus, the main difference with Bernoulli is the application framework (a current tube versus a VC) and the possibility of studying transient flows and including internal compensation reservoirs in the study [3].

4 CASE STUDY

The above is applied to a case study that has a supply source with a height of 84 m, from which the pumping station in charge of supplying the sector draws water. The physical data for the sector are as follows:

- Total length of the sector = 45.1 km
- Height of the critical junction (coincides with the highest point), $z_c = 120,66$ m
- pump suction elevation, $z_s = 84,00$ m
- Height of the lowest node, $z_1 = 35,64$ m.
- Distance between the source and the most demanding junction = 4,5 km.
- Minimum operating pressure $p_o/\gamma = 20$ m.

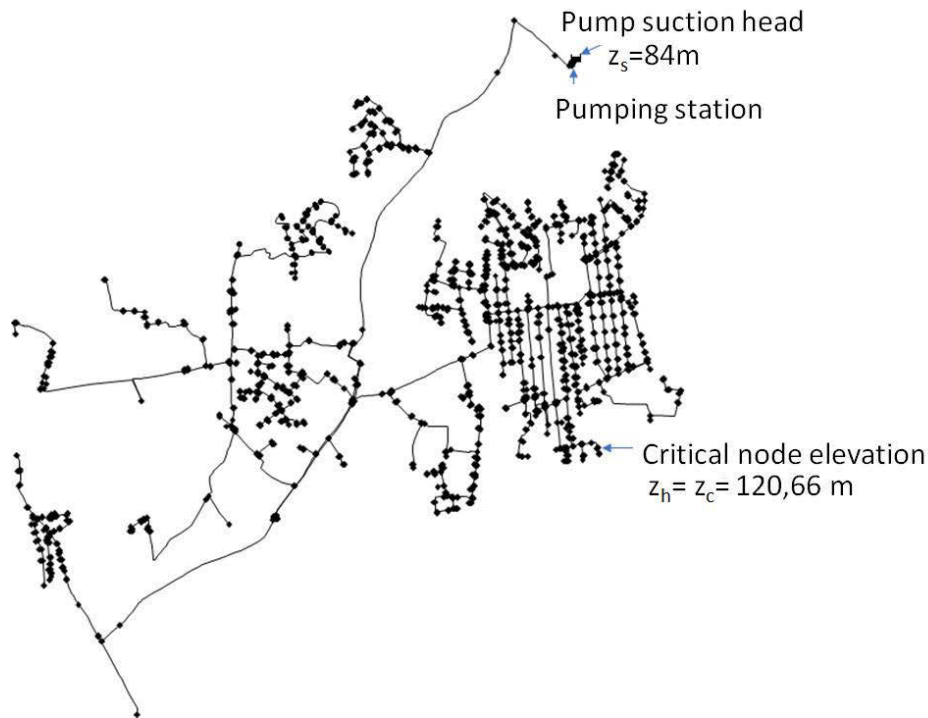


Figure 1. Case study network

Sector operating data:

- Volume injected = 15386 m³/month
- Volume registered = 8994 m³/month
- Real water efficiency (including real and apparent losses) $\eta_{lr} = 0,58$
- Energy consumed by the pumping unit $E_p = 3902$ kWh/month
- Average efficiency of the pumping unit, $\eta_{pr} = 0,70$
- As far as the head loss is concerned, a value of 1.4 m/km is assumed (Cabrera et al., 2018), resulting in an estimated friction height, h_{fe} , equal to 6.3 m.

With this information we obtain:

$$\begin{aligned}
 I_{ee} &= \frac{0.002725}{\eta_{pe}\eta_{le}} [(z_c - z_s) + (z_s - z_l)\eta_{pe} + h_{fe} + \frac{p_o}{\gamma}] \\
 &= \frac{0.002725}{0,7 \cdot 0,58} [(120,66 - 84) + (84 - 35,64)0,7 + 6,3 + 20] \\
 &= 0,64 \text{ kWh/m}^3
 \end{aligned}$$

After analysing the energy efficiency of the studied sector, it is desired to improve it. To this end, the energy intensity that could be achieved by improving the volumetric efficiency of the network and the efficiency of the pumping stations will be estimated. To do this, target values are set that are achievable in an urban supply network of these characteristics ($\eta_{pe} = 0.75$ and $\eta_{le} = 0.80$) and the volume demanded by the subscribers, V_r , is maintained. It can be seen how a large margin of improvement in water efficiency is imposed, from 0.58 to 0.80. This results in the following:

$$I_{ee} = \frac{0.002725}{\eta_{pe}\eta_{le}} [(z_c - z_s) + (z_s - z_l)\eta_{pe} + h_{fe} + \frac{p_o}{\gamma}] = 0,45 \text{ kWh/m}^3$$

There is a large margin for improvement between the stipulated target efficiencies and the actual efficiency of the system.

The above analysis does not include the spatial distribution of network demands. Consequently, nothing is known about the weight of structural losses, as well as the specific breakdown of other operational losses. For this it would be necessary to carry out an energy audit.

The following table shows the breakdown of the energy audit carried out on the basis of the data from the mathematical model:

Table 1. Energy audit.

Total energy input	3059.13	kWh/5days
Energy supplied by the pumps	1522.573	kWh/5days
Natural energy	1536.557	kWh/5days
Total energy consumed	3059.064	kWh/5days
Energy delivered to users	2170.936	kWh/5days
Minimum energy required	1528.718	kWh/5days
Energy excess of minimum pressure	643.045	kWh/5days
Energy dissipated by friction	41.672	kWh/5days
Energy dissipated in valves	70.647	kWh/5days
Energy lost through leakage	395.166	kWh/5days
Energy lost in pumps	380.643	kWh/5days

As can be seen, the information extracted from the energy audit made from the integral energy equation is much more extensive than that obtained with the diagnosis made using the Bernoulli equation.

5 CONCLUSIONS

In order to analyse the efficiency of a pressurised water conveyance, it is necessary to define the physical framework to which the analysis is to be made and to apply the energy equation of choice correctly. Otherwise, the results obtained will be inconsistent.

The Bernoulli equation is simple to use and works well for simple systems, in complex systems its use is not recommended due to its limitations. Bernoulli is useful for a first diagnosis of the energy state of the network. In complex systems it is better to use the Bernoulli integral equation, whose main limitation is the need to have a series of concrete data extracted from the mathematical model. However, it allows a detailed energy audit to be carried out.

6 REFERENCES

- [1] Grundfos, 2014. “High efficiency motor technology that reduces energy waste in pump applications”. Grundfos. Denmark.
- [2] WW (Water in the West), 2013 Water and Energy Nexus: A Literature Review. Stanford University. USA.
- [3] Cabrera E, Pardo MA, Cobacho R, Cabrera E. Jr. 2010. Energy audit of water networks. *J Water Resour Plan Manag* Vol. 136, pp 669 - 677.
- [4] Book review: *Groundwater around the World: A Geographic Synopsis*, by Jean Margat and Jac van der Gun. 2013, CRC Press.
- [5] Cabrera E., Gómez E., Soriano J., del Teso R., 2019. Towards eco-layouts in water distribution systems. *J. Water Resour Plan Manag* 145(1):04018088.
- [6] Cabrera E, Gómez E, Cabrera E, Soriano J, Espert V., 2015. Energy assessment of pressurized water systems. *J Water Resour Plan Manag* 141(8):04014095.
- [7] Cabrera E., Gómez E., Cabrera E., Jr., Soriano J., 2018. Calculating the economic level of friction in pressurized water systems. *Water*, 9, 763. DOI: 103390/w10060763.
- [8] Del Teso R., Gómez E., Estruch M.E., Cabrera E., 2019. Topographic energy management in water distribution systems. *Water Resources Management*. Vol 33, nº 12, pp 4385-4400
- [9] White F. M., 1979. *Fluid mechanics*. New York, USA: McGraw-Hill, Inc. ISBN 10: 0070696675.
- [10] Cabrera, E.; del Teso, R.; Gómez, E.; Cabrera, E., Jr.; Estruch-Juan, E., 2021. Deterministic Model to Estimate the Energy Requirements of Pressurized Water Transport Systems. *Water* 2021, 13, 345. <https://doi.org/10.3390/w13030345>