2nd International Joint Conference on Water Distribution Systems Analysis & Computing and Control in the Water Industry
Valencia (Spain), 18-22 July 2022
doi: https://doi.org/10.4995/WDSA-CCWI2022.2022.14802

OPTIMIZATION OF RESERVOIR TREATMENT LEVELS CONSIDERING UNCERTAINTY IN MIXING AT CROSS JUNCTIONS IN WATER DISTRIBUTION SYSTEMS USING INFO-GAP DECISION THEORY

Sriman Pankaj Boindala[1], G Jaykrishnan[2] and Avi Ostfeld, F. ASCE[3]

[1] PhD Student, Faculty of Civil and Environmental Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel
[2] PhD Student, Faculty of Industrial Engineering and Management, Technion – Israel Institute of Technology, Haifa 32000, Israel
[3] Professor (Corresponding Author), Faculty of Civil and Environmental Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel

[1] *srimanpankaj@gmail.com*, [2] *jaykrishnang@hotmail.com* , [3] *ostfeld@technion.ac.il*

Abstract

Water distribution systems are affected by several uncertainties in multiple stages. This uncertainty makes solving the optimal design and management of WDS a multifaceted problem. Past research has focused only on solving design and management problems of system hydraulics. There have been very few studies that involve considering uncertainties that affect the water quality aspect of WDS. One of the major assumptions in solving the design and management problems of WDS is considering uniform and instantaneous mixing at the cross junctions. However, in reality, this is not true. This assumption is made due to the lack of computational power to accurately estimate the level of mixing at every junction in a water distribution network. This study focuses on considering this level of mixing as uncertain/unknown and provides the optimal treatment levels required at the reservoirs to ensure the system is immune to the level of mixing occurring at the junctions to satisfy the water quality requirements at the customer level. Info-gap decision theory-based optimization approach combined with the cuckoo search metaheuristic is proposed in this study to handle the uncertainty. The proposed methodology is applied to a 4x4 grid hypothetical network example. The study's objective is to provide the best designs that can handle the maximum variation of the level of mixing at junctions within the given budget by the designer. The maximum variation of the level of mixing is reported for different budget levels. The designs are compared with the deterministic case using Monte-Carlo simulations.

Keywords
Water Quality Uncertainty, Incomplete Mixing, Epanet-BAM, Info-Gap Decision Theory, Treatment Level Design Optimization, Water Distribution System (WDS).

1 INTRODUCTION

A system consisting of reservoirs (sources), tanks, pipes, pumps, and valves that work together to provide high quality (clean, odourless, and clear) water at sufficient pressure head for the customers downstream is called a water distribution system (WDS). With growing urban migration, the demanding stress on these systems increases, and hence more centralized systems are being developed. With limited potable water sources available, these systems' optimal resilient design and management are inevitable. Optimal design and management of WDS have always been focused on satisfying hydraulic requirements like demand and pressure constraints in the past. The passing of the safe drinking water act (SDWA) in 1990 motivated research to include water quality constraints in the optimal design and management of WDS. Today the importance of maintaining the water quality is equivalent to satisfying the hydraulic constraints (Pasha and Lansey 2010). Even with including water quality constraints, the performance of the WDS is not as expected. Not accounting for the uncertainty involved in the optimal design and management problem of WDS and the assumptions made for simplifying the problem are the major reasons for the low performance of the systems when applied practically.

(Lansey et al. 1989)were pioneers that proposed an optimal WDS design methodology that considered uncertainty in hydraulic parameters. The uncertain parameters like future demand are assumed to follow a probability distribution function (PDF) with a mean and standard deviation and formulated the hydraulic constraints as chance constraints. They used a non-linear programming method with a generalized reduced gradient-II (GRG-II) technique for optimization. They applied this method to a small network and proposed multiple designs with corresponding reliability values. (Babayan et al. 2004; Kapelan et al. 2005), proposed methodologies to solve optimal single and multi-objective WDS design problems under uncertainty. They used Latin hypercube sampling (LHS) for sampling and PDF generation instead of assuming. They used genetic algorithms as optimization algorithms. (Babayan et al. 2005) proposed a methodology that eliminates sampling techniques by adding a margin of safety factor in the required minimum pressure heads. Then the stochastic problem became a simple deterministic problem with additional constraints and was solved using a standard genetic algorithm (GA). (Kang et al. 2009)proposed a methodology to find the optimal design of WDS under uncertainty by using approximation techniques like first-order second moment (FOSM) and combined that with LHS to simplify the computational time for the stochastic problem.

Due to the high computation time of these probabilistic approaches, the research shifted towards using a non-probabilistic approach to handle uncertainty in WDS parameters. Info gap decision theory (IGDT) and robust optimization (RO) are a few of the popular non-probabilistic uncertainty handling techniques. (Chung et al. 2009)implemented robust optimization (RO) approach to solve the design of municipal WDS, considering demand is uncertain. (Perelman et al. 2013) used robust counterpart (RC) to solve the least-cost design problem of WDS with demand uncertainty. (Ghelichi et al. 2018; Naderi and Pishvaee 2017) also used investigated RO formulations to handle uncertainties in the water resources and stated their advantages over the probabilistic approach. (Korteling et al. 2013) stated the benefits of using info-gap decision theory in water resources planning under severe uncertainty. They showed the effectiveness of the info-gap decision theory in supporting adaptive management of water systems under severe uncertainty in supply or demand. (Roach et al. 2015) compared the use of RO and IGDT for water resource management under deep uncertainty. They applied both methods to a case study approximating the Susexx north water resource zone in England. They concluded that the IGDT produced more expensive designs than RO as it has a more rigorous robustness analysis.

The research works in the past related to WDS design are highly concentrated on hydraulic uncertainty. Very few studies have considered uncertainty related to water quality parameters. (Pasha and Lansey 2005, 2010) examined the effect of a few water quality uncertain parameters like bulk and wall reaction coefficients and pipe diameters in the distribution system's water quality analysis. In addition to these water quality parameters, the uncertainty in solute mixing at cross junctions can also affect the consumer nodes' water quality requirements.

Fowler and Jones (1991)were the first to investigate the practicality of complete and instantaneous mixing assumptions at junctions in WDSs. They stated that among many other concerns regarding the accuracy of the water quality (WQ) models, the assumption of instantaneous, complete mixing at junctions was regarded as a significant cause of erroneous outcomes in water quality modelling of WDSs. (Romero-Gomez et al. 2009) investigated the impact of incomplete mixing on sensor network designs. They found the perfect mixing assumption inefficient, which led to wrong sensor placements that led to some locations in the system not being covered. (Ho and Khalsa 2008)developed a Bulk Advective Mixing model (BAM) for addressing the non-uniform mixing behaviour at cross junctions. They incorporated a mixing parameter "$s \in [0,1]$" which governs the extent of mixing from incomplete mixing being the lower bound and complete mixing being the upper bound. (Song et al. 2009)also developed a non-uniform mixing model named (AZRED) to model incomplete mixing phenomena for specific junction types like T and Y junctions. (Paez et al. 2017)compared the water quality at the consumer nodes assuming complete mixing (EPANET) and non-uniform mixing BAM model for WDS of two cities and two large grid networks. They concluded that the mixing uncertainty is more predominant in a grid-type network containing many cross junctions than in the conventional city WDSs in the study.

The present design of WDS does not consider various uncertainties in water quality affecting parameters, especially the mixing uncertainty at junctions. This assumption may lead to unsatisfactory designs that may not satisfy the practical water quality constraints. Realizing the exact mixing phenomenon is quite complex and requires complex CFD simulation. The complexity only increases with the size of the WDS and the number of cross junctions. The current study incorporates the mixing uncertainty in designing the water distribution system. Present work emphasizes quantifying the significance of non-uniform mixing uncertainty in WDS design and using non-probabilistic uncertainty handling techniques like info-gap decision theory to solve the complex, uncertain design problem.

2 METHODOLOGY

2.1 Info Gap Decision Theory

Info Gap Decision Theory is a non-probabilistic decision-making technique that helps in prioritizing alternatives and making decisions under deep uncertainty. The IGDT analysis is governed by three components, system model, desired model performance, and uncertainty model. The system model defines the problem, and the understanding of the system. The performance requirements are the answer to the question: What do we need to achieve in order for the outcome of the decision to be acceptable? i.e. our goal and the conditions it should satisfy to say we reached our goal. Uncertain model is to realize the uncertainty involved in either our understanding (system model) or our uncertainty in our performance criteria.

For illustration:

Consider the problem $\mathcal{P}$. Assume that some information about ξ is known. For example, assume that ξ lies in the polyhedral set U, where $\mathrm{U} = \{\xi \in \mathbb{R}^{\mathrm{n}} \ : \ \mathrm{A}\xi \leq \mathrm{b}\}$ where A is a matrix and b is a vector. The problem is now defined as:

$$\min_{\mathrm{x}} \mathrm{f}(\mathrm{x}, \xi) \tag{1}$$

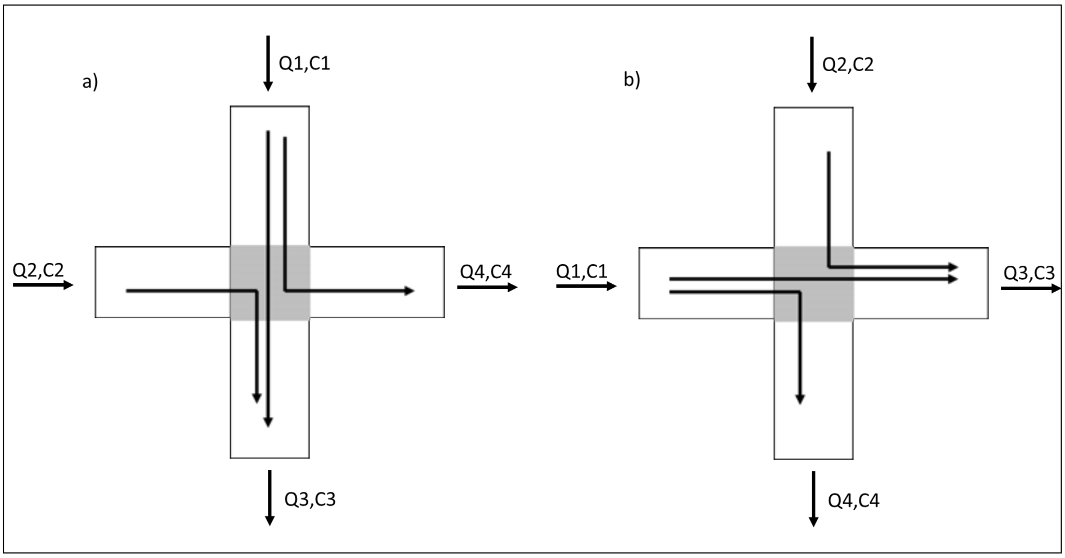
$$\mathrm{s.t.}\, \mathrm{g}(\mathrm{x}, \xi) \leq 0, \forall \xi \in \mathrm{U}. \tag{2}$$

The ideal case of an uncertain problem is that the constraint should be satisfied for all possible realizations of ξ. This uncertain version of the problem is not tractable since there are infinite constraints (Eq. 2).

Let the system model be problem P. This problem is solved by assuming both x, ξ are variables that can take any value, and the problem P is solved to obtain the minimum objective value $\#obj$, optimal decision variable $\#x$ and optimal parameter $\#\xi$. Using these as a base, we build our decision model. The uncertain model is created by taking the $\#\xi$ as a base. The performance criteria satisfy the constraints, and the maximum value the objective function can be allowed or acceptable. The result from the info-gap model is the uncertainty set, i.e. the maximum variation the system model can handle such that the objective function is within the performance criteria. As the maximum value of the objective function is increased, the uncertain sets will also increase, and the sets will be nested in each other (Ben-Haim 2006).

2.2 BAM Model

BAM model that was developed by (Ho et al. 2008) is used as the WQ model in the present study. Ho et al. (2008) assumed that when two fluid streams with different flows and concentrations enter a cross junction, the distribution of solute concentrations in the fluid is between complete and incomplete mixing/bulk mixing. The extent of deviation from the complete mixing is controlled by a mixing parameter "s" generally obtained through experimentation. In this present study, this mixing parameter is treated as uncertain as we cannot always find it through experimentation on real networks. We then aim to obtain optimal designs within the designer's budget that are immune to set of maximum variations of mixing. Equations associated with BAM



model are described below

Legend: Q_1, Q_2, Q_3, Q_4 – Flows; C_1, C_2, C_3, C_4 – Concentrations where $Q_1 > Q_2$

Figure 1 Pictorial representation of bulk mixing (Ho et al, 2008), the shaded region is the non-uniform mixing region

The bulk mixing equations for the case are shown in Fig 1 where $Q_1 > Q_2$ can be written easily with mass balance,

$$C_4 = C_1 \tag{3}$$

$$C_3 = \frac{Q_2C_2 + (Q_1 - Q_4)C_1}{Q_3} \tag{4}$$

Whereas complete mixing equations are,

$$C_3 = C_4 = \frac{Q_1C_1 + Q_2C_2}{Q_3 + Q_4} = \frac{Q_1C_1 + Q_2C_2}{Q_1 + Q_2} \quad (5)$$

The final BAM model equation is between

$$C_{BAM} = C_{incomplete} + s * (C_{complete} - C_{incomplete}) \quad (6)$$

3 TREATMENT COST OPTIMIZATION PROBLEM

Assume that every source for the WDS is associated with a treatment plant, the extent of treatment required at the source level is governed by the water quality requirements at the customer nodes and level of mixing at the junctions. The objective of the problem is to obtain optimal treatment levels that satisfy the water quality constraints at the customer nodes and minimize the overall treatment cost at the sources. We assume steady state conditions and the contaminant is non-reactive.

The problem structure can be written as follows equation (7-8):

$$\min \mathrm{TC}(\mathrm{IRC}, \mathrm{RCAT}) \quad (7)$$

$$\mathrm{s.t.}\ \mathrm{C^j}_{\mathrm{LL}} \leq \mathrm{C^j_{out}}\big(\mathrm{RCAT}, \mathrm{Q}, \mathrm{s}(\xi)\big)_{\mathrm{BAM}} \leq \mathrm{C^j_{UL}}, \forall \mathrm{j} \in \mathrm{J}, \xi \in \mathrm{U} \quad (8)$$

Here, TC -unit treatment cost, IRC – initial reservoir concentration before treatment, RCAT-reservoir concentration after the treatment, $\mathrm{C^j_{LL}}, \mathrm{C^j_{UL}}$ – lower limit and upper limit of desired outlet concentration at junction j, $\mathrm{C^j_{out}}$- outlet concertation at node j, Q- flows in pipes. Its states that the outlet concentration is dependent on the pipe flow, mixing parameter (which is uncertain) and reservoir concentration after treatment.

The explicit formulation of C^J_{out} for every node considering mixing parameter "s" as uncertain gets complex with increase in number of cross junctions, in order to eliminate the complexity, a data driven linear surrogate model is developed. The surrogate model is of the form shown in equation 7 where $\mathrm{C^j_{out}}$ is the outlet concentration and $\mathrm{a_j}$ is a vector and $\mathrm{E_j}$ is a matrix are obtained from regression modelling.

$$C^j_{out}\big(RCAT, Q, s(\xi)\big)_{BAM} = \boldsymbol{a_j^T} RCAT + RCAT^T \boldsymbol{E}_j s \quad (9)$$

Any convex model can be used instead of linear model if the accuracy is not significant

Using this surrogate model, the constraint (9) in treatment cost optimization problem can be re-written as

$$\mathrm{s.t.}\ \mathrm{C^j}_{\mathrm{LL}} \leq \boldsymbol{a_j^T} * RCAT + RCAT^T \boldsymbol{E}_j s \leq \mathrm{C^j_{UL}}, \forall \mathrm{j} \in \mathrm{J}, \xi \in \mathrm{U} \quad (10)$$

3.1 IGDT problem formulation

As mentioned earlier, for IGDT formulation, we need first to define the system model. The system model can be represented as shown in Equations 11-13. We also know that the mixin parameter can take any value in [0,1] where 1 -represents complete mixing and 0- represents bulk mixing (incomplete mixing).

$$\min_{\mathrm{RCAT}} \mathrm{co}^{\mathrm{T}}(\mathrm{IRC} - \mathrm{RCAT}) \tag{11}$$

$$\mathrm{s.t.}\, \mathrm{C}^{\mathrm{j}}_{\min} \leq \mathrm{a}_{\mathrm{j}}^{\mathrm{T}}\mathrm{RCAT} + \mathrm{RCAT}^{\mathrm{T}}\mathrm{E}_{\mathrm{j}}\mathrm{s} \leq \mathrm{C}^{\mathrm{j}}_{\max}\,, \forall \mathrm{j} \in \mathrm{J} \tag{12}$$

$$0 \leq \mathrm{RCAT} \leq \mathrm{IRC} \tag{13}$$

IRC-Initial reservoir concentration before treatment, RCAT – reservoir concentration after treatment. co – the cost of unit treatment of water per unit volume.

Suppose s is considered as an uncertain parameter. In that case, the constraint in (Eq. 12) becomes (Eq. 14), which should satisfy every realization of the parameter s∈U., which leads to an infinite number of constraints. The problem then becomes a semi-infinite optimization problem that is intractable.

$$\mathrm{C}^{\mathrm{j}}_{\min} \leq \mathrm{a}_{\mathrm{j}}^{\mathrm{T}}\mathrm{RCAT} + \mathrm{CR}^{\mathrm{T}}\mathrm{E}_{\mathrm{j}}\mathrm{s} \leq \mathrm{C}^{\mathrm{j}}_{\max}\,, \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{s} \in \mathrm{U} \tag{14}$$

As the most common phenomenon and the previous assumption is complete mixing, and we need the system to satisfy the complete mixing case definitely, we solve the system model (equations 11-13) by assuming (s=e). Once we assume this, the problem is linear and can be easily solved to obtain a base objective value ($\#Obj$).

The second component is to define the uncertainty model. The uncertainty model is assumed to be an envelope-based model, $\mathrm{U}(\delta) = \{\mathrm{s} \in \mathbb{R}^{\mathrm{I}} : (1 - \delta\sigma)\mathrm{e} \leq \mathrm{s} \leq \mathrm{e}\}, \delta \geq 0$ (Ben-Haim, 2006). The size of which is controlled by the scaling parameter δ. Notice that the uncertainty set is a function of δ, and as δ is increased, the set increases in size, but the sets are all nested. Since there are bounds on s, i.e., $0 \leq s \leq 1$, correspondingly, the bounds obtained for δ are:

$$1 - \delta\sigma \geq 0 \ \Rightarrow \delta \leq \frac{1}{\sigma} \tag{15}$$

The uncertain model can be equivalently written as $\mathrm{U}(\delta) = \{\mathrm{s} \in \mathbb{R}^{\mathrm{I}} : \mathrm{As} \leq \mathrm{b}\}, \delta \geq 0$ where $\mathrm{A} = \begin{bmatrix} \mathrm{I}^{\mathrm{I}\times\mathrm{I}} \\ -\mathrm{I}^{\mathrm{I}\times\mathrm{I}} \end{bmatrix}$ and $\mathrm{b} = \begin{pmatrix} \mathrm{e} \\ (1 - \alpha\sigma)\mathrm{e} \end{pmatrix}$, I is the identity matrix, e is the vector of all 1.

Now, based on the budget the decision maker can provide, the maximum objective value which is allowed is incorporated as a constraint $\mathrm{obj}' > \#\mathrm{obj}$, IGDT can be used to develop an uncertainty set and a solution such that the objective value corresponding to the robust solution is at most obj′. Thus, the info-gap formulation is below.

$$\max \delta \tag{16}$$

$$\mathrm{s.t.}\ \mathrm{co}^{\mathrm{T}}(\mathrm{IRC}-\mathrm{RCAT}) \leq \mathrm{obj}' \quad (17)$$

$$\mathrm{C}_{\min}^{\mathrm{j}} \leq \mathrm{a}_{\mathrm{j}}^{\mathrm{T}}\mathrm{RCAT}+\mathrm{RCAT}^{\mathrm{T}}\mathrm{E}_{\mathrm{j}}\mathrm{s} \leq \mathrm{C}_{\max}^{\mathrm{j}}\ \forall \mathrm{s} \in \mathrm{U}(\delta), \forall \mathrm{j} \in \mathrm{J} \quad (18)$$

$$0 \leq \mathrm{RCAT} \leq \mathrm{IRC} \quad (19)$$

$$0 \leq \delta \leq \frac{1}{\sigma} \quad (20)$$

This formulation (Eq. 16-20) can be reformulated using the dualization technique,

Consider LHS of equation 18,

$$a_j^T CR + CR^T E_j s \geq C_{min}^j\ \forall s \in U(\delta), \forall j \in J \quad (21)$$

This constraint is equivalent to

$$\begin{gathered} \min a_j^T CR + CR^T E_j s \geq C_{min}^j \\ s.t.\, s \in \mathbb{R}^I : As \leq b, A = \begin{bmatrix} I^{I\times I} \\ -I^{I\times I} \end{bmatrix} and\ b = \begin{pmatrix} e \\ (1-\alpha\sigma)e \end{pmatrix} \end{gathered} \quad (22)$$

Writing lagrangian for the above problem,

Writing the Lagrangian and minimizing, the dual objective is:

$$\min_{\mathrm{s}\in\mathbb{R}^{\mathrm{I}}} \mathrm{a}_{\mathrm{j}}^{\mathrm{T}}\mathrm{CR}+\mathrm{CR}^{\mathrm{T}}\mathrm{E}_{\mathrm{j}}\mathrm{s}+\mu_{\mathrm{j}}^{\mathrm{T}}(\mathrm{As}-\mathrm{b}) \quad (23)$$

$$= \begin{cases} \mathrm{a}_{\mathrm{j}}^{\mathrm{T}}\mathrm{CR}-\mu_{\mathrm{j}}^{\mathrm{T}}\mathrm{b}, \text{if } \mathrm{CR}^{\mathrm{T}}\mathrm{E}_{\mathrm{j}}+\mu_{\mathrm{j}}^{\mathrm{T}}\mathrm{A}=0 \\ -\infty, \text{else} \end{cases} \quad (24)$$

Using strong duality:

$$\mathrm{C}_{\min}^{\mathrm{j}} \leq \min_{\mathrm{s}\in\mathrm{U}} \mathrm{a}_{\mathrm{j}}^{\mathrm{T}}\mathrm{CR}+\mathrm{CR}^{\mathrm{T}}\mathrm{E}_{\mathrm{j}}\mathrm{s} = \max_{\mu\in\mathbb{R}_{+}^{2\mathrm{I}}} \mathrm{a}_{\mathrm{j}}^{\mathrm{T}}\mathrm{CR}-\mu_{\mathrm{j}}^{\mathrm{T}}\mathrm{b} :\ \mathrm{CR}^{\mathrm{T}}\mathrm{E}_{\mathrm{j}}+\mu_{\mathrm{j}}^{\mathrm{T}}\mathrm{A}=0 \quad (25)$$

Then, by asserting that since the maximum of the argument is at least $\mathrm{C}_{\min}^{\mathrm{j}}$ there should exist at least one value of the dual variable (μ_{j}) that makes the argument at least $\mathrm{C}_{\min}^{\mathrm{j}}$. We drop the maximization. Thus, the infinite number of constraints can be replaced with constraints (26), (27), and (28) for each $\mathrm{j} \in \mathrm{J}$.

$$\mathrm{C}_{\min}^{\mathrm{j}} \leq \mathrm{a}_{\mathrm{j}}^{\mathrm{T}}\mathrm{CR}-\mu_{\mathrm{j}}^{\mathrm{T}}\mathrm{b} \quad (26)$$

$$\mathrm{CR}^{\mathrm{T}}\mathrm{E}_{\mathrm{j}}+\mu_{\mathrm{j}}^{\mathrm{T}}\mathrm{A}=0 \quad (27)$$

$$\mu_{\mathrm{j}} \geq 0 \quad (28)$$

Similarly, the upper bound constraint (for some $\mathrm{j} \in \mathrm{J}$) can be reformulated to:

$$\mathrm{a}_{\mathrm{j}}^{\mathrm{T}}\mathrm{CR}+\nu_{\mathrm{j}}^{\mathrm{T}}\mathrm{b} \leq \mathrm{C}_{\max}^{\mathrm{j}} \quad (29)$$

$$CR^T E_j + \nu_j^T A = 0 \quad (30)$$

$$\nu_j \geq 0 \quad (31)$$

So the final IGDT problem of equations 16-20 can be reformulated into equations (32-40)

$$\max_{\delta, RCAT, \mu_j, \nu_j, j \in J} \delta \quad (32)$$

$$s.t.\ co^T(IRC - RCAT) \leq obj' \quad (33)$$

$$a_j^T RCAT - \mu_j^T b \geq C_{min}^j, \forall j \in J \quad (34)$$

$$CR^T E_j + \mu_j^T A = 0, \forall j \in J \quad (35)$$

$$a_j^T RCAT + \nu_j^T b \leq C_{max}^j, \forall j \in J \quad (36)$$

$$CR^T E_j - \nu_j^T A = 0, \forall j \in J \quad (37)$$

$$\mu_j, \nu_j \geq 0, \forall j \in J \quad (38)$$

$$0 \leq RCAT \leq IRC \quad (39)$$

$$0 \leq \delta \leq \frac{1}{\sigma} \quad (40)$$

where: $\mu_j, \nu_j J \in J$ is a dual variable that is required for writing the dual problem. The above formulation gives a larger and larger set as obj' is increased since a more robust solution will increase the objective value but will be more immunized against larger realizations of the mixing parameter s. The obtained formulation is non-convex because of the terms $\mu_j^T b$ and $\nu_j^T b$. The cuckoo search algorithm is used to solve this problem.

3.2 Cuckoo Search Algorithm

Cuckoo search is a metaheuristic based on swarm intelligence inspired by the breeding behavior of a few species of cuckoos. Yang and Deb introduced this algorithm in the year 2009. The optimization algorithm has been applied in multiple areas in engineering as well as in the sciences. This algorithm and its variations have effectively solved the WDS design problem (Naveen Naidu et al. 2020; Pankaj et al. 2020; Wang et al. 2012).The working mechanism of the cuckoo search algorithm in (Figure 2) is extracted from (Pankaj et al. 2020). And the detailed explanation of the mechanism can be referred from (Yang and Deb 2009). This cuckoo search algorithm is used to obtain the robustness α for the info-gap problem formulation.

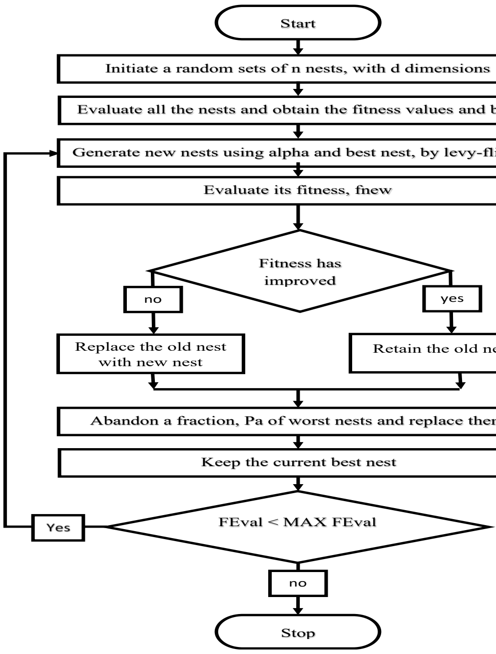


Figure 2 Cuckoo search algorithm flow chart

3.3 Illustrative example

A 4x4 grid network is considered for this study, as shown in (Figure 3), where all the possible junctions where incomplete mixing is feasible are highlighted. The possibility of incomplete mixing is dependent on flow direction. The objective is to obtain optimal treatment costs. Sources 1 and 2 are at 0m elevation, and source-3 is 70m. The pump linking source-1 is of power 2, and the pump linking source 2 is of power 1.5. The base demand, elevation, and concentration limits of all the nodes are mentioned in Table 1.

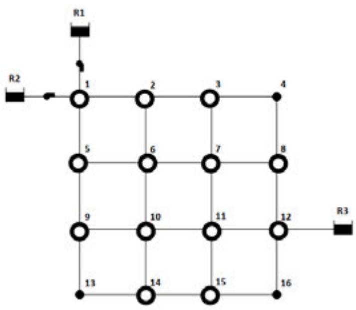


Figure 3 A 4x4 grid network that is considered for this study with three reservoirs. The nodes where the non-uniform mixing can occur are highlighted

Initial concentrations at the sources before treatment IRC = (100,200,150). Treatment costs for each source co = (5000, 200,10000).

<u>Objective function:</u>

$$\min_{i=1,2,3} \sum co^{T}\,(IRC - RCAT) \tag{56}$$

Here RCAT is the reservoir concentration after treatment entering the network.

Constraints:

$$C_j^{min} \le C_{out}^{j}\big(RCAT, Q, s(\xi)\big)_{BAM} \le C_j^{max} \tag{57}$$

C_j = concentration of the flow from node j, C_j^{min}, C_j^{max} are the lower and upper bounds for the concentration at node j.

Surrogate model:

As mentioned earlier, to apply RO and IGDT, a surrogate model is needed to replace the BAM model-based equations for outlet concentrations. A unique surrogate equation is developed for each outlet concentration (10 nodes- Illustrative example). After evaluating various forms, $a_j^T RCAT + CR^T E_j s$ form has been selected based on the model accuracy. Here, a_j is a vector and E_j is a matrix corresponding to node j. These equations are developed for a particular design of WDN (i.e., constant flow direction and flow values). The procedure to obtain the surrogate model is explained as a flowchart in (Figure 4).

Table 1 Illustrative example node data

Node-ID	Base Demand (GPM)	Min Co-Limit	Max Co-Limit
1	0	0	0
2	10	40	70
3	5	40	70
4	10	40	70
5	50	20	50
6	0	0	0
7	0	0	0
8	10	20	50
9	30	30	60
10	0	0	0
11	0	0	0
12	0	0	0
13	40	30	60
14	80	30	60
15	60	20	50
16	60	40	70

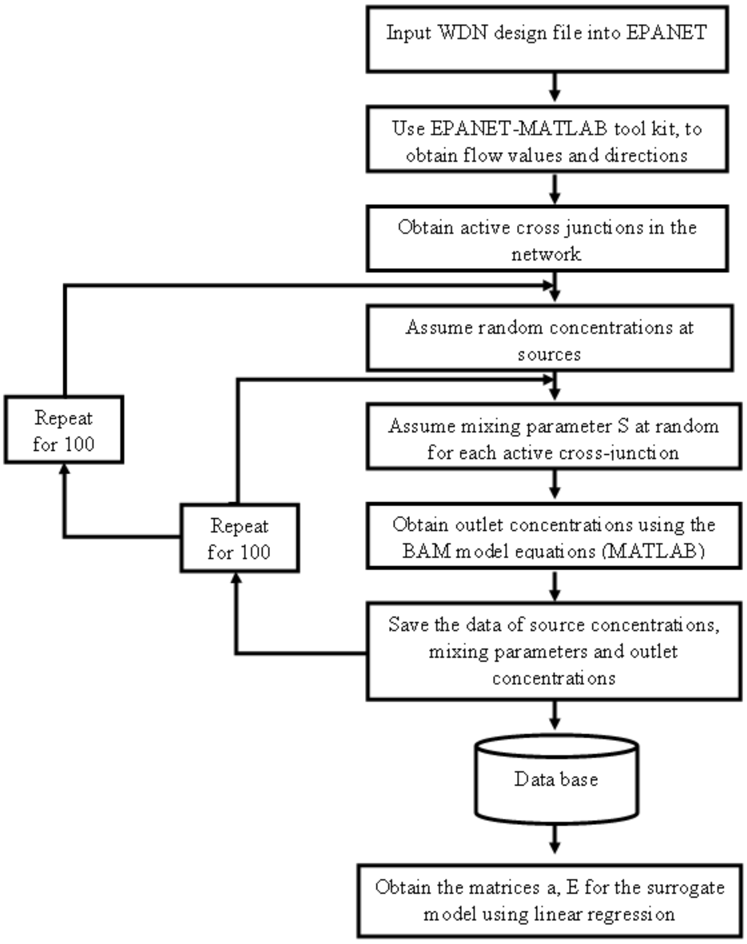


Figure 4 Procedure for obtaining surrogate model

The surrogate model's accuracy depends on the flow directions, and the node location, the maximum RMSE obtained for the cases studied is 2.876, with a minimum being R^2 = 0.989. This shows that the linear approximation is decently accurate. Monte Carlo simulations are used to evaluate the reliability of the obtained design.

After obtaining the surrogate models for each scenario, the nominal or deterministic problem is evaluated assuming complete mixing (i.e., s=e) for all cross junctions. The obtained treatment costs are assumed to be base treatment costs, and the mixing parameter set (s=e) as the nominal mixing parameter value. Using this nominal treatment cost, the maximum budget which can be provided is approximated by linearly increasing the nominal budget (10 times). For each of these approximated budgets, the info-gap problem is solved to obtain the least cost and the corresponding maximum robustness (δ). For solving this problem, a cuckoo search optimization algorithm is used. The initial nests (search agents) are taken to be 25, and the Lévy Flight parameters $\alpha_{\text{levy}} = 0.01$, $P_a = 0.25$ and $\beta_{\text{levy}} = 2/3$ as suggested in (Yang and Deb 2009) are used. The algorithm is run for 200 iterations or 25*2*200 = 10000 function evaluations. For all the cases, the problem converged at (100 ± 10) iterations. The uncertainty set obtained from this (δ) is reported. For each scenario, ten different linearly increasing budgets (Green) are solved, and their corresponding maximum treatment cost (Blue) and robust uncertainty set (X-axis) are reported in the graphs (Figure 5).

4 CONCLUSIONS

The present work is to understand the effect of non-uniform mixing as a water quality uncertain parameter. A simple treatment cost optimization problem is devised to understand the optimal design of the non-uniform mixing in a water distribution system. Non-uniform mixing uncertainty cannot assume any probability distribution. Handling this uncertainty requires a non-probabilistic approach. IGDT methodology is explored in this study. The complete methodology is explained with a simple grid network with three water sources. The complexity of the non-uniform mixing is clearly explained.

With varying flow patterns, the outlet concentration patterns have changed. The worst-case cannot be assumed as it changes with a change in flow directions and values. Considering incomplete mixing at all junctions did not lead to worst-case (i.e., maximum constraint violations). The change in flow patterns changes the locations of cross junctions, changing the non-uniform mixing junctions. A new set of surrogate models are required for a change in each flow direction and flow value. A convex surrogate model which includes flow values and direction with reasonable accuracy is difficult to achieve.

The treatment cost increases with an increase in the uncertainty set is observed from the results. Changes in flow direction changed the optimal treatment levels. The Monte Carlo simulation results show a small percentage of infeasibility is due to the surrogate model's approximation. This IGDT approach can be easily applied to any network and can solve this optimal treatment problem. Further study is to combine the water distribution network design problem with this treatment cost design problem. The emphasis was more on the theory, and less on using real data. This is why unitless cost data were used, and conservative water quality constituents. Applications to non-conservative water quality parameters, real cost data, and more complex systems are suggested as future work, as well as efficient extension to extended period simulations (EPS) loading conditions.

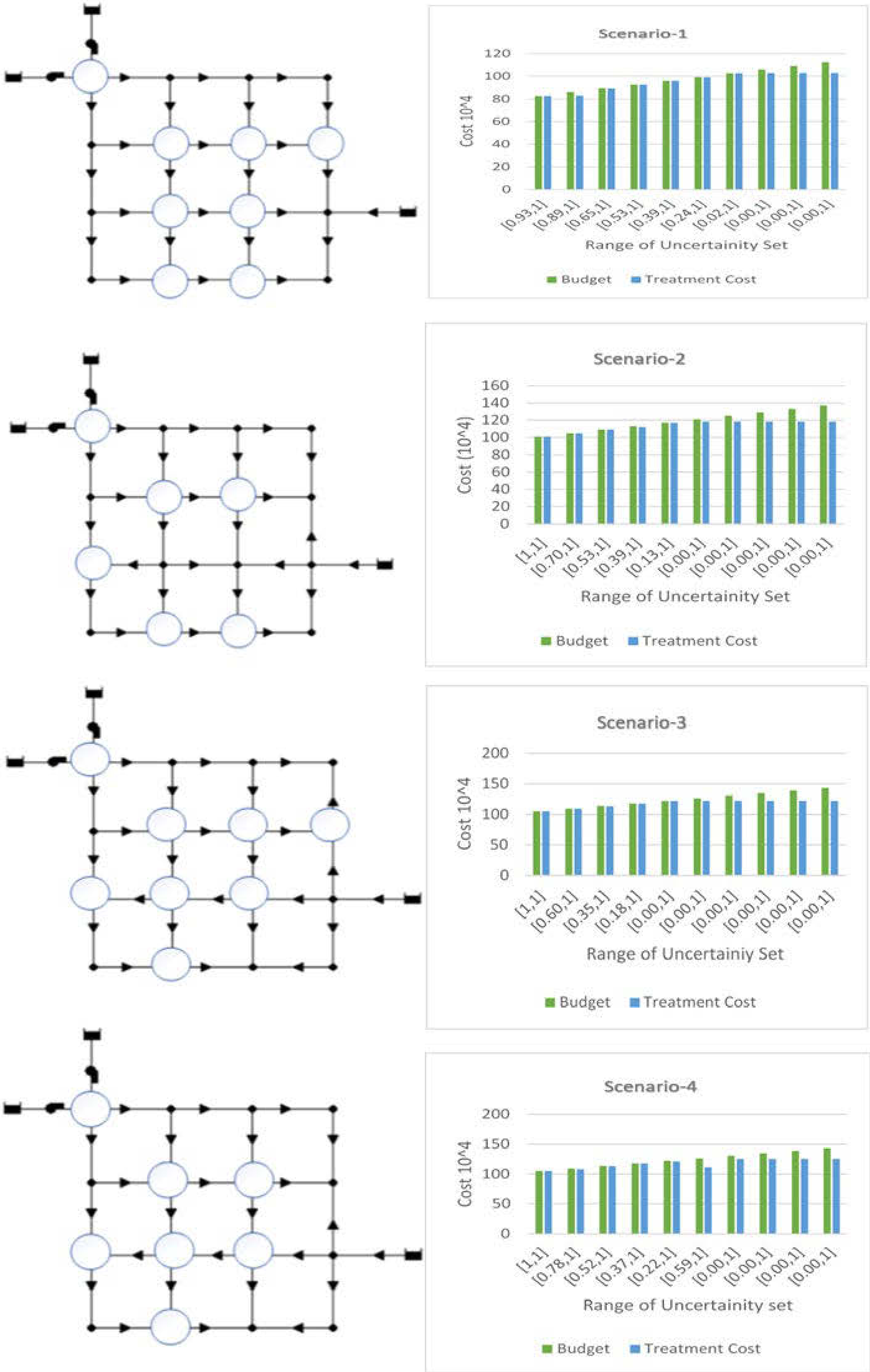


Figure 5 Results show the variation of uncertainty sets with an increase in budget values for four different flow directions, as shown in the network diagrams.

5 ACKNOWLEDGEMENT

This research was supported by the Israel Science Foundation (Grant No. 555/18).

6 REFERENCES

[1] Babayan, A., Kapelan, Z., Savic, D., and Walters, G. (2005). "Least-Cost Design of Water Distribution Networks." (October), 375–382.

[2] Babayan, A. V, Kapelan, Z. S., Savic, D. A., and Walters, G. A. (2004). "Comparison of two approaches for the least cost design of water distribution systems under uncertain demands." Critical Transitions in Water and Environmental Resources Management, 1–10.

[3] Ben-Haim, Y. (2006). Info-gap decision theory: decisions under severe uncertainty. Elsevier.

[4] Chung, G., Lansey, K., and Bayraksan, G. (2009). "Reliable water supply system design under uncertainty." Environmental Modelling and Software, Elsevier Ltd, 24(4), 449–462.

[5] Fowler, A. G., and Jones, P. (1991). "Simulation of water quality in water distribution systems." Proc., Water Quality Modeling in Distribution Systems, AWWA/EPA, Cincinnati

[6] Ghelichi, Z., Tajik, J., and Pishvaee, M. S. (2018). "A novel robust optimization approach for an integrated municipal water distribution system design under uncertainty: A case study of Mashhad." Computers and Chemical Engineering, Elsevier B.V., 110, 13–34.

[7] Ho, C. K., and Khalsa, S. S. (2008). "EPANET-BAM: water quality modeling with incomplete mixing in pipe junctions." Water Distribution Systems Analysis 2008, 1–11.

[8] Ho, C. K., Orear Jr, L., Wright, J. L., and McKenna, S. A. (2008). "Contaminant mixing at pipe joints: Comparison between laboratory flow experiments and computational fluid dynamics models." Water Distribution Systems Analysis Symposium 2006, 1–18.

[9] Kang, D. S., Pasha, M. F. K., and Lansey, K. (2009). "Approximate methods for uncertainty analysis of water distribution systems." Urban Water Journal, 6(3), 233–249.

[10] Kapelan, Z. S., Savic, D. A., and Walters, G. A. (2005). "Multiobjective design of water distribution systems under uncertainty." Water Resources Research, 41(11), 1–15.

[11] Korteling, B., Dessai, S., and Kapelan, Z. (2013). "Using Information-Gap Decision Theory for Water Resources Planning Under Severe Uncertainty." Water Resources Management, Kluwer Academic Publishers, 27(4), 1149–1172.

[12] Lansey, K. E., Duan, N., Mays, L. W., and Tung, Y. (1989). "Water Distribution System Design Under Uncertainties." Journal of Water Resources Planning and Management, 115(5), 630–645.

[13] Naderi, M. J., and Pishvaee, M. S. (2017). "Robust bi-objective macroscopic municipal water supply network redesign and rehabilitation." Water Resources Management, 31(9), 2689–2711.

[14] Naveen Naidu, M., Boindala, P. S., Vasan, A., and Varma, M. R. R. (2020). Optimization of Water Distribution Networks Using Cuckoo Search Algorithm. Advances in Intelligent Systems and Computing, Springer Singapore.

[15] Paez, N., Saldarriaga, J., and Bohorquez, J. (2017). "Water Quality Modeling Considering Incomplete Mixing in Extended Periods." Procedia Engineering, The Author(s), 186(571), 54–60.

[16] Pankaj, B. S., Naidu, M. N., Vasan, A., and Varma, M. R. (2020). "Self-Adaptive Cuckoo Search Algorithm for Optimal Design of Water Distribution Systems." Water Resources Management, Water Resources Management, 34(10), 3129–3146.

[17] Pasha, M. F. K., and Lansey, K. (2005). "Analysis of uncertainty on water distribution hydraulics and water quality." World Water Congress 2005: Impacts of Global Climate Change - Proceedings of the 2005 World Water and Environmental Resources Congress, 10.

[18] Pasha, M. F. K., and Lansey, K. (2010). "Effect of parameter uncertainty on water quality predictions in distribution systems-case study." Journal of Hydroinformatics, 12(1), 1–21.

[19] Perelman, L., Housh, M., and Ostfeld, A. (2013). "Robust optimization for water distribution systems least cost design." Water Resources Research, 49(10), 6795–6809.

[20] Roach, T., Kapelan, Z., and Ledbetter, R. (2015). “Comparison of info-gap and robust optimization methods for integrated water resource management under severe uncertainty.” Procedia Engineering, Elsevier B.V., 119(1), 874–883.

[21] Romero-Gomez, P., Choi, C. Y., Lansey, K. E., Preis, A., and Ostfeld, A. (2009). “Sensor network design with improved water quality models at cross junctions.” Proceedings of the 10th Annual Water Distribution Systems Analysis Conference, WDSA 2008, (2004), 1056–1066.

[22] Song, I., Romero-Gomez, P., and Choi, C. Y. (2009). “Experimental Verification of Incomplete Solute Mixing in a Pressurized Pipe Network with Multiple Cross Junctions.” Journal of Hydraulic Engineering, 135(11), 1005–1011.

[23] Wang, Q., Liu, S., Wang, H., and Savić, D. A. (2012). “Multi-objective cuckoo search for the optimal design of Water Distribution Systems.” Civil Engineering and Urban Planning 2012 - Proceedings of the 2012 International Conference on Civil Engineering and Urban Planning, 2012(Ceup), 402–405.

[24] Yang, X. S., and Deb, S. (2009). “Cuckoo search via Lévy flights.” 2009 World Congress on Nature and Biologically Inspired Computing, NABIC 2009 - Proceedings, IEEE, 210–214.