

The Pentagon as the Constructed Form of the City

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Abstract

The effectiveness of fortresses, until the fifteenth century, was tied to their height, as the most effective defensive actions involved the pouring of boiling objects and liquids from above. The subsequent development of heavy portable artillery systems questioned the traditional form of fortification, characterized by walls perpendicular to the ground. This recognition, found in the writings of Leon Battista Alberti in "*Re Aedificatoria*," emphasized that to increase effectiveness, defenses should be constructed along irregular lines, like the teeth of a saw. With Antonio and Giuliano da Sangallo, military architecture became a branch of geometry, and debates regarding the correct number of bastions to provide the best defensibility sparked numerous discussions in the transition between the fourteenth and fifteenth centuries. The radiocentric form, typical of the Renaissance city, was one of the reasons that led to the choice of the pentagon as a design solution for fortifications, as obtuse angles, unlike right angles, allowed for greater resistance to splaying.

This very form became the generative element of the urban development of the city of Livorno, coinciding with the urban plan drafted by Bernardo Buontalenti. In the mid-1500s, Francesco I de' Medici decided to expand the town of Livorno, incorporating a new inhabited center within a system of powerful fortifications surrounded by a canal, giving the city a pentagonal shape, with the hypothetical center being the Cathedral. Thus, the pentagon, part of Buontalenti's design, now visible in the course of the canals surrounding the core in the water, built by Venetian craftsmen, became the city's form.

Keywords: Mediterranean, Buontalenti's pentagon, Livorno, 1500.

1. Introduction

Until the 1400s, the effectiveness of fortresses was primarily linked to their height: the higher a defensive system, the more difficult it became to physically overcome it. Additionally, the most effective defensive actions involved the pouring of hot objects and liquids from above. The 15th century, with its portable artillery, was a period of great upheaval that questioned the traditional fortified systems consisting of walls perpendicular to the terrain, whose height exposed them further to innovative defense techniques. Some of the names of the most

famous architects in architectural design, pondered at length about the fundamental geometric issues in designing fortified structures. While Leon Battista Alberti had hypothesized an increase in defense effectiveness through the use of irregular lines, like "the teeth of a saw," the da Sangallo brothers transformed military architecture into pure geometry, articulating the debate on what was, mathematically and geometrically, the most appropriate number of bastions to include in order to enhance the effectiveness of a defensive structure.

1.1. The Design of the Five Corners

The research focused on urban issues, aiming to ensure the functional continuity of the Renaissance city conceived as radiocentric. These were therefore the main reasons that led to the choice of the five-sided shape, whose obtuse angles also provided better resistance to the chamfering of the wall corners. However, the geometric-constructive difficulties of the pentagon remained evident, despite the elucidations provided by Euclid. We had to wait for Ptolemy, about 500 years later, to discover that the side of the pentagon is the golden section of its diagonal, in order to simplify its reproduction and speak of a regular polygon. Furthermore, Luca Pacioli (1), Leonardo Da Vinci, Albrecht Dürer (2), Fibonacci, using Arabic literature that had provided the most important foundations for the study of the geometric world, allowed, with their discoveries and writings, to achieve increasing precision and simplification in the representation of regular polygons created through golden ratios. Through the Fibonacci sequence, we realize that it is possible to construct it starting from the golden triangle and its gnomon, as Luca Pacioli had already discovered in his "De Divina Proportione." Considering the numbers of the Fibonacci sequence for determining the relationship between the sides, for example, 5 and 8, we would have 5 units as the length of the base and 8 units as the length of the equal sides. In the second isosceles triangle, which is the gnomon of the first one, the base will, on the contrary, be 8 units long and the equal sides 5 units long, and the half of this isosceles triangle is the Pythagorean triangle (3,4,5). This characteristic of golden triangles allows us to construct a pentagon using only a straightedge and compass, resulting in an extremely simplified geometric construction.

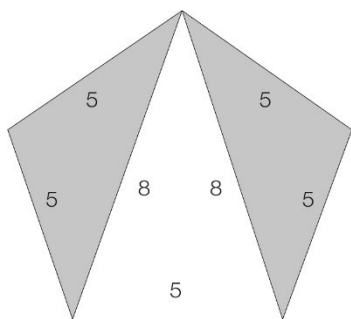


Fig. 1 - The construction of the pentagon using golden triangles.

2. Angles as "Teeth of a Saw"

In the fifteenth century, following the development of artillery, a new type of military architecture emerged, known as "Italian fortification" (3), which led to the development of new urban and architectural planning systems, as well as the advancement of new construction techniques.

As in many of the existing fortifications that were remodeled during that time, there was often the presence of a bulky pre-existing structure located at the center, which necessitated the construction of the city walls, now almost exclusively of a pentagonal shape, using the system of three golden triangles, in order to ensure the centrality of the existing structure.

We could cite, for example, the layout design of the new pentagonal walls equipped with bastions that were built between 1559 and 1565 around Castel Sant'Angelo in Rome. The project was entrusted to Francesco Laparelli (4) from Cortona, who had always been dedicated to the study of mathematics, architecture, design, and weaponry.

Or even, though not geometrically regular, the Fortezza da Basso in Florence, designed by Francesco Fiorenzuoli (5) and Antonio da Sangallo the Younger, between 1534 and 1537.

Italian fortification thus found its form in the polygonal system with obtuse angles, reinforced by the typical spade-shaped bastions, aligning with the idea advocated by Leon Battista Alberti in his writings collected in "De re aedificatoria". According to Alberti, the constructed form of military architecture should be jagged in order to become difficult to penetrate and make enemy passage as complicated as possible.

Therefore, the five sides, created with increasingly geometrically precise tracing systems, allowed for the breaking of the straightness of military constructions. At the same time, the pentagonal layout allowed for greater structural strength of the walls, making it the best form for the design of urban protection systems.

3. Buontalenti's Medici Port

The construction of the pentagonal urban system in the city of Livorno (Frattarelli, Fisher, 2011) is a clear example of how military architecture was influenced by the study and development of

geometry and advanced design systems during the Italian Renaissance.

In 1577, Bernardo Buontalenti was commissioned by Francesco I de' Medici to design the new urban layout of the city of Livorno, taking into account the relevant and important fortifications, as it was the main port of the Grand Duchy of Tuscany. Already in the time of Cosimo I, due to the decline of the Pisan port, which was almost entirely silted up, Livorno was seen as a major maritime hub under the jurisdiction of Florence. However, the idea fully materialized with his son Francesco I, who commissioned the Court Architect Bernardo Buontalenti in 1576 to design a grand city.

The port of Livorno, also known as the Medici Port, was designed and built on the initiative of the Medici family during the 16th century. This work was an integral part of the broader development project of the city of Livorno as an important port and commercial center for the Grand Duchy of Tuscany, capable of becoming a hub for the deposit and exchange of goods between the Muslim Mediterranean and the ports of Northern Europe.

The port was designed to accommodate ships of various sizes, allowing for the flourishing trade that facilitated the development of commercial and maritime activities in the region. Thanks to its strategic location on the Tyrrhenian Sea, the port of Livorno became an important stopover point for trade, giving rise to a bustling movement of merchants from all over the continent who settled in Livorno, creating a cosmopolitan city by the early 17th century.

This was the place where the urban and defensive design of what was becoming the commercial hub of the Grand Duchy took shape. The port of Livorno represents a remarkable example of how the Medici family invested in infrastructural and architectural projects to promote the economic development of the territory. Bernardo Buontalenti took charge of fortifying the newly-built port, designing a defense system that surrounded the port area, ensuring a high level of security. The fortifications included walls, bastions, and moats that provided an advanced defensive structure for the time.

Once again, the choice of the pentagonal military architecture proved effective, capable of encompassing the heart of the city within it, and at the same time, effectively responding to the

defense needs and technological advancements of the era.

4. The Shape of the City

Livorno became the epitome of pentagonal fortification design. The angles of the pentagon were capable of offering an optimal panoramic view, providing defenders with unobstructed lines of sight over the entire landscape (Fig.2)

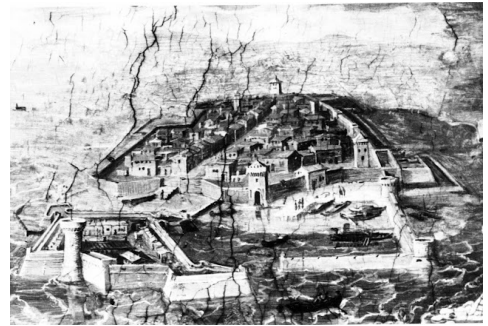


Fig. 2 - Livorno in a Giorgio Vasari's painting, 1556.

The pentagonal shape, slightly flattened and not perfectly regular, stemmed from the need to incorporate the pre-existing residential nucleus into the new defense system. The non-geometrically regular pentagon had one of its bastions extending into the fortification at the southeast vertex, in the new fortress that, along with the older fortress, protected the city's settlement. Therefore, the settlement was at the center, and man was at the center of the pentagon.

Livorno was one of the four Renaissance "Ideal cities" (6) of new foundation and, besides being the largest, it is the only one that succeeded, not remaining just a noteworthy experiment but becoming for centuries one of the most important cities in the Mediterranean, as envisioned by the foresighted vision of the Medici family.

The Renaissance ideal city was, in fact, the embodiment in stone and brick of the Renaissance philosophy that placed man at the center of creation. Thus, the city, enclosed in a pentagon naturally inscribed with the human body, with regular and straight streets as in ancient Roman cities, surrounded by a moat filled with seawater, sealed the marriage between the Medici state and the Mediterranean.

However, the pentagonal shape required reinforcement at each vertex.

Within the pentagon, each vertex was fortified with the presence of a bastion, capable of providing both tactical vantage points for the defense of the port, and defensive platforms where artillery could be stationed. This defensive system thus functioned like a network of concentric defenses, capable of protecting the urban center. Buontalenti ingeniously exploited these geometric features to enhance the defensive capabilities of the Livorno fortress, making it a city that was difficult to conquer. The system of water channels that became the distinctive feature of the Medici fortification, giving a completely new design and appearance to the city's urban plan.

The pentagonal shape of the city influenced the entire urban layout, determining the layout of roads and squares and the layout of the citizens who still inhabit this city (Fig. 3)

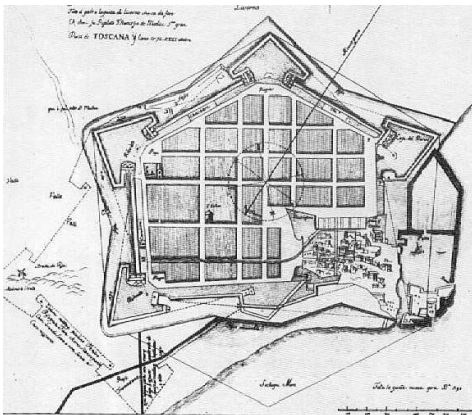


Fig. 3 – The pentagon project in a copy of Buontalenti's project 1575.

4.1 The New Waterfront Pentagon

Before long, the population, drawn by a series of benefits and privileges promoted by Grand Duke Ferdinando I and the reform of the customs regulations that made Livorno a duty-free port, began to grow rapidly, and the residential area quickly proved insufficient to accommodate everyone. In less than 40 years, it became necessary to provide the city with a new neighborhood that could meet the housing needs of a rapidly rising merchant class (Di Giacomo, 2004) whose businesses were in direct communication with the port. It was around 1630

that Grand Duke Cosimo III decided to raise a new neighborhood, entrusting the design to the Sienese architect Giovanni Battista Santi. The chosen area was located between the Navicelli Canal (7) and the outer moat of the Buontalenti fortifications.

Construction work on the new city on the water began, with warehouses and residences located right behind the port, capable of accommodating the interests of the many merchants who transferred the management of their commercial activities to Livorno. The challenge of building this new neighborhood in an area partially occupied by the sea made it necessary to employ techniques and labor imported directly from the city built on water in the Venetian lagoon.

Once again, the pentagonal shape became the form of the city, this time not protected by additional bastions in the shape of spades arranged at the corners, but built like an island in the middle of the sea. This choice remained in the toponymy of the places, which eventually led to the neighborhood being identified with the name "Venezia Nuova".

However, the new neighborhood still needed more space, so at the end of the 17th century, more than half of the New Fortress (8) was demolished to free up space that could be used for large warehouses located below street level. These warehouses were reached through the canals that separated the new neighborhood from the city, called "Fossi" in the local dialect, where large barges (the "navicelli") loaded with goods arrived from the nearby port. The pentagon, surrounded by an additional defensive system, thus became the shape of the new city, urbanistically determining the posthumous development that has characterized the Livorno area up to the present day.

In the 18th century, the New Venice became the quintessential mercantile district (Viganoni 2007). Consuls and important international merchants chose it as their residence. The spacious cellars of their palaces were overflowing with goods of all kinds and origins, waiting to be packaged and shipped by sea to the most diverse destinations.

Livorno experienced its period of greatest splendor when Cosimo III allowed the establishment of some Religious Orders in the city: the Trinitarians and the Dominicans, who, with the help of their respective mother houses

and wealthy patrons, built beautiful churches adorned with furnishings, statues, and Baroque-style stuccoes, such as the Church of San Ferdinando Re and the octagonal-planned one of Santa Caterina da Siena, known as the Dominicans. In the 19th century, with the advent of steam navigation and the development of the bourgeois city beyond the surrounding moat, the neighborhood inevitably transformed. The wealthy merchants abandoned Venice and were replaced by families of dockworkers and porters who gave the neighborhood a more picturesque and popular character. Deteriorating sanitary conditions led to a series of cholera epidemics that, in 1889, led to the decision to bury the ancient Navicelli Canal, the source of disease contagion, and transform it into what became Viale Caprera (9). World War II heavily impacted the Livorno port, and only at the end of the 20th century (Marini, 2021) did some recovery and enhancement operations begin in the waterfront neighborhood through the restoration of ancient palaces and, above all, with the project, not yet completed, that envisaged the reopening of the initial section of the Navicelli Canal (Fig.4).



Fig. 4 – The port of Livorno, “Lo Stato presente di tutti i paesi e popoli del mondo” published in Venice at Albrizzi. The engraver is F.Zucchi (1692-1764).

5. Conclusions

Bernardo Buontalenti's pentagonal masterpiece in the district of Venezia in Livorno, along with other historical pentagonal cities, emphatically affirm that the pentagon is the quintessential built form for fortified cities. Its geometric mastery, strategic brilliance, and proven legacy underscore its unparalleled status in the annals of defensive architecture. While Buontalenti's legacy endures and the echoes of history resonate through pentagonal fortresses, it is undeniable that the

pentagon reigns supreme as the perfect design for fortified cities, to the extent that it remained in the subsequent designs for the planning of the new Livornese neighborhood, the offspring of the port that Bernardo Buontalenti had protected with his project and whose geometric layout it emulates (Fig.5).



Fig. 5 - Aerial view of the city of Livorno 2023, google earth.

Notes

(1) Fra Luca Bartolomeo de Pacioli, also known as Paciolo (1445-1517), was an Italian friar, mathematician, and economist, known for his works *Summa de Arithmetica, Geometria, Proportioni et Proportionalita* and *Divina Proportione*. He is recognized as the founder of accounting.

(2) Albrecht Dürer (1471-1528) was a German painter, engraver, mathematician, and treatise writer.

(3) Modern fortification (or Italian-style fortification) is a type of fortification developed from the 15th century in Italy to address the issue posed by the development of artillery, highlighted by the Italian Wars and conflicts against the Ottomans.

(4) Francesco Laparelli (1521-1570) was an Italian architect. Assistant to Michelangelo and one of the most important architects of the 16th century in the field of military works, he served as an engineer for Pope Pius IV and Cosimo I de' Medici.

(5) Pier Francesco Fiorenzuoli da Viterbo (1470-1537) is a lesser-known figure who, nevertheless, assumed a significant role in the Italian panorama of the early 16th century. The Viterbo-born architect carried out intense design and consulting activities in a crucial phase for the evolution of fortification techniques.

(6) The expression "Ideal City" refers to the texts and drawings by Leonardo da Vinci on urban planning found in the so-called Manuscript B of the Institut de France in Paris. They aim to rationalize the structure and improve the efficiency of the city after the plague of 1484-1485. Significant importance is given to the canalization of watercourses to make them suitable for transporting goods or to the construction of stables with measures to ensure greater hygiene.

(7) The Canale dei Navicelli is a canal built between 1563 and 1575 that connects Pisa with the port of Livorno. It is named after the so-called "navicelli," characteristic small Tuscan boats used for transporting goods from the Pisan plain,

Lake Bientina, and the Empoli area, as the Arno was navigable up to Porta di Mezzo at the time.

(8) The Fortezza Nuova is a fortification in Livorno. It represents the ancient "Baluardo di San Francesco" of the pentagonal city designed by Bernardo Buontalenti. It was part of the defensive system located to the North-East of the city, along with the symmetrical Forte San Pietro d'Alcantara and the Rivellino di San Marco, located between the two fortifications, where the ancient Porta San Marco (Piazza dei Domenicani) was situated.

(9) The section between the Church of San Ferdinando and the Church of Luogo Pio (Livorno).

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