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Research Paper

Comparison and selection of experimental designs for the characterization of scroll and reciprocating compressors: Adjustment of polynomial models with compact experimental samples

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ABSTRACT

The use of refrigeration equipment and heat pumps is a widely used solution by manufacturers in the industry. Selecting components for this equipment and simulating their behavior require a thorough understanding and effective characterization of the compressor. In this sense, the standard AHRI-540 is the most used one to characterize compressor performance ($\dot{W}_c$ and $\dot{m}_{ref}$) considering polynomial models of 10 coefficients. Unfortunately, this standard does not offer information about where to perform the required experimental measurements for the model adjustment. Drawing on these precedents, this paper investigates various Design of Experiments methodologies to identify the most efficient approaches for characterizing both scroll and reciprocating compressors with minimal experimental points. The study aims to develop a straightforward methodology for optimal point selection, ensuring precise compressor characterization while minimizing experimental costs. For this purpose, two datasets from two Copeland scroll compressors – ZP21K5E-PFV and ZS21KAE-PFV – with massive test campaigns have been employed. Both compressors were subjected to a substantial number of experimental points (1097 and 866) across different suction conditions ($SH = 11$ K, $SH = 22$ K, $T_{suc} = 18$ °C) and different refrigerants (R134a, R32, R410A, R404a, …). In a first step, this study presents the design of experimental matrices for characterizing efficiently the energy consumption and mass flow rate in scroll compressors. Additionally, the energy consumption is also characterized from the specific energy consumption ($\dot{W}_{esp}$), i.e., the energy consumption divided by the mass flow rate. Considering that the specific energy consumption obtains equal dependencies between scroll and reciprocating compressors, the matrices performed for the specific energy consumption are also suitable for the characterization of reciprocating compressors, also allowing to obtain compact experimental matrices with a low number of points.

1. Introduction

Currently, modeling and simulation of units and systems are extensively used in research and industry. It allows, among many other applications, to reproduce the behavior of prototypes and industrial equipment with the greatest accuracy helping to optimize the process that they entail.

Focusing on the refrigeration and the building conditioning field, the modeling of compressors has been thoroughly analyzed in different studies due to its importance in the evaluation of heat pumps and refrigeration equipment performance. For example, several theoretical and semi-empirical models have been reported in Winandy and Lebrun [1], Shao et al. [2], Navarro-Peris et al. [3] or Aute et al. [4] among others. Additionally, the articles of Byrne et al. [5] and Hermes et al. [6] show an exhaustive review of compressors models published in the last years.

Although a succession of theoretical and semi-empirical models have been suggested in the last years, currently empirical models are the most extended approach. In fact, the current standard that manufacturers shall adopt in order to characterize their compressors (AHRI-540) includes a uniquely empirical model: a 10-term and third-degree polynomial to characterize the electrical consumption ($\dot{W}_c$) and the mass flow rate ($\dot{m}_{ref}$) based on the evaporation temperature (T_e) and the condensation temperature (T_c):

$$\dot{W}_c = a_0 + a_1 T_e + a_2 T_c + a_3 T_e T_c + a_4 T_e^2 + a_5 T_c^2 + a_6 T_e^3 + a_7 T_e^2 T_c + a_8 T_e T_c^2 + a_9 T_c^3 \quad (1)$$

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$$\dot{m}_{ref} = b_0 + b_1 T_e + b_2 T_c + b_3 T_e T_c + b_4 T_e^2 + b_5 T_c^2 + b_6 T_e^3 + b_7 T_e^2 T_c + b_8 T_e T_c^2 + b_9 T_c^3 \quad (2)$$

As it is shown in Cheung and Wang [7], if empirical models are compared with semi-empirical ones, they show a better adjustment and predictive power when the first are adjusted with many experimental points and they cover completely the working range of the compressor. Because of this, we can understand why the standard AHRI-540 specifies the use of empirical models for the characterization of compressors.

Regarding empirical models, recently, in Marchante-Avellaneda et al. [8,9], a comprehensive analysis was performed on the most suitable empirical models to characterize fixed-speed scroll and reciprocating compressors. In Marchante-Avellaneda et al. [8] was concluded that, for scroll compressors, the suggested model on the standard AHRI includes an excessive number of coefficients, and a second-order polynomial is more appropriate to characterize the energy consumption and mass flow rate in this type of compressors with more extrapolation capabilities. On the other hand, in Marchante-Avellaneda et al. [9], it was observed that for reciprocating compressors, the characterization of the mass flow rate is similar to scroll compressors, but it is not possible to fix a general polynomial equation for the characterization of the energy consumption. However, this study also reported that if the specific energy consumption is characterized rather than the energy consumption, it is possible to use a simple polynomial model with a lower number of terms. Another interesting result reported was that scroll and reciprocating compressors obtain similar response surfaces – 3D representation of response variables as a function of independent variables – for the specific energy consumption. Therefore, the same polynomial model allows us to characterize both technologies.

However, the use of empirical models brings up two important questions: "How many experimental points do we need for the modeling adjustment?" "Where should we locate them on the working domain of the compressor?"

These questions are critical because poor planning of the experimental matrices to test, which information will be needed for the model adjustment, will lead to important prediction errors [see 10].

Despite that large amount of studies address the empiric modeling of compressors, just few papers focus on studying and determining an appropriate methodology that allows specify where and how many experimental points are needed for the adjustment, see e.g. Aute et al. [11], Aute and Martin [12] and Cheung and Wang [7]. Moreover, the standard AHRI-540 raises a concern as it fails to provide any indications on addressing these questions, creating a gap in guidance for users. Undoubtedly, properly addressing these types of issues would benefit from obtaining more robust models since the experimental information used in their fitting would be more representative, and the reduction in sample size would also allow a significant reduction in costs at the experimental stage.

One of the reasons why these issues are not normally broached is the need of featuring a database that includes an exhaustive characterization with a great number of points for different compressors. From this point of view, some years ago, the project "Low-GWP Alternative Refrigerants Evaluation Program", sponsored by the AHRI Institute, obtained a massive experimental campaign where different compressors were evaluated working with different refrigerants and suction conditions.

Against this background, this paper pioneers the assessment of various Design of Experiments (DoE) methodologies to identify the most efficient approach for defining an experimental sample with minimal points in both scroll and reciprocating compressors. The key focus lies in the strategic selection and placement of these points within the experimental domain to achieve a precise characterization of both compressor types. The primary objective is to establish a straightforward methodology for optimal point selection, aiming to accurately characterize the compressors while minimizing experimental costs. For this purpose, two scroll compressors have been selected with the densest test matrices (around 65 experimental points) from the abovementioned database. The different evaluated experimental designs include classical Design of Experiments methodologies and more sophisticated computer-aided methods. Finally, two types of empirical models have been considered to evaluate the experimental designs. The first one includes the polynomial models for the prediction of $\dot{W}_c$ and $\dot{m}_{ref}$ referenced in the characterization standard as they are commonly used, but including only the second-order terms according to the recommendations reported in Marchante-Avellaneda et al. [8]. From the results obtained with this first model, the most advantageous experimental designs are obtained in terms of accuracy and sample size for the characterization of scroll compressors. Moreover, this work will also evaluate the experimental designs by using the polynomial models reported by Marchante-Avellaneda et al. [9] to characterize the specific energy consumption ($\dot{W}_{esp}$) and mass flow rate ($\dot{m}_{ref}$). Therefore, using this second approach will extend the present study to reciprocating compressors by evaluating proper experimental designs to characterize the specific energy consumption and mass flow rate.

2. Methodology

The Design of Experiments (DoE) methodologies is a branch of statistics aimed to prepare and plan the experimental matrices and, along with the Response Surface Methodologies (RSM), they provide researchers with powerful tools when selecting experimental samples for empirical model adjustments. Therefore, the main objective of the DoE is to specify where to take the tests within the experimental domain to define an optimal ratio between the number of points to test and experimental information with statistical inference to make the model adjustment.

These experimental design methodologies can be divided into classical experimental designs and computer-aided designs. The first group defines experimental designs for mainly orthogonal domains, while the second one presents the advantage of defining designs for irregular domains. In Fig. 1-left, it can be seen an example of a non-orthogonal domain illustrated in Atkinson and Donev's book [see 13, chap. 12, pg. 180].

It describes the experimental design proposed to characterize the torque of an internal combustion engine based on the ignition advance of the spark plug. It can be seen that the shown experimental domain is similar to the working area of a compressor (Fig. 1-right). They are both characterized by having areas where it is not possible to work, creating a convex polygon in contrast with other types of processes without limitations and with orthogonal domains. In the case of compressors, they have two areas of no operation, one limited by the high discharge temperatures, where the integrity of the compressor would be compromised, and another area limited by a low-pressure ratio with a considerable loss of efficiency [14].

Taking into account the experimental domain of compressors, several experimental designs from the existing literature have been selected to determine a suitable methodology for selecting experimental samples in this field. In this sense, various classical experimental designs have been chosen for their simplicity and ease of use to assess their predictive power when applied to non-orthogonal domains. Additionally, some extra computer-aided experimental designs have been evaluated to address the challenges posed by irregular experimental domains.

In this research, these methodologies are applied to three datasets of compressor calorimetric tests with many points allowing to obtain experimental samples of different sizes. In a second step, these samples are evaluated according to the following procedure:

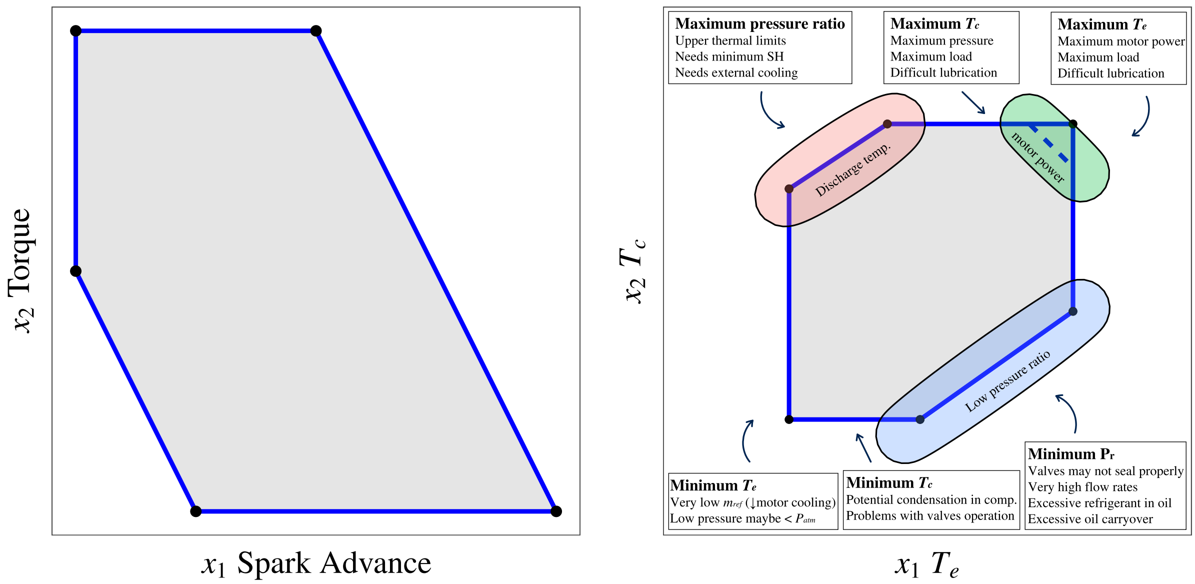

Fig. 1. Non-orthogonal experimental domains. Internal combustion engine (left-hand) and refrigeration compressor (right-hand).

- The experimental points included in the various samples are selected and used to fit different empirical models suitable for characterizing scroll and reciprocating compressors.
- Considering all available test points in the datasets, these models are also fitted to obtain reference models for comparison.

The comparison focused on the prediction errors obtained with the models fitted with the different samples concerning the prediction errors of the reference models. For the models fitted with samples the reported errors are the errors obtained when comparing the whole dataset. The errors used on the comparative have been the Maximum Relative Error (*MRE*), the Root Mean Square Error (*RMSE*) and the Coefficient of Variation of the RMSE (CV_{RMSE}). The latter is calculated as the ratio of the RMSE to the mean of the response variable characterized. Furthermore, the sign of the regression coefficients obtained from fitting the models with the corresponding samples have been analyzed, comparing them with those obtained from the reference models. Any change of sign in the coefficients should be considered as a reduction in the experimental information obtained by the samples since it does not correspond to the trend obtained in the coefficients of the reference models.

Finally, it is important to note the adjustment method used by the authors to obtain the regression coefficients in this work. Commonly, the most widespread method involves considering the Ordinary Least Squares (OLS) adjustment. However, when using experimental information, it is unavoidable that the experimental results obtained in the laboratory have a certain measurement error. So, the tested points on the operation limits of the compressor usually have a higher measurement error than central points. In order to further consider the experimental error of the data in this study, the authors have used the weighted regression method selecting the Inverse-Variance Weighting (IVW) for the adjustment instead of the classical adjustment by ordinary least square. It includes a weight vector with the same length as the experimental sample. This vector must be constructed as the inverse of the experimental variance, i.e., the inverse-square of the combined standard uncertainty [15].

The following two subsections provide further details on the experimental data used in this analysis, as well as the selected experimental design methodologies for defining the different samples and the empirical models used for their evaluation.

2.1. *Compressor performance data*

Some years ago, the AHRI Institute published a series of reports within the project "Low-GWP Alternative Refrigerants Evaluation Program", in which they evaluated the performance of scroll and reciprocating compressors. The experimental information includes a huge number of test points and compressors with common refrigerants and new mixtures, and different suction conditions.

From all the revised experimental information, the authors selected the reports AHRI 11 [16], AHRI 21 [17], and AHRI 33 [18] as the most appropriate for the present study because they include an exhaustive experimental characterization with a great number of tests. The compressors analyzed are two models of Copeland scroll compressor: ZP21K5E-PFV (AHRI 11 and 33) with R410A as the base refrigerant and ZP21KAE-PFV (AHRI 21) with R404A as the base refrigerant.

Table 1 summarizes the principal characteristics of every compressor, the base refrigerants and new evaluated mixtures, the tested suction conditions, and the total number of test points.

Finally, Table 2 shows the composition of the new mixtures tested on these compressors. The thermophysical properties of these mixtures have been obtained with the Refprop database [19], considering the evaporation and condensation temperatures on the dew point.

2.2. *Selected models and evaluated experimental designs*

The evaluated experimental designs include classical and computer-aided typologies. The objective will be to determine the most suitable methodology to obtain an accurate characterization with fewer experimental points. Based on the irregular experimental domain of compressors, the authors selected the following classical and computer-aided experimental designs alternatives for two factors.

1. Classical designs:
   - Full factorial test plan considering two independent variables and three levels (3^2).
   - Central Composite Design, CCD [21].
   - Small Composite Design, SCD [22].
   - Hexagonal Design HD [see 23, chap. 7, pg. 331].

Table 1
Calorimeter data (AHRI reports)

Report	Compressor model	Manufacturer	Displacement cm^3/rev	Refrigerants tested	Test cond.[a] °C	Test points[b]	Total tests
AHRI 11	ZP21K5E-PFV	Copeland	20.32	R410A/R32/DR5/L41a	a/b/c	196/166/189/186	737
AHRI 21	ZS21KAE-PFV	Copeland	50.96	R404A/ARM31a/D2Y65/L40/R32+R134a	c/a/b	191/186/183/173/133	866
AHRI 33	ZP21K5E-PFV	Copeland	20.32	R410A/R32+R134a	a/b/c	196/168	364

[a] Test conditions: **a.** SH = 11 K; **b.** SH = 22 K ; **c.** T_s = 18 °C;
[b] Total test points: **1967**;

Table 2
New refrigerants composition (Low-GWP HFC mixture)

Reported name	ASHRAE designation[abc]	Company	Refrigerant composition	(% mass)
DR5	≈ R454B	Chemours	R32/R1234yf	72.5/27.5
L41a	≈ R459A	Honeywell	R32/R1234yf/R1234ze(E)	73.0/15.0/12.0
ARM31a	–	Arkema	R32/R134a/R1234yf	28.0/21.0/51.0
D2Y65	R454A	Daikin	R32/R1234yf	35.0/65.0
L40	–	Honeywell	R32/R152a/R1234yf/R1234ze(E)	40.0/10.0/20.0/30.0
R32+R134a[d]	–	–	R32/R134a	50.0/50.0
R32+R134a[e]	–	–	R32/R134a	94.1/5.9

[a] Refrigerant designation according to ANSI/ASHRAE Standard 34 [20];
[b] – : Development mixture;
[c] ≈ : Development mixture with a similar composition to an ASHRAE-designated mixture;
[d] Mixture used in AHRI 21 report;
[e] Mixture used in AHRI 33 report;

2. Computer-aided designs:
   - Optimal Designs, OD [24,25].
   - Cluster Design, CD [11, section 8.1.1].
   - Polygonal Designs, PD [11, section 8.1.2].

The total number of points goes from 7 to 9 tests in the classical designs and from 6 to 12 in the computer-aided designs. The classical designs were built trying to cover the greatest possible experimental domain among the analyzed compressors and selecting available points on the experimental matrices included in the AHRI reports. Open-source statistical packages have been selected for the computer-aided designs to facilitate their applicability. Appendices A and B can be consulted for more details on the selected experimental designs.

Regarding the empirical models selected, the experimental designs will be evaluated by applying two types of polynomial models:

1. The models reported on the standard AHRI-540 for characterizing $\dot{W}_c$ and $\dot{m}_{ref}$, but removing the third order terms (Eqs. (3) and (4)). These high-order terms were removed based on the previous work of Marchante-Avellaneda et al. [8], in which it was stated that they are non-significant in scroll compressors characterization.
2. The models reported on Marchante-Avellaneda et al. [9] for the characterization of $\dot{W}_{esp}$ and $\dot{m}_{ref}$ (Eq. (5), Eqs. (6) and (7)). These models are also suitable for reciprocating compressors, and the results obtained can be extrapolated to this technology. The mass flow rate is characterized as a first-order polynomial model depending on the evaporation and condensation pressures. Then, the specific energy consumption is fitted to a simple linear model depending on a corrected pressure ratio (Eq. (7)). The coefficients z_c and z_e are obtained in the regression fit with the coefficients k_0 and k_1 and adjusted by nonlinear regression to the experimental data.

Polynomial models for $\dot{W}_c$ and $\dot{m}_{ref}$:

$$\dot{m}_{ref} = b_0 + b_1 T_e + b_2 T_c + b_3 T_e T_c + b_4 T_e^2 + b_5 T_c^2 \tag{3}$$

$$\dot{W}_c = a_0 + a_1 T_e + a_2 T_c + a_3 T_e T_c + a_4 T_e^2 + a_5 T_c^2 \tag{4}$$

Polynomial models for $\dot{W}_{esp}$ and $\dot{m}_{ref}$:

$$\dot{m}_{ref} = c_0 + c_1 P_e + c_2 P_c + c_3 P_e P_c \tag{5}$$

$$\dot{W}_{esp} = \frac{\dot{W}_c}{\dot{m}_{ref}} = k_0 + k_1 P'_r \tag{6}$$

$$P'_r = \frac{P_c - z_c}{P_e - z_e} \tag{7}$$

Finally, a brief summary of the following sections is presented in order to assist the reader in understanding how this work is structured.

Section 3 briefly describes how the samples generated with the selected experimental design methodologies were obtained. Subsequently, Section 4 includes the analysis of results by comparing the models selected to evaluate the experimental designs fitted with the different samples and comparing them with the fit of these models, considering all the points available in the datasets (reference models). In the first subsection (Section 4.1), a visual analysis of how the sample points are distributed within the experimental domain will be performed, evaluating if they are able to cover the whole envelope of the compressor. To conclude the analysis of results, the prediction errors between the reference models and the models fitted with the different samples will be compared, and the regression coefficients will be evaluated to see if they show the same trend (Section 4.2). From this analysis it will be possible to determine which methodology is most advantageous for the characterization of scroll and reciprocating compressors and which is the minimum sample size to ensure good accuracy with low experimental cost. Finally, Section 5 summarizes the most relevant findings of the present study.

3. Experimental sample selection

As the experimental data disclosed in the AHRI 11, 21, and 33 reports include many tests resulting in a very thin mesh of points over the entire working map, it has been possible to compare all the methodologies mentioned above.

In order to obtain the experimental samples with the classical experimental designs, it has been sought to draw the designs centering them on the compressor working map and trying to cover the largest

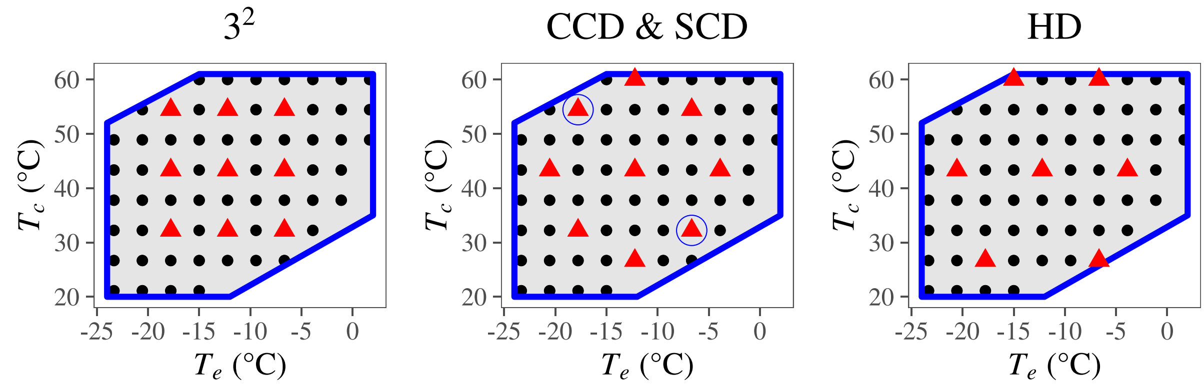

Fig. 2. Classical designs: 3^2, CCD, SCD y HD (AHRI 21 R404A and $SH = 11K$).

possible experimental area. No additional tools have been needed as designs were of common use and perfectly documented in the technical literature.

On the other hand, for the use of designs assisted by computer, the authors considered using a specific open source software with pre-programmed functions of experimental design. The objective is to facilitate the use of the methodologies described here.

Particularly, it has been used the statistical software R [26] together with the package **AlgDesign** [27].

For the experimental samples generated with the optimal designs methodology, it has been used the function *optFederov()*, which is an implementation of the Fedorov's algorithm [28]. This algorithm automatically obtains the optimal experimental designs needing:

1. A set of candidate points on coordinates of T_e and T_c.
2. A rescaling of said coordinates from −1 to 1.
3. To know the mathematical functional of the model to apply.
4. To specify the desired number of points for the sample.
5. To select the optimality criterion to apply and run the algorithm.
6. To rescale the coordinates of the design to the original range.

This study has considered three designs with 6, 9, and 12 points. All the experimental points available for each dataset were provided to the algorithm as candidate points, and the Optimal-D criterion has been selected. This criterion is the most widespread, allowing the sample to be defined to obtain greater precision in the calculation of the regression coefficients when fitting the model.

The experimental samples performed with the cluster design has been obtained with the *k-means* function of the base package **stats**. This methodology needs the following steps to get the designs:

1. To select a set of candidate points on coordinates of T_e and T_c.
2. A rescaling of said coordinates from −1 to 1.
3. To specify the number of clusters. This number will be the same as the number of tests desired for the design.
4. To execute the clustering algorithm.
5. To rescale the coordinates of the design to the original range.

The obtained designs include the centroid of every cluster. As these points are not included in the revised reports, it has been necessary to estimate their value by a smooth interpolation. A non-parametric model, the Thin-Plate-spline [29,30], was used for this purpose.

Finally, the experimental samples generated with the polygonal design have been obtained by manually selecting the vertexes of the compressor envelope (6 points) and applying the steps above described on the cluster designs to define the remainder points. The designs included in this study for the polygonal and cluster designs also include 6, 9, and 12 points.

Appendix E includes an easy example of how to obtain experimental designs assisted by computer applying the optimal designs and the cluster designs methodologies described in this study.

4. Analysis of results

As the extension of this article was limited, only the results using the candidate points as tests in $SH = 11K$ to generate the different experimental designs will be shown below. These designs were proposed for every refrigerant included in the AHRI 11, 21, and 33 reports. Similar results were obtained, taking the $SH = 22K$ and $T_s = 18$ °C tests as candidate points. As mentioned before, classical designs will include a total of 7 to 9 points. On the other hand, for the designs assisted by computer, three designs per methodology were considered, including a total of 6, 9, and 12 points. The objective is to evaluate the prediction power of the adjusted models with the different samples. Section 4.1 includes a visual analysis of how the different samples are distributed within the experimental domain. Finally, Section 4.2 will evaluate the predictive power of the selected models when fitting them with the experimental designs.

4.1. Distribution of the experimental points in the compressor envelope

Fig. 2 shows an example of how the points were distributed in the envelope for the AHRI 21 compressor considering its reference refrigerant (R404A) and $SH = 11K$ as suction conditions. In this figure, the black points include the set of given points in the AHRI report, and the red points are the selected ones to build the different samples. With the SCD methodology, the selected points are the same ones as in the CCD design, eliminating the two highlighted points on the diagonal.

The main limitation detected is that they are not able to adapt to irregular experimental domains. Hence, centering the designs on the experimental domain, we will be able to cover a larger or smaller working area depending on the limits of the compressor envelope. Fig. 3 shows a second example of how the working range of the compressor AHRI 11 is modified using R410A and R32 as refrigerants. We can observe that having higher discharge temperatures, the refrigerant R32 shows a lower range when having high T_c and low T_e values.

As can be seen in Fig. 3, depending on the compressor, the refrigerant, and the fixed suction conditions, these methodologies cannot be appropriate in the case of not covering a significant area of the working domain.

Analyzing the cluster designs in the second place, Fig. 4 shows an example of the selected points for a design of 6, 9, and 12 points using the data from the AHRI 11 report and the reference refrigerant (R410A).

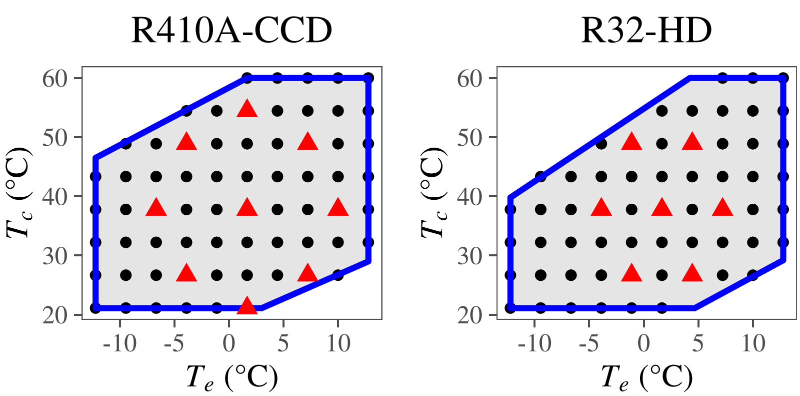

Fig. 3. AHRI 11 R410A-CCD vs R32-HD ($SH = 11K$).

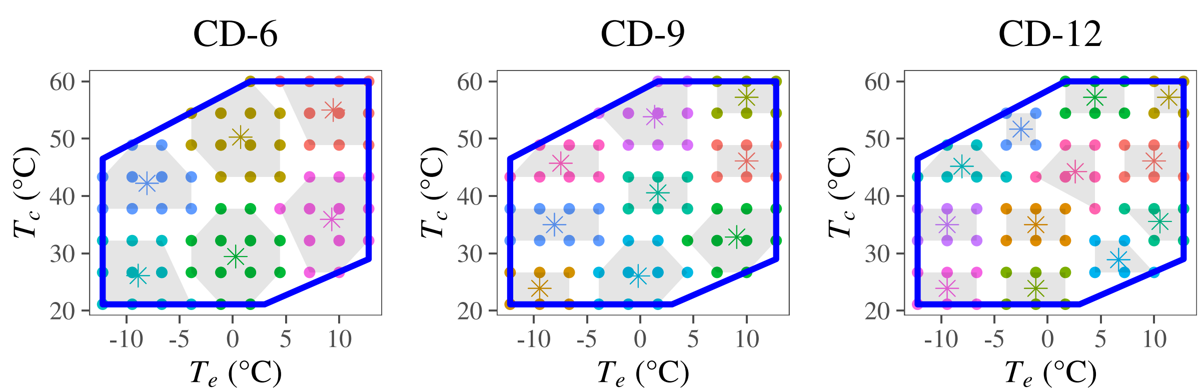

Fig. 4. Clustering 6, 9, and 12 points (AHRI 11 R410A and $SH = 11K$).

In Fig. 4, it can be seen how the clustering algorithm operates. Having a number n of points to include in the design, the algorithm classifies the candidate points in n clusters. The centroid obtained from every cluster will be the point to include in the experimental design. Therefore, this methodology obtains an equidistant distribution of points to test within the experimental design and presents the advantage of being able to adapt to an irregular experimental domain. It is possible to notice that, considering a low number of clusters when building the design, the points obtained show a high distance from the compressor's operation limits. However, this distance is minimized as we include a larger number of points and, in contrast with the classical designs, they always cover a larger area in the experimental domain.

Additionally, Fig. 5 presents the experimental designs generated with the optimal approach methodology and Fedorov's algorithm for an Optimal-D criterion. These designs have been obtained considering the second-order polynomials for the prediction of $\dot{W}_c$ and $\dot{m}_{ref}$. They are unsuitable for the selected $\dot{W}_{esp}$ and $\dot{m}_{ref}$ models because they can only be applied in purely linear models. Therefore, this methodology will only be analyzed using the selected $\dot{W}_c$ and $\dot{m}_{ref}$ models.

For the optimal design, it can be noticed that the algorithm makes the selection of points mainly on the compressor's operation limits and 1 or 2 central points. These results are coherent with the model that we wish to adjust (Eqs. (3) and (4)), which contains only linear terms (T_e and T_c) and quadratic terms (T_e^2 and T_c^2) along with a first-order interaction term ($T_e \times T_c$). When placing the points on the limits of operation, we obtain a greater accuracy in the adjustment of linear terms. Concerning the addition of central points, these are needed to adjust the quadratic terms, where a higher number of levels are required to characterize the curvature of the response variable ($\dot{W}_c$ or $\dot{m}_{ref}$). In contrast with the cluster design, it has the advantage of including in the design the 100% of the experimental domain.

Finally, Fig. 6 shows the selection of points for the polygonal design. Selecting the envelope's vertexes of the compressor and adding the remaining points with the clustering methodology could be considered a hybrid methodology between the previous two. However, with the borderline case of considering the most compact design (6 tests), this methodology does not include central points, which complicates the estimation of quadratic terms in the model.

4.2. Prediction errors with the selected models adjusted to the experimental designs

As mentioned in Section 2.2, two models have been selected in order to evaluate the proposed experimental designs. The first one – Eqs. (3) and (4) – will allow evaluating the experimental designs suitable for the characterization of scroll compressors, and the second one – Eqs. (5) and (6) – will allow to identify which models are also suitable for reciprocating compressors. For this purpose, both models have been fitted with the test points defined by the experimental designs in order to check the prediction errors with the remaining available points in each compressor and refrigerant. These points include the three suction conditions tests for the prediction of $\dot{W}_c$, and the $SH = 11K$ tests for the prediction of $\dot{m}_{ref}$. It was already confirmed in Marchante-Avellaneda et al. [8] that scroll compressors' suction conditions mainly affect the mass flow rate and have no apparent effect on the electrical consumption.

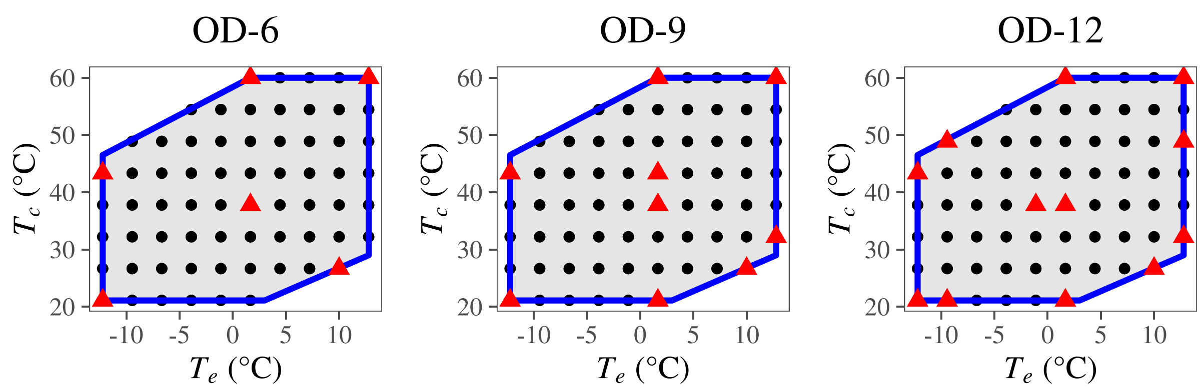

Fig. 5. Optimal design using Fedorov's algorithm for 6, 9, and 12 points (AHRI 11 R410A and $SH = 11K$).

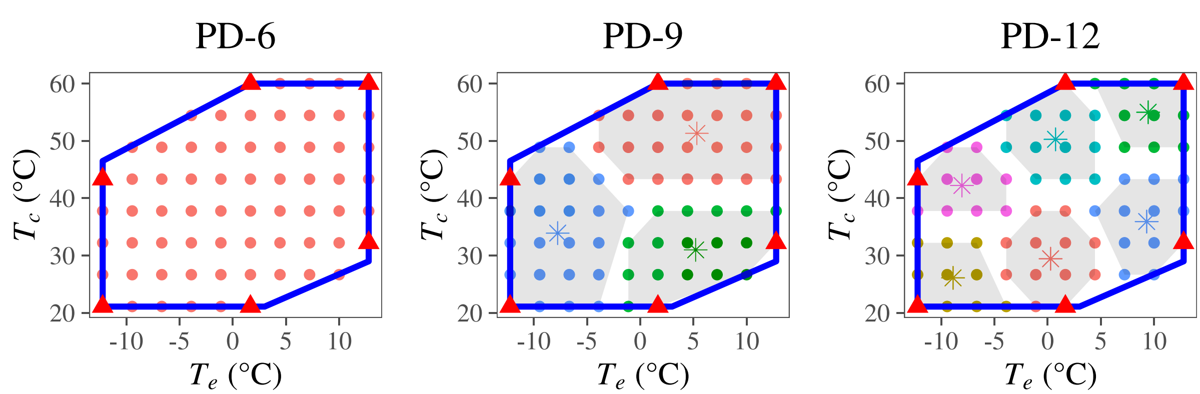

Fig. 6. Polygon Design 6, 9, and 12 points (AHRI 11 R410A and $SH = 11K$).

Table 3
AHRI 11 (R410A). Energy consumption. Temperatures as independent variables.

	All tests	3k	CCD	SCD	HD	OD6	OD9	OD12	CD6	CD9	CD12	PD6	PD9	PD12
$(Int.)$	7.7e-01 ***	7.1e-01 ***	7.0e-01 ***	7.1e-01 *	8.0e-01 *	7.6e-01	7.4e-01 ***	7.5e-01 ***	−1.4e+00	7.6e-01 ***	7.6e-01 ***	9.9e-01	7.3e-01 ***	7.4e-01 ***
T_e	−3.9e-03 ***	−5.3e-03 **	−5.2e-03 +	−5.7e-03	−3.0e-03	−3.7e-03	−3.8e-03 **	−3.6e-03 **	−7.8e-02	−4.5e-03	−4.9e-03 *	4.9e-03	−4.0e-03 +	−4.3e-03 *
T_c	1.0e-03 *	4.9e-03 *	5.3e-03 +	5.0e-03	−3.2e-04	1.8e-03	2.4e-03 *	2.0e-03	1.2e-01	2.0e-03	2.2e-03	−1.4e-02	3.4e-03	2.6e-03
$({T_e}^2)$	−7.6e-05 ***	−1.0e-04	−1.2e-04	−1.2e-04	−1.4e-04	−7.0e-05	−3.2e-05	−4.9e-05	−1.2e-03	−5.8e-05	−7.3e-05	3.2e-04	−4.3e-05	−4.0e-05
$({T_c}^2)$	4.5e-04 ***	3.9e-04 ***	3.8e-04 ***	3.9e-04 +	4.6e-04 *	4.4e-04	4.3e-04 ***	4.4e-04 ***	−1.0e-03	4.3e-04 **	4.3e-04 ***	6.5e-04	4.2e-04 ***	4.3e-04 ***
$T_e \times T_c$	−2.1e-05 *	5.2e-06	7.9e-06	1.2e-05	−4.0e-05	−2.9e-05	−2.6e-05	−3.8e-05	1.9e-03	−1.1e-05	1.5e-06	−2.7e-04	−2.1e-05	−1.1e-05
Num.Obs.	196	9	9	7	7	6	9	12	6	9	12	6	9	12
MRE (%)	1.21	2.77	2.46	2.27	1.90	1.24	1.33	1.13	39.56	2.06	1.98	6.32	1.31	1.30
RMSE (W)	7.26	17.76	14.84	13.43	8.96	8.78	8.79	8.14	161.72	10.22	9.28	44.29	9.15	8.04
CV_{RMSE} (%)	0.46	1.13	0.94	0.85	0.57	0.56	0.56	0.52	10.28	0.65	0.59	2.82	0.58	0.51

a + $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$;
b Temperatures (°C);
c Energy consumption (kW); Range: [973, 2454] (W);

As mentioned above, the errors used on the comparative have been the *MRE*, *RMSE* and the CV_{RMSE} and the regression coefficients have been obtained by considering the Inverse-Variance Weighting (IVW) for the regression adjustment. The results obtained for each of the selected models are shown below.

4.2.1. Polynomial models for $\dot{W}_c$ and $\dot{m}_{ref}$

Tables 3 and 4 show as example the results obtained for the AHRI 11 compressor and the base refrigerant (R410A). The first columns have added, as a reference model, the results obtained by performing the adjustment on every available point.

It can be seen that the adjustment with the different samples obtains a similar error to the one obtained considering the adjustment with all the available points (reference model).

In the case of the electrical consumption, we have an $RMSE = 7.3$ W and an $MRE = 1.2\%$, adjusting with 196 points. The variation range of the electrical consumption is between 973 W and 2454 W. Taking these values of error as a reference, we can see how the prediction of these 196 points gets an error very similar to the optimal designs.

We can point out the optimal design of 6 points (OD6) where we obtain an $RMSE = 8.8$ W and an $MRE = 1.2\%$ with only using 6 points for the adjustment. This sample of 6 points is the most possible compact design considering the number of terms to adjust in the model.

On the other hand, cluster designs and polygonal designs obtain similar results except for the designs of 6 points (CD6 and PD6). In this case, these methodologies obtain a considerable error compared to the other designs.

Table 4
AHRI 11 (R410A). Mass flow rate. Temperatures as independent variables.

	All tests	3k	CCD	SCD	HD	OD6	OD9	OD12	CD6	CD9	CD12	PD6	PD9	PD12
$(Int.)$	1.2e+02 ***	1.2e+02 ***	1.2e+02 ***	1.2e+02 *	1.2e+02 ***	1.2e+02	1.2e+02 ***	1.2e+02 ***	2.7e+02	1.2e+02 ***	1.2e+02 ***	1.5e+02	1.2e+02 ***	1.2e+02 ***
T_e	4.1e+00 ***	4.1e+00 ***	4.1e+00 ***	3.9e+00 *	4.0e+00 **	4.1e+00	4.1e+00 ***	4.2e+00 ***	9.3e+00	4.1e+00 ***	4.1e+00 ***	5.2e+00	4.0e+00 ***	4.0e+00 ***
T_c	1.2e-01 **	−1.8e-01	5.8e-03	5.8e-03	1.8e-01 *	8.1e-03	1.1e-01	2.7e-02	−8.3e+00	1.1e-01	8.1e-02	−2.0e+00	2.1e-01	2.3e-01
$(T_e{}^2)$	5.6e-02 ***	6.1e-02 **	6.1e-02 **	6.2e-02 +	6.7e-02 **	5.5e-02	5.6e-02 ***	5.7e-02 ***	1.4e-01	5.8e-02 ***	5.7e-02 ***	1.0e-01	5.4e-02 ***	5.2e-02 ***
$(T_c{}^2)$	−6.6e-03 ***	−2.1e-03	−4.7e-03 +	−4.9e-03	−7.3e-03 **	−5.4e-03	−6.6e-03 **	−5.6e-03 **	1.0e-01	−6.2e-03 **	−5.8e-03 ***	2.0e-02	−7.9e-03 *	−8.2e-03 **
$T_e \times T_c$	4.0e-03 ***	2.9e-03	2.0e-03	6.0e-03	3.5e-03 *	3.6e-03	4.6e-03 +	2.8e-03	−1.3e-01	2.7e-03	3.1e-03 +	−2.6e-02	6.3e-03 +	6.4e-03 *
Num.Obs.	66	9	9	7	7	6	9	12	6	9	12	6	9	12
MRE (%)	1.42	3.84	4.11	3.20	5.44	1.80	1.74	1.66	28.94	2.65	2.17	9.75	1.49	1.17
RMSE (W)	0.40	1.03	0.76	0.76	1.02	0.62	0.56	0.55	11.78	0.49	0.46	5.94	0.52	0.50
CV_{RMSE} (%)	0.34	0.88	0.64	0.65	0.87	0.53	0.48	0.47	10.04	0.41	0.39	5.06	0.45	0.42

a + p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001;
b Temperatures (°C);
c Mass flow rate (kg/h); Range: [66, 178] (kg/h);

Finally, classical designs show an error lightly higher than the rest of the designs, without considering the cluster and polygonal designs with 6 points. Regarding the mass flow rate prediction, we obtained similar results to the ones described in the energy consumption.

As we can see from the results, the errors obtained on the different designs are low except for those obtained in CD6 and PD6. However, if we take a closer look at the results, we will notice in some designs that the regression coefficients reverse the sign when comparing them with the reference model. These coefficients have been highlighted in red in the tables. Considering that in the reference model, all of the regression coefficients are significant (p-$value < 0.05$) and that this model has been adjusted with a large number of experimental points, this change in trend is an effect that we must try to avoid. This effect is motivated by the reduction of experimental information in the selected samples, which can lead to:

- A change in trend in second-order effects.
- A change in trend in first-order effects.

In the first case, we only lose some accuracy being terms of low weight in the model. For example, despite reversing the sign of the term $T_e \times T_c$ for classical designs in Table 3, this just means a slight rise in the prediction error. Another example would be the term T_c of the design 3^2 in Table 4. In this case, the model for the mass flow rate depends mainly on T_e instead of T_c [8].

In the second case, the prediction error may increase considerably. Considering that the electrical consumption in scroll compressors depends mainly on T_c [8], it can be observed in the CD6 and PD6 designs that the terms T_c and T_c^2 invert their sign concerning the ones obtained in the reference model with a considerable increase in the prediction error. In the case of the HD design, we only obtain a change of tendency in T_c without a significant increase in the error because it is compensated with the quadratic term T_c^2.

Considering what has been described above, Appendix C includes a summary table per analyzed AHRI report (Tables C.1–C.3). These tables show the error in the $\dot{W}_c$ and $\dot{m}_{ref}$ models for the whole set of refrigerants. They have been adjusted with the different experimental designs next to an additional column (*"sign"*). This column shows if there is a change in trend, or not, in some of the regression coefficients when compare them with the model fitted for all the available points (reference model). This evaluation has only been performed for those coefficients that are significant in the reference model (p-$value < 0.05$). Additionally, to facilitate the analysis, a column with the design's label has been colored according to the obtained value of *RMSE*.

According to the results on the summary tables, it can be seen that similar results to the ones described above for the AHRI 11 compressor and the reference refrigerant are obtained. CD6 and PD6 methodologies always get high values of *RMSE* and *MRE*. Concerning the rest of the designs, classical methodologies generally have acceptable prediction errors but are higher than computer-aided methodologies. On the other hand, having analyzed the tendencies in the coefficients, methodologies with optimal designs get a better prediction of the regression coefficients showing the same trend if they are compared with the results obtained when adjusting them to all available points per compressor and refrigerant. Regarding cluster and polygonal designs with more than 6 points in the design, despite not getting the same trend in all coefficients for some of the analyzed refrigerants, this does not become a significant increase in the prediction error. Therefore, it can be concluded that all methodologies assisted by computer generally obtain the lower prediction errors if more than 6 points on the experimental design are included. They are the most suitable methodologies to characterize scroll compressors by applying Eqs. (3) and (4), with lower prediction errors and not dependent on the shape of the compressor's envelope. A sample size of 9 points has proved adequate in all cases, being a rational number of required points and accuaracy.

4.2.2. Polynomial models for $\dot{W}_{esp}$ and $\dot{m}_{ref}$

A similar analysis to the previous one was also performed for the models presented in Eqs. (5) and (6). As reported in Marchante-Avellaneda et al. [9] these models are also suitable for characterizing reciprocating compressors. The characterization of the specific energy consumption instead of the energy consumption will allow us to evaluate the experimental designs selected in this work and extrapolate the results to reciprocating compressors. This approach aims to identify the most suitable sample sizes and methodologies for both technologies (scroll and reciprocating). Table 5 and Table 6 shows again the results obtained for the AHRI 11 compressor and the base refrigerant (R410A) by applying the models defined in Eqs. (5) and (6). As reported in Marchante-Avellaneda et al. [9], the model to be applied for the specific energy consumption can be considered linear (Eq. (6)) or increase the degree of the polynomial if observing a significant reduction in the prediction errors. By including the quadratic term in Eq. (6) ($k_2 P_r'^2$), it could be seen that lower prediction errors were obtained for the ZP21K5E-PFV compressor (AHRI 11 and 33). In the case of compressor ZP21KAE-PFV (AHRI 21), no significant reduction in the prediction error was obtained, and the linear model considered in Eq. (6) was maintained.

Summarizing the results of Tables 5 and 6 we can see that the new models also obtain a low prediction error for the experimental designs analyzed in this work. As previously mentioned, the optimal design methodologies have not been included in this second part because a purely linear model is not considered for the characterization of the specific energy consumption.

In this case, for the specific energy consumption and mass flow rate models, the reduction in the number of terms in Eqs. (5) and (6), if we compare them with the second-order models (Eqs. (3) and (4)), results in a more stable behavior when fitting the model to the experimental samples. In all samples, the same trend in the regression coefficients has been observed if we compare them with the adjusted model with all available points. Only the intercept term for the mass flow rate model using the SCD in Table 6 shows an inverse trend, but in that case the intercept term is not significant in the reference model (p-$value < 0.05$).

Table 5
AHRI 11 (R410A). Specific energy consumption. Pressures as independent variables.

	All tests	3k	CCD	SCD	HD	CD6	CD9	CD12	PD6	PD9	PD12
k_0	−5.7e-01	−3.6e+00 ***	−3.6e+00 **	−1.8e+00 +	−1.7e+00	−6.0e+00	−2.9e+00 +	−2.6e+00 +	−3.0e+00	−1.4e+00	−2.2e+00
k_1	8.2e+00 ***	9.7e+00 ***	9.5e+00 ***	8.7e+00 **	7.9e+00 **	1.2e+01	9.2e+00 **	9.6e+00 ***	1.0e+01 *	8.8e+00 **	9.5e+00 ***
z_c	−6.8e+00 ***	−7.9e+00 ***	−8.1e+00 ***	−7.6e+00 **	−8.3e+00 *	−6.2e+00	−7.8e+00 **	−6.8e+00 ***	−5.8e+00 +	−6.5e+00 **	−6.3e+00 ***
z_e	−9.0e-01 ***	−1.0e+00 *	−1.2e+00 ***	−1.0e+00 *	−1.4e+00 *	−7.8e-01	−1.2e+00 *	−8.4e-01 *	−6.1e-01	−7.9e-01 +	−7.5e-01 *
k_2	1.8e+00 ***	1.5e+00 ***	1.6e+00 ***	1.7e+00 **	2.0e+00 **	1.1e+00	1.7e+00 **	1.5e+00 ***	1.4e+00 +	1.7e+00 **	1.5e+00 ***
Num.Obs.	196	9	9	7	7	6	9	12	6	9	12
MRE (%)	1.63	2.37	2.45	2.24	3.00	2.55	2.15	1.93	1.86	1.52	1.64
RMSE (W)	8.46	15.00	13.59	10.76	12.01	14.26	10.10	10.50	15.75	10.96	10.29
CV_{RMSE} (%)	0.54	0.95	0.86	0.68	0.76	0.91	0.64	0.67	1.00	0.70	0.65

a + $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$;
b Pressures (bar);
c Energy consumption (kW); Range: [973, 2454] (W);

Table 6
AHRI 11 (R410A). Mass flow rate. Pressures as independent variables.

	All tests	3k	CCD	SCD	HD	CD6	CD9	CD12	PD6	PD9	PD12
$(Int.)$	−1.0e+00	−1.6e+00	−8.6e-01	4.6e+00	−1.2e+00	−1.7e+00	−4.2e-01	−1.1e+00	−4.8e-01	−3.9e-01	−9.6e-01
P_e	1.6e+01 ***	1.6e+01 ***	1.6e+01 ***	1.5e+01 **	1.6e+01 ***	1.6e+01 ***	1.6e+01 ***	1.6e+01 ***	1.6e+01 **	1.6e+01 ***	1.6e+01 ***
P_c	−8.2e-01 ***	−7.6e-01 **	−7.5e-01 *	−1.0e+00	−8.6e-01 +	−7.3e-01 *	−7.9e-01 ***	−7.9e-01 ***	−8.4e-01 *	−8.4e-01 **	−8.1e-01 ***
P_e: P_c	1.7e-02 **	1.9e-02	1.6e-02	4.5e-02	1.7e-02	7.3e-03	1.6e-02	1.4e-02	1.5e-02	1.5e-02	1.2e-02
Num.Obs.	66	9	9	7	7	6	9	12	6	9	12
MRE (%)	1.37	2.24	1.91	1.62	1.82	1.56	1.47	1.39	1.41	1.29	1.24
RMSE (W)	0.57	0.79	0.74	0.77	0.67	0.68	0.63	0.59	0.68	0.65	0.63
CV_{RMSE} (%)	0.49	0.67	0.63	0.65	0.57	0.58	0.53	0.50	0.58	0.56	0.54

a + $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$;
b Pressures (bar);
c Mass flow rate (kg/h); Range: [66, 178] (kg/h);

Appendix D also includes a summary table per analyzed AHRI report (Tables D.1–D.3) with the corresponding prediction errors for all the analyzed data. From these results, it is not possible to identify a single methodology that always obtains the lowest prediction error. We can see that depending on the compressor and refrigerant used, sometimes lower prediction errors are obtained with classical methodologies and others with computer-aided methodologies. One possible explanation is that the reduction in the number of terms included in the models results in a lower sensitivity in the location of the experimental points. It is only noticed that the more compact samples of 6 points continue to obtain higher prediction errors, and therefore a suitable sample size must include a higher number of tests.

Against this background, the authors recommend using Polygonal Designs with a sample size of 9 points as the most promising approach in order to characterize scroll and reciprocating compressors by using Eqs. (5) and (6). Despite the lower sensitivity of Eqs. (5) and (6) to the test's location, the polygonal design includes the vertexes of the compressor envelope. These are experimental points commonly measured when characterizing compressors as they inform us of the working limits. In addition, as shown in Section 4.1, these points' locations are similar to those obtained with the optimal designs. Therefore, the designs obtained are also suitable to characterize the compressor with the models used in the first part (Eqs. (3) and (4)).

5. Conclusion

In a pioneering effort, this paper evaluates diverse Design of Experiments methodologies to discern the most suitable one for defining an experimental sample with the least number of points in scroll and reciprocating compressors. The focus is on strategically selecting and situating these points within the experimental domain to ensure an accurate characterization of both scroll and reciprocating compressors. The main goal of this study is to articulate a straightforward methodology for optimal point selection, with the ultimate aim of characterizing the compressor accurately while keeping the experimental costs at a minimum. A total of 7 methodologies of experimental designs have been considered, including classical methodologies and more sophisticated methodologies assisted by computer. These are the main conclusions obtained from this study:

- The response surfaces for the energy consumption and mass flow rate are smooth for scroll compressors, and generally, all of the analyzed designs obtain acceptable prediction errors by using Eqs. (3) and (4), except for the CD6 and PD6 designs.
- A polynomial of 2nd degree are appropriate for the characterization of scroll compressors. These models allow the construction of more compact experimental designs than the ones needed for the adjustment of the original AHRI polynomial. The results for the costs of experimentation are significantly lower. From a mathematical point of view, the second-order model need a minimum of 6 points to obtain the values of the regression coefficients. On the contrary, the 3-order model need a minimum of 10 points. Moreover, as reported in Marchante-Avellaneda et al. [8], the response surfaces of scroll compressors' power consumption and mass flow rate are simple and smoothly trending. A high-degree polynomial does not obtain a significant decrease in the prediction error and can overfit the model and obtain significant extrapolation and interpolation errors for non-training points.
- The models reported in Marchante-Avellaneda et al. [9] (Eqs. (5) and (6)) for the characterization of the specific energy consumption and mass flow rate are less dependent on the experimental test location. They are suitable models in order to characterize both technologies, scroll and reciprocating compressors.
- Classical designs are not able to adapt to irregular experimental domains. They do not generally obtain errors as low as the computer-aided designs. However, they do not show high errors of prediction, which makes it appropriate if they can cover a significant area within the limits of the compressor envelope. These designs should be used only in the case of not having the necessary tools to use computer-aided designs.
- In most cases, computer-aided designs have obtained the lowest errors of prediction. The cluster and polygonal designs considering 6 test points shall be excluded. These methodologies allow to adapt the designs to irregular experimental domains minimizing extrapolation errors. Therefore, computer-aided designs are more appropriate than classical methodologies in irregular experimental domains.
- Optimal designs show the advantage of selecting an optimal sample for adjusting the target model, being more recommendable in case of knowing the functional to adjust. They show the

limitation of considering the same functional for the prediction of $\dot{W}_c$ and $\dot{m}_{ref}$ and can only be used in purely linear models in its terms. Considering different functional, we must consider the model with a higher number of terms in order to generate the experimental design by using the optimal methodology.

- In order to characterize scroll compressors by using Eqs. (3) and (4), the use of 9 experimental points in the design has shown to be an appropriate number of tests balancing accuracy and experimentation costs. This sample size shows similar results non-depending on the computer-aided design methodology selected.
- For the specific energy consumption and mass flow rate models – Eqs. (5) and (6) – a suitable experimental sample includes 9 experimental points and by using the Polygonal Design it is possible to know the working limits of the compressor as additional information. This sample size of 9 points is also suitable to characterize reciprocating compressors allowing to reduce the required experimental tests by using the specific energy consumption. This sample size includes a lower number of test than the required for the characterization of the energy consumption and by using the original AHRI polynomials reported in the AHRI standard.
- The use of the three methodologies of computer-aided designs allows to define the total number of points to include in the sample. This allows the adjustment of models considering a greater number of terms in the functional. Therefore, they may be appropriate for adjusting the AHRI polynomial [31], which includes 10 coefficients in the functional. Unfortunately, due to the lack of a suitable dataset including a large number of experimental points for reciprocating compressors, it has not been possible to evaluate the performance of these designs by using the 10-term and third-degree polynomials of the AHRI standard.

CRediT authorship contribution statement

Javier Marchante-Avellaneda: Conceptualization, Methodology, Software, Validation, Writing – original draft, Writing – review & editing, Data curation. **Emilio Navarro-Peris:** Conceptualization, Project administration, Supervision, Writing – review & editing. **Som S. Shrestha:** Data curation, Resources, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Many thanks as well to the late Dr. José Miguel Corberán, without whom this work would never have been possible. Sadly, Dr. José Miguel Corberán passed away in July of 2022. I wish to give my wholehearted support to José Miguel's family. I hope we did you proud.

Appendix A. Classical experimental designs

Nowadays, in the literature exists a huge diversity of methodologies and experimental designs which can be used to obtain a good experimental plan as well as relevant statistical results. The use of one methodology or another will be established according to the experimentation capacity (number of tests), the quantity of independent variables that we shall control during the experiment, and the complexity of the response surface to be characterized.

In the compressors field, we only have two factors of control (T_e and T_c) and two response variables to characterize (e.g. $\dot{W}_c$ and $\dot{m}_{ref}$). Based on this scenario, one of the most extended methodologies is the use of X^k designs. A full factorial design X^k with k factors of control is obtained by selecting a determined number of levels X for each factor. These levels are considered discrete values selected at the continuous range of the control factors. The selection of 2 or more levels will depend on whether the effects of the factors over the response variable are or not linear.

Normally this modest methodology is very useful when completing experimental orthogonal matrices in a great number of scenarios. Said propriety of orthogonality is especially interesting because it grants, in regression models, to estimate the effects of every factor and interaction among factors free from influences with other factors or interactions.

On the other hand, in the 1950s George E.P. Box and K.B. Wilson proposed an alternative to the X^k factorial designs. Starting first from a 2^k design and with the addition of central and axial points, it can be built the Central Composite Design (CCD) [21]. Currently, this design is the most used one when adjusting second-order polynomial models. The addition of central and axial points allows the estimation of quadratic terms and interactions. Furthermore, it allows the use of a great number of levels for the factors, reducing the number of total points if it is compared to a complete factorial design with the same number of levels.

Additionally, with the objective of obtaining even more compact designs, exists a variant of the CCD known as Small Composite Design (SCD) [22]. These designs come originally from a CCD eliminating some points and trying to lose a lesser amount of information, which can be justified in the analysis of processes with elevated experimentation costs, where an agreement between the accuracy of the model and the experimentation costs is sought.

Finally, in the event of having just two factors, other alternatives such as Hexagonal Designs (HD) exist [see 23, chap. 7, pg. 331]. Having CCD as an octagonal equiradial design, this alternative shows an alternative design, which is also equiradial and could be of greater interest depending on the experimental domain that one tries to cover.

Based on the irregular experimental domain of compressors, the authors selected the following classical experimental design alternatives for two factors: 3^2, CCD, SCD, and HD. The total number of points goes from 7 to 9 tests for the four methodologies. Said designs were built trying to cover the greatest possible experimental design among the analyzed compressors and selecting available points on the experimental matrices included in the AHRI reports.

Fig. A.1 shows an example of how the points were distributed in the envelope for the AHRI 21 compressor considering its reference refrigerant (R404A) and $SH = 11K$ as suction conditions. In this figure, the black points include the set of given points in the AHRI report, and the red points are the selected ones to build the different samples. With the SCD methodology, the selected points are the same ones as in the CCD design, eliminating the two highlighted points on the diagonal.

Appendix B. Computer-aided experimental designs

Nowadays, the development of informatics has allowed us to have a great processing power on problem-solving at our disposal. In the field of experimental designs, the increase in this potential for calculation has allowed researchers to use sophisticated algorithms to plan

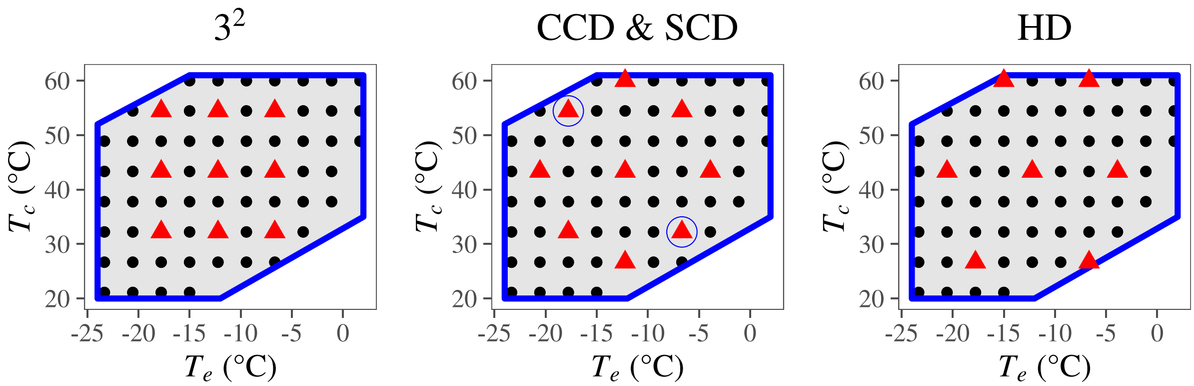

Fig. A.1. Classical designs: 3^2, CCD, SCD *y* HD (AHRI 21 R404A and $SH = 11K$).

and generate experimental matrices. These kinds of methodologies are known as computer-aided experimental designs because they generate the samples from algorithms and calculations by computer [see 32, chap 5, sec. 5]. In contrast with classical factorial designs, these methodologies are used in particular situations. For example, two of the most common situations are:

- The physical or chemical phenomena analyzed include a great number of factors. In this situation, classical designs with completed or fractionated factorials result in a high number of tests.
- Not all combinations of factors are possible. Therefore, we have an irregular experimental domain instead of an orthogonal one.

Focusing on our case of study, experimental planning of fixed-speed compressors, it might be possible that the first situation does not justify using these methodologies. We only have two control factors (T_e and T_c), so the samples for classical methodologies will be compact. As it is mentioned above, the selected designs only include 7 to 9 experimental points.

However, we must keep in mind that the compressor envelope is irregular and, as can be seen in Fig. A.1, classical methodologies do not cover it completely. Inscribing the designs in the central area, there are two remaining excluded areas of the design (high T_e, T_c and low T_e, T_c). This can lead to extrapolation errors if these excluded areas are significant.

For this last reason, the present study also includes the use of experimental designs assisted by computer in the comparative analysis. The selected methodologies are two of the most used ones in experimental planning, the Optimal Designs (OD) and Cluster Design (CD). Additionally, one more typology has been added; it derives from Cluster Designs, the Polygonal Designs (PD), [11, section 8.1.2].

The first methodology – optimal experimental designs – was firstly proposed by Kieffer and Wolfowitz in the 1950s [24,25] and later was published as a more relevant work in the Atkinson & Donev's book [33]. This type of methodology assumes knowing the mathematical function of the model to apply, able to reproduce the response variable accurately. Having this function and knowing the experimental domain (set of candidate points), these designs lay out the selection of points depending on an optimality criterion. Hence, the obtained design normally will be optimal for a specific model. Using matrix notation, the adjustment of a linear regression model is given by:

$$Y = \hat{\beta} X \quad \text{and} \quad X = \begin{bmatrix} f_1(x_1) & \cdots & f_m(x_1) \\ \vdots & \ddots & \vdots \\ f_1(x_n) & \cdots & f_m(x_n) \end{bmatrix} \tag{B.1}$$

Being able to calculate the regression coefficients $\hat{\beta}$ as:

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y \tag{B.2}$$

Within this context, it is common to use the determinant of the matrix of covariance as a measure (scalar) of the adjustment precision. This generalized variance for the estimation of $\hat{\beta}$ is calculated as:

$$VG(\hat{\beta}) = \left|Var(\hat{\beta})\right| = \left|\left(X^T X\right)^{-1}\right| \sigma^2 \tag{B.3}$$

Based on the previous equations, the optimality criteria will select the experimental points to maximize or reduce a statistical estimator of the matrix of information. In this article, we have selected one of the most used ones, the Optimal-D criterion, which intends to maximize the determinant of the matrix of information $|X^T X|$. Therefore, it minimizes the generalized variance (Eq. (B.3)) for the regression coefficients ($\hat{\beta}$). In other words, it selects the points intending to obtain the minimum error for the prediction of the regression coefficients ($\hat{\beta}$).

On the other hand, the cluster design is based on the automatic grouping of points considering their location in the experimental domain. Therefore, when talking about compressors, it just needs as main input a series of candidate points on evaporation and condensation temperature coordinates. With this information, this methodology is able to analyze the experimental domain and classify in a number of k clusters the complete set of candidate points. Once the clustering is made, the experimental design will be considered as the set of k points located in the center of each cluster. This study has used the algorithm *k-means* to obtain the clustering. In this case, the algorithm calculates the average of T_e and T_c in the k clusters assigning the candidate points to the cluster whose average value of T_e and T_c is closer.

Finally, the polygonal design methodology [11, sec 8.1.2] combines the manual selection of points with cluster design. In a first stage, the vertexes of the polygon defining the compressor envelope are selected. The rest of the points to be included in the experimental design will be selected using the clustering methodology.

Appendix C. Polynomial models for $\dot{W}_c$ and $\dot{m}_{ref}$

See Tables C.1–C.3.

Table C.1
AHRI 11. Summary table DoE samples results.

Energy consumption								mass flow rate						
DoE	MRE (%)	RMSE (W)	CV_{RMSE}(%)	Sample	Range (W)	sign	Fluid	sign	Range (kg/h)	Sample	CV_{RMSE}(%)	RMSE (kg/h)	MRE (%)	DoE
All tests	1.21	7.26	0.46	196		✓		✓		66	0.34	0.40	1.42	All tests
3k	2.77	17.76	1.13	9		✗		✗		9	0.88	1.03	3.84	3k
CCD	2.46	14.84	0.94	9		✗		✓		9	0.64	0.76	4.11	CCD
SCD	2.27	13.43	0.85	7		✗		✓		7	0.65	0.76	3.20	SCD
HD	1.90	8.96	0.57	7		✗		✓		7	0.87	1.02	5.44	HD
OD6	1.24	8.78	0.56	6		✓		✓		6	0.53	0.62	1.80	OD6
OD9	1.33	8.79	0.56	9	[973, 2454]	✓	R410A	✓	[66, 178]	9	0.48	0.56	1.74	OD9
OD12	1.13	8.14	0.52	12		✓		✓		12	0.47	0.55	1.66	OD12
CD6	39.56	161.72	10.28	6		✗		✗		6	10.04	11.78	28.94	CD6
CD9	2.06	10.22	0.65	9		✓		✓		9	0.41	0.49	2.65	CD9
CD12	1.98	9.28	0.59	12		✗		✓		12	0.39	0.46	2.17	CD12
PD6	6.32	44.29	2.82	6		✗		✗		6	5.06	5.94	9.75	PD6
PD9	1.31	9.15	0.58	9		✓		✓		9	0.45	0.52	1.49	PD9
PD12	1.30	8.04	0.51	12		✓		✓		12	0.42	0.50	1.17	PD12
All tests	3.12	14.43	0.93	166		✓		✓		59	0.66	0.54	2.16	All tests
3k	5.66	26.86	1.73	9		✓		✓		9	1.47	1.21	3.88	3k
CCD	4.68	20.61	1.33	9		✓		✓		9	1.77	1.45	7.48	CCD
SCD	4.53	20.26	1.30	7		✓		✓		7	1.64	1.35	7.37	SCD
HD	5.52	25.89	1.67	7		✓		✓		7	1.80	1.48	6.35	HD
OD6	4.00	26.97	1.74	6		✓		✓		6	0.82	0.67	2.73	OD6
OD9	2.67	20.53	1.32	9	[1005, 2607]	✓	R32	✓	[46, 123]	9	0.72	0.59	2.25	OD9
OD12	2.80	20.05	1.29	12		✓		✓		12	0.70	0.58	2.33	OD12
CD6	12.83	57.22	3.68	6		✗		✓		6	1.75	1.43	3.82	CD6
CD9	3.45	21.39	1.38	9		✓		✓		9	0.81	0.66	1.94	CD9
CD12	2.85	17.04	1.10	12		✗		✓		12	0.68	0.56	2.04	CD12
PD6	31.30	288.88	18.59	6		✗		✗		6	6.56	5.38	12.23	PD6
PD9	3.12	19.95	1.28	9		✓		✓		9	0.80	0.66	2.43	PD9
PD12	3.64	23.79	1.53	12		✓		✓		12	0.74	0.61	2.13	PD12
All tests	4.62	13.00	0.87	189		✓		✓		66	0.21	0.19	0.65	All tests
3k	6.15	20.37	1.37	9		✗		✓		9	0.38	0.33	1.30	3k
CCD	5.72	17.19	1.16	9		✗		✓		9	0.45	0.40	2.21	CCD
SCD	5.51	16.85	1.13	7		✗		✓		7	0.43	0.38	2.01	SCD
HD	4.17	15.15	1.02	7		✓		✓		7	0.44	0.39	2.41	HD
OD6	4.15	18.52	1.25	6		✓		✓		6	0.27	0.23	1.03	OD6
OD9	4.06	15.62	1.05	9	[929, 2401]	✓	DR5	✓	[49, 134]	9	0.25	0.22	1.06	OD9
OD12	4.17	16.94	1.14	12		✓		✓		12	0.24	0.21	0.61	OD12
CD6	60.36	225.08	15.14	6		✗		✗		6	5.79	5.10	16.21	CD6
CD9	5.27	14.83	1.00	9		✗		✓		9	0.22	0.20	0.80	CD9
CD12	5.00	14.08	0.95	12		✗		✓		12	0.23	0.20	0.59	CD12
PD6	24.74	191.08	12.86	6		✗		✗		6	2.39	2.11	4.89	PD6
PD9	4.39	15.15	1.02	9		✗		✓		9	0.25	0.22	1.05	PD9
PD12	4.60	13.96	0.94	12		✗		✓		12	0.24	0.21	0.96	PD12
All tests	1.25	6.33	0.45	186		✓		✓		65	0.32	0.25	0.99	All tests
3k	3.39	10.90	0.78	9		✓		✗		9	0.81	0.65	1.90	3k
CCD	2.46	8.47	0.61	9		✓		✓		9	0.56	0.45	2.08	CCD
SCD	2.81	8.80	0.63	7		✓		✓		7	0.76	0.61	3.28	SCD
HD	3.90	11.41	0.81	7		✓		✗		7	0.60	0.48	1.41	HD
OD6	1.47	9.90	0.71	6		✓		✓		6	0.38	0.30	1.06	OD6
OD9	1.33	9.13	0.65	9	[888, 2211]	✓	L41a	✓	[43, 122]	9	0.39	0.31	0.95	OD9
OD12	1.41	9.43	0.67	12		✓		✓		12	0.33	0.27	0.99	OD12
CD6	6.99	28.14	2.01	6		✓		✗		6	2.14	1.72	6.00	CD6
CD9	1.89	7.74	0.55	9		✓		✓		9	0.33	0.27	0.96	CD9
CD12	1.84	7.20	0.51	12		✓		✓		12	0.32	0.26	0.97	CD12
PD6	1.90	9.57	0.68	6		✗		✗		6	1.60	1.29	3.94	PD6
PD9	1.29	8.37	0.60	9		✓		✓		9	0.34	0.28	1.01	PD9
PD12	1.08	7.11	0.51	12		✓		✓		12	0.34	0.27	1.00	PD12

Table C.2
AHRI 21. Summary table DoE samples results.

Energy consumption								mass flow rate						
DoE	MRE (%)	RMSE (W)	CV_{RMSE}(%)	Sample	Range (W)	sign	Fluid	sign	Range (kg/h)	Sample	CV_{RMSE}(%)	RMSE (kg/h)	MRE (%)	DoE
All tests	2.19	14.04	0.48	191		✓		✓		63	0.24	0.49	0.75	All tests
3k	4.88	26.57	0.92	9		✓		✓		9	0.26	0.52	0.75	3k
CCD	4.81	22.46	0.77	9		✓		✓		9	0.29	0.58	0.91	CCD
SCD	4.83	22.38	0.77	7		✓		✓		7	0.30	0.60	0.97	SCD
HD	3.95	18.95	0.65	7		✓		✓		7	0.27	0.54	0.78	HD
OD6	2.64	16.49	0.57	6		✓		✓		6	0.31	0.63	1.00	OD6
OD9	2.50	15.69	0.54	9	[1856, 4172]	✓	R404A	✓	[124, 308]	9	0.29	0.58	0.87	OD9
OD12	2.62	15.83	0.55	12		✓		✓		12	0.28	0.56	0.90	OD12
CD6	5.07	44.57	1.54	6		✓		✗		6	1.34	2.69	4.85	CD6
CD9	3.58	21.72	0.75	9		✓		✓		9	0.29	0.59	0.87	CD9
CD12	3.51	19.68	0.68	12		✓		✓		12	0.26	0.52	0.79	CD12
PD6	2.64	16.73	0.58	6		✓		✗		6	2.83	5.69	5.54	PD6
PD9	2.78	17.81	0.61	9		✓		✗		9	0.38	0.76	1.18	PD9
PD12	2.79	18.36	0.63	12		✓		✓		12	0.30	0.60	0.98	PD12
All tests	1.65	9.73	0.39	186		✓		✓		64	0.22	0.28	0.92	All tests
3k	4.05	19.35	0.78	9		✓		✓		9	0.34	0.43	0.92	3k
CCD	3.13	15.54	0.63	9		✓		✓		9	0.25	0.32	1.17	CCD
SCD	3.65	16.19	0.65	7		✓		✓		7	0.27	0.34	0.97	SCD
HD	2.83	15.08	0.61	7		✓		✓		7	0.26	0.32	0.99	HD
OD6	1.71	14.54	0.59	6		✓		✓		6	0.38	0.48	0.96	OD6
OD9	1.75	16.10	0.65	9	[1582, 3615]	✓	ARM31a	✓	[73, 200]	9	0.32	0.40	0.87	OD9
OD12	1.83	12.87	0.52	12		✓		✓		12	0.27	0.35	0.60	OD12
CD6	3.06	15.56	0.63	6		✓		✓		6	0.35	0.44	1.93	CD6
CD9	2.78	15.13	0.61	9		✓		✓		9	0.25	0.32	0.75	CD9
CD12	2.63	14.08	0.57	12		✓		✓		12	0.24	0.30	1.18	CD12
PD6	4.76	50.34	2.03	6		✗		✗		6	1.40	1.76	2.63	PD6
PD9	1.86	13.62	0.55	9		✓		✗		9	0.30	0.38	1.02	PD9
PD12	1.98	12.88	0.52	12		✓		✗		12	0.38	0.48	1.22	PD12
All tests	1.85	11.98	0.44	183		✓		✓		64	0.32	0.44	1.33	All tests
3k	5.06	24.99	0.92	9		✓		✓		9	0.39	0.54	1.38	3k
CCD	3.66	17.73	0.66	9		✓		✓		9	0.36	0.50	1.55	CCD
SCD	4.77	20.00	0.74	7		✓		✓		7	0.37	0.51	1.78	SCD
HD	3.42	18.18	0.67	7		✓		✓		7	0.39	0.54	1.76	HD
OD6	1.91	16.26	0.60	6		✓		✓		6	0.35	0.48	1.43	OD6
OD9	1.88	17.74	0.66	9	[1724, 3988]	✓	D2Y65	✓	[81, 214]	9	0.35	0.49	1.30	OD9
OD12	1.99	14.68	0.54	12		✓		✓		12	0.34	0.47	1.40	OD12
CD6	3.55	18.67	0.69	6		✓		✓		6	0.52	0.71	2.98	CD6
CD9	3.25	17.84	0.66	9		✓		✓		9	0.38	0.53	1.50	CD9
CD12	3.01	16.30	0.60	12		✓		✓		12	0.34	0.46	1.41	CD12
PD6	3.26	30.33	1.12	6		✗		✗		6	1.93	2.66	4.19	PD6
PD9	2.08	15.53	0.57	9		✓		✓		9	0.43	0.59	1.05	PD9
PD12	2.19	14.47	0.54	12		✓		✓		12	0.41	0.56	1.49	PD12
All tests	1.29	9.12	0.37	173		✓		✓		61	0.29	0.32	1.04	All tests
3k	4.65	22.01	0.90	9		✓		✓		9	0.64	0.71	2.88	3k
CCD	2.33	11.73	0.48	9		✓		✓		9	0.37	0.42	1.74	CCD
SCD	3.87	16.51	0.67	7		✓		✗		7	0.44	0.49	1.93	SCD
HD	2.79	16.27	0.66	7		✓		✗		7	0.78	0.87	3.74	HD
OD6	1.49	17.04	0.70	6		✓		✓		6	0.33	0.36	0.81	OD6
OD9	1.34	15.35	0.63	9	[1570, 3655]	✓	L40	✓	[64, 175]	9	0.37	0.41	0.78	OD9
OD12	1.37	13.63	0.56	12		✓		✓		12	0.31	0.34	0.86	OD12
CD6	2.74	13.84	0.56	6		✓		✓		6	0.88	0.98	2.70	CD6
CD9	2.02	12.68	0.52	9		✓		✓		9	0.35	0.39	1.49	CD9
CD12	2.11	11.99	0.49	12		✓		✓		12	0.32	0.35	1.13	CD12
PD6	1.41	13.59	0.55	6		✓		✗		6	0.72	0.80	1.65	PD6
PD9	1.53	12.54	0.51	9		✓		✗		9	0.50	0.56	1.13	PD9
PD12	1.76	11.22	0.46	12		✓		✓		12	0.32	0.35	0.72	PD12
All tests	1.28	13.62	0.51	133		✓		✓		48	0.44	0.53	1.33	All tests
3k	2.58	27.31	1.02	9		✓		✓		9	0.47	0.56	1.44	3k
CCD	6.03	35.10	1.31	9		✗		✓		9	0.87	1.04	3.56	CCD
SCD	4.47	40.94	1.52	7		✗		✓		7	1.55	1.86	7.92	SCD
HD	4.31	28.11	1.05	7		✓		✓		7	1.44	1.72	7.89	HD
OD6	2.39	21.16	0.79	6		✓		✓		6	0.67	0.81	1.61	OD6
OD9	1.21	15.38	0.57	9	[1740, 4268]	✓	R32/R134a	✓	[68, 179]	9	0.55	0.66	1.44	OD9
OD12	1.24	15.71	0.58	12		✓		✓		12	0.54	0.64	1.50	OD12
CD6	3.78	37.30	1.39	6		✓		✗		6	1.96	2.35	7.03	CD6
CD9	2.40	16.04	0.60	9		✓		✓		9	0.50	0.60	1.74	CD9
CD12	2.17	16.38	0.61	12		✓		✓		12	0.48	0.57	1.67	CD12
PD6	43.17	346.18	12.89	6		✗		✗		6	15.37	18.47	33.55	PD6
PD9	2.18	20.96	0.78	9		✓		✓		9	0.89	1.07	3.32	PD9
PD12	1.39	15.01	0.56	12		✓		✓		12	0.81	0.97	2.32	PD12

Table C.3
AHRI 33. Summary table DoE samples results.

Energy consumption								mass flow rate						
DoE	MRE (%)	RMSE (W)	$CV_{RMSE}(\%)$	Sample	Range (W)	sign	Fluid	sign	Range (kg/h)	Sample	$CV_{RMSE}(\%)$	RMSE (kg/h)	MRE (%)	DoE
All tests	1.70	7.72	0.50	196		✓		✓		66	0.22	0.26	0.75	All tests
3k	3.52	14.32	0.92	9		✓		✓		9	0.29	0.34	1.23	3k
CCD	2.32	13.74	0.89	9		✓		✓		9	0.31	0.37	1.47	CCD
SCD	3.38	12.56	0.81	7		✓		✓		7	0.32	0.38	1.83	SCD
HD	3.19	10.24	0.66	7		✗		✓		7	0.38	0.46	2.51	HD
OD6	2.37	12.64	0.81	6		✓		✓		6	0.32	0.39	0.64	OD6
OD9	2.38	11.94	0.77	9	[945, 2432]	✓	R410A	✓	[68, 181]	9	0.30	0.36	0.77	OD9
OD12	1.97	10.16	0.65	12		✓		✓		12	0.27	0.33	0.66	OD12
CD6	60.02	233.14	15.01	6		✗		✗		6	11.98	14.39	33.59	CD6
CD9	2.97	9.86	0.63	9		✓		✓		9	0.28	0.33	0.93	CD9
CD12	2.76	9.60	0.62	12		✓		✓		12	0.25	0.30	0.79	CD12
PD6	3.61	19.60	1.26	6		✗		✗		6	5.86	7.04	11.39	PD6
PD9	2.42	12.55	0.81	9		✓		✓		9	0.28	0.33	0.74	PD9
PD12	2.09	10.42	0.67	12		✓		✓		12	0.28	0.33	0.56	PD12
All tests	1.42	8.31	0.56	168		✓		✓		59	0.27	0.23	1.10	All tests
3k	2.23	12.34	0.83	9		✓		✓		9	0.32	0.26	1.15	3k
CCD	2.12	11.29	0.76	9		✓		✓		9	0.32	0.26	1.62	CCD
SCD	2.19	10.46	0.70	7		✓		✓		7	0.31	0.26	1.54	SCD
HD	2.42	12.64	0.85	7		✓		✓		7	0.47	0.39	1.89	HD
OD6	2.20	13.22	0.89	6		✓		✓		6	0.35	0.29	1.51	OD6
OD9	1.74	11.92	0.80	9	[943, 2452]	✓	R32/R134a	✓	[46, 123]	9	0.33	0.28	0.96	OD9
OD12	1.85	12.04	0.81	12		✓		✓		12	0.35	0.29	1.00	OD12
CD6	3.18	10.96	0.74	6		✗		✓		6	0.73	0.60	1.99	CD6
CD9	1.72	10.27	0.69	9		✓		✓		9	0.34	0.28	1.10	CD9
CD12	1.97	9.20	0.62	12		✓		✓		12	0.30	0.24	1.28	CD12
PD6	8.97	63.77	4.29	6		✗		✗		6	3.65	3.02	6.89	PD6
PD9	1.96	12.19	0.82	9		✓		✓		9	0.38	0.31	1.26	PD9
PD12	1.79	10.99	0.74	12		✓		✓		12	0.33	0.28	0.91	PD12

Appendix D. Polynomial models for $\dot{W}_{esp}$ and $\dot{m}_{ref}$

See Tables D.1–D.3.

Table D.1
AHRI 11. Summary table DoE samples results.

Specific energy consumption								mass flow rate						
DoE	MRE (%)	RMSE (W)	CV_{RMSE}(%)	Sample	Range (W)	sign	Fluid	sign	Range (kg/h)	Sample	CV_{RMSE}(%)	RMSE (kg/h)	MRE (%)	DoE
All tests	1.63	8.46	0.54	196		✓		✓		66	0.49	0.57	1.37	All tests
3k	2.37	15.00	0.95	9		✓		✓		9	0.67	0.79	2.24	3k
CCD	2.45	13.59	0.86	9		✓		✓		9	0.63	0.74	1.91	CCD
SCD	2.24	10.76	0.68	7		✓		✓		7	0.65	0.77	1.62	SCD
HD	3.00	12.01	0.76	7		✓		✓		7	0.57	0.67	1.82	HD
CD6	2.55	14.26	0.91	6	[973, 2454]	✓	R410A	✓	[66, 178]	6	0.58	0.68	1.56	CD6
CD9	2.15	10.10	0.64	9		✓		✓		9	0.53	0.63	1.47	CD9
CD12	1.93	10.50	0.67	12		✓		✓		12	0.50	0.59	1.39	CD12
PD6	1.86	15.75	1.00	6		✓		✓		6	0.58	0.68	1.41	PD6
PD9	1.52	10.96	0.70	9		✓		✓		9	0.56	0.65	1.29	PD9
PD12	1.64	10.29	0.65	12		✓		✓		12	0.54	0.63	1.24	PD12
All tests	5.82	27.46	1.77	166		✓		✓		59	0.86	0.70	1.67	All tests
3k	5.40	23.80	1.53	9		✓		✗		9	1.75	1.44	4.89	3k
CCD	4.98	18.75	1.21	9		✓		✗		9	1.53	1.25	3.78	CCD
SCD	5.20	20.11	1.29	7		✗		✗		7	1.45	1.19	3.35	SCD
HD	5.15	21.65	1.39	7		✓		✗		7	1.71	1.41	4.50	HD
CD6	18.42	74.04	4.77	6	[1005, 2607]	✗	R32	✗	[46, 123]	6	1.18	0.97	3.42	CD6
CD9	5.55	30.18	1.94	9		✓		✓		9	0.97	0.80	2.44	CD9
CD12	5.75	29.32	1.89	12		✓		✓		12	0.90	0.74	1.96	CD12
PD6	7.18	71.20	4.58	6		✓		✓		6	1.09	0.90	2.38	PD6
PD9	4.19	37.01	2.38	9		✓		✓		9	1.06	0.87	2.34	PD9
PD12	4.02	36.73	2.36	12		✓		✓		12	0.95	0.78	2.30	PD12
All tests	5.33	15.00	1.01	189		✓		✓		66	0.33	0.29	1.02	All tests
3k	5.90	18.75	1.26	9		✓		✓		9	0.44	0.39	1.58	3k
CCD	5.42	16.75	1.13	9		✓		✓		9	0.45	0.40	1.42	CCD
SCD	5.44	16.38	1.10	7		✓		✓		7	0.46	0.41	1.42	SCD
HD	4.19	16.00	1.08	7		✓		✓		7	0.39	0.35	1.06	HD
CD6	5.29	17.20	1.16	6	[929, 2401]	✓	DR5	✓	[49, 134]	6	0.34	0.30	0.91	CD6
CD9	5.16	15.39	1.04	9		✓		✓		9	0.34	0.30	1.07	CD9
CD12	5.55	16.32	1.10	12		✓		✓		12	0.34	0.30	1.09	CD12
PD6	4.04	22.99	1.55	6		✓		✓		6	0.49	0.43	1.07	PD6
PD9	4.74	18.19	1.22	9		✓		✓		9	0.39	0.35	0.92	PD9
PD12	4.88	16.76	1.13	12		✓		✓		12	0.38	0.34	0.89	PD12
All tests	2.31	7.30	0.52	186		✓		✓		65	0.33	0.27	1.22	All tests
3k	2.31	8.51	0.61	9		✓		✓		9	0.45	0.36	1.24	3k
CCD	2.90	8.54	0.61	9		✓		✓		9	0.44	0.36	1.46	CCD
SCD	2.98	10.98	0.78	7		✓		✗		7	0.69	0.56	3.03	SCD
HD	3.90	11.52	0.82	7		✓		✓		7	0.44	0.36	1.63	HD
CD6	2.97	8.96	0.64	6	[888, 2211]	✓	L41a	✓	[43, 122]	6	0.38	0.31	1.08	CD6
CD9	2.19	7.09	0.51	9		✓		✓		9	0.36	0.29	1.12	CD9
CD12	2.29	7.29	0.52	12		✓		✓		12	0.35	0.28	1.08	CD12
PD6	1.75	10.67	0.76	6		✓		✓		6	0.37	0.30	1.05	PD6
PD9	1.51	9.02	0.64	9		✓		✓		9	0.36	0.29	0.95	PD9
PD12	1.42	8.10	0.58	12		✓		✓		12	0.35	0.28	0.95	PD12

Table D.2
AHRI 21. Summary table DoE samples results.

Specific energy consumption								mass flow rate						
DoE	MRE (%)	RMSE (W)	$CV_{RMSE}(\%)$	Sample	Range (W)	sign	Fluid	sign	Range (kg/h)	Sample	$CV_{RMSE}(\%)$	RMSE (kg/h)	MRE (%)	DoE
All tests	2.84	24.07	0.83	191		✓		✓		63	0.26	0.53	0.85	All tests
3k	3.04	33.42	1.15	9		✓		✓		9	0.29	0.58	1.03	3k
CCD	2.62	28.43	0.98	9		✓		✓		9	0.29	0.58	0.97	CCD
SCD	2.52	27.25	0.94	7		✓		✓		7	0.28	0.57	0.96	SCD
HD	2.36	27.96	0.96	7		✓		✓		7	0.29	0.58	0.89	HD
CD6	3.07	32.57	1.12	6	[1856, 4172]	✓	R404A	✓	[124, 308]	6	0.33	0.67	1.11	CD6
CD9	2.78	29.40	1.01	9		✓		✓		9	0.30	0.60	1.05	CD9
CD12	2.47	27.90	0.96	12		✓		✓		12	0.29	0.59	0.94	CD12
PD6	2.60	34.85	1.20	6		✓		✗		6	0.55	1.12	1.31	PD6
PD9	3.05	30.24	1.04	9		✓		✓		9	0.35	0.70	0.97	PD9
PD12	2.64	29.88	1.03	12		✓		✓		12	0.28	0.56	0.78	PD12
All tests	2.18	16.73	0.68	186		✓		✓		64	0.23	0.29	0.92	All tests
3k	1.89	19.13	0.77	9		✓		✗		9	0.37	0.46	1.03	3k
CCD	1.65	17.25	0.70	9		✓		✓		9	0.32	0.40	0.87	CCD
SCD	1.81	16.80	0.68	7		✓		✗		7	0.34	0.43	0.78	SCD
HD	1.73	19.92	0.80	7		✓		✗		7	0.32	0.41	0.64	HD
CD6	3.46	26.67	1.08	6	[1582, 3615]	✓	ARM31a	✓	[73, 200]	6	0.31	0.39	1.50	CD6
CD9	2.09	18.59	0.75	9		✓		✓		9	0.26	0.33	0.79	CD9
CD12	1.98	20.26	0.82	12		✓		✓		12	0.23	0.30	1.05	CD12
PD6	2.43	29.92	1.21	6		✓		✓		6	0.44	0.56	1.04	PD6
PD9	2.98	25.35	1.02	9		✓		✓		9	0.31	0.39	0.76	PD9
PD12	2.67	23.24	0.94	12		✓		✓		12	0.26	0.32	0.57	PD12
All tests	2.48	20.85	0.77	183		✓		✓		64	0.37	0.50	1.62	All tests
3k	2.33	22.23	0.82	9		✓		✗		9	0.56	0.77	1.56	3k
CCD	1.73	19.05	0.70	9		✓		✗		9	0.46	0.64	1.39	CCD
SCD	2.10	22.10	0.82	7		✓		✓		7	0.40	0.56	1.72	SCD
HD	2.11	25.02	0.92	7		✓		✓		7	0.40	0.55	1.40	HD
CD6	3.81	31.30	1.16	6	[1724, 3988]	✓	D2Y65	✓	[81, 214]	6	0.44	0.61	2.36	CD6
CD9	2.18	22.94	0.85	9		✓		✓		9	0.40	0.55	1.76	CD9
CD12	1.94	22.21	0.82	12		✓		✓		12	0.37	0.51	1.47	CD12
PD6	2.64	41.14	1.52	6		✓		✓		6	0.59	0.81	1.10	PD6
PD9	3.66	31.41	1.16	9		✓		✓		9	0.49	0.67	0.94	PD9
PD12	3.05	27.95	1.03	12		✓		✓		12	0.41	0.56	1.13	PD12
All tests	2.02	16.78	0.68	173		✓		✓		61	0.36	0.41	1.29	All tests
3k	1.78	16.45	0.67	9		✗		✗		9	0.50	0.56	1.13	3k
CCD	1.65	13.14	0.54	9		✓		✗		9	0.45	0.50	1.13	CCD
SCD	3.06	18.90	0.77	7		✓		✓		7	0.37	0.41	1.53	SCD
HD	2.57	19.27	0.79	7		✓		✓		7	0.42	0.47	1.92	HD
CD6	3.73	30.01	1.22	6	[1570, 3655]	✓	L40	✓	[64, 175]	6	0.43	0.47	2.02	CD6
CD9	1.78	17.61	0.72	9		✓		✓		9	0.38	0.42	1.41	CD9
CD12	2.02	17.56	0.72	12		✓		✓		12	0.37	0.41	1.45	CD12
PD6	2.88	38.45	1.57	6		✓		✓		6	0.49	0.54	0.94	PD6
PD9	2.17	25.22	1.03	9		✓		✓		9	0.45	0.50	0.91	PD9
PD12	2.00	23.37	0.95	12		✓		✓		12	0.40	0.44	0.98	PD12
All tests	2.03	22.48	0.84	133		✓		✓		48	0.51	0.61	1.75	All tests
3k	2.84	29.40	1.09	9		✗		✓		9	0.83	0.99	2.85	3k
CCD	3.35	24.12	0.90	9		✓		✓		9	1.37	1.65	4.67	CCD
SCD	6.90	46.35	1.73	7		✓		✓		7	2.24	2.69	8.43	SCD
HD	3.86	28.31	1.05	7		✗		✓		7	0.93	1.12	3.12	HD
CD6	3.29	29.35	1.09	6	[1740, 4268]	✓	R32/R134a	✓	[68, 179]	6	0.56	0.68	2.06	CD6
CD9	2.46	26.24	0.98	9		✓		✓		9	0.54	0.65	1.51	CD9
CD12	1.93	26.26	0.98	12		✓		✓		12	0.54	0.65	1.69	CD12
PD6	3.07	27.93	1.04	6		✓		✓		6	1.38	1.66	2.61	PD6
PD9	2.61	30.48	1.14	9		✓		✓		9	0.64	0.77	1.31	PD9
PD12	2.18	29.79	1.11	12		✓		✓		12	0.56	0.67	1.48	PD12

Table D.3
AHRI 33. Summary table DoE samples results.

Specific energy consumption								mass flow rate						
DoE	MRE (%)	RMSE (W)	CV_{RMSE}(%)	Sample	Range (W)	sign	Fluid	sign	Range (kg/h)	Sample	CV_{RMSE}(%)	RMSE (kg/h)	MRE (%)	DoE
All tests	2.46	12.31	0.79	196		✓		✓		66	0.41	0.49	1.14	All tests
3k	2.33	12.18	0.78	9		✓		✓		9	0.61	0.73	1.80	3k
CCD	2.24	11.60	0.75	9		✓		✓		9	0.58	0.69	1.67	CCD
SCD	2.69	13.76	0.89	7		✓		✓		7	0.54	0.65	1.87	SCD
HD	3.63	16.75	1.08	7		✓		✓		7	0.47	0.57	1.27	HD
CD6	3.29	17.16	1.10	6	[945, 2432]	✓	R410A	✓	[68, 181]	6	0.44	0.53	1.34	CD6
CD9	1.82	11.78	0.76	9		✓		✓		9	0.44	0.53	1.19	CD9
CD12	2.28	13.27	0.85	12		✓		✓		12	0.42	0.51	1.20	CD12
PD6	3.52	32.46	2.09	6		✓		✓		6	0.57	0.69	0.88	PD6
PD9	2.11	17.56	1.13	9		✓		✓		9	0.48	0.57	1.05	PD9
PD12	1.96	16.04	1.03	12		✓		✓		12	0.44	0.53	1.09	PD12
All tests	3.04	16.44	1.11	168		✓		✓		59	0.55	0.46	1.33	All tests
3k	1.77	9.06	0.61	9		✓		✗		9	0.98	0.81	2.82	3k
CCD	1.52	8.11	0.55	9		✓		✗		9	0.87	0.72	2.28	CCD
SCD	2.30	12.06	0.81	7		✓		✗		7	0.67	0.56	1.82	SCD
HD	2.07	10.94	0.74	7		✓		✗		7	0.90	0.74	2.47	HD
CD6	5.85	27.71	1.87	6	[943, 2452]	✗	R32/R134a	✓	[46, 123]	6	0.60	0.50	1.69	CD6
CD9	2.59	15.27	1.03	9		✓		✓		9	0.58	0.48	1.38	CD9
CD12	2.82	16.51	1.11	12		✓		✓		12	0.57	0.47	1.51	CD12
PD6	3.87	33.94	2.28	6		✓		✓		6	0.90	0.74	1.70	PD6
PD9	2.55	19.40	1.31	9		✓		✓		9	0.68	0.56	1.32	PD9
PD12	2.34	18.67	1.26	12		✓		✓		12	0.62	0.51	1.08	PD12

Appendix E. Source code optimal & cluster designs

```
library(AlgDesign)
library(geoR)
library(scales)

#Define data.frame with compressor envelope
df_env = data.frame(Te = c(-24, -24, -15, 2, 2, -12),
                    Tc = c(20, 52, 61, 61, 35, 20)
                    )
#Define number of points to include in the experimental design
test_points = 9

#Extra variables to build the mesh grid inside the compressor envelope
dTe = 1
dTc = 1

#Generate the mesh grid
grid = polygrid(xgrid = seq(from = min(df_env$Te), to = max(df_env$Te), by = dTe),
                ygrid = seq(from = min(df_env$Tc), to = max(df_env$Tc), by = dTc),
                borders = df_env
                ) ; names(grid) = c("Te", "Tc")

#Rescale Tc, Te coordinates to -1, 1 range
grid$Te_N = rescale(grid$Te, to = c(-1,1))
grid$Tc_N = rescale(grid$Tc, to = c(-1,1))

#Optimal design Fedorov D-optimal criteria
Fedorov = optFederov(~Te_N + Tc_N + I(Te_N^2) + I(Tc_N^2) + Te_N:Tc_N,
                     data = grid,
                     nTrials = test_points,
                     criterion = 'D'
                     )

df_Fedorov  = Fedorov$design[, c("Te", "Tc")]

#Cluster design
cluster = kmeans(x = grid[, c("Te_N", c("Tc_N"))],
                 centers = test_points,
                 iter.max = 10000,
                 nstart =  nrow(grid)
                 )


df_cluster = data.frame(Te = rescale(c(-1, 1, cluster$centers[, "Te_N"]),
                        to = c(min(df_env$Te), max(df_env$Te)))[-c(1:2)],
                        Tc = rescale(c(-1, 1, cluster$centers[, "Tc_N"]),
                        to = c(min(df_env$Tc), max(df_env$Tc)))[-c(1:2)]
                        )


#Experimental matrices
print(df_Fedorov, row.names = F)


##  Te  Tc
## -24  20
## -12  20
##   02  35
## -24  36
## -11  42
## -11  43
## -24  52
## -15  61
##   02  61


print(df_cluster, row.names = F)


##      Te   Tc
## -01.59  41.8
## -20.31  37.8
## -13.36  27.2
## -11.42  42.2
## -06.23  32.6
## -02.00  55.0
## -19.40  50.2
## -11.11  55.3
## -20.73  25.3


#Model adjustment. The df data.frame includes the experimental results
#(including the experimental error)

Wc = lm(data = df, formula = Wc ~ Te + Tc + I(Te^2) + I(Tc^2) + Te:Tc,
        weights = 1/(df$Wc_error^2)
        )

m = lm(data = df, formula = m ~ Te + Tc + I(Te^2) + I(Tc^2) + Te:Tc,
       weights = 1/(df$m_error^2)
       )
```

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