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# Perspectives of Experts in the Selection of Contents for a Course of Differential Equations in Engineering

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## Abstract

This study investigates the teaching of Differential Equations in engineering education, focusing on expert opinions from diverse institutional backgrounds. The primary objectives were to understand expert perspectives on essential course content, justify their content choices, measure institutional gaps, and highlight the range of professional profiles. Through qualitative interviews with nine experts from academia and industry, we explored their views on the integration of Differential Equations in engineering curricula. The findings reveal a consensus on the importance of basic concepts to support physical models. Notably, experts highlighted variational calculus and energy representations as critical yet underrepresented areas in Differential Equations courses, essential for modeling complex, nonlinear problems. The study also states on the growing importance of computational methods and software in engineering education, advocating for an integrated approach aligning theoretical concepts with real-world applications. The expert opinions revealed a significant institutional influence on content perception, demonstrating a gap between traditional theoretical focus and modern, application-oriented approaches. The study suggests a need for curricular innovation in differential equations education, emphasizing practical applications and computational methods to bridge this gap between the two institutions considered in which the experts are adhered to.

**Keywords:** Anthropological Theory of Didactics; Differential Equations; Organizations; Teaching; Institutional Gap.

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## 1. Introduction

Differential Equations represent a field of applied mathematics of significant importance in engineering education. A sound understanding of this subject area leads to an enhanced ability among students to comprehend the physics of continuous media, which is vital in mechanical, industrial, aeronautical, and related engineering disciplines. Attaining fundamental competencies in the physical interpretation of Differential Equations directly equips future engineers with the capacity to model the reality surrounding them that connects with continuum media like fluid or solid mechanics.

Differential Equations are typically taught in intermediate courses, typically in the second year of engineering programs. Consequently, students possess a solid foundation in mathematics and physics, allowing them to approach Differential Equations from a relational perspective, seeking connections between mathematics, models, and the physics of continuous media. We can argue that the scientific maturity resulting from this connection via Differential Equations constitutes a convergence point where the basic quantitative and static notion of mathematics, previously grounded in analysis, calculus, algebra, and geometry, is extended. In Differential Equations, we encounter a significant area associated with modeling continuous objects and understanding the physical laws that culminate in the formulation of a Differential Equation. The techniques and solutions thereof must be scrutinized critically. This is likely the first area of mathematics where students can observe Blum and Leiss's (2007) modeling cycle. According to this cycle, it is particularly important to comment on how the mental representation of reality is established as a mathematical model. This is where Differential Equations find their primary purpose in an engineering context. Subsequent mathematical work typically requires the application of suitable techniques for solving the formulated equation, whether analytically and/or numerically. However, the work of an engineer does not conclude there; it is important to interpret the solution and to identify its limitations (generally due to assumptions made during the resolution phase). Finally, the validity or invalidity of the solution depends on an evaluation of its calibration with reality, which determines whether the assumptions made and the limitations identified are acceptable in the real-world scenario being modeled.

Additionally, we should mention other authors who have addressed the need for mathematical modeling from a tangible perspective. Freudenthal (1983) proposed the idea of inversion and conversion to describe how a universal mathematical model can be constructed based on tangible and real experiences. He emphasized that the mathematical model evolves into a more abstract conceptual form as it is better understood and gradually eliminates initial limitations. It is worth noting that models leading to mathematical equations are, above all, dynamic and subject to continuous improvement by the scientific community. Notable examples include the Navier-Stokes equations describing fluid mechanics, proposed by their creators over two centuries ago. A minimal scientific search reveals that these equations are still being adapted to various realities, with key terms being modified, making the equations a living entity (as seen in microfluidics or non-Newtonian fluids).

Furthermore, as a theoretical pedagogical reference in this work, we consider the Anthropological Theory of Didactics (ATD), as proposed by Chevallard (1986). This theory has provided valuable insights into the didactics of mathematics and into its surrounding environment, emphasizing the significant roles of educators, institutions, praxeologies, and society. It essentially shapes the formation of a Noosphere around mathematics and its didactics. It is worth noting that Baquero, Bosch, and Gascón (2007) contend that in university education, mathematical science is often constructed in isolation and loosely connected to the model and its ideal or real representation it aims to satisfy. This notion promotes the prominence of mathematics as a self-sufficient science based on theoretical or algorithmic concepts, with limited grounding in the reality being modeled, and to which future engineers cannot remain indifferent.

Throughout this paper, we aim to establish a fundamental understanding of the topics taught in a Differential Equations course in various engineering studies. To achieve this, we will employ the techniques provided by ATD. We seek to understand the perceptions of different educators and professionals regarding what should be included in a Differential Equations course in engineering. Consequently, we will analyze institutional and professional influences and biases by examining the responses provided by a group of highly qualified experts with scientific and technical backgrounds. Our focus of study is particularly relevant given the fact that teachers who often teach ordinary differential equations are engineers or mathematicians with limited knowledge in didactics (Lozada et al., 2021). There exists a call for these teachers to adopt diverse teaching methodologies, to innovate in contents and to introduce experiment in their classes to better support students in meeting academic and professional challenges (Lozada et al., 2021).

## 2. Theoretical Background

The increasing integration of computing in engineering applications has significantly transformed the approach to deriving mathematical models of real-world phenomena, as noted by Weintrop et al. (2016) and Devany et al. (2020). This technological shift raises questions about the alignment of current teaching methods in differential equations, typically offered as service courses in mathematics, with some basics modeling requirements of engineering applications. Acording to Lozada et al. (2021), such modeling exercises correspond with basic laws concerning biomathematics, electrics, mechanics, kinematics and quemical kinetics. The extent to which these educational approaches supported by basic models from applied sciences in Differential Equations facilitate transferable skills for students remains uncertain, as discussed in the works of Schumacher & Siegel (2015), Neuper (2017) and Pennell et al. (2009). This uncertainty calls for a reassessment of curriculum and teaching methodologies to better bridge the gap between theoretical mathematical concepts and their practical application in engineering. It suggests the need for a more integrated educational approach, where the teaching of differential equations is closely aligned with real-world engineering problems.

The teaching of Differential Equations typically commences with the delivery of theoretical information accompanied by standard examples and questions closely related to the presented theory. Some previous research indicates that the way students frame problems in engineering could be linked to their understanding of the usefulness of Differential Equations concepts, methods, and tools (Crandall, 2022). Furthermore, the application of Differential Equations knowledge in engineering contexts might be influenced by factors like the degree to which mathematics and engineering fields allow for the selection of appropriate tools, the separation of knowledge domains, and the fact that many undergraduate engineering students are still acquiring fundamental algebra and calculus skills during and after their studies (Crandall, 2022).

Subsequently, students often work independently on solving typical problems, similar to those covered by the instructor in class, with perhaps a brief reference to immediate physics models (Widjaja, Dolk, Fauzan, 2010). This may lead to an approach to learning Differential Equations that is somewhat detached from their contextual nature in engineering, i.e., their capacity to model real situations and foster the critical thinking of future engineers. Nonetheless, the importance of introducing mathematical knowledge in a natural and relatable manner remains a key area of study and research in the present day and has indeed been a driving force in didactic and pedagogical scientific activity (Lozada et al. 2021). For instance, consider the elementary education example introduced by Fauzan, Plomp, and Gravemeijer (2013) or the early works of Chevallard (1986), which aimed to align mathematics didactics with the natural way people learn this science, in connection with its epistemological and ultimate natures. Certainly, building new pedagogical approaches to Differential Equation are required with focus on social constructivism and students natural learning. Rasmussen & Keene (2019) explored five distinct ways students use the concept of rate of change to solve first order autonomous differential equations graphically identifying five "waypoints" in student reasoning about rate of change. These waypoints allow educators to approach the students' mathematical understanding and hence to promote learning-oriented pedagogies. In addition, we remark the study by Ortiz, Camacho & Velasco (2010) where the authors examined the challenges engineering students faced in interpreting the solutions of Ordinary Differential Equations in both graphical and algebraic forms. Their findings indicated that although students were proficient in algebraic methods for solving ODEs, they struggled to use these methods to understand solution behaviors in a graphical context. Additionally, when working with direction fields to analyze ODE solutions, their focus was limited to small, localized areas of the field, indicating a limited scope in their interpretative approach (Ortiz, Camacho & Velasco, 2010). The study of Zeynivandnezhad & Bates (2018) revealed that students employed diverse, non-sequential mathematical thinking processes when learning Differential Equations. The authors proposed the importance of supporting the continued use of technology, like computer algebra systems, in undergraduate mathematics education and suggested further research to explore the various processes students use in mathematical problem-solving (Zeynivandnezhad & Bates, 2018).

In addition to the topics mentioned, we shall remark that the learning and teaching of Differential Equations in engineering studies are heavily influenced by institutional criteria, as exemplified by Zhao (2022) (that connects with institutions like Massachusetts Institute of Technology) which, in turn, shape the curriculum approved by each institution with competent academic and quality authorities. If we consider a perspective close to didactic transposition, we can understand that all scientific knowledge (in this case, Differential Equations) is conditioned by social, professional, and institutional considerations. Throughout this work, we will observe how the transposition of content in a Differential Equations course is significantly influenced by the social, professional, and institutional environment, as each institution has a unique working philosophy, certain praxeologies, and, ultimately, an epistemology. Initially, within the field of engineering, we can think of two distinct institutions: the University and the Enterprise, each of which plays a significant role in shaping engineers. On one hand, we observe the social significance of the University as a center of education and knowledge reference, where future engineers have their first encounter with engineering. This initial exposure is critical in shaping their perception of engineering, which will accompany them, at least during the early years of their professional life. On the other hand, we can think of a second institution, the Enterprise, where engineers develop and shape themselves professionally by concretely and economically executing projects that respond to a societal demand that the Enterprise seeks to satisfy. Both institutions are epistemologically different; their intentions and praxeological environments vary. This aspect leads to an institutional gap or norm that is challenging to measure but necessary to establish clear strategies that foster their alignment beyond good intentions, which in many cases manifest as efforts external to the classroom. Bridging this gap would entail an optimal understanding between both institutions, with the University generating well-established knowledge and the Enterprise applying that knowledge to provide services that meet societal needs.

Considering these two vastly different institutions necessitates a scientific theory characterized by its multidimensionality and the establishment of metric principles to guide our research. For this reason, we believe that Chevallard's Anthropological Theory of Didactics (ATD) (1986, 1999) serves as a well-established theoretical reference from which to analyze our problem related to different contents perspectives in a Differential Equations course. ATD has been extensively used to analyze the realities that influence mathematics and its didactics and, more recently, the development of mathematical models used in applied sciences (Barquero, et al. 2007). A key aspect of ATD concerns the importance it places on constructing a mathematical representation in the students' minds that aligns with an intuitive understanding. It fosters a natural and constructivist acquisition of knowledge. The development of a didactic model and its subsequent use for meaningful learning is crucial (Gascón, 2001). From this perspective, we provide a common thread for Differential Equations and their didactics, where the theories proposed by ATD can be applied, even if they have not been extensively applied to this branch of mathematics before. Under this context, it is key to consider the idea of modeling as relevant for extending ATD into Differential Equations. Indeed, we interpret modeling as a human attitude (in the sense of the ATD) aimed at rigorously understanding reality, with the model being intrinsic to mathematics (stemming from its theoretical concepts) or

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extrinsic to it (rooted in applied sciences or engineering, as is the case here). ATD posits that a given Institution I is understood as a social organization formed by professionals who possess a specific way of thinking and doing (Chevallard, 2003, p.82). Members associated with Institution I carry out their activities based on a set of praxeologies (work methods and task typologies) typical of that institution. Additionally, the technological foundation associated with these activities, necessary for performing the tasks, justifies the choice of specific techniques over others. The technological framework that an institution provides to its members significantly shapes the praxeologies and, in each professional, defines a different way of understanding a particular science. This applies to the case of Differential Equations, which exhibits various perspectives depending on the members belonging to different institutions subject to different epistemological paradigms. An such variations in the epistemological perceptions towards Differential Equations have not been measured before, being hence a motivation for our presented study. Another relevant concept introduced by Chevallard (2001) is the idea of isomorphism to relate the themes and didactic organizations through hierarchies of study that link didactics with the topics and content to be developed. Focusing purely on the social hierarchy, we question how society, with its knowledge-demanding needs, directly influences the themes (contents) that must be developed in institutions and how these themes eventually become didactic knowledge.

## 3. Objectives

The pursued objectives are summarized as follows:

1. To understand the different opinions of experts affiliated with various institutions regarding the content that should be taught in a Differential Equations course in engineering programs.
2. To ascertain a minimal justification for why experts consider the identified content should be taught. In this way, we seek to determine if there is an influence from the institution and the professional trajectory of the experts on their perception of the content that a Differential Equations course should cover.

3. Based on the last objective, to measure the institutional gap in connection with the contents to be taught in a course of Differential Equations.
4. To highlight the opinions of different experts with varying professional profiles.

## 4. Methodology

The first step undertaken was the selection of institutions for our study. The Polytechnics University of Madrid was chosen as Institution I1, given its offering of a wide range of engineering studies and its extensive history in education. On the other hand, the technological company Airbus Group was selected as Institution I2, as it maintains a complete department focused on the modeling of continuous media: aerodynamics, fluids, structures, etc. It is worth noting that there are connections between these two institutions through collaborations such as the creation of chairs, research groups, and scholarship programs for students. Furthermore, Institution I2 often serves as the place where graduates from Institution I1 commence their professional careers. Considering the fact that only two institutions have been selected, the generalizability of the study's findings must be further discussed. Indeed, the representativeness of these two institutions, which are significant in their respective fields, lends weight to the external validity of the research. Hence, our findings can provide valuable insights provided that we consider similar educational and industrial contexts compared to the Institutions I1 and I2, so that we can reliably admit the transferability of the outcomes to other settings. The robustness of the research methodology followed based on a qualitative approach (that will be described after) is also important in ensuring that the results are reliable and valid (Gerring, 2017). We claim generability under the following conditions:

- To generalize to a university other than the Polytechnic University of Madrid, it is necessary to consider that such a university should have properties similar to the Polytechnic University of Madrid. This implies paying attention to the size of the university, the social and cultural context in which it is immersed, the number of students, its penetration in society, and its social impact. These details can be observed directly on the website of the Polytechnic University of Madrid.

- To generalize to a company other than Airbus Group, attention should be paid to aspects related to the size of the company, the number of employees, the presence of a department focused on the modeling of continuous media, and its focus on markedly technological aspects.

Once the institutions associated with our study were chosen and the generability of our research discussed, we aimed to identify a group of experts representative of each institution. A rubric, validated by peers and logical content analysis, was developed. This rubric considered the following search criteria:

- Job function and years of experience in roles related to the modeling of continuous systems.
- In the case of experts in the academic domain, the number of scientific contributions indexed in prestigious databases (mainly Scopus and Web of Science) was requested.
- For those not associated with the academic domain, an approximate number of engineering projects in which they had participated was requested. These projects needed to have a minimum duration of three months and focus on the modeling of continuous systems.
- In the case of those not involved in academia, a minimum of one year of experience as training instructors in non-academic institutions, such as companies, institutes or training centers, was required.
- Details about projects in which the expert had participated, supported by external funding from sources like state, regional, European, or foundation grants.

Considering the aforementioned institutions and the rubric provided, we conducted a search for experts via Google Scholar, ResearchGate, and LinkedIn. It is important to mention that the primary objective is to capture the opinions of highly qualified experts, hence the sample size is relatively small. In this regard, it is worth noting that the use of small samples for comparing responses among experts has been considered scientifically acceptable within the ATD framework in studies such as Artigue and Winsløw (2010) and Trouche, et al. (2019) regarding mathematical curriculum. Additionally, in qualitative research, the common practice of working with a smaller number of participants, often referred to as a "low number of sampling," is influenced by several key factors that distinguish it fundamentally from quantitative research methodologies. One of the primary reasons for this approach is the focus on depth over breadth. Qualitative research aims to obtain

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detailed and rich data, prioritizing the depth and quality of information over the quantity. This approach allows us to delve into more context-specific information, which is often lost in larger samples. This depth of data is important in qualitative studies, as explained by Creswell and Poth (2018), who emphasize the importance of detailed exploration in understanding complex phenomena. Another significant aspect of qualitative research is its resource intensiveness. Methods such as interviews, ethnographies, and focus groups are time-consuming and require considerable resources for data collection, transcription, analysis, and interpretation. A smaller sample size makes this intensive process more manageable and ensures that the collected data can be analyzed thoroughly. Merriam and Tisdell (2015) highlight this aspect, noting the importance of manageable data volumes for effective qualitative analysis. Qualitative research often targets specific, niche populations or phenomena, where a smaller, specialized sample is more appropriate and feasible than a large, random one. This specificity, as Patton (2015) points out, allows for a more detailed exploration of the phenomena of interest, which is particularly important in studies aiming to understand unique or specialized contexts. Lastly, the inductive nature of qualitative research focuses on generating insights and theories from the data, as opposed to testing hypotheses. This approach, as described by Charmaz (2006), is more suited to a smaller, more focused sample that allows for in-depth exploration and theory development. It enables researchers to build theories grounded in the rich, detailed data that qualitative methods are known to provide.

Nonetheless, we shall argue on the potential limitations introduced by considering a small number of highly qualified experts. One of the primary limitations is the risk of large differences between the sample and the population. Small samples often do not adequately represent the broader population, making it difficult to generalize findings with a high degree of confidence (Stuart *et al.*, 2017; Westfall *et al.*, 2014). Hence, to avoid this burden, we shall consider that our research is applicable under the concept of similar institutions (as discussed previously), and under similar professional profiles within the institutions. Finding profiles similar to the experts mentioned in our study is not difficult as all professors in university institutions are evaluated under the same metrics by the quality agencies paying attention to professors' teaching and research mainly. However, this is not the case if we consider a business institution, where there will be different paths among professionals. In this latter case, we indicate that our study will be valid considering that the professionals have similar curricular lines (these curricular lines will be presented in the results section).

Following the outlined principles, we followed a qualitative interview technique, as it allows for an open and comfortable environment in which experts can reflect upon and express their opinions based on one or more guiding questions. A total of nine experts from both institutions, working in different environments but consistently with models related to Differential Equations, were interviewed. Four of these experts came from Institution I1. They possess notable teaching experience, with over 400 hours of teaching in applied mathematics, including Differential Equations. On average, they have 24 scientific contributions indexed in quality databases (Scopus and Web of Science), and they have contributed to or served as principal investigators on an average of four projects with external funding beyond Institution I1. In addition, we counted with five experts from Institution I2. It is important to mention that these experts are graduates who were former students of Institution I1 and work in the field of fluid modeling and fluid-structure interaction. Their average professional experience is five years, and, on average, they have worked on a total of nine projects with a minimum duration of three months. Notably, one of the experts from Institution I2 also serves as an associate professor at Institution I1, which is a significant detail as it provides a strong point of comparison to understand the disparities and constraints imposed on the same professional by each of the mentioned institutions.

Under the qualitative research involving interviews with experts, ethical considerations are important in order to ensure informed consent and confidentiality of participants (Flick, 2018). All the participants were previously informed about the study's nature, objectives and methods before they agree to take part. This involved providing them with clear and understandable information, emphasizing that their participation was voluntary and that they could withdraw at any time without any repercussions. It is also important to remark that informed consent was considered as an ongoing process, not just a one-time agreement, and was maintained throughout the research. Another key ethical issue involved safeguarding the information provided by participants. In the course of our research, we ensured the confidentiality of personal data and interview details. In this publication, identities of the participants have been anonymized to safeguard their privacy. This commitment to confidentiality fostered a trustful environment, encouraging them to share openly and honestly. The management and safeguarding of data in this study adhered to the institutional guidelines of the author's university, focusing on the security of sensitive information. We employed a dual password protection system, involving both the author's institutional access credentials and a secure key for a privately hosted remote server maintained by the author's institution.

### 4.1. Expert Profiles

In the following lines, a detailed description of the experts considered for our study is provided, based on the information extracted from the rubric.

Expert 1 (E1):

Expert E1 has an important academic profile with a profound knowledge of Differential Equations and their applications in continuous media. His/her main research focus lies in finding solutions to various mathematical-physics models formulated in terms of Differential Equations. He/she has published works related to nanofluids, combustion, and asymptotic and numerical methods for solving Differential Equations. His/her academic position is that of a University Professor. In summary, his/her professional trajectory includes:

- Aeronautical engineer, with a Ph.D. in the same field.
- University Professor.
- Numerous publications on the modeling of continuous systems.
- Participation in externally funded projects from both the public sector, foundations, and companies.

It is observed that the relationship of Expert E1 with Differential Equations is strongly influenced by the working methods of Institution I1.

Experts 2 (E2), 3 (E3), and 4 (E4):

These experts are grouped together due to their similar academic backgrounds. They form a group of experts with extensive experience in the academic domain at Institution I1 as educators and researchers in the field of modeling continuous systems. They specifically work on turbulence, combustion, nanofluids, and jet dynamics. Although they are strongly influenced by Institution I1, they occasionally collaborate with other corporate institutions as consulting experts.

- Aeronautical engineers and Ph.D. holders in the same field.
- Associate Professors at the University.

- Occasional collaborations with other institutions, with a strong connection to Institution I1.

Expert 5 (E5):

Expert E5 is an aeronautical engineer, holding a Ph.D. in technology and sciences. His/her primary professional role is within Institution I2, where he/she is responsible for simulation and modeling of fluid systems. Additionally, he/she has worked as a par time lecturer in applied mathematics (including Differential Equations) at various institutions, including Institution I1. In summary:

- A background in aeronautical engineering and a Ph.D. in technology and sciences.
- Responsibilities as an engineer in charge of simulating fluid systems.
- A primary focus on Institution I2, with concurrent roles as a part time lecturer at Institution I1.

The personal relationship of this expert is particularly interesting as he/she has professional experience in both institutions, which can shed light on how the same scientific area is developed in both institutions, the contradictions and restrictions imposed on the same professional by each of these institutions, and how they can be integrated.

Experts 6 (E6), 7 (E7), 8 (E8), and 9 (E9):

Within this group of experts, a diverse foundational background is presented. Experts 6 and 7 are Aeronautical Engineers, Expert 8 is an Industrial Engineer, and Expert 9 is a Civil Engineer, all of them previously trained at Institution I1. Importantly, since they completed their university studies, they have had no further contact with the university. Therefore, they are a group of professionals strongly influenced by Institution I2, where they work in the department of continuous systems modeling involving fluid-structure interaction. The trajectory of this group of experts can be summarized as follows:

- Engineers with backgrounds in Industrial, Civil, and Aeronautical Engineering.
- No contact with university institutions after finalizing their studies and exclusively carrying out their work at Institution I2.

- Experience as instructors within Institution I2, where they have conducted internal courses on continuous systems modeling. This point is noteworthy because they possess a minimum level of critical thinking and experience that allows them to participate in our study.

### 4.2. Questions to Experts

For each of the experts, as mentioned earlier, the interview methodology was followed, focusing on the formulation of two sequential questions. The interviews lasted for a maximum of 45 minutes. A cycle of interviews was conducted, involving either a single interviewee or multiple. Given the previously mentioned curricular similarities, the interviews were conducted in accordance with the division of experts as indicated. The two guiding questions introduced sequentially were:

- Question 1: What content do you believe should be taught in an Engineering Differential Equations course?

- Question 2: Why that content?

These questions are open-ended to avoid conditioning the potential responses of the experts. It is important to note that the subjectivity in this type of interview process is common, and rather than being a limitation, it has been viewed as a source of richness, a way to connect with the personal experiences of each interviewee concerning Differential Equations and their noosphere (the sphere of human thought and knowledge). Subsequently, a particular and comparative analysis was conducted for each of the contributions of the interviewees with the aim of drawing conclusions regarding the guiding question, especially concerning institutional and personal distances.

## 5. Results

To compile the interview data, we began with the transcription of interviews, creating a detailed, verbatim written record of each conversation (Flick, 2014). Following transcription, the data underwent thematic analysis, a method that identifies and categorizes recurring themes within the data. This technique is particularly useful in discussing the complexity and varied contexts of interview data (Flick, 2014). The next phase involved a thorough familiarization with the data. Multiple readings of the transcripts were undertaken to gain a deep understanding of the content and context of each interview. A systematic coding of the data was followed, where significant or interesting features within each transcript were identified and labeled. These initial codes served as the foundational elements for identifying potential themes. The process of clustering related codes into coherent patterns was undertaken to form these themes, each representing broader ideas or concepts emerging from the data. A review and refinement of these potential themes were then conducted. This step ensured that the themes were consistent with the coded extracts and accurately represented the entire data set. The coherence and representation of each theme were critically assessed and adjusted as necessary. After ensuring the themes accurately reflected the underlying data, each theme was clearly defined and named. This phase was essential for the clarity and precision of the thematic representation. The final stage of the thematic analysis involved compiling the findings into a concise text. This text presented a coherent and logical narrative, supported by relevant data extracts, effectively illustrating the identified themes and patterns within the interview data. The final text was sent to the experts who participated in the interviews with the intention of obtaining their impressions and possible comments. Hence, we ensured that the experts validated the results and that the final text succinctly captured the themes present in the interviews. Throughout this process, maintaining objectivity is important to ensure that conclusions are firmly rooted in the data, minimizing personal biases (Flick, 2014). The ultimate goal is to distill the large amount of data into manageable, comprehensible information, identifying patterns, and deriving inferences or conclusions (Flick, 2014).

We present the summarized text after the thematic analysis for each expert (or group of experts), based on the text agreed upon with each interviewee.

**5.1. Responses to Question 1**

Expert E1:

Ordinary Linear Equations, Systems of Equations, Fourier Series, Sturm-Liouville Problems.

Experts E2 and E3 (interviewed together):

We believe that classic topics should be included, along with variational calculus due to its utility in energy-based formulations.

Expert E4:

First-order linear and nonlinear equations, second-order and "n"-order equations. Qualitative analysis of solutions and their regularity. Self-similar solutions. Separation of Variables method.

Expert E5:

It is important to pay attention to modeling. Continuous models. Types of differential equations and their solutions. Perturbation method. Computational methods for nonlinear problems. Optimization and finding optimal solutions.

Experts E6 and E7 (interviewed together):

We believe that students should learn classical models because every modeling engineer should start from them to understand the physics of the problem. Afterward, they should be able to find solutions and discuss their physics using computer-aided methods and finite elements.

Experts E8 and E9 (interviewed together):

Every engineer should begin with real-world problems and understand where they come from and what they respond to. In engineering, it is of little use to know how to solve equations "by hand" if you do not understand their origin and meaning. Knowing this, the analysis of perturbations and the use of computational software should be studied as subjects to be taught.

**5.2. Responses to Question 2:**

Expert E1:

I believe that the Bologna Plan (This is a European Plan of common application in Spain) has significantly reduced the teaching hours for Differential Equations, thus diminishing powerful tools such as Fourier analysis and Laplace transforms, which were more extensively studied before the Bologna reforms.

Experts E2 and E3 (interviewed together):

Variational calculus is important for energy-based formulations that can be employed in computational calculations.

Expert E4:

I think it is essential to emphasize qualitative methods as they often provide more intuitive insights into the problem. Additionally, self-similar solutions are a simple tool for understanding the dynamics of equations and can complement the Separation of Variables method.

Expert E5:

Because it is necessary for engineers to learn how to model and subsequently solve problems using computational methods. It is also important to be able to find the optimal solution in terms of economics and effort.

Experts E6 and E7 (interviewed together):

We believe that students should be well-versed in classical models as every modeling engineer should start with them to comprehend the physics of the problem. Afterward, they should be capable of finding solutions and discussing their physics using computer-assisted methods and finite elements.

Experts E8 and E9 (interviewed together):

Every engineer should begin with real-world problems and understand their origins. In engineering, it is not very useful to know how to solve equations "by hand" if you do not understand where they come from and what they respond to. If you understand this, it will be more intuitive to seek solutions. In this context, perturbation analysis and the use of computational software should be studied as the content to be imparted.

## 6. Discussion

One relevant aspect in classifying the responses is to categorize the teaching perspective that has dominated the answers. To adequately code these perspectives, we have employed Gascón's (2001) theory of teaching moments. This theory is based on different epistemological conceptions of mathematics (in our case, Differential Equations) and aims to construct a category of educators whose activities will be strongly influenced by their epistemological approach towards Differential

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Equations. A teacher who conceives this science from an experimental perspective, closely related to reality, sees Differential Equations epistemologically differently than a purely theoretical teacher or one focused solely on typical equation-solving methods.

Regarding the responses to Question 1, there is an initial comparison between Experts E1, E2, E3, and E4 (members of Institution I1) and Experts E5, E6, E7, E8, and E9 (members of Institution I2). This initial comparison might seem trivial as it establishes the fact that experts associated with Institution I1 consider the content to be taught as intrinsic to Differential Equations. They propose content derived from the theory and specific exercises of these equations. Therefore, in a preliminary approach, experts from Institution I1 present content influenced by theoretical and algorithmic teaching perspectives (Gascón, 2001). This approach has been already discussed in the literature as stated in the study by Lozada *et al*. (2021) where the tradicional teaching and learning methodology is still presented in a considerable extend (see as Arslan 2010, 2010b). In contrast, experts from Institution I2 emphasize the need to introduce models that enable the understanding of different types of equations and their solutions. They do not focus as much on classic algorithmic techniques for solving equations but rather on the use of qualitative and computational tools that help comprehend solution behavior with the aim of selecting the most optimal solution based on economic considerations in a specific project context. This approach to the teaching and learning has been already discussed in the literature and a complete analysis is provided in Section 7 Lozada *et al*. (2021). From this perspective, there is room for quasi-empirical epistemologies of Differential Equations that encourage exploratory activities regarding models and their solutions in the line of Czocher (2017). Following Gascón (2001), a modern teaching perspective can be established, possibly influenced by constructivist approaches since knowledge can be generated from situations closely related to reality (Czocher, 2017), which are more attractive and challenging for engineering students.

On the other hand, it is interesting to delve into the responses to Question 2, as certain technological justifications for the proposed content are introduced. Notably, Expert E1 highlights the importance of continuing to teach theoretical material about Fourier analysis and Laplace transforms. Their primary justification is based on a comparison with curricula before the implementation of the Bologna Plan in tertiary education. However, within the same Institution I1, we observe a different rationale in the responses of Experts E2 and E3, who advocate the introduction of Variational Calculus as a theoretical tool to incorporate energy-based and computational methods in Differential Equations. This perspective is more aligned with the views of experts from Institution I2. Nevertheless, it does not specify the purpose of introducing energy-based and computational methods, and it is in this purpose where we find a more profound justification in the experts from Institution I2, as we will see later. Additionally, the response of Expert E4 suggests the importance of developing intuition on a larger scale through qualitative analysis of solutions and self-similar solutions. This expert is more concerned with understanding the dynamics of solutions than with obtaining the solutions themselves. This conception has been explored in the existing litetature as reported in Lozada *et al*. (2021). Furthermore, he argues that knowledge of these dynamics will further develop students' intuition towards the problem at hand, which can indeed have a positive impact on the development of physical intuition typically characteristic of engineering professionals.

Regarding the responses to Question 2 provided by experts affiliated with Institution I2, Expert E5 encapsulates the justification upheld by the teaching perspectives of this group of experts. Specifically, the importance of proposing models through Differential Equations that can provide solutions to real-world challenges and finding precise solutions through computational strategies with a focus on selecting the optimal solution that also takes into account economic and effort-related factors. Here, computational strategies allow modifying parameters in the model equations and understanding how they influence the solutions. The variation of these parameters can respond to economic or other questions, enabling a direct comparison through graphs or three-dimensional digital representations of reality. It is also interesting to note that in the responses of Experts E6 and E7, references are made to the knowledge of classical models in mathematical physics and the importance of interpreting more complex models based on these fundamentals. This is an intriguing teaching perspective with strong links to constructivism, as it starts from simpler models that can be intuitively understood as a preliminary step to more comprehensive models. This is, in fact, the underlying idea that permeates all scientific progress. Finally, explicit references to the reality to be modeled are once again found in the responses of Experts E8 and E9. They stress the importance of understanding solutions with an intuitive perspective. They emphasize the perturbations that a problem may have and the need to introduce knowledge of computational software as an alternative to solutions obtained through standard manual procedures.

Now we aim to analyze our findings by comparing them with those existing in the literature. To do this, we focus on section 7.2 of Lozada et al. (2021), where a bibliographic search is conducted to answer the question *What Topics of Ordinary Differential Equations Have Been Explored in the Previous Studies?*, which has an intrinsic connection with our study. The summary provided by Lozada et al. (2021) of 120 selected articles on the teaching of Differential Equations provides a diverse array of thematic groups, each addressing different facets of this mathematical field. The first group focuses on the basic concepts of Differential Equations, where studies delve into the foundational definitions and solutions. This includes analysis of students' understanding and their approaches to solving, with a specific emphasis on both analytic and qualitative methods. Notable within this category are investigations into how students' responses evolve in solving ODEs, their grasp of equilibrium solutions, and their comprehension of graphical and numerical solutions. Another significant theme reported by Lozada et al. (2021) revolves around biomathematical models, highlighting articles that introduce differential equation-based models with applications to biomathematics. This encompasses a range of topics from population growth to epidemic transmission models, with a strong emphasis on the qualitative analysis of solutions. Key models discussed include scalar models like the Malthus, Gompertz, and Verhulst models, along with systems of differential equations such as Lotka-Volterra. As stated in Lozada et al. (2021) scalar-based models form another category, featuring works that employ scalar differential equations to elucidate physical realities. This segment covers a spectrum of models based on first-order scalar equations, such as freefall, Newton's law of cooling, and Kirchoff and Ohm laws. In addition, Lozada et al. (2021) states that the theme of systems based on mechanical theory concentrates on second-order systems arising in Mechanical Vibration Theory. This includes models like a two-mass two-spring vibration system, where concepts like amplitude, modes of vibration, period, and frequency are central. Lastly, Lozada et al. (2021) reports that a group of studies is dedicated to exploring other concepts in differential equations, encompassing studies on a variety of topics like theorems of existence and uniqueness, Laplace transform, and bifurcation concepts. These studies delve into the more complex aspects of differential equations and their varied applications.

Now, comparing the thematic groups identified in the 120 selected articles as discussed by Lozada et al. (2021) with the contents suggested by the nine experts reveals some key similarities and differences in the approach to teaching differential equations. Let us split them based on their thematic scopes:

- Basic Concepts and Classical Models: The articles discussed by Lozada et al. (2021) and the experts of our study agree on the importance of basic concepts in Differential Equations. Experts E1, E4, and E6-E7 emphasize classical models and basic types of equations (linear, nonlinear, first-order, second-order, etc.), which align with the "Basic Concepts of Ordinary Differential Equations" theme in the articles discussed by Lozada et al. (2021). This indicates a consensus on the importance of foundational knowledge in differential equations.

- Advanced Topics and Specialized Methods: Experts E1, E4, and E5 mention more advanced topics such as Fourier Series, Sturm-Liouville problems, self-similar solutions, and the perturbation method. These are less explicitly covered in the thematic groups from the articles discussed by Lozada et al. (2021) but align with the "Other Concepts" group where advanced topics like Laplace transforms and bifurcation concepts are discussed.

- Computational Methods and Software Usage: There is a clear emphasis from experts E5, E6-E7, and E8-E9 on the importance of computational methods and the use of software, which is echoed in the articles discussed by Lozada et al. (2021) under "Scalar-based Models" and "Biomathematical Models." These areas highlight the relevance of computer-aided methods and software in understanding and solving Differential Equations.

- Real-world Applications and Modeling: Experts E5, E8, and E9 stress the significance of real-world problem understanding and modeling. This is mirrored in the "Biomathematical Models" and "Systems Based on Mechanical Theory" categories from the articles, where real-world applications and modeling are central themes.

- Variational Calculus and Energy-Based Formulations: Experts E2 and E3 suggest including variational calculus due to its utility in energy-based formulations. This specific topic does not directly correspond to any of the thematic groups identified in the articles, indicating a unique perspective or a less commonly discussed area in the existing literature.

In summary, while there is a certain overlap between the experts' opinions and the themes identified in the literature, particularly regarding the importance of basic concepts, classical models, computational methods, and real-world applications, other areas like variational calculus and energy-based formulations are less represented in the articles. This comparison highlights a multifaceted

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approach to teaching Differential Equations, combining foundational knowledge with advanced topics and practical applications. Additionally, it shows us that the opinion of the experts highlights an area with limited representation in Differential Equations courses reported in the literature, which concerns the relevance of variational calculus. It is important to note that variational calculus, along with energy representations, constitute a fundamental tool for the numerical modeling of differential problems of high complexity and nonlinearity.

## 7. Highlights & Concluding Remarks

This study sought to explore the current state of Differential Equations education in engineering programs, with a focus on understanding the perspectives of experts from different institutions. The objectives were to comprehend expert opinions on the content that should be taught in a Differential Equations course, to ascertain their justifications for these content choices, to measure the institutional gap in content, and to highlight the diversity of professional profiles among these experts. Our findings reveal a multifaceted approach to teaching contents of Differential Equations, which combines basic knowledge with advanced topics and practical applications. The experts' insights, when compared to the thematic groups identified in the literature as discussed by Lozada et al. (2021), show a consensus on the importance of basic concepts as prior step to introduce modeling scopes. This fact substantially varies from the didactic approach of first constructing the real model, and from this model establishing resolution patterns, which can be globally defined as the modernist approach to the didactics of mathematics Gascon (2001). The study also uncovers an area less represented in differential equations courses as reported in the literature: the relevance of variational calculus. Experts have highlighted variational calculus, alongside energy representations, as fundamental for the numerical modeling of complex and nonlinear differential problems. This area, though not prominently featured in the existing literature, is important for engineering students, particularly for those specializing in fields in connection with energy-based formulations and numerical modeling. Furthermore, the experts emphasized the importance of computational methods and software usage, reflecting a growing trend in engineering education. This perspective resonates with the need for an integrated educational approach, where differential equations are taught in close alignment with real-world engineering problems. The experts' suggestions align with the call for innovation in teaching methodologies and the introduction of experiment-based learning, as advocated by Lozada et al. (2021).

In terms of meeting our objectives, the study successfully gathered diverse opinions from experts across different institutions, providing a rich understanding of what content should be included in a Differential Equations course. The experts' justifications for their content choices revealed a strong influence of their institutional affiliations and professional trajectories, thereby highlighting the institutional gap in the selection of thematic contents. The study also provided the variety of professional profiles and their unique perspectives on the teaching of differential equations.

The findings suggest the need for a more integrated and real-world-focused approach to teaching differential equations in engineering programs. To bridge the institutional gap, there is a need for curricula that balance theoretical mathematical concepts with their practical application in engineering. This balance can be achieved by introducing real-world modeling problems, employing computational tools, and focusing on the interpretative aspects of solutions. The study reinforces the importance of continuous improvement and adaptation in the teaching of differential equations. As the field of engineering evolves, so must the educational approaches to ensure that future engineers are well-equipped with the necessary skills and knowledge. The inclusion of variational calculus and a stronger emphasis on computational methods in differential equations courses could be vital steps towards achieving this goal.

In conclusion, the study has provided valuable insights into the current state and potential future directions of Differential Equations education in engineering. It has highlighted the need for a curriculum that is responsive to the demands of modern engineering practice, blending theoretical aspects with practical applications, and is informed by the insights of experts from diverse professional backgrounds.

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