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CAD-integrated parametric design and analysis of lightweight shell structures

Ann-Kathrin Goldbach [a,*], Carlos Lázaro [b]

[a] Technical University of Munich, Arcisstr. 21, Munich, 80333, Germany
[b] Universitat Politècnica de València, Camino de Vera, València, 46022, Spain

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ABSTRACT

In the digitalization of planning, analysis and building processes, a streamlined interaction between the conceptual phase and the structural design is of crucial importance to overcome the gap between architecture and engineering. In this paper we show the advantages of performing parametric CAD-integrated design of shell structures by means of an integrated Isogeometric B-Rep Analysis (IBRA) tool that allows to perform Finite Element Analysis directly on trimmed multipatch CAD models. This allows for non-linear analyses and form-finding techniques to be directly applied to complex shapes within the CAD environment —without the need to convert to a simple polygonal mesh— and thus enables structurally informed design iterations at any stage of the design process. We use different hyperbolic-paraboloid geometries and combinations inspired in exemplary works of the shell master Félix Candela to highlight the design steps and the potential of the IBRA tool. The examples show that the smooth integration of parametric CAD and analysis model allows the designer to perform a powerful exploration of different configurations and their structural behavior and improve the shape and performance through numerical hanging models.

1. Introduction

The decisions made in early design stages of buildings have a major effect on their performance, in a structural as well as architectural sense. Thanks to the digitalization of planning, analyzing and building processes, influential parameters can easily be identified and changed during the conceptual phase. The structural behavior, including load transfer mechanisms and resistances, as well as deformations and stresses, is one of the core aspects of a building's performance and needs to be considered in its interaction with all steps of planning. Especially for structures characterized by complex geometries, like lightweight shell structures, finding the best solution for a multitude of requirements is a challenging task that requires several iterations. The key to a design that satisfies these requirements lies in the communication of the involved parties.

However, cumbersome translations from design to analysis software are still part of most engineering processes and create a clear deficit in the collaboration between architects and engineers. Overcoming this gap clearly holds advantages for the workflow, but also enables performance based design, which is defined as "the potential of an integration of evaluative simulation processes with digital 'form generation' and 'form modification' models" by [1]. In this paper we show parametric models that can process both geometrical and structural data variations, which are equally relevant for large shell structures.

Linking geometrical choices with structural behavior can lead to optimized and therefore sustainable structures with a minimal material input. In lightweight design, the minimization of material and hence resource efficiency has been pursued for decades. Pioneers of lightweight structures like Frei Otto and Félix Candela created an impressive variety of structures that followed an integrated approach to design and analysis, e.g. through form-finding. The design-to-production workflow developed by Félix Candela provides an inspirational example of creating shell structures at the interface of design, analysis and engineering. Performance based design has been proposed for a variety of structures by e.g. [2,3] and linked to parametric design [4]. Before the term "performance" was introduced for built structures, shape optimization had already been established to reach highly efficient systems by e.g. [5,6]. The application of optimization techniques for the built environment has recently been investigated and summarized in [7] and the importance of the geometry to this end has long been acknowledged [8]. A comprehensive overview of architectural shells and the prominent role of finding their shapes was provided by [9]. With the development of Isogeometric B-Rep Analysis (IBRA), geometrical modeling and structural analysis could be unified into a single process, rendering a powerful and integrated method for the exploration of a multiplicity of designs [10,11]. Focusing on tensile and hybrid structures with prestressed surface elements, the advantages of

* Corresponding author.
E-mail address: ann-kathrin.goldbach@tum.de (A.-K. Goldbach).

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a unified parametric CAD-integrated design and analysis framework on the basis of IBRA were presented in e.g. [12–14]; these works were conducted with isogeometric Kirchhoff–Love shell elements, as presented in [11]. It should be noted that the development of IGA shell element formulations has been a prominent topic in research, leading to a large variety of formulations, see e.g. [15–22].

The aim of this work is to show that the use of an integrated IBRA tool in a parametric CAD environment provides an ideal setup to interactively design architectural shells, extending therefore the application range of this method beyond the design of tensile structures. To demonstrate the possibilities of this unified workflow in the realm of thin shells, we have selected three case studies based on structures designed by the Spanish-Mexican shell master Félix Candela. Parametric models are created at the interface of engineering and architecture, allowing for the inclusion of structural properties and parameters within the unified framework and eliminating the need for converting smooth design geometries to simple polygonal meshes for analysis. Smooth CAD-models are preserved throughout an analysis process, creating very flexible structural design models and facilitating the exploration of various parameter combinations. The IBRA approach thus leads to a continuous model development through multiple design and analysis steps and iterations in an integrated workflow, with the advantage of not being limited to simple geometries but including and preserving trimmed multipatch surfaces for complex shapes from pre- until postprocessing. Supporting architectural design through structural information within the CAD-environment has the potential to change the design paradigm from “form making” to “form finding” as intended by Frei Otto, [5].

The paper is structured as follows: Section 2 starts providing the context of shape creation and form-finding of lightweight structures. After that, the fundamentals of Isogeometric B-Rep Analysis are briefly reviewed and the requisites of CAD-integrated design-through-analysis workflow are summarized. Section 3 is introduced by describing Félix Candela’s unique method of creating large concrete shell structures. Three case studies based on Candela’s shells exemplify the CAD-integrated approach. Finally, Section 4 includes the conclusions.

2. The CAD-integrated design-through-analysis workflow

The importance of successful collaboration between architects and engineers has been highlighted in the previous section. It is evident that this is especially important in the early design phases (see e.g. [1]). The integration of Computer Aided Engineering (CAE) and Computer Aided Design (CAD) leads to a collaborative workflow based on a digital model. Isogeometric Analysis by Hughes et al. [23] is one of the techniques that facilitates the integration of CAD and CAE. Since the advances of IGA with Isogeometric B-Rep Analysis (IBRA, see e.g. [11]), Finite Element Analysis (FEA) can be performed directly on the CAD model within the CAD software environment. Geometric changes can be made at any stage of the analysis and the possible consequences on the performance of a structure can be directly investigated. An iterative design-through-analysis workflow follows as all relevant parameters – geometrical and mechanical – can be optimized (as described e.g. in [14,24]). Section 2.2 summarizes the basic components of Isogeometric B-Rep Analysis to provide a basic understanding of the technique while Section 2.3 revises the Kiwi!3D plugin for a mechanically enhanced parametric design environment.

One way to minimize material use and thus create resource-efficient shell structures is to find shapes that carry given loads with purely in-plane membrane stress states. If only membrane forces are present, the cross-section can be reduced to a minimum. Even for very large surfaces, tensile structures such as membranes can be built from prestressed fabrics (about 1 mm thick), and concrete shells can be built under predominant compression with only a few centimeters of thickness, as demonstrated in e.g. [25–27]. Because lightweight structures can only be designed by taking into account the interaction of shape and forces, the benefits of CAD integration are most evident in these structures. To find shapes that satisfy the condition of transferring external loads in pure tension or compression, a number of numerical methods can be used, [28]. A brief overview of these methods is given in the next section. It should be noted, that they can be used for a variety of structures to find optimal shapes. Furthermore, all of the mentioned techniques can be performed with the CAD-integrated approach.

2.1. Creating shapes and finding form

The earliest reinforced concrete shell structures were built from mathematically motivated surfaces. Their designers knew the closed formulations of the stress states for such surfaces under various loads and boundary conditions, see e.g. [29]. Therefore, they created shapes from a limited range of geometries or combinations of those, to make sure that they could handle the calculations of internal forces for a feasible design.

With Heinz Isler’s advances in possible shell geometries determined by hanging models, a new era of creating mechanically motivated shapes evolved, [30,31].

Architects and engineers developing surface structures soon coined the term *form-finding*. Form-finding has the goal of creating a mechanically motivated shape for given load and boundary conditions (i.e. supports). Form-found shapes will have a desired stress state in equilibrium with the chosen loading conditions, either in pure tension for membrane structures or in pure compression for shell structures. As Isler’s hanging models lead to equilibrium surfaces in pure compression (at least for self-weight), they could already be characterized as form-found. The creation of numerical hanging models mimicks the physical experiment with a geometrically non-linear analysis performed on a model subjected to the expected dominant loads. Following the setup of the physical experiment, the digital model’s hanging shape can also simply be turned upside down to achieve a compression-only shell structure, as was shown in [13] for simple one-patch geometries. Form-finding techniques to generate these equilibrium shapes have been developed with different approaches since the 1970s and can be divided into forward and inverse methods (see also [13]). Some numerical methods that are commonly used for form-finding include the Force Density Method, e.g. [32] and the Updated Reference Strategy, e.g. [33], which mostly focus on tensile structures. Another well-established form-finding technique is Dynamic Relaxation [34], which needs additional input-parameters for solving a dynamic problem. Graphical Statics, e.g. [35,36] are closely related to Combinatorial Equilibrium Modeling, e.g. [37] and use the form and force duality to explore discrete equilibrium shapes. Thrust Network Analysis has also been established as a method to find discretized shell geometries that work in compression only, e.g. [28,38]. Recently, the membrane equilibrium analysis has been presented for shell form-finding of simple single-patch continuous surfaces, see [39,40]. Form-finding for shell structures based on NURBS geometries has also been presented in [41, 42]. [43] presented the advantages of automatic differentiation for the form-finding of pin-jointed truss structures.

Non-linear FEA can be used to create numerical hanging models as shown in Section 3.4.2 of this paper. In contrast to the other form-finding approaches, the IBRA based numerical hanging models can be built from complex trimmed and coupled NURBS geometries. Furthermore, these models are not limited to form-finding but can be linked to consecutive analysis steps and therefore facilitate the iterative nature of lightweight design and analysis processes.

The role of architects and engineers is to identify opportunities to optimize the performance of the design in terms of load bearing behavior, appearance, and other requirements and to balance the input parameters to find the best solution. One of the advantages of the presented CAD-integrated framework for shell design and analysis is the unified workflow based on one design and analysis model that can capture the complexity of the shape as well as the structural behavior.

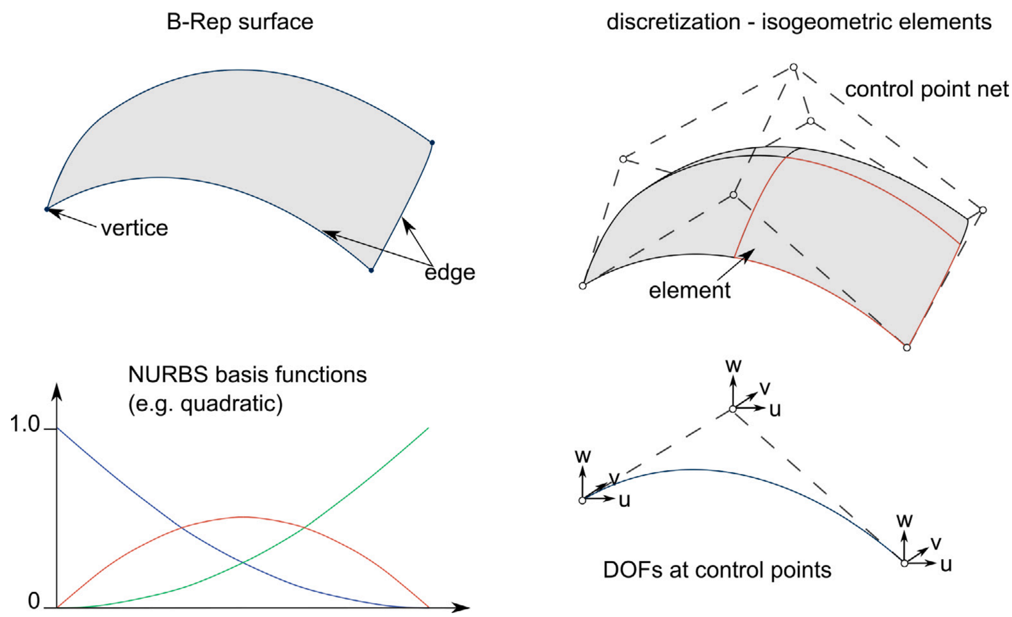

Fig. 1. B-Rep model: vertices, (curved) edges and surfaces, the control point net and NURBS basis functions for a simple geometry.

2.2. Isogeometric B-Rep analysis

In Computer Aided Design, geometric models are commonly stored as B-Reps (Boundary Representations), as shown in Fig. 1: The curves and surfaces of these B-Reps are defined with NURBS (Non-Uniform Rational B-Splines). NURBS curves and surfaces can represent nearly any geometry and thus provide a very powerful tool for flexible design, see [44]. A NURBS curve $\mathbf{C}(t)$ is described by its control points $\mathbf{P}_i$, their weights w_i and corresponding B-Spline basis functions $N_{i,p}$:

$$\mathbf{C}(t) = \sum_{i=1}^{n} R_{i,p}(t)P_i \quad ; \quad R_{i,p}(t) = \frac{N_{i,p}(t)w_i}{\sum_{i=1}^{n} N_{i,p}(t)w_i} \tag{1}$$

Isogeometric B-Rep Analysis (IBRA by [11] as an extension of Isogeometric Analysis by [23]) uses geometric B-Rep models from CAD for structural analysis by assigning the necessary mechanical properties and boundary conditions to the B-Rep entities (vertices, edges, surfaces) and using the NURBS basis functions for the elements, which are defined in the parameter space. The smoothness of the geometry description is thus kept for analysis. Some of the most prominent advantages of CAD-integrated analysis with IBRA for conceptual lightweight design are the concepts of refinement, trimming and coupling. Refinement, i.e. introducing more design handles to ensure a sufficient solution space, can be applied to NURBS curves and surfaces without affecting the geometry itself. Especially for form-finding tasks, this is an important property, since the target geometry might be more complex than its initial setup. Furthermore, as in any FEA workflow, the results depend on the discretization, i.e. the mentioned refinement. Fig. 5 shows the different results for the vertical displacement of a shell constructed from four trimmed hypar leaves inspired by Candels's church roof in Mexico (shown in Fig. 3) for a uniformly distributed dead load of 5 kN/m^2. Trimming is a functionality in CAD that separates the geometry into visible and non-visible parts. Trimmed surfaces can be modeled in order to avoid complicated geometry descriptions. Coupling methods can be applied in order to "connect" different NURBS patches to each other while satisfying the necessary continuity requirements and therefore allow the user to implement boundary conditions at all B-Rep entities. Current research on coupling strategies for isogeometric approaches is invested in both weak and strong coupling methods. The mentioned concepts are described in detail in e.g. [14].

The analyses shown for the examples in this paper are geometrically non-linear and equilibrium is defined with the principle of virtual work of the physical domain Ω and its boundary Γ:

$$\delta W = \delta W_{\text{int}} + \delta W_{\text{ext}} = 0 \tag{2}$$

The internal virtual work δW_{int} contains stresses $\mathbf{S}$ and conjugated virtual strains $\delta\mathbf{E}$

$$\delta W_{\text{int}} = -\int_{\Omega} \mathbf{S} : \delta\mathbf{E}\; d\Omega \tag{3}$$

and the external virtual work δW_{ext} entails body forces $\mathbf{p}$, boundary forces $\mathbf{t}_\Gamma$ and virtual displacements $\delta\mathbf{u}$.

$$\delta W_{\text{ext}} = \int_{\Omega} \mathbf{p} \cdot \delta\mathbf{u}\; d\Omega + \int_{\Gamma} \mathbf{t}_\Gamma \cdot \delta\mathbf{u}\; d\Gamma \tag{4}$$

To fulfill equilibrium, Eq. (2) has to be fulfilled for any kinematically admissible virtual displacement field $\delta\mathbf{u}$ and thus the well-known system equation of the Finite Element Method can be formulated, [45], i.e.

$$\delta W = \frac{\partial W}{\partial \mathbf{u}} \delta\mathbf{u} = 0 \tag{5}$$

is linearized so that it is solvable with an iterative scheme as

$$\mathbf{K}\Delta\mathbf{u} = \mathbf{R}. \tag{6}$$

The expressions for the residual force vector components R_r and stiffness matrix entries K_{rs} for $r, s = 1 \ldots n_{\text{DOF}}$ can be composed from

$$R_r = -\frac{\partial W}{\partial u_r} \tag{7}$$

and

$$K_{rs} = \frac{\partial R}{\partial u_r} = -\frac{\partial W^2}{\partial u_r u_s}. \tag{8}$$

In [19], the derivation of an isogeometric Kirchhoff–Love element is explained in detail. The necessary additions for Isogeometric B-Rep Analysis are described in [11], as well as the numerical integration of NURBS surfaces by Gauss quadrature performed in the parameter space.

Fig. 2. Hypar umbrellas at the Bacardi factory near Mexico City. (Image: C. Lázaro).

Fig. 3. "San Antonio de las Huertas" church in Mexico City. A succession of groined vaults with free edges. (Image: C. Lázaro).

2.3. CAD-integrated parametric design and analysis with Kiwi!3D

The application of IBRA in a parametric CAD environment leads to a conceptual phase that is as flexible as possible and thus provides the largest design space. Both geometric and structural properties can be defined as parameters to be varied, either by manually trying different variations or by adding external optimization loops to certain parameters in order to exploit the design space (see e.g. [46]). Recent research deals with the inclusion of artificial intelligence for design parameters and the question of how to deal with and assess the amount of possible solutions (see e.g. [47,48]). Parametric design has become very popular and there are numerous analysis tools available, offering various simulation possibilities within the CAD-environment. However, the majority of plugins that are currently available to link parametric CAD with FEA software only provide preprocessing and a one-way connection into the FEA environment where analysis and postprocessing are handled. There are some plugins that work in a CAD-integrated manner and provide form-finding tools (e.g. [49–51] among others), but they all perform analyses on a traditional polygonal mesh (and thus lose geometrical continuity) rather than directly utilizing the CAD-model. Any iteration in a design process thus necessitates a new mesh to be generated before an analysis can be repeated.

This publication will focus on the Kiwi!3D framework, that was developed to close the gap between CAD and FEA and to make the IBRA technology (see Section 2.2) available to architects, designers and engineers, [52]. The freeware plugin can be appended to Grasshopper within Rhinoceros [53]. It is based on the research FE code Carat++ [54] and allows for the performance of all design, analysis and optimization steps with one model. Since there is no conversion to a classical FE mesh of simple planar elements, a unified design and analysis workflow is possible with IBRA, meaning that manipulations and alterations of the model are possible at any stage without compromising the analysis setup and the result model is a fully intact CAD-model itself.

3. Applying CAD-integration to Félix Candela's shell development: a proposed design and analysis workflow

In this section, the application of the CAD-integrated workflow to shell structures is shown by means of three examples of shell structures designed by the The Spanish-Mexican architect Félix Candela (1910–1997). Firstly, the context of Candela's achievements is introduced, with emphasis on presenting his own pre-digital era workflow; although it is not directly applicable nowadays, it exemplifies the potential of interweaving the geometric design with the analysis. Three different structures are then modeled and analyzed within the interactive CAD-Analysis environment.

3.1. Félix Candela's design-to-production process

Félix Candela is considered one of the master designers and builders of the 20th century [55]. He showed an early interest in applying geometry to thin reinforced concrete shell structures and self-studied the mechanics of shells in French and German publications of the 1930s [26,56]. He went into exile after the end of the Spanish Civil War and arrived in Mexico in 1939. In 1950, Candela founded "Cubiertas Ala", a shell construction firm where he could combine the acquired knowledge of geometry and shell mechanics with his experience working for other architects. He soon recognized the extraordinary advantages of building shells with hyperbolic paraboloid (*hypar*) shapes, namely the rigidity and stability against buckling provided by double curvature and the simplicity of assembling the formwork for a spatially curved structure with narrow straight timber laths thanks to the fact that hypars are ruled surfaces. A crucial point in Candela's success was the efficient structural analysis and design method he perfected: given a shell geometry and loading, the analytic membrane equilibrium solution included two indeterminate functions; a suitable choice of these, allowed to have no normal forces along two straight edges (or one curved edge). In this way, the designer *selected* a stress state in the shell and designed the reinforcement and boundary conditions accordingly. The formulas were translated to a systematic process that could be performed by trained workers. The method is described in [26,57,58].

Candela's firm offered shell roofs as a finished product. Final clients were accessed through the collaboration with generalist architects from the very early stages of each project. As shell roofs were the trend, architects often tried shell-like designs intuitively. Candela worked with them to transform the initial roof sketches into a buildable and structurally sound shell structure. Once the project was commissioned, the shell was built rapidly and economically, taking advantage of the streamlined design process, the proven construction method, and the availability of experienced local workers.

Starting from small-scale experimental ruled-surface shells [57], the company developed a diversity of hypar arrangements to shape roofs [26], which considerably extended the basic combinations of hypars proposed by Aimond [59]. Roof dimensions were not bounded to standardized values; they were only limited by the structural performance of the shell. The standard solutions referenced in [57] allowed very rational setups —excellent examples of which are the roofing

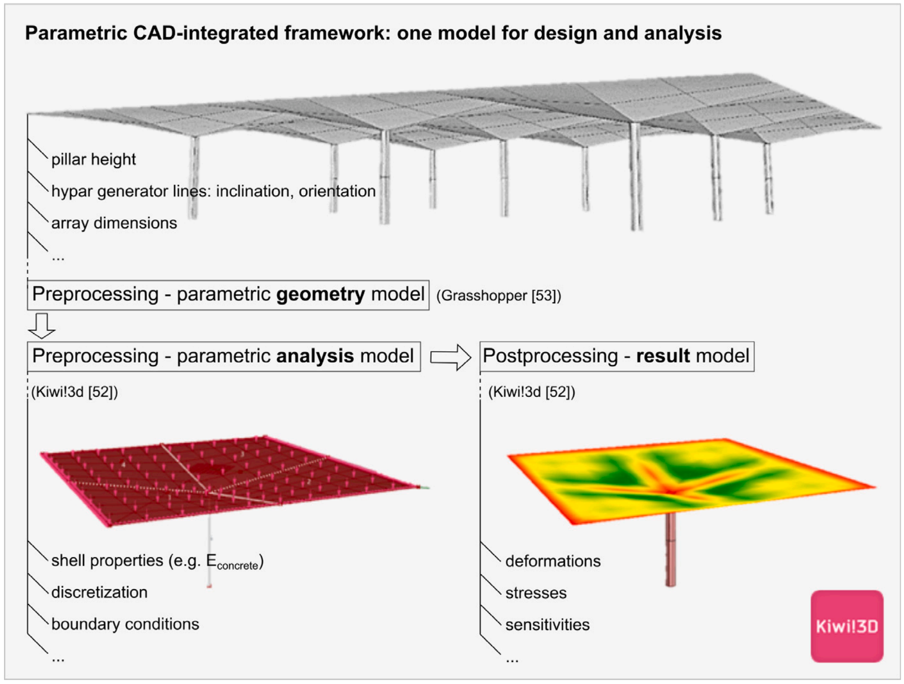

Fig. 4. Parametric geometry and structural analysis model of a roof inspired in the Bacardi factory near Mexico City built with the framework Kiwi!3d for Grasshopper and Rhino: some Pre- and Postprocessing parameters are listed in grey and qualitative results portrayed for the shell's vertical deformation under an exemplary dead load of $5\,\mathrm{kN/m^2}$.

of public markets and factories (e.g. Fig. 2)— but also opened up the way to shape complex roofs adapted to a variety of architectural solutions by combining hypars of different sizes and shapes by means of geometric operations such as replication, juxtaposition and trimming of the surface (Fig. 3).

Candela's process provides valuable inspiration for present-time workflows: the importance of collaboration between architect and structural designer from the start; the capacity to define a variety of structurally sound geometries in a fast (today interactive) way; the implementation of a streamlined flow: geometry–forces–design–quotation–production, and the readiness of the design to be directly translated to building instructions.

3.2. Case 1: Parametrization and analysis of an umbrella roof

The first example (Fig. 4) shows the application of Candela's toolset of geometric operations within the parametric environment Grasshopper for Rhinoceros [53] to define a structure inspired in the Bacardi factory near Mexico City (Fig. 2), and provides an overview of the whole integrated process. To create the geometry, a first "umbrella" is constructed from four hypars with straight outer edges and a lowered center point. This shape can then be used to build an array (portrayed on top) like the one built for the Bacardi factory near Mexico city. To define the CAD-integrated analysis model, the "umbrella" model is enhanced by structural information via Kiwi!3d: boundary conditions, element types and properties (i.e. shell elements with a material model for concrete defined for the surface) and an exemplary load of $5\,\mathrm{kN/m^2}$. The magnitude of the load reproduces the effect of the permanent loads (self-weight plus waterproofing) plus an additional uniform live load on the whole structure. The distribution is oversimplified, but exemplifies the type of loading that was used in Candela's procedure [26]. We are adopting this magnitude also in subsequent examples. A geometrically non-linear analysis is also defined with the plugin Kiwi!3D, which performs IBRA on the NURBS surfaces with Carat++. Finally, the results can again be studied within the CAD environment in postprocessing, as shown in Fig. 4 qualitatively for the vertical displacement under the given surface load.

3.3. Case 2. Refinement of the discretization

In this example, the geometry of a 4-lobe groined vault similar to the modules of the "San Antonio de las Huertas" church (see Fig. 3) has been selected to analyze the influence of the discretization in the results. IBRA models can be subjected to different refinement techniques such as h- and p-refinement, without altering the geometry itself (see [11,60] for a detailed explanation of refinement in IGA and IBRA). Fig. 5 shows the different results corresponding to an increasing number of elements, i.e. applying h-refinement to the integrated shell model. Interpolation polynomials of degree 3 have been used in all cases. IBRA is able to get results from the extremely coarse mesh that is directly provided by the CAD model (first row, 1 element). However, these results are not reliable, and further refinement is required. In this case, to reach a good accuracy, 25 elements in u and v directions are needed. This example also portrays the IBRA concept for analyzing trimmed geometries: the discretization is available and shown for the complete untrimmed geometry, but only the trimmed part of the geometry model is considered for analysis (see the red triangular surface in the discretization view of Fig. 5).

As expected, CAD models may not be directly suitable to perform the analysis, and a proper refinement is required. The refinement can be performed within the same CAD-parametric environment, and is a straightforward operation. However, users must be knowledgeable enough to assess the required refinement operations. This case thus highlights the fact that further research is needed to streamline the refinement process.

3.4. Case 3: Applying CAD-integration shell design: a proposed design and analysis workflow

In this section we showcase the CAD-integrated design-analysis workflow on the basis of Félix Candela's last shell for the restaurant

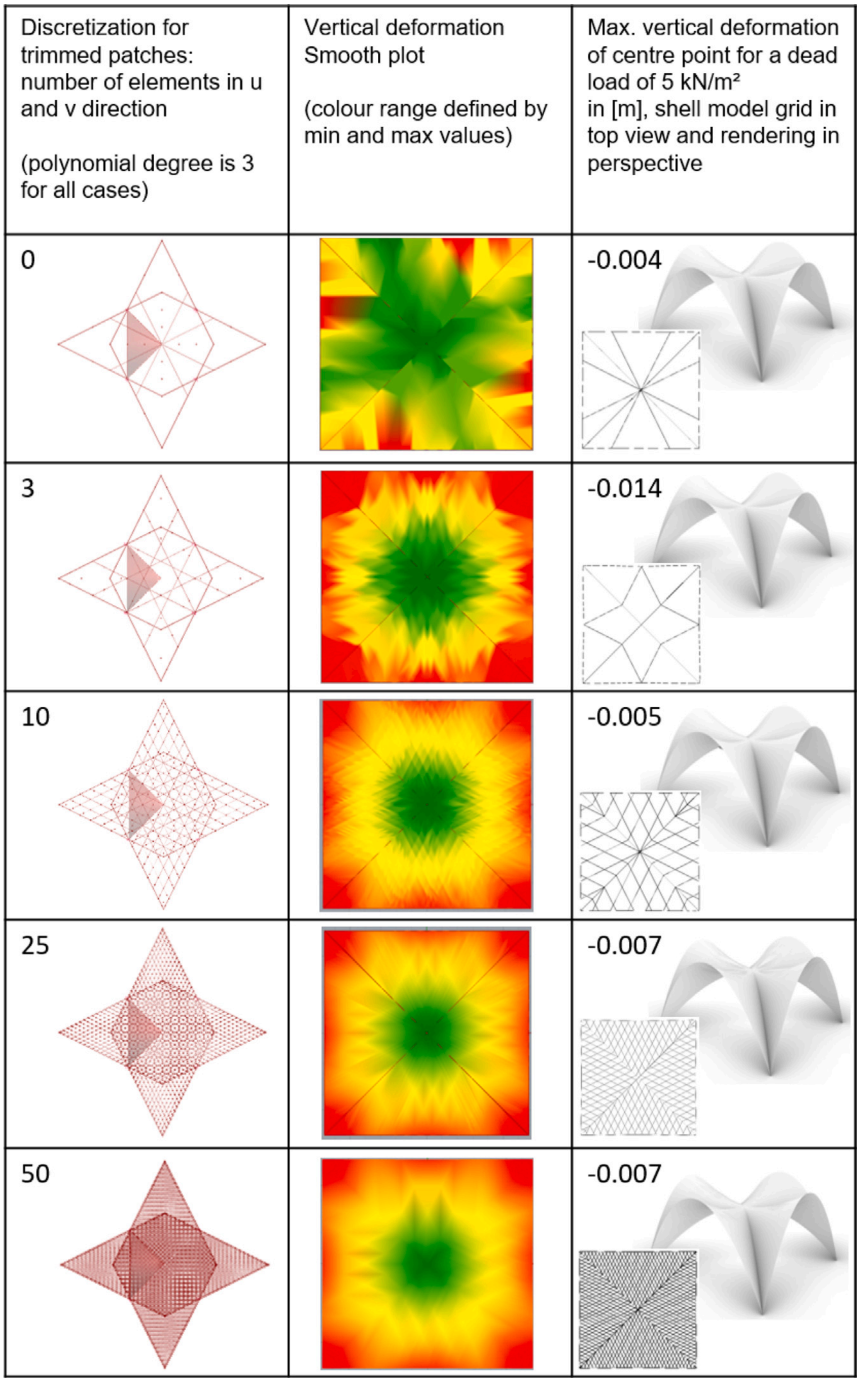

Fig. 5. Refinement of the discretization of a 4-lobe groined vault. The third column shows the B-Rep geometry model of the vault which does not change, as refinement can be performed without affecting the geometry.

at the center of the aquarium-park “L’Oceanogràfic” in Valencia, which was built in 2000 (see Fig. 6). From the point of view of the geometry, it is similar to Candela’s “Los Manantiales” shell in Xochimilco (México), built in 1959. The roof is a groined vault resulting from the intersection of four hyperbolic paraboloids (*hypars*) sharing the origin and vertical z axis. The following data of the shell are taken from [61]. The angle ω between the x and y oblique horizontal axes (depicted in Fig. 8) of each hypar is equal to 22.5°. The elevation is given by

$$z = k\,xy, \tag{9}$$

where $k = -0.00351$ is the hypar constant. The intersections of the four hypar surfaces define the groins of the vault: they are curves embedded in vertical planes at 45°. This geometric operation generates eight leaves (two opposite leaves belonging to the same hypar surface.) The free outer boundary is the result of trimming the hypar leaves with inclined planes, forming an angle of 62.6° with the horizontal plane. The free boundaries and the groins intersect in points defining the (eight) supports of the roof shell. The distance between opposite supports is 35.5 m, and the distance between adjacent supports is 13.6 m. The apex of the vault has an elevation of 8.12 m, and the elevation of the tip of the cantilever leaves is 12.9 m.

The shell was built with steel-fiber reinforced concrete. The base thickness is 6 cm. The thickness increases in the transitions to the groins and near the apex. The groins, with near-triangular sections, have variable thickness, ranging between 0.8 m at the supports and 0.25 m at

Fig. 6. "L'Oceanogràfic" restaurant in Valencia. (Image: C. Lázaro).

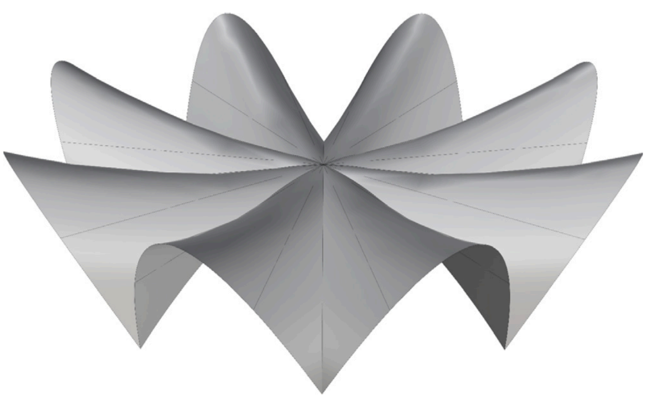

Fig. 7. Final CAD model of the concrete shell at l'Océanografic in València.

the apex. The characteristic strength of concrete is 30 MPa. 50 kg/m^3 of galvanized steel fibers were added in the concrete mix. The passive base reinforcement was a single ∅8 @ 15 cm mesh of 500 MPa yield-strength reinforcing steel following the parabolas resulting from intersecting the surface with vertical planes parallel to the symmetry planes of each hypar.

In modern CAD programs, mechanically motivated geometries like the shells developed by Candela can be created in a number of ways. For the shell at l'Océanografic (see Fig. 7), a parametric model was built with Grasshopper in Rhino [53], leading to a very flexible design model that easily allows to study geometric changes. Furthermore, the CAD model can be used for numerical analysis with IBRA with the plugin Kiwi!3D [52]. This way, analysis models are also defined within Grasshopper, the advantages of a parametric environment are thus also available for the exploration of structural properties and capabilities.

3.4.1. Parametric shape creation

The process for the generation of the geometry is depicted in Fig. 8. From a set of four given points matching the analytic definition of the hypar, four generator lines are created that lead to a surface with the desired ruled shape (a). The desired distance between opposite supports of the roof defines the diameter of a circle. It is extruded in order to find the intersection points with the straight generators of the hypar that define the positions of the support points (b). Rotating the hypar by a 45° angle clock and counterclockwise creates the intersection curves providing the desired opening angle of the hypar segment (c). A plane is inclined by a given angle to cut out the final hypar segment (d), which is then copied in a polar array to build the complete shell geometry (e).

Fig. 9 shows the changes appearing for some simple parameter variations, that do not affect the process of creating the shell geometry as shown in the previous Figure, but still yield completely different geometries. The inclination of the intersecting plane creating the opening arches β (step (d) in Fig. 7) is altered –(1) and (2)–. Different rotation angles α lead to different intersections of the hypars (step (c) in Fig. 7) and thus change the number of leaves –(3) to (5)–.

3.4.2. CAD-integrated analysis and shell form-finding with IBRA

In the context of Finite Element Analysis, the creation of a CAD model belongs to the preprocessing stage and would be followed by meshing to create an analysis model. This is not necessary in the CAD-integrated approach, since the geometry description is used directly for the analysis, as described in Section 2.2. Fig. 10 shows how the isogeometric analysis model is defined within the parametric CAD environment using the Kiwi!3D plugin. The NURBS-based B-Rep geometry from the previous section, which provides the parametric definitions of faces, edges and vertices, is shown on the lower left picture. The element types, support and load conditions are assigned to these geometric entities, providing the model with all the required mechanical properties, as indicated by the turquoise "Analysis" box. The analysis input is completed by defining the type of analysis and specifying accuracy, and other parameters. The lower middle picture shows the analysis model that uses the inherent NURBS-based discretization from CAD. The displacements are constrained at the support points (highlighted in pink) and the intersection curves between the hypars automatically define the edges to be coupled for the analysis and hence ensure watertightness of the model (white beaded lines). Finally, the deformed shape is available for post-processing (green "Result" box) along with the stress and deformation results, as shown in the lower right picture.

As the model now possesses structural as well as geometrical parameters, their interaction can easily be tested. Fig. 11 shows how a change in the height parameter z of the initial hypar is investigated. The model framework of Fig. 10 remains intact, the parameter study is performed by simply changing the z component of the hypar created in the first step of geometry definition and re-evaluating the analysis model. Beyond the obvious changes to the shell geometry, the CAD-integrated model inherently provides information relevant to the evaluation of the structural behavior. For this simple example, the results are shown for the vertical displacements under an exemplary uniform dead load of 5 kg/m^2 at the midpoint and highest point of the outer arches. Naturally, changing and evaluating the effect of structural element properties (e.g. Young's modulus) can be investigated in the same straight-forward manner through the alteration of these parameters.

As this example shows, the CAD-integrated parametric model allows the user to calibrate both geometric and structural requirements and facilitates close collaboration among design team members to arrive at a performance-based design solution. The model parameters can also be fed into additional optimization loops to achieve design goals, as shown for a hybrid structure's size to be maximized within a given design frame in [46].

The CAD-integrated analysis model can be used for a variety of different investigations, including form-finding by creating a numerical hanging model. Fig. 12 shows the construction of such a model used to find the equilibrium surface for a compression-only shell under prescribed loading conditions. A geometrically nonlinear analysis is performed for the geometrically inverted structure under self-weight. In order to enforce a membrane state, the thickness of the shell is significantly reduced so that the bending stiffness becomes negligible and the hanging shape evolves — resulting in a shell in pure compression under self-weight when turned upside down. A comparison between the initial geometry and the hanging model shows an upward deformation of the vault in the middle and lower outer arch edges. As can be seen in Fig. 12, the maximum difference in shape is in the magnitude of a few cm and hardly distinguishable in some areas. However, subjecting the form-found shell geometry to a dead load in a subsequent analysis results in even smaller displacements than shown in Fig. 11.

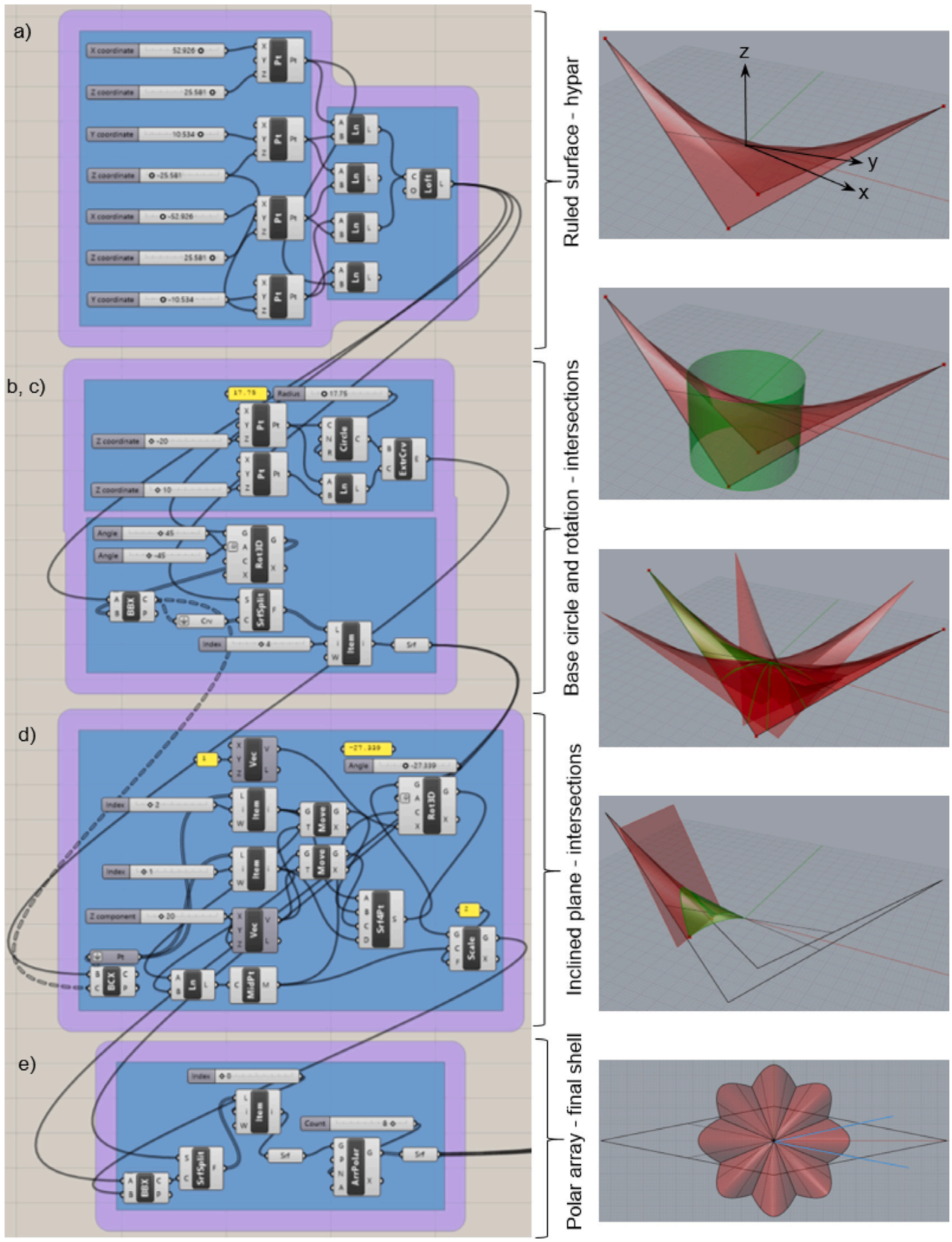

Fig. 8. Building a parametric geometry model in Grasshopper [53], following Candela's tools: (a) ruled surface from two sets of generator lines; (b) rotating the hypar to generate intersection curves; (c) trimming along intersection curves; (d) trimming with an inclined plane to generate final shell segment; (e) shell geometry after rotating the segments.

3.5. Design to production

In order to include aspects of a design-to-production process, the shown example would have to be enhanced with further information regarding the construction stages. Again, the proposed CAD-integrated approach provides the opportunity to not only consider geometry-related production requirements and to derive e.g. scaffolding plans from the model, but also enables the design team to perform structural analyses of the construction stages and their implications for consecutive steps. As the IBRA model preserves the CAD geometry throughout these steps, parameter variations can easily be investigated in order to find the best production sequence and plan.

4. Conclusions and outlook

In this paper, the advantages of a CAD-integrated workflow for the conceptual design and the analysis of thin shell structures have been highlighted. Using shell structures designed by the Spanish-Mexican architect Félix Candela as a motivating examples, we have shown that similar variations of a basic structural geometry can be explored in a performance-informed environment by means of an integrated parametric CAD-FEM framework.

The present CAD-integrated design-analysis workflow resembles the design process used by Candela, described in Section 3.1, in the control of the interplay between the geometric definition of the structure and

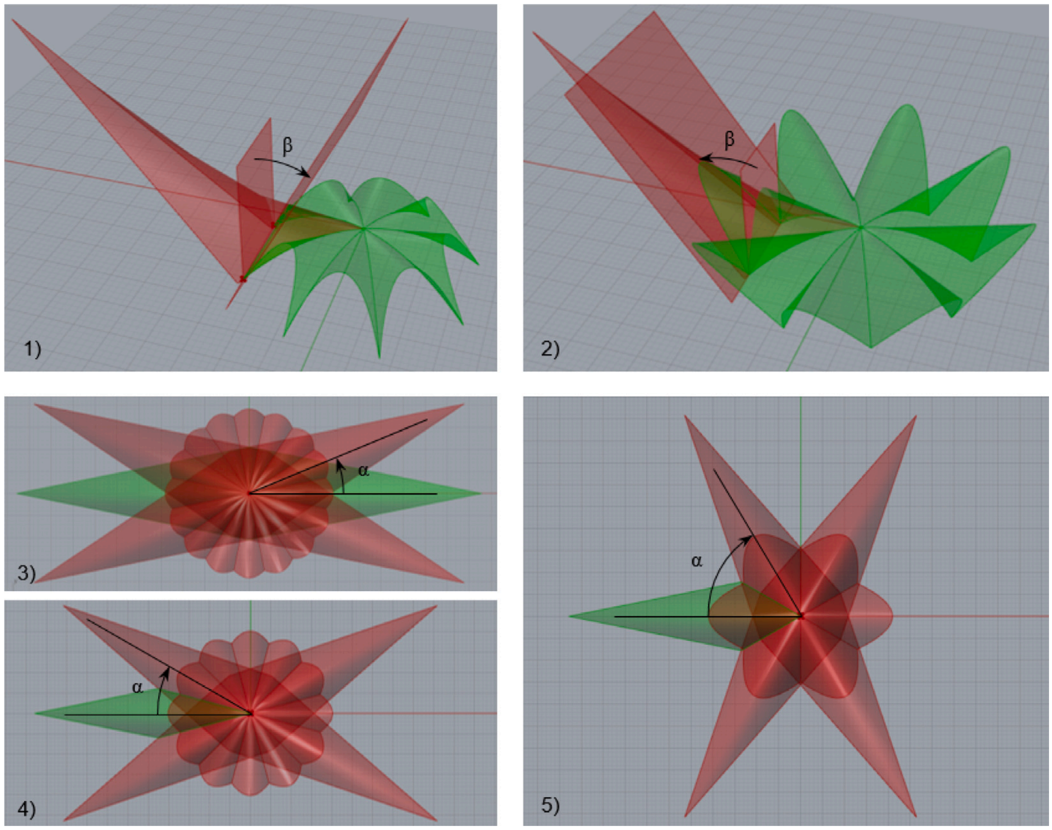

Fig. 9. Varying geometric parameters α and β and different resulting shells.

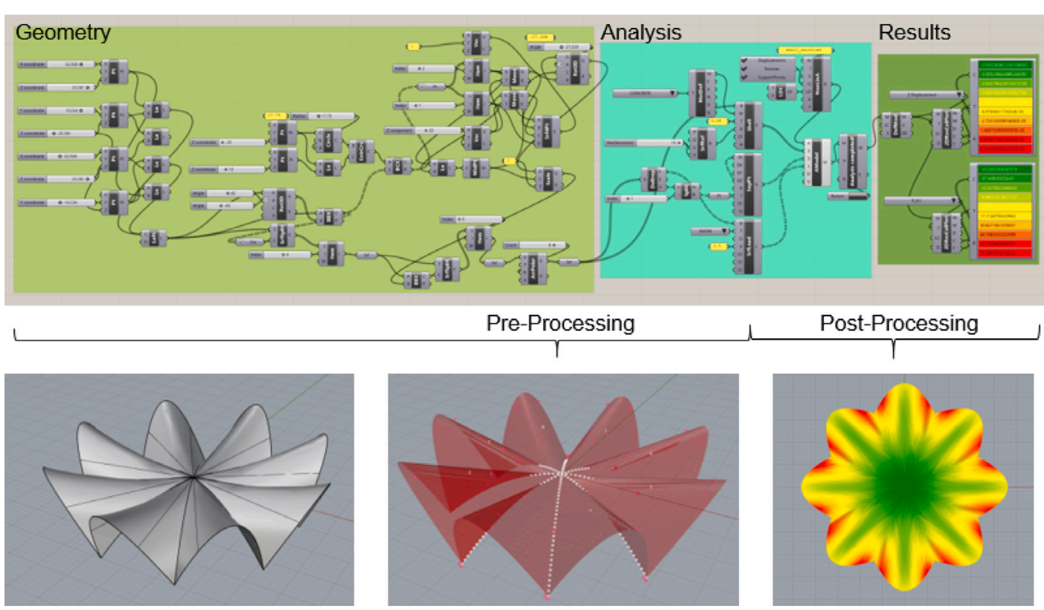

Fig. 10. Building a parametric isogeometric analysis model with Kiwi!3D in Grasshopper and Rhino: Pre- and Post-Processing as well as the IBRA solver are encapsulated within the CAD-environment. The results to evaluate structural performance directly available. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

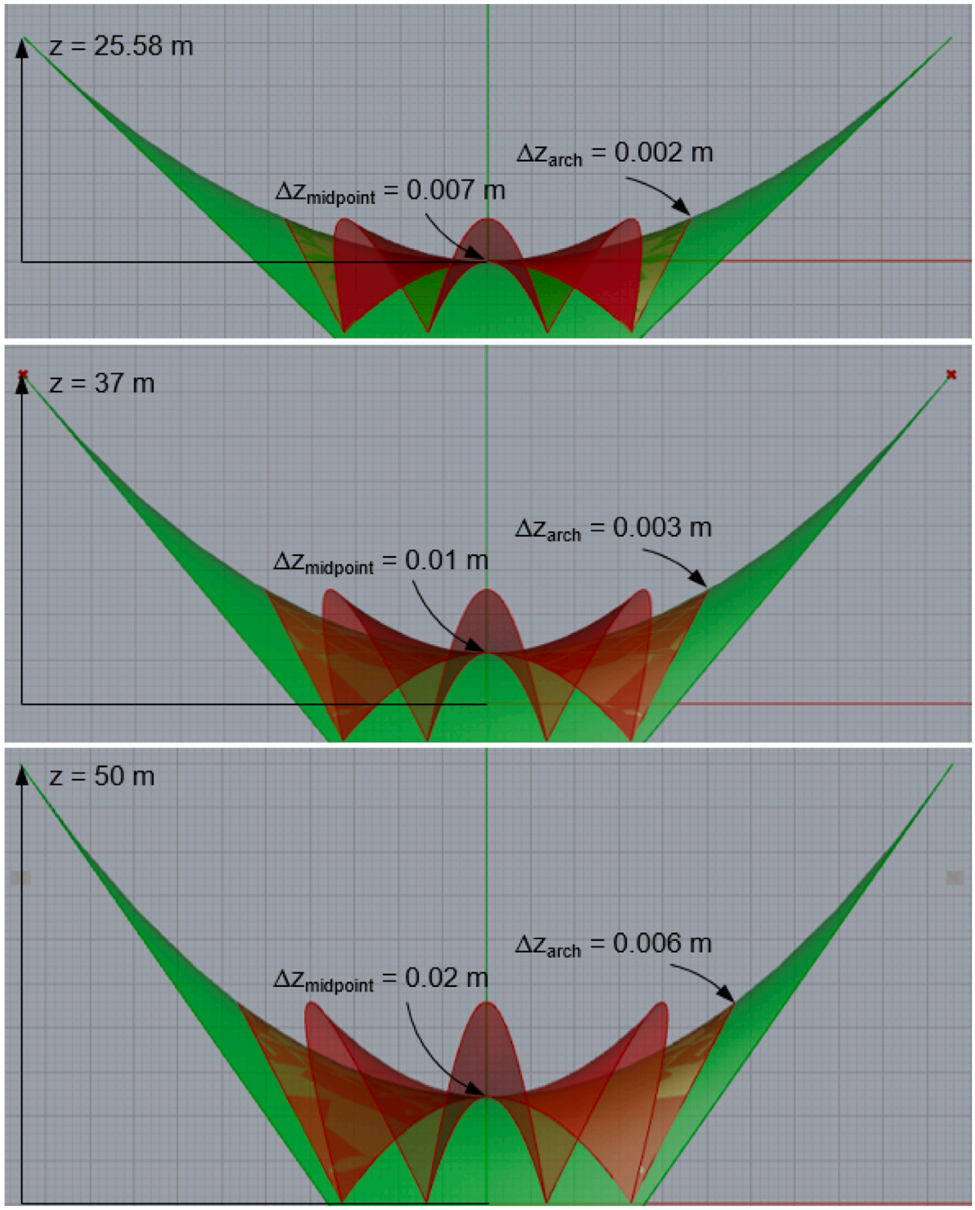

Fig. 11. Changing the height parameter z and observing the vertical deformation Δz at the midpoint and highest point of the outer arches for a uniformly distributed dead load of 5 kN/m^2.

its mechanical performance. Our workflow updates and expands the possibilities of formal exploration thanks to the use of parametric tools, integrated analysis and form-finding capabilities in a common CAD environment.

The integration of the mechanical analysis into the CAD model provided by Isogeometric B-Rep Analysis (IBRA) eliminates the need of meshing the geometry, sparing thereby a time-consuming step in a normal workflow. The key ingredient is the ability to perform mechanical analyses within the same CAD parametric environment that is used to generate different geometries. Exploring the design space by changing different parameters becomes a straightforward task for the user, making it easier to produce suitable design alternatives. The design space is not restricted to any particular simple geometric shape, but can include trimmed multi-patch geometries, taking advantage of the straightforward structural analysis provided by IBRA. In addition, the presented framework provides form-finding capabilities that guide the user in the search of better performance by letting the interaction of form and forces steer design choices.

Future developments may include optimization capabilities (e.g. related to environmental performance or modularity for end-of-life disassembling scenarios) to orient the design to sustainable solutions. The concept of CAD-integrated design and analysis is not limited to lightweight structures and shell elements, but can just as well be applied to other structural systems. Integrating parametric design and analysis into Computer Aided Design (CAD) environments can be interpreted as adding a layer of structural properties and results onto an existing CAD-model — and thus as a structural digital twin. Additional data can be added in the same manner, with the CAD or Building Information Model (BIM) model as a basis. In addition, current research towards monitoring and related maintenance decisions for buildings holds the promise of creating feedback-loops from on-site monitoring to structural models that can accompany and possibly prolong a building's life cycle. Relating this to CAD-integration means that the creation of interfaces between monitoring data and the parametric environment is needed in order to move towards a structural digital twin that can process this data and hence provides as-built post-processing results. It should be mentioned once more, that for lightweight structures

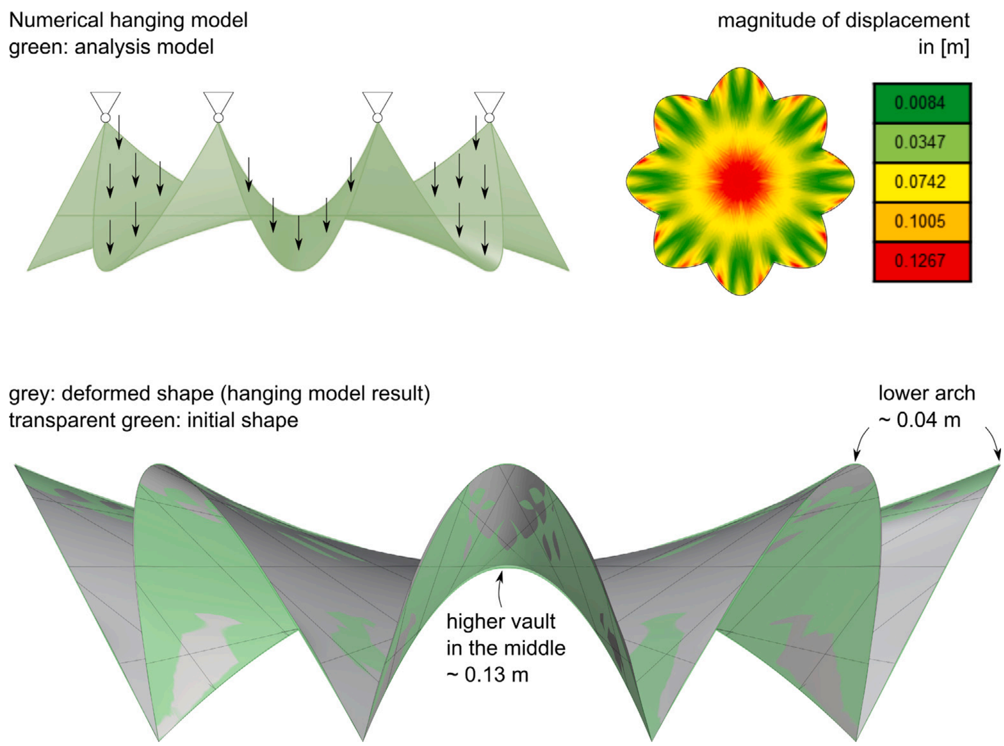

Fig. 12. CAD-integrated hanging model compared to the initial shell geometry.

like shells, geometrical and mechanical properties and/or changes are equally important for an accurate assessment of structural health.

CRediT authorship contribution statement

Ann-Kathrin Goldbach: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Carlos Lázaro:** Conceptualization, Methodology, Validation, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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