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A proposal of analytical formulations to calculate safety lead times under demand variability. A case study

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A R T I C L E I N F O

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A B S T R A C T

This paper deals with the issue of safety lead time (SLT) calculations in production-inventory systems in presence of both demand and replenishment lead time variability. We provide some formulations of the SLT and numerically show their performance as compared to a benchmark in the literature. Thus the main objective of this paper is to provide analytical formulations to calculate the SLT that contemplate demand variability. To this end, a literature review was done to analyze the approaches and justifications of the different revised research works to identify reference formulations according to the objectives of this work. A supply chain from the automotive sector was used as the study frame and to validate the proposed formulations. This supply chain involved two companies: a car manufacturer and a first-tier supplier. In order to compare the proposed formulations with one another, and with that currently used by the first-tier supplier and is the study object, three parameters were used: safety stock, the number of times stockout occurs and the mean stock. They allowed the final selection of the most suitable SLT formulation for each case study.

Glossary of acronyms:	
EOQ:	Economic order quantity.
ERP:	Enterprise resource planning.
JIT:	Just in time.
KPI:	Key performance indicator.
LT:	Lead time
MRP:	Material requirement planning.
PLT:	Planned lead time.
POQ:	Periodic order quantity.
SLT:	Safety lead time.
SS:	Safety stock.

1. Introduction

The classical material requirement planning (MRP) [1] approach in the supply chain management context [2] is based on the reasoning that demand and supply time or lead time (LT) are known and follow a deterministic pattern. In the real world however, there are many forms of uncertainty that affect production processes, such as uncertainty in demand or supply [3–8]. Given such uncertainty, stabilization techniques like safety stock (SS) and safety lead time (SLT) should be considered before loading the MRP system. The objectives of the SS are to absorb fluctuations in supply and demand; e.g., unexpected demand and short supplies, and to stabilize any errors in stock records that might occur during production. The objective of the SLT is to absorb fluctuations in the supply schedule by conferring production planner flexibility under uncertainty; the LT of components is rarely forecast reliably; poor supply planning leads to situations with excess stock or, conversely with low stocks. In certain cases, the uncertainty of LTs does not essentially cause any effect, and can therefore be ignored. However, in most cases, fluctuations in LTs considerably degrade system performance.

Boutsioli [9] distinguishes between variability and uncertainty in demand by determining that both concepts are interrelated, but not identical. Variability in demand is the sum of the forecast part, plus the part of demand that cannot be forecast. Hence, uncertainty in demand is defined as the part that cannot be forecast in demand variability. This author also considers that demand variability, and demand uncertainty in particular, more strongly impacts costs. It has also been observed that most studies employ aggregate data and lose relevant daily information about the nature of the variability in demand and uncertainty.

Given the optimal SS level is adequate for a certain inventory system and inventory control [10] and for measuring supply chain performance

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[11], here we present alternative SLT calculations under a demand variability context. The main contributions of this paper are: (i) identifying existing approaches to calculate SLTs; (ii) proposing new analytical formulations to calculate the SLT which contemplate demand variability; (iii) applying and validating these analytical formulations in the multiproduct assembly context, specifically in a supply chain in the automotive sector. In this research, demand variability is considered as the daily difference between estimated demand and real demand by taking demand variability as a function of normal distribution where calculations are done with disaggregate data on a daily basis. This study also includes the interrelation between variability and uncertainty in demand, and both concepts are not considered identical, but divided by a narrow frontier by taking uncertainty to be lack of knowledge about the root source that causes the brusque changes which result in daily variations between estimated demand and real demand.

The rest of the paper is organized as follows: Section 2 analyzes the concepts and modelling approaches to calculate SLTs. Section 3 contains the problem description and the SLT calculation process in the supply chain that is the study object. Section 4 describes the analytical model formulation and Section 5 applies and simulates the proposed formulations. Section 6 evaluates the results. Finally, Section 7 offers the conclusions and future research lines.

2. Literature review

New [12] states, after a conceptual analysis, that using the SLT probably proves more useful when raw material is purchased from an external company. Whybark and Williams [13] consider that both the SLT and SS are two stock stabilization techniques which can be used when uncertainty appears. These authors demonstrate that the SLT is preferable to the SS if there is uncertainty in the LT, especially for made-to-order systems where end products can be highly adapted to customer requirements.

Lambrecht et al. [14] formulate a Markov model to fix production quantities and to identify the quantity of the SS and the SLT in a serial two-level system with constant processing times. Melnyk and Piper [15] define that the planned lead time (PLT) is the sum of the LT and SLT, and suggest that prolonging the LT as an effective method to face uncertain LTs. Chang [16] considers that the SS can stabilize shortages caused by delayed raw material and production processes. The SLT can stabilize fluctuations in process times with lower levels. This author also considers interchanging the SS and the SLT by representing the one-level production process with a deterministic production time per produced unit and, therefore, considers the SS to be the equivalent to the SLT.

Kanet [17] demonstrates that changing the PLT can have fleeting and stationary effects. Indeed, the objective of cutting stock is only achieved temporarily when reducing the PLT is contemplated. Yano [18–20] presents a generalized process for a two-level system based on a continuous single-period stock control model by taking the LT and the processing time as stochastic variables.

Vargas and Dear [21] propose that the SS should be robust enough by comparing it with the SLT to face brusque changes in demand. Buzacott and Shanthikumar [22] determine that when excessive production capacity is lacking, both the SS and SLT are useful measures to face supply variability in those occasions in which demand information is not very reliable. These authors conclude that the SS is more robust to ease changes in customer requirements of the LT or other fluctuations when it comes to forecasting the demanded LT. However, if the intention is to forecast demand, then the models suggest that either of the two parameters can be used.

Gupta and Brennan [23] identify that the amount of LT uncertainty strongly influences the cost. Keaton [24] determines a stockout function and describes the LT as a stochastic variable which, when increasing LT variability, increases the number of stockouts.

Lambrecht and Vandaele [25] combine lot-sizing models and single-product queues with aleatory LTs. These authors follow an approach with expected values and the LT variance of lots, and bring together the LT distribution probability and a logarithmic function to meet the customer's service level. Fujiwara and Sedarage [26] consider an EOQ model (economic order quantity) in which the LT of the components is aleatory to minimize the mean total cost per time unit, made up of the fitting cost, the stock holding for components and the assembled product, and the stockout cost of the assembled product.

Tang [27] studies a multiperiod single-level production-inventory system based on the SLT, which would function better than one based on the SS when stochastic influences result from supply or production times. Hegedus and Hopp [28] develop a practical method by considering the service level to fix the SLT of the components in an assembly system with stochastic LTs. Their results indicate that the optimum SLT reduces supply variability and determine that the SS is used to face demand uncertainty. Koh et al. [29] identify how the SLT is a widely used tool to ease LT uncertainty. Chopra et al. [30] base their work on two management prescriptions with normal distribution: (i) for service levels above 50 %, reducing LT variability reduces the reorder point and the SS; (ii) for service levels above 50 %, reducing LT variability is more effective than reducing the LT because this considerably reduces the SS.

Koh and Saad [31] simulate a production system with increasing demand, LT variations and breakdowns of resources to evaluate the system's capacity to meet their delivery objectives by contemplating four scenarios: (1) assigning an LT; (2) assigning capacity; (3) assigning the SS; (4) flexibility in grouping. Lin and Lin [32] consider a supply chain that consists in many retailers and one supplier, and they examine the ways in which the supplier can reduce total demand variance by adjusting the size of orders with the Portfolio Theory. Song et al. [33] considers a multilevel made-to-order assembly system with stochastic production supply times to find optimum PLTs by minimizing the sum of stock holding costs and the costs caused by orders arriving late.

Hnaien and Dolgui [34] consider a multilevel assembly system in a supply chain with aleatory LTs to find an optimum supply plan for all levels and for the date that each component is released. Hnaien et al. [35] contemplate a multilevel supply chain with aleatory LTs to find optimum release dates and to meet the demand of the end products subject to a due date. These authors introduce the same assumptions as Yano [19] and Chu et al. [36].

Jakšič and Rusjan [37] evaluate the impact of forecasting demand and the LT to eliminate the bullwhip effect. Louly et al. [38] study the stock control problem for an assembly system where the LT of the components is aleatory. These authors propose a lot-for-lot policy to minimize the holding cost of the components according to a given service level. Louly and Dolgui [39,40] consider calculating the PLT for MRP systems according to a periodic order quantity (POQ) policy for multiple components, whose LTs can be modelled with all the possible distributions by minimizing the average holding costs, but also setup costs.

Nenni et al. [41] evaluate the effect of delaying orders in the customer service by considering an SLT in making deliveries. With the technique they present, it is possible to seek optimum compensation between the LT and the SS, and to also determine the increase in service level by considering the LT. Chatfield et al. [42] simulate a model that examines the effects that stochastic LTs, distributing information and the quality of this information have on a supply chain. They identify how LT variability worsens the extension of variance in the supply chain, and that distributing information and its quality are most significant.

The supply variability effect is reflected in delivery performance levels, which are measured as the fraction of the orders that meet the due date according to the production plan [43]. The literature reports similar relations [44,45]. Using the SLT to face this problem leads to a better scenario in which performance levels increase. Utilizing the SS to face supply variability leads to worse results compared to those obtained with the SLT technique. This is explained by inherent SLT-related flexibility, where products are not specified beforehand as they are in the SS.

Van Kampen et al. [43] investigate the effects of both the SS and SLT on the performance of deliveries in a multiproduct scenario with supply under uncertainty combined with three types of demand uncertainty caused by: a change in lot size, a change in order type and changes in the sequence of orders. These authors conclude that the reliability level of demand information and production performance forces the system to adopt a stabilization technique, either the SS or the SLT, to guarantee acceptable delivery performance.

Gansterer et al. [46] propose a simulation–optimization procedure in a hierarchical planning production system to optimally set the PLT, the SS and lot sizes. Recommendations for identifying good parameter settings are provided in the robust production planning context.

Altendorfer [47] studies the influence of lot sizes and the PLT on inventory levels and service level in a make-to-stock production system. In an uncertain demand environment, and by applying endogenous production time distribution, the proposed analytical procedure allows the optimization of both planning parameters to reach a service level constraint with minimum overall inventory. A similar approach that considers the relation between lot sizes and the LT is addressed by Boute et al. [48] who, by applying a Markov chain analysis and matrix analytic methods, determine the distribution of the LT and stocks that takes into account the correlation of these two planning parameters. Yuan and Graves [49] consider the simultaneous calculations of PLTs and production lot sizes in a job shop environment by implementing a nonlinear optimization model on a spreadsheet. Prak et al. [50] determine SSs for constant LTs and a stationary demand considering the uncertainty associated with the estimation of the mean and the variance of the demand. The authors address batch ordering policies by combining moving average and exponential smoothing forecast techniques. Ben-Ammar et al. [51] propose a general probabilistic model and a genetic algorithm to determine PLT and SS levels by minimizing backlogging and inventory holding costs. The authors prove that it could be better to optimize PLTs rather than apply SSs. Kania et al. [52] develop a solution method based on an evolutionary algorithm to integrate a lot sizing problem with the problem of determining the optimal values of SS and SLT.

De and Mahata [53] study a supply chain network with flow and raw materials with imperfect quality and under the effect of learning experiences in a fuzzy decision-making process with constant demand and a fixed rate replenishment. Additionally, De and Mahata [54] consider an EOQ inventory model for items with imperfect quality developing a fuzzy mathematical model, which considers demand cost, inventory system parameters and analyses a single type of product with instantly replenished, i.e., LT equals to zero. On the other hand, Barman and Mahata [55] develop a supply chain inventory model with a single-manufacturer and multi-retailers in which each retailer's demand is dependent on selling price of the product and LT, following a random normal distribution, is composed of several components that could be reduced by adding additional crashing cost. Also, Barman and Mahata [56] conceptualize a vendor–buyer supply chain production inventory model based on the advance payment phenomenon where retailers receive price discounts and considering a buyer stochastic LT. Finally, Barman and Mahata [57] study an integrated vendor–buyer inventory system with a controllable buyer LT which could be reduced by using a crashing cost from normal to minimum duration. They develop the study based on the idea that production control, cost reduction and a controllable LT is a strategic key performance indicator (KPI) for

Table 1
Survey of papers addressing SLT calculations with demand variability.

References	Demand	LT	PLT	SS	SLT	Aim
Whybark and Williams [13]	Uncertain	Uncertain		X	X	Uncertainty dampening
Lambrecht et al. [14]	Variable	Fixed		X	X	Uncertainty dampening
Melnyk and Piper [15]	Variable	Variable			X	Cost
Chang [16]	Stochastic	Fixed		X	X	Stockout
Kanet [17]	Fixed	Variable	X			Stockout
Yano [18–20]	Fixed	Stochastic				Cost
Buzacott and Shanthikumar [22]	Variable	Not considered		X	X	Stockout
Vargas and Dear [21]	Variable	Fixed		X	X	Stockout
Gupta and Brennan [23]	Uncertain	Uncertain				Cost
Keaton [24]	Stochastic	Stochastic				Stockout
Lambrecht and Vandaele [25]	Fixed	Variable				Service level
Fujiwara and Sedarage [26]	Fixed	Stochastic				Re-order point
Tang [27]	Fixed	Stochastic				Cost
Hegedus and Hopp [28]	Variable	Stochastic			X	Stockout
Koh et al. [29]	Stochastic	Uncertain		X	X	Uncertainty dampening
Chopra et al. [30]	Variable	Variable		X		Re-order point
Koh and Saad [31]	Variable	Variable		X		Service level
Lin and Lin [32]	Variable	Fixed				Bullwhip effect
Song et al. [33]	Fixed	Uncertain	X		X	Cost
Hnaien and Dolgui [34]	Fixed	Stochastic			X	Cost
Hnaien et al. [35]	Fixed	Stochastic	X		X	Cost
Jakšič and Rusjan [37]	Variable	Fixed				Bullwhip effect
Louly et al. [38]	Fixed	Uncertain			X	Cost
Louly and Dolgui [39,40]	Fixed	Stochastic			X	Cost
Nenni et al. [41]	Variable	Stochastic	X	X		Service level
Chatfield et al. [42]	Variable	Stochastic				Bullwhip effect
Van Kampen et al. [43]	Variable	Fixed				Service level
Gansterer et al. [46]	Variable	Stochastic		X		Correlation D, LT and SS
Altendorfer [47]	Stochastic	Stochastic	X			Service level
Boute et al. [48]	Variable	Variable		X		Correlation D, LT and SS
Yuan and Graves [49]	Variable	Variable				Cost
Prak et al. [50]	Variable	Variable		X		Re-order point
Ben-Ammar et al. [51]	Variable	Stochastic	X	X		Cost
Kania et al. [52]	Stochastic	Uncertain		X	X	Stockout
De and Mahata [53]	Fixed	Variable				Cost
De and Mahata [54]	Variable	Fixed				Cost
Barman and Mahata [55]	Variable	Stochastic				Cost
Barman and Mahata [56]	Stochastic	Stochastic				Cost
Barman and Mahata [57]	Stochastic	Variable				Cost
Our paper	Variable	Variable	X	X	X	Stockout

commercial enterprise fulfilment.

Table 1 compares the reviewed literature related to demand, LT, PLT, SS, SLT with our proposal. The papers that consider variable demand and LTs are, mainly, oriented to measure cost, service level and re-order points. In general, the literature reviewed focused primarily on costs and secondarily on stockouts. Nevertheless, 50 % of the cost-oriented studies consider the demand parameter as a fixed value. On the other hand, almost 40 % of the papers included SLT as an explicit parameter in their investigation but few are stockout oriented. As showed in Table 1, none of the reviewed papers consider directly the stockout indicator as a key factor when calculating SLT with demand and LT variability.

Relevant studies have shown the importance of measuring stockout cost, effects, consequences on factories, buyers, customers, and their relationship. For example, Dion et al. [58] investigate the consequences of a stockout for the buyer and the effects on vendor relationships. These authors carried out an exploratory study on 180 National Association of Purchasing Management members through a questionnaire. On the other hand, Kahn [59] argues that stockout avoidance is largely sufficient to explain the pertinent facts about inventories, based on that firms hold inventories not to smooth production but rather because stockouts are costly, and includes theory and evidence on the stockout-avoidance motive for inventory-holding. Also, Dion and Banting [60] study the buyer reactions to product stockouts in business-to-business markets through interviews and mail surveys. The authors identifies that product availability is viewed as a critical aspect of customer service and addresses the operational consequences of stockouts for the buyer firms and the actions taken due to that. Gallego and Moon [61] study the multiple product single facility stock avoidance problem (SAP) to find a schedule that avoids stockouts over a finite horizon. Finally, Andersen et al. [62] consider the understanding of a stockout cost is critical if retail managers want to implement an inventory model. Based on that, the authors conduct a field test to measure short and long run stockout cost.

Therefore, we conclude that in most of the reviewed research works the terms SS and SLT are jointly dealt with and address variability in demand, supply, and the LT. Nevertheless, and based on those conditions, few of them calculate SLT and measure its impact also in terms of stockouts.

This research works explicitly focus on optimizing the system by reducing stockouts, considering jointly PLT, SS and SLT with variable demand and LT. Variability in LTs does not appear to have been sufficiently studied, particularly in assembly systems whose components have variable LTs. Establishing LTs to control production systems with uncertainties is a complex problem. The literature provides models for certain cases that involve specific variables and exclusive conditions where reaching conclusions unusually differ. By considering all this information, this article develops an analytical formulation to calculate the SLT to face demand variability in a multiproduct assembly context.

3. Analytical formulations

In this section, we propose analytical formulations to improve SLT calculations in demand variability contexts. The nomenclature to be used throughout this section is provided below (Table 2).

Table 3 presents the four equations taken as the basis for the present work and proposed by Chatfield et al. [42], Jakšič and Rusjan [37], Lin and Lin [32] and Nenni et al. [41].

With the four equations in Table 3, we propose six new analytical formulations to calculate the SLT that incorporate demand variability, five of which are based on the aforementioned research works, and one on the experienced acquired from analysing data and through the research conducted for the present work. These six analytical formulations are:

(adapted from Chatfield et al. [42])

Table 2
Nomenclature.

I	Number of items (i = 1,…,I)
T	Time periods ($t = n, \ldots, T$), where n is a set positive or negative number
$D_{i,t}$	Demand of item i during time period t
$P_{i,t}$	Production of item i during time period t
$VD_{i,t}$	Demand variability of item i during time period t
$\beta_{Di,t}$	Normal distribution of the demand variability of item i during time period t
$\sigma_{Di,t}$	Standard deviation of the demand variability of item i during time period t
$\overline{VdD}_{i,t}$	Demand variability average of item i during time period t
n_i	Number of data readings of item i
$Z_{i,t}$	Constant corresponding to desired service level of item i during time period t
$R_{i,t}$	Number of times that deliveries are made weekly of item i during time period t
$k_{i,t}$	Number of deliveries made of item i during time period t
w_t	Number of weeks during time period t
$W_{cf_{i,t}}$	Reliability constant of on time delivery of item i during time period t
$SLT_{i,t}$	Safety lead time of item i during time period t
$PLT_{i,t}$	Theoretically planned lead time of item i during time period t
$LTp_{i,t}$	Real time it takes each placed order of item i to reach its destination during time period t
$\sigma_{LTp_{i,t}}$	Standard deviation of the real time it takes each placed order of item i to reach its destination during time period t
$LT_{i,t}$	Previously set lead time of item i during time period t
$\sigma_{DNi,t}$	Standard deviation of the normal distribution of the demand variability of item i during time period t
$\overline{DT_{i,t}}$	Average lead time of the deliveries made of item i during time period t
$O_{i,t}$	Placed order of item i during time period t
$Dy_{i,t}$	Theoretical deliveries made of item i during time period t
$S_{i,t}$	Stock of item i during time period t
$SS_{i,t}$	Safety stock of item i during time period t
$SA_{i,t}$	Mean stock of item i during time period t
$DyS_{i,t}$	Deliveries from the first-tier supplier of item i during time period t
$sof_{i,t}$	The variable used as counter of stockouts of item i during time period t takes a value of 1 when the stock during t is below zero
$SO_{i,t}$	Number of stockouts of item i during time period t
$S_{Li,t}$	Standard deviation of the lead time of item i during time period t
$\overline{DT}_{i,t}$	Average lead time of item i during time period t
$\sigma^2{}_{DNi,t}$	Standard deviation of the normal distribution of the demand variability of item i during time period t
WD	Number of working days per week

Table 3
Reference equations.

Equation by Chatfield et al. [42]	Equation by Lin and Lin [32]
$S_X{}^2 = (\overline{L}+R)*S_D{}^2 + \overline{D^2}*S_L{}^2$ $S_X{}^2$ = variance of demand during period $L + R$. $\overline{L}$ = average lead time R = review period S_D = standard deviation of demand $\overline{D}$ = average demand S_L = standard deviation of the lead time	$S_{i,t} = (l_R + R)*\widehat{\mu_{i,t}} + k_i*\sigma_{i,l_r+R}$ l_R = the lead time from the supplier to the customer $\widehat{\mu_{i,t}}$ = demand of customer i forecast for time period t. $S_{i,t}$ = customer's reorder point k_i = constant that determines the desired service level R = review period σ_{i,l_r+R} = standard deviation of the errors in forecasting in interval R and the lead time
Equation by Jakšič and Rusjan [37]	**Equation by Nenni et al. [41]**
$SS_t = k*\overline{D}*\sqrt{R+L}$ k = safety factor corresponding to desired service level $\overline{D}$ = forecasted demand SS_t = safety stock level R = review period L = lead time	$SS = k*\sqrt{\sigma^2{}_D*\overline{DT} + \sigma^2{}_{DT}*\overline{D}^2}$ SS = safety stock $\overline{D}$ = average demand $\overline{DT}$ = average lead time σ_D = standard deviation of demand σ_{DT} = standard deviation of the lead time k = safety factor corresponding to desired service level

$$SLT_{i,t} = \sqrt{Z_{i,t}{}^2 + \beta_D{}^2{}_{i,t} * S_{Li,t}{}^2} \tag{1}$$

(adapted from Lin and Lin [32])

$$SLT_{i,t} = \sqrt{(WD/R_{i,t}) * S_{Li,t} + Z_{i,t} * \beta_{Di,t}} \tag{2}$$

(adapted from Jakšič and Rusjan [37])

$$SLT_{i,t} = Z_{i,t} * \beta_{Di,t} * \sqrt{WD/R_{i,t} + S_{Li,t}} \tag{3}$$

(source: the authors)

$$SLT_{i,t} = W_{cf\,i,t} * \left(Z_{i,t} * \frac{WD}{R_{i,t}} + \beta_{Di,t} * S_{Li,t}\right) \tag{4}$$

(adapted no. 1 from Nenni et al. [41])

$$SLT_{i,t} = Z_{i,t} * \sqrt{\sigma^2{}_{DNi,t} * \overline{DT}_{i,t} + S_L{}^2{}_{i,t} * \beta_{Di,t}} \tag{5}$$

(adapted no. 2 from Nenni et al. [41])

$$SLT_{i,t} = Z_{i,t} * \sqrt{\sigma^2{}_{DNi,t} * LT_{i,t} + S_L{}^2{}_{i,t} * \beta_{Di,t}} \tag{6}$$

Demand variability can be determined with the above analytical formulations, and by using the demand and real production values (7).

$$VD_{i,t} = D_{i,t} - P_{i,t} \tag{7}$$

Once this value is obtained for each i and t, the normal distribution of demand variability is calculated as:

$$\beta_{Di,t}(\overline{VdD}_{i,t}, \sigma_{Di,t}) \tag{8}$$

The number of times that deliveries are made weekly is also considered and calculated with this equation:

$$R_{i,t} = k_{i,t}/w_t \tag{9}$$

Analytical formulation (4) considers the reliability that a delivery is made on time, $W_{cf\,i,t}$. This value is determined with the following equation:

$$W_{cf\,i,t} = 1 - \sum_{t=1}^{n_i} PLTt_{i,t} / \sum_{t=1}^{n_i} ABS(LTp_{i,t} - PLTt_{i,t}) \tag{10}$$

For analytical formulations (5) and (6), it is necessary to determine the value of the average lead time of the deliveries made of item i during time period t.

$$\overline{DT}_{i,t} = \left(\sum_{t=1}^{n_i} LTp_{i,t}\right) / n_i \tag{11}$$

Finally, the PLT for each proposed equation is calculated with the equation below:

$$PLT_{i,t} = LT_{i,t} + SLT_{i,t} \tag{12}$$

To validate the above equations, having determined the PLT, demand becomes an order as so:

$$O_{i,t} = \sum_{t=1}^{n_i} D_{i,t} ift = PLT_{i,t} \tag{13}$$

Therefore, deliveries are:

$$Dy_{i,t} = O_{i,\pm T - LT_{i,t}} \tag{14}$$

The "+" value for T is for the positive T values, and the negative sign is for the negative T values. In parallel, stock is calculated without considering the SS as follows:

$$S_{i,t} = S_{i,t-1} + Dy_{i,t} - P_{i,t} \tag{15}$$

The next equation is used to calculate the SS, which is the minimum value that appears in the stock calculation.

$$SS_{i,t} = min(S_{i,t}) \tag{16}$$

The next step is to calculate the mean existing stock with the following equation:

$$SA_{i,t} = \left(\sum_{t=1}^{n_i} S_{i,t}\right) / n_i \tag{17}$$

The stock that does not include the SS is slightly modified to be updated and is calculated as follows:

$$S_{i,t} = S_{i,t-1} + DyS_{i,t} - P_{i,t} \tag{18}$$

Finally, for the six proposed analytical formulations and for the first-tier supplier's equation performance, it is important to calculate the number of stockouts. This calculation is done as follows:

$$sof_{i,t} = 1 if S_{i,t} < 0 and S_{i,t} = 0 if SO_{i,t} = \sum_{t=1}^{n_i} sof_{i,t} \tag{19}$$

4. A case study

The supply chain considered herein is of a dual-type [63], formed by an automotive manufacturer and a first-tier supplier. The first-tier assembly supplier is in charge of its own logistics and independently selects its suppliers and components for its products. Its production plant is made up of four production plants, where finished goods are assembled and supplied by a just-in-time (JIT) production system. MRP is calculated by using the company's ERP (enterprise resource planning), which is based on a standard MRP system that employs the weekly and daily demand information provided by the automotive manufacturer. According to the levels of demand that the automotive manufacturer supplies, the first-tier supplier must plan transport in such a way that the required replenishment level is met to most efficiently cover the customer demand on its assembly lines. To do this, the logistics department plans with the suppliers of materials and transport the calendar with which to replenish materials. Each order placed with suppliers is made according to the net calculated requirements and is placed for a period depending on the contemplated supplier. These firm orders are included in the MRP system as scheduled arrivals. Once they have arrived, receiving orders are compared with the calculations of the aforementioned net requirements.

In order to determine the time when orders must be made for each product, stock is constantly checked by the MRP system. Some parameters, like the SS and the LT, are previously determined so that the MRP system knows when the time is right to place an order.

Five workdays per week are considered. In principle, it is established that all the orders placed by the first-tier supplier will reach the factory within 3 days (LT), regardless of the quantities ordered. However, the SLT is aggregated to the previously set LT to deal with possible uncertainty parameters. The first-tier supplier uses the following formula based on its experience in determining the SLT and calculating the PLT.

$$SLT_{i,t} = \left[0.3 \cdot \left(\frac{5}{R_{i,t}}\right) + LT_{i,t} \cdot 0.4\right] \cdot \alpha \tag{20}$$

where $SLT_{i,t}$ is the safety lead time of item i during time period t. $R_{i,t}$ is the number of deliveries made weekly, $LT_{i,t}$ is the previously set LT, which equals 3 days, and α is a safety coefficient to be defined by the planner, which takes values between 0 and 1. In this paper, α is considered 1. The number of deliveries made weekly is calculated using the list of deliveries made by each supplier, which varies depending on the use of components. If these two parameters are included in the Equation, the SLT for each product is obtained. The value obtained by equation (20) is the time that should be summed to the LT, whose sum results in the PLT that considers possible breakdowns or delays in the supplier's deliveries.

5. Evaluation of results

The model application considers information from 13 items on an approximate 3-month time horizon based on the background of demand and production, and on a six-month one for the deliveries made by the first-tier supplier. Table 4 presents a summary of these initial data. The information on deliveries provided by the first-tier supplier includes the date on which the order was sent and that on which the customer received it. In this way, it is possible to calculate the real LT that each order takes to reach its destination, and to then calculate is average. For future calculations it is worth determining the standard deviation of the LT. Another piece of useful information is the average number of deliveries made weekly, which can be obtained from the information about the number of deliveries made weekly and the number of weeks from the reading period. It is also possible to calculate the SLT with equation (20), and the corresponding PLT, this being $PLT(20)_{i,t}$. Thus, the average LT, the weekly deliveries made, the standard deviation and the reliability constant consider the total reading period of deliveries; i.e., 6 months. For the quantity of the delivered product, the period that coincides with the reading period of both demand and production is only considered.

Having viewed all the information as a whole, each particular item is analysed. Table 5 shows the demand variability calculation. The information there refers to item 1 and shows the demand, production and quantity of the delivered material for each day. In this case, we can see that the number of readings equals 54 (according to Table 4). Demand variability is calculated from the difference between demand and production. Real demand is considered the first-tier supplier's production because the company that produces vehicles determines and freezes the production sequence that the first-tier supplier must make on a daily basis (Scenario 1). To extend the analyses, another scenario is dealt with in which the demand variability of the moving averages of both demand and production is calculated (Scenario 2). Table 5 shows that the normal distribution of demand variability is also calculated.

Table 6 shows the process followed to evaluate all six proposed analytical formulations through item 1, which specifically indicates the time when daily demand must be requested according to each equation. Table 7 shows Scenario 2 for the same item 1, which considers the normal distribution of the demand variability obtained by the difference in the moving average of both demand and production.

The initial t' value for the moving averages depends on the equivalent value to t for $t' = t + 4$. It is worth mentioning that the objective is to obtain a 99 % customer service level in all items i for time period t. Hence the constant obtained to reach this level is $Z_{i,t}(99\%) = 2.327$.

A $PLT_{i,t}$ number exists that equals the number of proposed analytical formulations, from (1) to (6), for each scenario: i.e. 12 equations per item. Numbering is in accordance with that shown in Table 6 and Table 7: $PLT(1)_{i,t}$, $PLT(2)_{i,t}$, $PLT(3)_{i,t}$, $PLT(4)_{i,t}$, $PLT(5)_{i,t}$ and $PLT(6)_{i,t}$. Every day the PLT is calculated for the six analytical formulations, and how many days the quantity demanded each day must be determined beforehand.

Tables 8 and 9 show the time when the quantity demanded for the real values must be requested (Table 6), and also for the values that consider demand variability (Table 7), respectively. For placed orders, the value that results from the PLT is considered. The stock with and without the SS is also calculated. Daily demand considers the PLT and assigns it a placed order. This process is done from $t = 1$ to $T = n_i$ for demand and is assigned to the placed orders that consider the time interval from $t = -10$ to $T = n_i$. Delivered orders are made by considering

Table 4
Initial data analysis.

i	n_i(Days)	w_t(Weeks)	$D_{i,t}$(Units)	$P_{i,t}$(Units)	$DyS_{i,t}$(Units)	$k_{i,t}$(Deliveries)	$R_{i,t}$(Deliveries / Week)	$LTp_{i,t}$(Days)	$\sigma_{LTp_{i,t}}$	$W_{\sigma_{i,t}}$	$LT_{i,t}$(Days)	$SLT_{i,t}$(Days)	$PLT(20)_{i,t}$(Days)
1	54	25.2	17,000	17,155	16,384	94	4	5	2.819515609	0.580330707	3	2	5
2	54	25.2	16,020	1562	1080	30	1	5	2.330529086	0.741391941	3	1	4
3	50	25.2	1280	1562	1120	30	1	5	2.345183492	0.771916972	3	1	4
4	51	25.2	14,620	15,588	14,440	78	3	5	2.912904885	0.664140457	3	2	5
5	54	25.2	15,460	15,586	14,840	84	3	5	3.186708842	0.589093702	3	2	5
6	52	25.2	33,240	34,298	33,096	93	4	5	3.18823406	0.570945045	3	2	5
7	52	25.2	41,975	46,385	42,775	75	3	4	2.946483699	0.620498866	3	2	5
8	47	25.2	6500	5268	4800	13	1	3	1.2741378	0.432900433	3	1	4
9	23	25.2	500	99	500	1	0	0	0	0	3	1	4
10	52	25.2	30,050	29,064	25,940	44	2	6	3.260114402	0.750796373	3	1	4
11	52	25.2	56,050	60,866	53,100	69	3	5	2.980989105	0.675532197	3	2	5
12	52	25.2	22,000	22,678	22,470	72	3	4	2.56815138	0.652851264	3	2	5
13	51	25.2	16,000	17,880	16,129	62	2	4	3.016920179	0.608446377	3	1	4

Table 5
Data analysis for item $i = 1$.

t	$P_{i,t}$(Units)	$DyS_{i,t}$(Units)	$D_{i,t}$(Units)	$VD_{i,t}$(Units)	$\beta_{Di,t}$	Moving average			
						$P_{i,t}$(Units)	$D_{i,t}$(Units)	$VD_{i,t}$(Units)	$\beta_{Di,t}$
1	385	720	0	−385	0.056080	–	–	–	–
2	396	0	0	−396	0.051099	–	–	–	–
3	0	0	0	0	0.504760	–	–	–	–
4	0	0	0	0	0.504760	–	–	–	–
5	386	664	480	94	0.656416	–	–	–	–
6	329	360	400	71	0.620612	–	–	–	–
7	363	440	400	37	0.565822	–	–	–	–
8	293	520	320	27	0.549412	–	–	–	–
9	740	360	400	−340	0.080534	422	400	−22	0.403418
10	0	240	360	360	0.934286	345	376	31	0.666322
11	317	520	0	−317	0.095798	343	296	−47	0.289868
12	411	440	400	−11	0.486520	352	296	−56	0.249696
13	414	360	280	−134	0.292835	376	288	−88	0.139278
14	376	440	480	104	0.671577	304	304	0	0.516725
15	320	0	0	−320	0.093694	368	232	−136	0.046302
16	317	0	400	83	0.639444	368	312	−56	0.252117
17	381	400	440	59	0.601489	362	320	−42	0.311926

the LT previously established by the company that manufactures vehicles and the first-tier supplier, and is the equivalent to $LT_{i,t} = 3$.

In this case, the initial stock value corresponds to the deliveries made from $t = -10$ to $T = 1$. Thus there are no production values and the SS is null.

$$S_{i,1} = \sum_{t=-10}^{1} Dy_{i,t} - P_{i,1} \tag{21}$$

Next the stock that considers the SS is calculated and it is only necessary to make one modification to Equation (21), as so:

$$S_{i,1} = \sum_{t=-10}^{1} Dy_{i,t} + SS_{i,t} - P_{i,1,t} \tag{22}$$

where $S_{i,1}$ for $t = 1$ considers the made deliveries $Dy_{i,t}$ from $t = -1$ to $T = 1$, SS $SS_{i,t}$ by previously calculating Equation (16) and the production in $t = 1$. For the next stock values from $t = 2$ to $T = n_i$, the process follows the aforementioned pattern. Tables 8 and 9 consider the real deliveries made by the first-tier supplier, and the stock with and without the SS is also calculated.

6. Evaluation of the results

The parameters used to evaluate the results are based on the equations for the SS (16), stockout (19) and average stock (17). The results consider the 13 items, the seven equations and the six proposed analytical formulations, the equation presently used by the first-tier supplier, equation (20), and the two considered scenarios. We selected these parameters, which represent the cost of the stock, stockouts and their possible customer service effects. Table 10 offers the results of Scenario 1 with real data and Table 11 represents the results of Scenario 2, which considers the moving average of demand variability.

The framework that we selected as being the most suitable for each parameter is that which represents the lowest result of the total sum for each equation. This framework would represent a lower cost for the first-tier supplier given the objective of having the smallest stock quantity with the minimum number of stockouts. Table 12 represents the total values for Scenario 1 with each item for each equation and parameter. The total values are calculated as shown below:

$$TotSS_i = \sum_{t=1}^{T} SS_{i,t} \tag{24}$$

$$TotSA_i = \sum_{t=1}^{T} SA_{i,t} \tag{25}$$

$$TotSO_i = \sum_{t=1}^{T} SO_{i,t} \tag{26}$$

Thus, for Scenario 1, Table 12 identifies the best result with real data that represents the smaller number of stockouts obtained with analytical formulation (5) and also estimates the second better result for the SS, which only exceeds the best result for this parameter by 2.16 %, represented by analytical formulation (6). It is important to note that analytical formulation (5) considers a mean stock that represents the most deficient of the several proposed analytical formulations. The equation that shows the best result for the SS corresponds to the equation that the first-tier supplier employs, but it also shows the worst results for the number of stockouts and the SS. Additionally, Table 12 shows the total percentage that indicates the best results of the parameters for each equation in each item. The different items take seven values for all three parameters and a unit value is assigned if the result of the analysed equation is the lowest of the seven equations. Otherwise it is assigned zero if the value is above the best result of the analysed equation. Then the values for each equation and item are summed and divided by the total number of items. The total sum of these values among the equations does not equal 100 % because the result of each parameter for some items is equal or similar between two equations or more. Analytical formulation (5) shows the best results for stockouts and the SS. In this case however, the equation that gives the best result for the mean stock is no. (2). With this analysis sequence, where the equation and the value that best adapt to the case study were found, the intention was to compare the total results of the equations. Thus, Table 12 presents the percentage by which each equation exceeds the value which is considered the best (the smallest values of each parameter are taken as null). As expected, the best value for the parameter that measures stockouts corresponds to analytical formulation (5), while the equation used by the first-tier supplier, equation (20) determines that with the most deficient value. The best value for the parameter that measures the SS corresponds to analytical formulation (6), which is exceeded by 2.16 % by the value of analytical formulation (5). The most deficient value goes to the first-tier supplier's equation which, however, obtains the best value for the mean stock parameter.

For Scenario 2, which obtains the demand variability of the moving averages of demand and production, Table 13 is similarly created to Table 12, respectively. The obtained results are similar to Scenario 1. Table 13 shows how the fewest stockouts are obtained with analytical

Table 6
Calculation of the SLT and PLT for item 1.

$S_{Li,t}$	$\overline{DT}_{i,t}$(Days)	$R_{i,t}$(Deliveries / Week)	$W_{cf_{i,t}}$	$\sigma_{DNi,t}$				$Z_{i,t}$(99 %)	
2.819515609	2.819515609	3.73015873	0.580330707	0.29555				2.327	
t	**$D_{i,t}$(Units)**	**$P_{i,t}$(Units)**	**$\beta_{Di,t}$**	**$PLT(1)_{i,t}$(Days)**	**$PLT(2)_{i,t}$(Days)**	**$PLT(3)_{i,t}$(Days)**	**$PLT(4)_{i,t}$(Days)**	**$PLT(5)_{i,t}$(Days)**	**$PLT(6)_{i,t}$(Days)**
1	0	385	0.056079989	5	5	3	5	5	4
2	0	396	0.05109926	5	5	3	5	5	4
3	0	0	0.504760256	6	5	5	6	8	7
4	0	0	0.504760256	6	5	5	6	8	7
5	480	386	0.656416209	6	6	6	6	9	7
6	400	329	0.620611827	6	6	6	6	8	7
7	400	363	0.565821558	6	6	6	6	8	7
8	320	293	0.549411539	6	6	6	6	8	7
9	400	740	0.080534004	5	5	3	5	5	4
10	360	0	0.934286184	7	6	8	6	10	9
11	0	317	0.095797562	5	5	3	5	6	4
12	400	411	0.486519971	6	5	5	6	8	6

Table 7
Calculation of the SLT and PLT for item 1 and demand variability.

Equation parameters									
$S_{Li,t}$	$\overline{DT}_{i,t}$(Units)	$R_{i,t}$(Deliveries / Week)	$W_{cf_{i,t}}$	$\sigma_{DNi,t}$				$Z_{i,t}$(99 %)	
2.819515609	2.819515609	3.73015873	0.580330707	0.277092473				2.327	
t	**$D_{i,t}$(Units)**	**$P_{i,t}$(Units)**	**$\beta_{Di,t}$**	**$PLT(1)_{i,t}$(Days)**	**$PLT(2)_{i,t}$(Days)**	**$PLT(3)_{i,t}$(Days)**	**$PLT(4)_{i,t}$(Days)**	**$PLT(5)_{i,t}$(Days)**	**$PLT(6)_{i,t}$(Days)**
1	400	740	0.403417837	6	5	5	5	7	6
2	360	0	0.666322281	6	6	6	6	9	8
3	0	317	0.289867676	5	5	4	5	7	5
4	400	411	0.249695645	5	5	4	5	7	5
5	280	414	0.139277513	5	5	4	5	6	4
6	480	376	0.516725344	6	5	6	6	8	7
7	0	320	0.046302473	5	5	3	5	5	4
8	400	317	0.252116894	5	5	4	5	7	5
9	440	381	0.311926091	5	5	5	5	7	5
10	240	358	0.326413151	6	5	5	5	7	5
11	320	711	0.044131897	5	5	3	5	5	4
12	440	328	0.271083376	5	5	4	5	7	5

Table 8
Simulation of analytical formulation (1) for item 1.

t	$PLT(20)_{i,t}$				$PLT(1)_{i,t}$		
	$O_{i,t}$(Units)	$Dy_{i,t}$(Units)	$S_{i,t}$(Units)	$S_{i,t}withSS_{i,t}$(Units)	$DyS_{i,t}$(Units)	$S_{i,t}$(Units)	$S_{i,t}withSS_{i,t}$(Units)
−10	–	–	–	–	–	–	–
−9	–	–	–	–	–	–	–
−8	–			–	–	–	–
−7	–			–	–	–	–
−6	–			–	–	–	–
−5	–	–	–	–	–	–	–
−4	0	–	–	–	–	–	–
−3	0	–	–	–	–	–	–
−2	0	–	–	–	0	–	–
−1	480	–	–	–	0	–	–
1	400	720	335	1106	0	−385	440
2	400	0	−61	710	480	−301	524
3	320	0	−61	710	400	99	924
4	360	0	−61	710	400	499	1324
5	400	664	217	988	320	433	1258
6	–	360	248	1019	360	464	1289
7	400	440	325	1096	400	501	1326
8	–	520	552	1323	–	208	1033
9	760	360	172	943	400	−132	693
10	–	240	412	1183	–	−132	693
11	400	520	615	1386	760	311	1136
12	440	440	644	1415	–	−100	725
13	–	360	590	1361	400	−114	711
14	240	440	654	1425	440	−50	775

Table 9
Simulation of analytical formulation (1) for item 1 with demand variability.

t	$PLT(20)_{i,t}$				$PLT(1)_{i,t}$		
	$O_{i,t}$(Units)	$Dy_{i,t}$(Units)	$S_{i,t}$(Units)	$S_{i,t}withSS_{i,t}$(Units)	$DyS_{i,t}$(Units)	$S_{i,t}$(Units)	$S_{i,t}withSS_{i,t}$(Units)
−10	–	–	–	–	–	–	–
−9	–	–	–	–	–	–	–
−8	–			–	–	–	–
−7	–			–	–	–	–
−6	–			–	–	–	–
−5	400	–	–	–	–	–	–
−4	360	–	–	–	–	–	–
−3	–	–	–	–	400	–	–
−2	0	–	–	–	360	–	–
−1	400	–	–	–	0	–	–
1	760	360	−380	943	0	20	293
2	–	240	−140	1183	400	420	693
3	0	520	63	1386	760	863	1136
4	400	440	92	1415	0	452	725
5	680	360	38	1361	0	38	311
6	–	440	102	1425	400	62	335
7	320	0	−218	1105	680	422	695
8	760	0	−535	788	0	105	378
9	440	400	−516	807	320	44	317
10	920	240	−634	689	760	446	719
11	–	400	−945	378	440	175	448
12	0	400	−873	450	920	767	1040
13	400	0	−873	450	0	767	1040
14	440	1240	−27	1296	0	373	646

formulation (5), and also estimates the second better result for the SS, which exceeded analytical formulation (6) on this occasion. Analytical formulation (5) generates a mean stock that is the most deficient of the various equations. Likewise, the equation that provides the best result for the SS corresponds to the equation employed by the first-tier supplier, but it also obtains the most deficient results for the number of stockouts and the SS. Analytical formulation (5) in Table 13 displays the best results for stockouts and the SS, and analytical formulation (4) shows the same result as analytical formulation (5) for the SS. In this case, the equation with the best result for the mean stock is analytical formulation (2), which differs from that in Table 11. The best value obtained in the comparison made of the values in Table 13 for the parameter that measures stockouts corresponds to analytical formulation (5), and the most deficient value is determined by the first-tier supplier's equation. The best value for the parameter that measures the SS corresponds to analytical formulation (5). Although the first-tier supplier's equation obtains the most deficient value, it gives the best value for the mean stock parameter.

It is noteworthy that analytical formulations (5) and (6) in Scenarios 1 and 2 well exceed the equation used by the first-tier supplier for the parameter that measures stockouts and the SS. Indeed, the results of these two parameters in the first-tier supplier's equation are the most deficient ones. For the SS, analytical formulations (5) and (6) give the most deficient value. It is important to note that the first-tier supplier's equation for Scenario l has the best SS value, and the second better value is found in Scenario 2, which is exceeded by analytical formulation (2) by 16 %. Logically, and in line with the calculation process, a higher mean stock is necessary to obtain fewer stockouts. The first two parameters of analytical formulation are emphasized (5). This analytical formulation shows that a lower SS level is necessary to obtain fewer stockouts compared with the other equations. Thus, obtaining a minimum number of stockouts and SS is considered a very good result.

As determined in the previous section, the mean stock is drawn from the situation where zero stockouts is the aim, obtained by considering the SS. The reason why the mean stock of analytical formulations (5) and (6) is bigger than the others is because orders are placed, on average, earlier than with the other equations. This situation means that more stock accumulates at the beginning and the aggregate increases, which becomes a higher mean stock.

After analysing both scenarios, it was determined that analytical formulation (5), which was the first to be adapted from and Vollmann, Berry and Whybark [64] and McClain and Thomas [65], best fitted the case study. The second better analytical formulation to fit the case study was no. (6), which was the second of the aforementioned authors to be adapted.

The difference between the two adaptations lies in how the LT is used. Analytical formulation (5) uses the average real LT, obtained from the information provided by the first-tier supplier. Analytical formulation (6) uses the LT agreed on by the suppliers and the first-tier supplier.

Given the equation's characteristics, using an average LT is recommended and this value is updated with the new information provided by the first-tier supplier. For future calculations, a time period shorter than 6 months or one that equals 2 months can be used, and the result would be used during a period that equals that studied for updating purposes. In both cases, the analytical formulation (5) value corresponds to 7 days. Hence the SLT would be 5 days if we consider that the agreed LT is 3 days.

Results presented from Table 12 to Table 13 can be found in the Appendix.

7. Conclusions

This work commenced with a literature review to determine the equations that currently exist and are related with calculating the SLT (vendor–buyer context) and, in turn, to verify if any equation considered demand variability. We were unable to find an equation related to that context and under that condition, demand variability. However, we identified several equations that calculate the SLT and the SS, which mainly use variables directly related with different supply chain costs. We also observed that the SLT was generally included in stock-related calculations.

The present work considered the ideas and arguments of the different reviewed research works, and consequently proposes six new analytical formulations to calculate the SLT which depends on demand variability. A process to calculate the PLT was determined, which should be systematically followed to obtain values in accordance with the analysed time horizon.

Our objective was to propose an equation that would calculate the SLT and would formalize demand variability. The study framework was applied to the automotive industry, specifically in a first-tier supply chain. The results were compared with those provided by the equation that the first-tier supplier uses, which does not consider demand variability. The calculations were done using the real disaggregate data provided by the first-tier supplier. Three parameters were determined to measure the performance of each equation, and the proposed ones provided better results than that used by the first-tier supplier. The equation finally selected for the case study was analytical formulation (5). We emphasize that we were able to describe the SLT according to demand variability. The results of analytical formulation (5) were much better than those obtained with the first-tier supplier's equation, Eq. (20), which was exceeded considerably in the two scenarios. It is important to mention that the first-tier supplier's equation obtained worse results than the six proposed analytical formulations when considering the parameters that measure the number of stockouts and the quantity of the SS. These are important parameters because they indirectly represent costs in companies.

Limitations of this proposal are related to use different data sources from first-tier suppliers belonging to the same industry. Also, it could be incorporated the cost factor to this scenario to compare the behaviour of each equation result. On the other hand, a combinatorial approach could be used to calculate the SLT for a specific item, demand season or LT, among others.

The following future research lines in the SLT and demand variability domain have been identified throughout this work. Therefore, forthcoming works are to: (i) investigate whether a relationship exists between the SLT and the bullwhip effect. Should this link exist, it would be worth investigating if it is possible to modify, dominate or lead the bullwhip effect. To undertake this study, several continuous supply chain points and the performance of both the SLT and demand variability should be investigated. This could be done by systems dynamics-based simulation to: (ii) apply these formulations to calculate the SLT which depends on demand variability in other sectors; (iii) to build a multi-period calculation model; (iv) to consider other common distributions such as exponential distribution to provide other functionalities to the proposed formulations; wand (iv) to develop the current case study and to explicitly determine costs. The cost per stockout should be defined, as should the cost per inventoried unit per item, and whether this aspect defines a new solution should be observed. This same scenario could include a study that analyses if the SLT depends on the quantity of items and what would happen if interdependent items existed.

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Appendix

Table A1
The results of Scenario 1 for each item and parameter.

i	Parameter	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)
1	$TotSO_i$	18	38	49	37	39	3	10
	$TotSS_i$ (Units)	771	825	838	840	825	155	161
	$TotSA_i$ (Units)	932	738	449	733	693	793	477
2	$TotSO_i$	53	0	0	0	0	0	0
	$TotSS_i$ (Units)	482	929	929	1337	2709	1369	1337
	$TotSA_i$ (Units)	214	9098	9133	9829	11,647	10,029	9781
3	$TotSO_i$	49	48	48	47	50	47	47
	$TotSS_i$ (Units)	442	578	578	492	322	500	500
	$TotSA_i$ (Units)	198	244	244	212	120	221	213
4	$TotSO_i$	27	36	38	32	26	6	17
	$TotSS_i$ (Units)	1148	968	1004	1016	968	968	968
	$TotSA_i$ (Units)	1030	794	681	847	962	1458	1159
5	$TotSO_i$	10	20	32	20	19	3	3
	$TotSS_i$ (Units)	746	384	604	616	384	126	126
	$TotSA_i$ (Units)	931	501	552	725	518	1016	708
6	$TotSO_i$	4	40	46	42	42	5	15
	$TotSS_i$ (Units)	1202	1991	2450	1991	2232	1058	1058
	$TotSA_i$ (Units)	2174	1412	1298	1357	1549	2176	1502
7	$TotSO_i$	44	31	31	31	31	30	31
	$TotSS_i$ (Units)	3610	6552	7587	6437	6552	4410	4420
	$TotSA_i$ (Units)	2107	4871	5673	4899	5412	4830	4099
8	$TotSO_i$	44	17	11	5	2	16	17
	$TotSS_i$ (Units)	1124	417	379	251	71	417	417
	$TotSA_i$ (Units)	683	803	914	1201	1436	814	793
9	$TotSO_i$	12	10	10	12	12	12	12
	$TotSS_i$ (Units)	73	71	71	73	73	73	73
	$TotSA_i$ (Units)	226	268	268	226	226	226	226
10	$TotSO_i$	37	13	15	5	0	0	0
	$TotSS_i$ (Units)	3124	695	1096	521	970	747	399
	$TotSA_i$ (Units)	2482	1567	1807	1913	3736	3370	2615
11	$TotSO_i$	52	43	47	40	29	10	18
	$TotSS_i$ (Units)	9549	4816	4816	4816	4816	4816	4816
	$TotSA_i$ (Units)	4239	3147	2861	3444	4373	6032	5072

(continued on next page)

Table A1 (*continued*)

i	Parameter	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)
12	$TotSO_i$	26	26	38	26	12	7	10
	$TotSS_i$ (Units)	1731	1057	1260	979	781	678	678
	$TotSA_i$ (Units)	1650	1046	970	987	1106	1388	1051
13	$TotSO_i$	51	40	40	39	39	32	35
	$TotSS_i$ (Units)	2528	1880	1880	1880	1880	1880	1880
	$TotSA_i$ (Units)	1098	980	951	1147	1284	1823	1608

Table A2
The results of Scenario 2 for each item and parameter.

i	Parameter	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)
1	$TotSO_i$	37	6	13	10	6	0	4
	$TotSS_i$ (Units)	1323	273	286	877	273	110	276
	$TotSA_i$ (Units)	927	641	399	1124	6 00	1132	911
2	$TotSO_i$	41	0	0	0	0	0	0
	$TotSS_i$ (Units)	302	1422	1422	1742	2222	1742	1742
	$TotSA_i$ (Units)	185	8433	8526	8854	9908	9091	8887
3	$TotSO_i$	35	44	44	44	44	44	44
	$TotSS_i$ (Units)	262	521	518	480	294	472	518
	$TotSA_i$ (Units)	180	200	200	194	128	199	222
4	$TotSO_i$	32	12	18	16	6	4	6
	$TotSS_i$ (Units)	1256	390	412	704	376	376	376
	$TotSA_i$ (Units)	1038	679	590	968	867	1209	962
5	$TotSO_i$	46	0	1	4	0	0	0
	$TotSS_i$ (Units)	13,614	10	12	304	10	333	211
	$TotSA_i$ (Units)	6165	654	507	864	674	1624	1163
6	$TotSO_i$	26	12	28	15	14	1	5
	$TotSS_i$ (Units)	2229	1094	1146	1579	1146	136	902
	$TotSA_i$ (Units)	2225	1550	1114	1929	1497	2029	2155
7	$TotSO_i$	21	29	29	29	29	23	26
	$TotSS_i$ (Units)	1793	5770	5770	6695	4735	4735	5310
	$TotSA_i$ (Units)	2189	5439	5127	6402	4754	6004	5842
8	$TotSO_i$	37	10	4	4	0	9	11
	$TotSS_i$ (Units)	966	310	221	186	359	259	315
	$TotSA_i$ (Units)	650	829	973	1124	1994	813	822
9	$TotSO_i$	9	7	8	9	9	9	8

(*continued on next page*)

Table A2 (*continued*)

i	Parameter	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)
	$TotSS_i$ (Units)	53	51	52	53	53	53	52
	$TotSA_i$ (Units)	260	311	286	260	260	260	286
10	$TotSO_i$	38	20	20	18	1	4	9
	$TotSS_i$ (Units)	3691	2159	1633	2159	225	775	1633
	$TotSA_i$ (Units)	2464	2364	1731	2513	2199	2448	2651
11	$TotSO_i$	40	14	15	19	7	5	10
	$TotSS_i$ (Units)	6173	2482	2482	2819	1640	1640	1640
	$TotSA_i$ (Units)	3877	3470	3242	3650	3750	4908	3749
12	$TotSO_i$	10	9	16	13	5	5	9
	$TotSS_i$ (Units)	960	483	483	904	128	128	904
	$TotSA_i$ (Units)	1654	963	789	1297	912	1205	1623
13	$TotSO_i$	18	32	34	29	22	12	22
	$TotSS_i$ (Units)	921	1314	1314	1314	944	944	944
	$TotSA_i$ (Units)	1069	1162	1129	1140	1059	1359	1070

Table A3
The total results of each parameter and equation.

Total							
Parameter	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)
$\sum_i TotSO_i$	427	362	405	336	301	171	215
$\sum_i TotSS_i$(Units)	26,530	21,163	23,492	21,249	22,583	17,197	16,833
$\sum_i TotSA_i$(Units)	17,964	25,467	25,802	27,521	33,064	34,177	29,304

Table A4
The result that shows the total percentage which indicates the best results of the parameters for each equation in each item.

Results %							
Parameter	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)
$\sum_i TotSO_i$	7.69 %	15.38 %	15.38 %	15.38 %	23.08 %	76.92 %	30.77 %
$\sum_i TotSS_i$	15.38 %	30.77 %	23.08 %	15.38 %	38.46 %	53.85 %	53.85 %
$\sum_i TotSA_i$	30.77 %	15.38 %	46.15 %	7.69 %	15.38 %	7.69 %	7.69 %

Table A5
Comparison of the total results of each parameter in each equation with the results of the equation showing the best values.

% of separation							
Parameter	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)
$\sum_i TotSO_i$	149.71 %	111.70 %	136.84 %	96.49 %	76.02 %	0.00 %	25.73 %
$\sum_i TotSS_i$	57.61 %	25.72 %	39.56 %	26.23 %	34.16 %	2.16 %	0.00 %
$\sum_i TotSA_i$	0.00 %	41.77 %	43.63 %	53.20 %	84.06 %	90.25 %	63.12 %

Table A6
The total results for each parameter and in each equation. Scenario 2 (moving average).

Total							
Parameter	**Eq. (1)**	**Eq. (2)**	**Eq. (3)**	**Eq. (4)**	**Eq. (5)**	**Eq. (6)**	**Eq. (7)**
$\sum_i TotSO_i$	390	195	230	210	143	116	154
$\sum_i TotSS_i$(Units)	33,543	16,279	15,751	19,816	12,405	11,703	14,823
$\sum_i TotSA_i$(Units)	22,884	26,693	24,612	30,318	28,603	32,282	30,343

Table A7
The result that shows the total percentage which indicates the best results of the parameters for each equation in each item. Scenario 2 (moving average).

Results, %							
Parameter	**Eq. (1)**	**Eq. (2)**	**Eq. (3)**	**Eq. (4)**	**Eq. (5)**	**Eq. (6)**	**Eq. (7)**
$\sum_i TotSO_i$	15.38 %	23.08 %	7.69 %	7.69 %	38.46 %	61.54 %	15.38 %
$\sum_i TotSS_i$	30.77 %	15.38 %	0.00 %	7.69 %	38.46 %	38.46 %	15.38 %
$\sum_i TotSA_i$	30.77 %	0.00 %	53.85 %	7.69 %	23.08 %	7.69 %	0.00 %

Table A8
Comparison of the total results of each parameter in each equation with the equation results that show the best values. Scenario 2 (moving average).

% of separation							
Parameter	**Eq. (1)**	**Eq. (2)**	**Eq. (3)**	**Eq. (4)**	**Eq. (5)**	**Eq. (6)**	**Eq. (7)**
$\sum_i TotSO_i$	236.21 %	68.10 %	98.28 %	81.03 %	23.28 %	0.00 %	32.76 %
$\sum_i TotSS_i$	186.62 %	39.10 %	34.59 %	69.32 %	6.00 %	0.00 %	26.66 %
$\sum_i TotSA_i$	0.00 %	16.65 %	7.55 %	32.49 %	24.99 %	41.07 %	32.59 %

Table A9
The average PLT that results from this case study.

i	Moving average (Days)						Real Data (Days)					
	Eq. (2)	**Eq. (3)**	**Eq. (4)**	**Eq. (5)**	**Eq. (6)**	**Eq. (7)**	**Eq. (2)**	**Eq. (3)**	**Eq. (4)**	**Eq. (5)**	**Eq. (6)**	**Eq. (7)**
1	6	5	5	6	8	6	6	5	5	6	8	7
2	6	6	6	11	7	6	6	6	6	11	7	6
3	6	6	6	11	7	6	6	6	6	11	7	6
4	6	5	5	7	8	7	6	5	5	6	8	7
5	6	5	5	6	8	7	6	5	5	6	8	7
6	6	5	5	6	8	7	6	5	5	6	8	7
7	6	5	5	6	8	7	6	6	5	6	8	7
8	5	7	7	12	5	5	5	7	7	12	5	5
9	5	4	3	3	3	4	5	4	3	3	3	3
10	6	6	6	9	8	7	6	6	6	9	8	7
11	6	6	6	7	8	7	6	6	6	7	8	7
12	6	5	5	6	7	6	6	5	5	7	7	6
13	6	6	6	7	8	7	6	6	6	7	8	7
The average LT	6	5	5	7	7	6	6	6	5	7	7	6

Table A10
The average PLT that results from this case study, where the values of two items were eliminated since historic data were lacking.

i	Moving average (Days)						Real Data (Days)					
	Eq. (2)	**Eq. (3)**	**Eq. (4)**	**Eq. (5)**	**Eq. (6)**	**Eq. (7)**	**Eq. (2)**	**Eq. (3)**	**Eq. (4)**	**Eq. (5)**	**Eq. (6)**	**Eq. (7)**
1	6	5	5	6	8	6	6	5	5	6	8	7
2	6	6	6	11	7	6	6	6	6	11	7	6
3	6	6	6	11	7	6	6	6	6	11	7	6
4	6	5	5	7	8	7	6	5	5	6	8	7
5	6	5	5	6	8	7	6	5	5	6	8	7
6	6	5	5	6	8	7	6	5	5	6	8	7
7	6	5	5	6	8	7	6	6	5	6	8	7
8												
9												
10	6	6	6	9	8	7	6	6	6	9	8	7

(*continued on next page*)

Table A10 (*continued*)

i	Moving average (Days)						Real Data (Days)					
	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)
11	6	6	6	7	8	7	6	6	6	7	8	7
12	6	5	5	6	7	6	6	5	5	7	7	6
13	6	6	6	7	8	7	6	6	6	7	8	7
The average LT	6	5	5	7	8	7	6	6	5	7	8	7

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