Barometric characterization of a draining container

Isabel Salinas[1], Francisco M. Muñoz-Pérez[1], Juan C. Castro-Palacio[1], Luisberis Velazquez[2], Juan A. Monsoriu[1]

[1]Centro de Tecnologías Físicas, Universitat Politècnica de València, 46022 València, Spain.

[2]Departamento de Física, Universidad Católica del Norte, 1240000 Antofagasta, Chile

Corresponding author: jmonsori@fis.upv.es

Abstract

A characterization of a draining container is performed by measuring the pressure change at its bottom while the container drains through a small orifice. The Physics model is based on the continuity equation and Bernoulli's principle- The pressure is measured with the barometer of the smartphone which is placed inside a hermetically sealed bag and laid at the bottom of the container. The expected theoretical relationship between the pressure at the bottom of the container and time was observed. The value for the coefficient of discharge is also estimated. The results of a survey applied to students and teachers at secondary school level indicate that the use of the pressure sensor of the smartphones helped the students to understand the basic concepts of hydrostatics and hydrodynamics at the same time their motivation for physics was increased.

Keywords: Bernoulli's principle, smartphone, barometer, draining container.

1. Introduction

Bernoulli's principle and Torricelli's law of Fluid Mechanics topics are commonly included in the syllabus of General Physics at introductory and first year university level courses [1,2]. In line with this, an important number of works have been published on these topics in Physics teaching journals over the last decades [3-17].

One interesting, common problem within this topic that have been widely treated in the literature is the draining of a container through a small orifice. The Physics model in most of these works is developed within the frame of irrotational and inviscid fluids making use of the continuity equation, Bernoulli's principle, and Torricelli's law. The experiments and models have usually focussed on how the different quantities (i.e. fluid level, the speeds of the fluid level and at the exit orifice, drain time) change as a function of time while the draining of containers is produced [18,19].

One common approximation made in these works consists in considering the area of the orifice much smaller that the area of the open container. In Ref. 20, the authors present a detailed Physics model for this simple case where the different limits for the ratio between the areas of the cylindrical vessel and of the cylindrical drain hole are analysed. Some works have detailed on how the water jet leaves the container. For instance, in Ref. 21, the authors study the water jets flowing from three equidistant orifices on the side of a vertical cylindrical bottle. The water jets are analysed as a function of the geometrical features of the layout, that is, the height of the holes above the bottom, thickness of the block supporting the vessel, and considering the different shapes of the vessel profile. The problem of a draining container can be extended to include the possibility of filling it at the same time it drains through an orifice. This case has been analysed in Ref. 22 and the case of only emptying the container is derived as a limit case.

One of the issues that arises when modelling the draining of a container is the discrepancy observed when the calculated rate of efflux from the container (obtained by multiplying the Torricelli velocity by the cross-section area of the orifice) is compared with the flowrate derived by dividing the volume of the bucket (where the jet is collected) by the time it takes to be filled up. This leads to a careful consideration of the flow shape at the drain orifice. For this purpose, the discharge coefficient is usually defined as the ratio of the experimental value of the volumetric discharge Q_e, to the theoretical value of the discharge Q_{th} (velocity times the area), defined as $C_d = Q_e/Q_{th}$. In Ref. [23] a simple model is introduced to estimate the discharge coefficient from assuming that the velocity be symmetrical around its centre, perpendicular to the cross section at the centre of the slot, parallel to it at its edge, and vary linearly in between. Another work in this respect proposes a mathematical model for a draining open container as well as an experimental methodology to determine the coefficient of discharge [24]. The experimental procedure in this work is based on PVC test caps with holes drilled in the centre, considering the cases of including or not a grommet.

In this work we will revisit the problem of the characterization of an open draining container but from a different view, that is, measuring the pressure change at the bottom of the container with water while it drains through a small orifice. We will use water as the fluid and will model the system within the frame of irrotational and inviscid fluids and making use of the continuity equation and Bernoulli's principle. In order to measure the pressure, we will use the barometer incorporated in the smartphones. Some works report the use of this sensor in Physics teaching laboratory. For instance, it has been used explore the are pressure variations in the atmosphere [25, 26], to study the inflation of a rubber balloon [27] or as a Pitot tube speedometer [28]. Some recent teaching articles have also used a pressure sensor with Arduino to investigate, for example, the Torricelli's law [29] or the forces due to atmospheric pressure [30].

2. Experimental setup

Figure 1a shows a schematic representation of the experimental setup. The figure shows the geometry of the container and the variables considered in the physical model. The experiments use three containers with the same geometry ($a = 19.0\ cm$, $b = 29.0\ cm$) but different measures of the orifice diameter (container 1: $d = 8.0\ mm$; container 2: $d = 10.0\ mm$, and container 3: $d = 12.0\ mm$). The sides of the container were measured with a ruler and the diameter of the orifices with a caliper of 0.1 mm and 0.02 mm precision, respectively. Figure 2b shows a photo of the actual setup and Figure 2c, a photo from above of the smartphone barometer app. while the container is draining.

The experiment consisted in filling up the container up to a given height H_0 with the orifice closed. The smartphone was placed horizontally at the bottom of the container, at the same level as the center of the orifice. Then, it was set to measure pressure using the free Android app. Physics Toolbox Suit. Finally, the orifice was open, and the container started to drain. The experiments use a smartphone Samsung Galaxy S22 Ultra model.

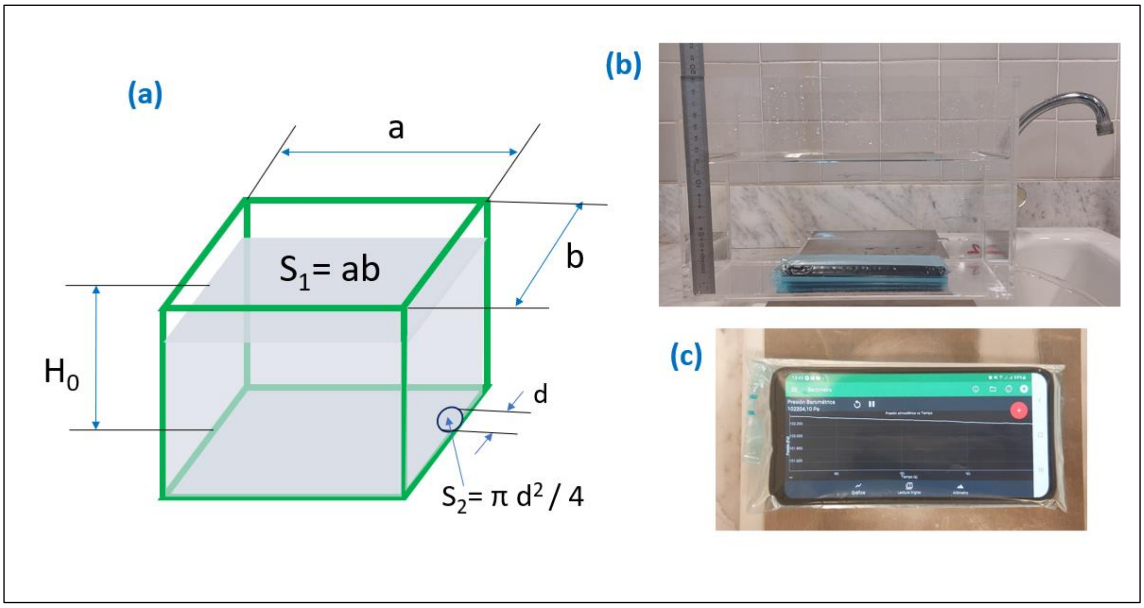

Figure 1. Schematic representation of the container showing the geometrical variables considered in the theoretical model (panel a). A photo of the actual setup is shown on panel b and, on panel c, a photo from above of the smartphone barometer app. while the container is draining.

3. Physics model

Bernoulli's principle and continuity equation are at the root of the physics model of this work,

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \, , \tag{1}$$

Where ρ is the density of the fluid. The variables p_1, h_1, and v_1 and p_2, h_2, and v_2 are the pressure, the height with respect to a reference and the fluid speed for two given points 1 and 2 of a streamline of the fluid, respectively. The continuity equation for the areas of the cross sections and the corresponding speeds is written as follows

$$v_1 S_1 = v_2 S_2 \, , \tag{2}$$

In our experimental setup $h_1 = h$, $h_2 = 0$ and $p_1 = p_2 = p_{at}$. The reference for the height has been taken at center of the orifice. Point 1 refers to a given point at the open surface of the water of area $S_1 = ab$ and point 2 refers to the center of the orifice of area $S_2 = \pi d^2/4$ (See figure 1a). Then, Eq. 1 becomes,

$$\rho g h + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2 \, , \tag{3}$$

Combining Eq.(s) 2 and 3, the velocity at the orifice is obtained,

$$v_2 = S_1\sqrt{2gh/(S_1^2 - S_2^2)} \, , \tag{4}$$

On the other hand, the volume of fluid that leaves the container can be expressed as follows,

$$-S_1 dh = S_2 v_2 dt \, , \tag{5}$$

Let us consider that at the initial moment $t = 0$ the initial height is $h = H_0$,

$$-\int_{H_0}^{h} \frac{dh}{\sqrt{h}} = S_2\sqrt{2g/(S_1^2 - S_2^2)} \int_0^t dt \,, \tag{6}$$

Integrating at both sides,

$$\sqrt{H_0} - \sqrt{h} = \frac{S_2}{2} \sqrt{\frac{2g}{S_1^2 - S_2^2}} \, t \,, \tag{7}$$

Given that $S_1 \gg S_2$, the pressure at the bottom of the container as a function of h (water level) can be expressed as follows,

$$P(h) = p_{at} + \rho g h \,, \tag{8}$$

Eq. 8 for $h = H_0$ it becomes,

$$P_0 = P(H_0) = p_{at} + \rho g H_0 \,, \tag{9}$$

Taking h and H_0 apart in Eq.(s) 8 and 9 and substituting in Eq. 7,

$$\sqrt{\Delta P} = -\frac{S_2 g}{S_1} \sqrt{\frac{\rho}{2}} \, t + \sqrt{\Delta P_0} \,, \tag{10}$$

where $\Delta P = P(h) - p_{at}$ and $\Delta P_0 = P_0 - p_{at}$. Eq. 10 represents a linear equation of $\sqrt{\Delta P}$ *versus* time.

4. Results and discussion

Figure 2 shows the experimental data of the pressure measured with the barometer of the smartphone *versus* time during the draining of the containers. As shown in figure 1, the smatphone is placed at the bottom of the container. Figure 2 represents the decrease of the pressure as the water level decreases upto a minimum corresponding to the atmospheric pressure.

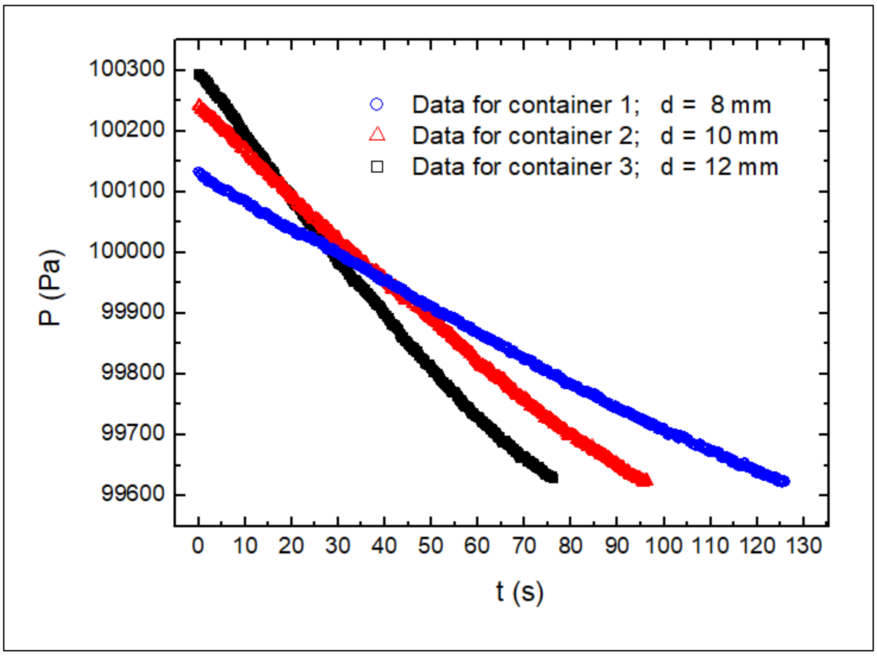

Figure 2. Experimental data of the pressure measured with the barometer of the smartphone *versus* time during the draining of the containers.

Table 1. The diameter of the container's orifice as meassured with a caliper is shown in column 2. The parameters of a Least-Squares linear fitting of Eq. 10 to the experimental data of pressure and time are shown in column 3. The discharge coefficient as calculated using Eq. 11 is shown in the last column.

Container	d (mm)	$P(Pa) = A + B\,t(s)$ R is the determination coefficiente.	Discharge Coefficient (C_d) (from Eq. 11)
1	8.0	$A = 27.29$	0.486
		$B = -0.09703$	
		$R^2 = 0.9986$	
2	10.0	$A = 29.32$	0.475
		$B = -0.1482$	
		$R^2 = 0.9986$	
3	12.0	$A = 30.20$	0.444
		$B = -0.1996$	
		$R^2 = 0.9996$	

Table 1 shows the results of a Least-Squares linear fitting of Eq. 10 to the experimental data of pressure and time. The coefficients of determination for the three cases indicate the good quality of the fit. Figure 3 shows the experimental data of $\sqrt{\Delta P}$ *versus* t and the corresponding fit curves. This result confirms the validity of the physics model developed in section 3 and the quality of the smartphone's barometer data for this type of experiments.

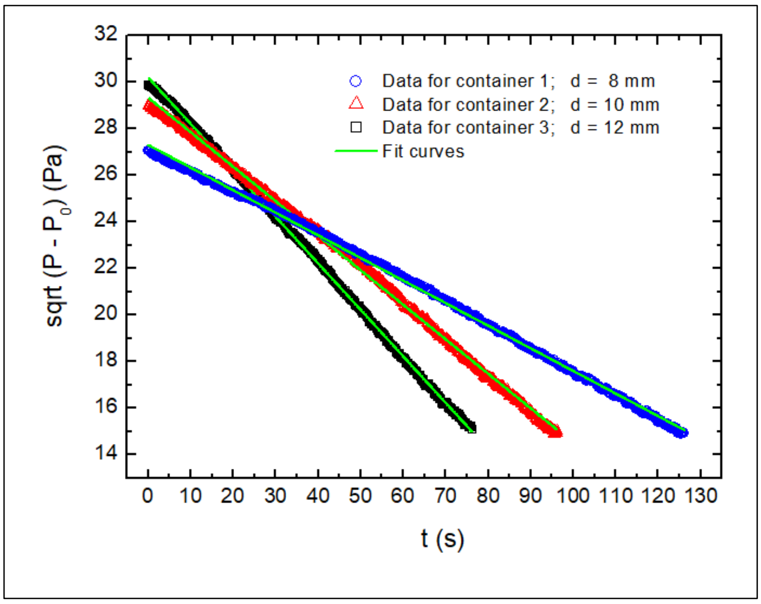

Figure 3. Experimental data of $\sqrt{\Delta P}$ *versus* t and the corresponding fit curves as the container drains.

However, the theorical slope in Eq. 10 does not match the slope derived from the fit, B (table 1). This is because the effective area of the orifice results smaller than its actual area. As commented in the Introduction section, this is one of the issues that arises when modelling the draining of a container through a small orifice. Therefore. the shape of the flow in the vicinity of the orifice needs careful consideration. The coefficient of discharge definition takes into account the deviation between the volumetric discharge Q_e, to the theoretical value of the discharge Q_{th} (velocity times the area) as $C_d = Q_e/Q_{th}$. In this work we use the alternative expression for the coefficient of discharge derived in Ref. 24,

$$C_d = \frac{S_{2exp}}{S_2}\sqrt{\frac{1-S_2^2/S_1^2}{1-S_{2exp}^2/S_1^2}} , \tag{11}$$

The authors in Ref. 24 determine $S_{2_{exp}}$ by means of a fitting to the experimental data collected with a standard ultrasonic motion detector placed at the bottom of the container to follows the water level change. This work reports an experimental discharge coefficient ranging between (0.650 ± 0.003) and (0.666 ± 0.005) for diameters of a sharp-edged orifice of 8.92 mm and 12.5 mm, respectively. Other experimental works report coefficients of discharge of 0.61 [31] and 0.59-0.61 [32].

In this work we estimate $S_{2_{exp}}$ from making the slope resulting from the fitting of $\sqrt{\Delta P}$ *versus* t, B (see table 1), equal to the theoretical slope in Eq. 10 which yielding, $S_{2_{exp}} = \sqrt{\frac{2}{\rho}}S_1 B/g$. The resulting values are included in the last column of table 1. Similar values are reported in Ref. [23] where a coefficient of discharge of $\sim$ 0.44 was calculated with a simple model that assumes the velocity to be symmetrical around its centre, perpendicular to the cross section of the orifice and varying linearly in between.

This work has been used as laboratory experiment for introductory physics courses at first year university level and in workshops developed with secondary school students (age range from 13 to 18). We could see that the use of smartphones clearly increased the motivation of the students as their own devices became measurement instruments. In this respect, we have carried out a simple survey with 25 secondary school students and with 10 secondary school teachers, both involved in our workshops for secondary level developed during the year 2023. The students and teachers were asked to rate the following statements from 0 (not at all agree) to 10 (very much agree).

Questions for the students.

1) The use of smartphones as measurement instrument increased your motivation for physics concepts.

2) The use of the pressure sensor of a smartphone helped you understand and explore hydrostatics and hydrodynamics concepts.

3) You would be happy to carry out this workshop at home.

4) Based on this experience with the smartphone pressure sensor, you would like to see more physics workshops that use other smartphones sensors.

Questions for the teachers.

1) The use of smartphones as measurement instrument increased the students' motivation for physics concepts.

2) The pressure sensor of smartphones helped the students understand and explore hydrostatics and hydrodynamics concepts.

3) The smartphones can be a low-cost alternative in front of the traditional measurement equipment that can be found at the laboratory classroom.

4) Based on this experience with the smartphone pressure sensor, you would use smartphones as measurement instruments in your laboratory classes.

About 95% of the students responded with 8-10 to all three questions. In the case of the teachers more than 90 % responded with 8-10 to the first two questions and with 6-9 to the last two questions. Even when teachers acknowledge that students can benefit from using smartphone sensors as measurement instruments, they can be still a bit reluctant to use them as an alternative in front of the traditional lab equipment.

5. Conclusions

The predicted functional relationship of the pressure at the bottom of the container with water as a function of time while the container drains through a small orifice has been compared with the experimental data collected with a smartphone pressure sensor. Results show that a Physics model based on Bernoulli's principle and the continuity equation is fair enough to reproduce the experimental results. Likewise, the quality of the smartphone's barometer data can be considered good for this type of experiments. A survey applied to students and teachers at secondary school level yielded that the use of smartphones resulted very motivating and of great help for the students to understand the basic concepts of hydrostatics and hydrodynamics.

6. Acknowledgments

The authors would like to thank the *Instituto de Ciencias de la Educación* (Institute of Education Sciences) at the *Universitat Politècnica de València* (Technical University of Valencia), Spain, for its support to the teaching innovation group MSEL.

References

[1] Lamb H, Hydrodynamics, 6th ed. (Dover, 1945), p. 23.
[2] Resnick R, Halliday D and Krane K, Physics 4th edn (Mexico, DF: CECSA, 1999).
[3] Lindsay G A 1952 Pressure Energy and Bernoulli's Principle *Am. J. Phys.* 20 86; doi: 10.1119/1.1933123
[4] Seeger R J 1953 On Fluid Dynamics in Physics Teaching *Am. J. Phys.* 21 29; doi: 10.1119/1.1933340
[5] Glover F 1968 Bernoulli Effect and Kinetic Theory *Am. J. Phys.*36 811; doi: 10.1119/1.1975153
[6] Bedford D and Lindsay R 1977 A misinterpretation of Bernoulli's theorem *Phys. Educ.* 12 311
[7] Murphy A B and Brusca S 1986 Bernoulli effect *Phys. Educ.* 21 262
[8] Greene N R and Dworsak M R 2001 Bernoulli at the gas pump *Phys. Teach.* 39 346; https://doi.org/10.1119/1.1407128

[9] Cohen H and Horvath D 2003 Two Large-Scale Devices for Demonstrating a Bernoulli Effect *Phys. Teach.* 41 9; https://doi.org/10.1119/1.1533954
[10] Kamela M 2007 Thinking About Bernoulli *Phys. Teach.* 45 379; https://doi.org/10.1119/1.2768700
[11] Faulkner B E and Marty Ytreberg F 2011 Understanding Bernoulli's principle through simulations, *Am. J. Phys.* 79 214; doi: 10.1119/1.3533216
[12] Mungan C E 2011 The Bernoulli equation in a moving reference frame *Eur. J. Phys.* 32 517–520, doi:10.1088/0143-0807/32/2/022
[13] Greenslade T B 2013 Water Spout, *Phys. Teach.* 51 82; https://doi.org/10.1119/1.4775524
[14] Ivanov D et al, Testing Bernoulli's law, 2014 *Phys. Educ.* 49 436
[15] Santos L S F 2018 The energy density distribution of an ideal gas and Bernoulli's equations, *Eur. J. Phys.* 39 035102 (16pp), https://doi.org/10.1088/1361-6404/aaa34c
[16] Koumaras P and Primerakis G 2018 Flawed Applications of Bernoulli's Principle, Phys. Teach. 56 235; https://doi.org/10.1119/1.5028240
[17] D'Alessio S 2021 Torricelli's law revisited *Eur. J. Phys.* 42 065808 (11pp) https://doi.org/10.1088/1361-6404/ac279a
[18] Njock Libii J 2003 Mechanics of the slow draining of a large tank under gravity, *Am. J. Phys.* 71 1204; doi: 10.1119/1.1545764
[19] Guerra D, Plaisted A and Smith M, A 2005 Bernoulli's Law Lab in a Bottle *Phys. Teach.* 43 456; https://doi.org/10.1119/1.2060646
[20] Blasone M, Dell'Anno F, De Luca R, Faella O, Fiore O and Saggese A 2015 Discharge time of a cylindrical leaking bucket *Eur. J. Phys.* 36 035017 (11pp), doi:10.1088/0143-0807/36/3/035017
[21] Lopac V, Water Jets from Bottles, Buckets 2015 Barrels, and Vases with Holes *Phys. Teach.* 53 169; https://doi.org/10.1119/1.4908088
[22] Otto J and Mungan C E 2022 Filling and emptying a tank of liquid, *Eur. J. Phys.* 43 055003 (8pp), https://doi.org/10.1088/1361-6404/ac7e8b
[23] Horsch G M 2020 A Simple Model for the Calculation of the Fluid Discharge from a Small Orifice *Phys. Teach.* 58 113; https://doi.org/10.1119/1.5144793
[24] Hicks A and Slaton W 2014 Determining the Coefficient of Discharge for a Draining Container *Phys. Teach.* 52 43; https://doi.org/10.1119/1.4849155
[25] Monteiro M, Vogt P, Stari C, Cabeza C, and Marti A C 2016 Exploring the atmosphere using smartphones *Phys. Teach.* 54 308; doi: 10.1119/1.4947163
[26] Monteiro M and Martí A C 2017 Using smartphone pressure sensors to measure vertical velocities of elevators, stairways, and drones *Phys. Educ.* 52 015010
[27] Vandermarlière J 2016 On the inflation of a rubber balloon *Phys. Teach.* 54 566; https://doi.org/10.1119/1.4967901
[28] Dorsel D, Staacks S, Heinke H, and Stampfer C 2022 Using a Smartphone Pressure Sensor as Pitot Tube Speedometer *Phys. Teach.* 60 273; https://doi.org/10.1119/5.0025899
[29] Atkin K 2018 Investigating the Torricelli law using a pressure sensor with the Arduino and MakerPlot *Phys. Educ.* 53 065001
[30] Ángel Moya A, and Merino A 2022 Building a digital barometer with Arduino to study forces due to atmospheric pressure and weather maps at schools *Phys. Educ.* 57 045018 (6pp)
[31] Judd H and King R S 1906 Some experiments on the frictionless orifice *Eng. News.* 56 326–331
[32] Vennard J K and Street R L, Elementary Fluid Mechanics, 5th ed. (Wiley, New York, 1975), p. 558.