ACTA IMEKO
ISSN: 2221-870X
June 2024, Volume 13, Number 2, 1 - 9

Ancient architectural design interpretation: a framework based on Alexandrian manuals

Silvia Bertacchi[1], Francisco Juan Vidal[2], Filippo Fantini[3]

[1] *Dipartimento di Ingegneria, Università degli Studi della Campania Luigi Vanvitelli, Via Roma 29, 81031 Aversa, Italy*
[2] *Instituto Universitario de Restauración del Patrimonio, Universitat Politècnica de València, Camino de Vera, s/n, 46022 Valencia, Spain*
[3] *Dipartimento di Architettura, Alma Mater Studiorum - Università of Bologna, Viale del Risorgimento 2, 40136 Bologna, Italy*

ABSTRACT
The study of ancient architecture is a wide field that allows the contribution of various disciplines. One of the key issues is the archaeological aspect, which mainly consists of reconstructing the history of the events affecting a particular building and its changes over time. The discipline focusing on design analysis, which starts from the measurement of a building, has a twofold purpose: the definition of the patron's requirements and objectives, and the solution provided by the ancient professional through the application of specific operational protocols, typical of his time. In this paper we intend to analyse a series of sequential steps based on both graphs and calculations from the technical manuals of the Alexandrian area (1st century AD). Although neglected for a long time and lacking recent critical editions, they can be effectively integrated into the operational flow of various scholars studying ancient architecture from the perspective of measurement. Through some case studies from the Roman imperial period, documented with state-of-the-art devices, authors intend to illustrate the results of this approach integrating new measurement technologies, reverse modelling methods and verification of the results in the light of the texts of Heron of Alexandria focusing on construction.

Section: RESEARCH PAPER

Keywords: Design analysis; historical treatise; Heron of Alexandria; Hadrian's Villa; Vitruvius

Citation: S. Bertacchi, F. J. Vidal, F. Fantini, Ancient architectural design interpretation: a framework based on Alexandrian manuals, Acta IMEKO, vol. 13 (2024) no. 2, pp. 1-9. DOI: 10.21014/actaimeko.v13i2.1833

Section Editor: Fabio Leccese, Università Degli Studi Roma Tre, Rome, Italy

Received February 28, 2024; **In final form** April 4, 2024; **Published** June 2024

Corresponding author: Silvia Bertacchi, e-mail: silvia.bertacchi@unicampania.it

1. INTRODUCTION

The analysis of the ancient project entails an initial awareness: it is not possible to understand a structured and evolved operational method through the eyes of the contemporary world alone [1]. This means that it is necessary to "immerse oneself" in a world characterised by a precise scientific, technical, political, and social context before tackling the analysis of the ancient project. The critical editions of classical and late antique literary works dealing with architecture and art, which have only survived in small numbers, provide a clear example of the contemporary epistemological problem underlying this branch of study. The terminological issue, in this sense, is emblematic.

As an example, considering the term *ichnographia* used by Vitruvius (I, 2): in many texts, this is assimilated to the contemporary concept of "plan" of the building [2]. Other authors, more aware of the cultural distance between "us" and the Latin architect, prefer to describe it as the "science of drawing the plan of a building" [3]. It is therefore reasonable to hypothesise that the architect's training included a specific focus on how to design the building's plan: a particular discipline in which the current concepts of typological and distributive characteristics were combined with other disciplines presumably concerning calculation and, more generally, a functional conception of the architectural project. Similar considerations may be extended to the other two *species dispositionis* that together with *ichnographia* contribute to forming the triad of tools that the Latin author names *dispositio*, namely *orthographia* and *scaenographia*. These, respectively, should be read as the discipline of how to design elevations – with the consequent structural and rhythmic issue – and as the art of planning and managing the perception of the building from one or more major viewpoints, in order to optically correct the structural elements of the design (e.g., from the curved stylobate, to the entasis in the column, and so on). These three words are relevant to the ancient architect because through them, he could express his work of elaborating and adapting formal matrices, or schemes, standardised and measured to make them congruent with the specific needs of the client. Could this problem be related to the "archaeological issue"? Definitely yes, but the cultural distance between our mathematical, geometric and technological tools and those used in ancient times, most of which have been lost or have fallen into disuse, makes this task even more complex. Moreover, the best-known and most sophisticated buildings of the ancient world, although well studied, cannot be considered as the product of a single "ordinary" architect, perhaps assisted by one or more engineers or "mechanical engineers" [4]. In fact, for works of such commitment, which were innovative compared to the more common constructing scenario, the project team included mathematicians, surveyors, and physicists who, with their specific contribution, provided essential advice to solve the requirements of ambitious and demanding clients, thus achieving a stylistic and technological advancement that we cannot find in average buildings.

In this sense, Vitruvius' contribution is not very helpful, as he describes the principles of the conceptual and applicative "tools" available to the architect with rare but useful exceptions (temple and theatre design). In addition to this, there is a lack of direct evidence in the form of drawings or graphic diagrams (even engraved in stone) that could show us the specific design system carried out by the designer in his laboratory. Last but not least, it should be noted that Vitruvius wrote his work in the 1st century BC, while the history of ancient design begins to develop new and unprecedented structural and compositional solutions only from the second half of the 1st century AD, about one hundred years later [5]. Fortunately, there exist other written sources compatible with the *De Architectura* to integrate the dearth of the numerical and quantitative component that, today as in the past, characterises the architect's profession. Indeed, it is impossible to approach any design analysis without even a minimal knowledge of how numbers and formulae were used in the past, or in which way anthropometric units of measurement could be used to the advantage of the designer, even to simplify operations that were often only mnemonic.

Vitruvius dedicates little space to these topics, and this is a deliberate choice because the purpose of his text is to train high officials of the Roman state about the suitable execution of this novel architecture: consistent with the image of the newly established Empire, similar but different from the older sources of inspiration coming from Greece and Asia. The aim of intellectual autonomy of the *De Architectura* is also combined with the desire to dignify the architect's profession in order to make it independent from the numerical component, which was considered applicative and therefore not worthy of being considered with the same importance as *artes liberales*.

2. AIMS OF THE RESEARCH

Our aim is to integrate a metrological interpretation centred on calculations, to obtain linear measurements and areas within the complex system of relations expressed – but hidden by the state of ruin of the building –, in order to highlight the causal link between the technical-scientific context of the time and the solutions proposed by the architect.

As Giuliani [6] reminds us, some geometric forms only made their appearance in the Roman world following the reception of formulas derived from Hellenistic scientific experiences that allowed the calculation of areas and volumes resulting from the intersection of solids, namely the cylinders that form the basis of the geometric genesis of cross and pavilion vaults.

However, the large number of formulae one can find in the texts of the mathematician Heron of Alexandria, incorporated in the critical edition by the Danish philologist Johan Ludvig Heiberg (1854 -1928) entitled "Heronis Alexandrini opera quae supersunt omnia", at least partially compensated for this evident lack. Heiberg's meticulous work, collected in different volumes, is based on the earlier work by Hultsch [7] and Schmidt [8], and illustrates numerous engineering problems applied to mechanics, hydraulics and, of course, architecture through calculation examples and drawings.

The integration of theoretical aspects with applicative ones is the main aim of this study, which is based on the interdisciplinary collaboration carried out by architectural scholars from the Department of Architecture of the University of Bologna, the Instituto Universitario de Restauración del Patrimonio (IRP) and the Department of Applied Mathematics of the UPV and public administrators at Villa Adriana and Villa d'Este Institute.

Since 2010 [9], a customised approach has been improved based on the use of reality-based digital models derived from laser scanning and photogrammetry, used as the basis for a series of experiments aimed at tracing the patterns of those design methods formerly employed and which can now be recognised within such virtual replicas.

The aim of such studies is therefore to develop a system that extracts from reality-based digital models the material on which to perform tests and comparisons: hence a scientific purpose of digital representations [10], that goes beyond the concept of the mere documentation and description of the state of preservation of a cultural asset in order to critically investigate the traces of the *species dispositionis* implicitly contained in what has been measured.

3. THE MODEL AS A WORKING TOOL

Although the traces of the design method to be reconstructed are present in the ruins of the ancient building, they are difficult to read in the absence of an archaeological study that would allow the original parts to be discarded from the restoration or the modification also made in the past (reinforcements necessary to limit the effects of earthquakes, structural and foundation failure, expansions, etc.).

Fortunately, there are a rare number of archaeological finds that represent the architecture to be erected in the form of a model or drawing (engravings on stone or marble) [11]: these make it possible to reveal those features that are "concealed" in the built architecture, but which constituted the conceptual framework imagined by the classical architect.

An emblematic case is the so-called "model of stadium" [12] that was found in fragments near the Great Baths complex at Hadrian's Villa, near Rome. Although it was never built, this project is one of the very few that survived from the ancient world and was most likely made to illustrate to the imperial patrons a new entertainment building, smaller in size than the large spectacle buildings in Rome, as well as in the most important cities of the period.

The model was carved on a marble slab (one of those to be used as a wall covering) and expresses, through a partial three-dimensionality, the plan of a small amphitheatre, but not of a stadium (Figure 1). The aspects most clearly emphasised by the artifact are addressed to an architecturally experienced patron, because it focuses on a basic technical issue of ancient design, namely the presence of a rigid modular structure. Both Cassius Dio and Aelius Spartianus [13]-[15] highlight Hadrian's

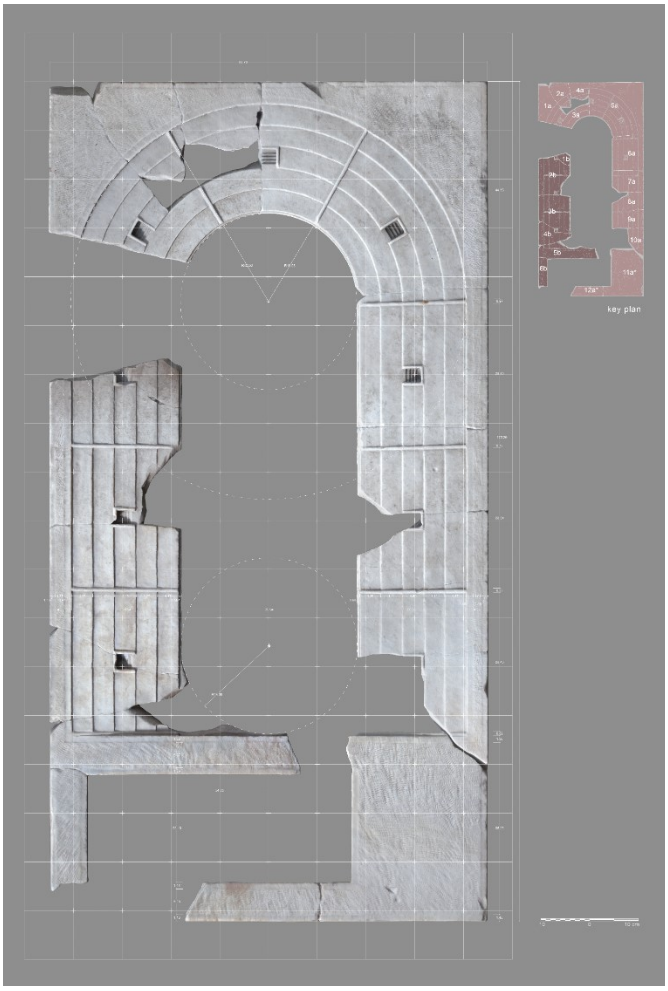

Figure 1. Plan view of the maquette carved in a marble slab representing the model of a building for spectacles. It is to be noted how the chisel work is surrounded by a smooth edge running along all the main parts of the layout. Illustration by F. Fantini.

fascination and expertise in architecture, and therefore the hybrid nature of this maquette, which resembles a partially three-dimensional planimetry (Figure 2 and Figure 3), as is the case of another model found at Ostia Antica [16]-[18] (Figure 4 and Figure 5), should not be surprising.

The purpose of these representations seems to foster the debate with the patron on the main issues of the building: distribution of functions, paths, access walkways to the rows of seats, relationships between the fundamental elements (tiers of seats, arena, adjoining spaces) and above all the modular grid, that gives metric coherence to each part – the *koilòn métron* from Heron's "Definitiones" [19] – as well as facilitating the reading of the scale of the model.

The presence of a repeated module in every part of the described architectural representation is the main evidence that this maquette from Hadrian's Villa can be considered a model with technical value and not only an ornamental artefact, as some authors had hypothesised [20]. In particular, the author of the maquette wanted to underline the relationship that existed between the constructed building and the allocated overall space. In addition to this, it is evident that the area available for the new project was a standard dimension divisible by the module that is repeated in the building.

A distinct "graphic" treatment characterised each element: the perimeter of the marble slab is emphasised by a smooth edge, which makes it recognize from the tiers of seats; the same happens to the shape of the tiers of seats that contain the elongated arena, and finally an additional uncovered rectangular space, which is eccentric to the general composition and could be interpreted as a porticoed space.

In short, the model shows to the patron the size of the overall area necessary for the building, the covered area (tiers of seats, *vomitoria*) and lastly the uncovered area.

Modularity is a necessary condition for any acceptable construction of the architectural project and has two different meanings:

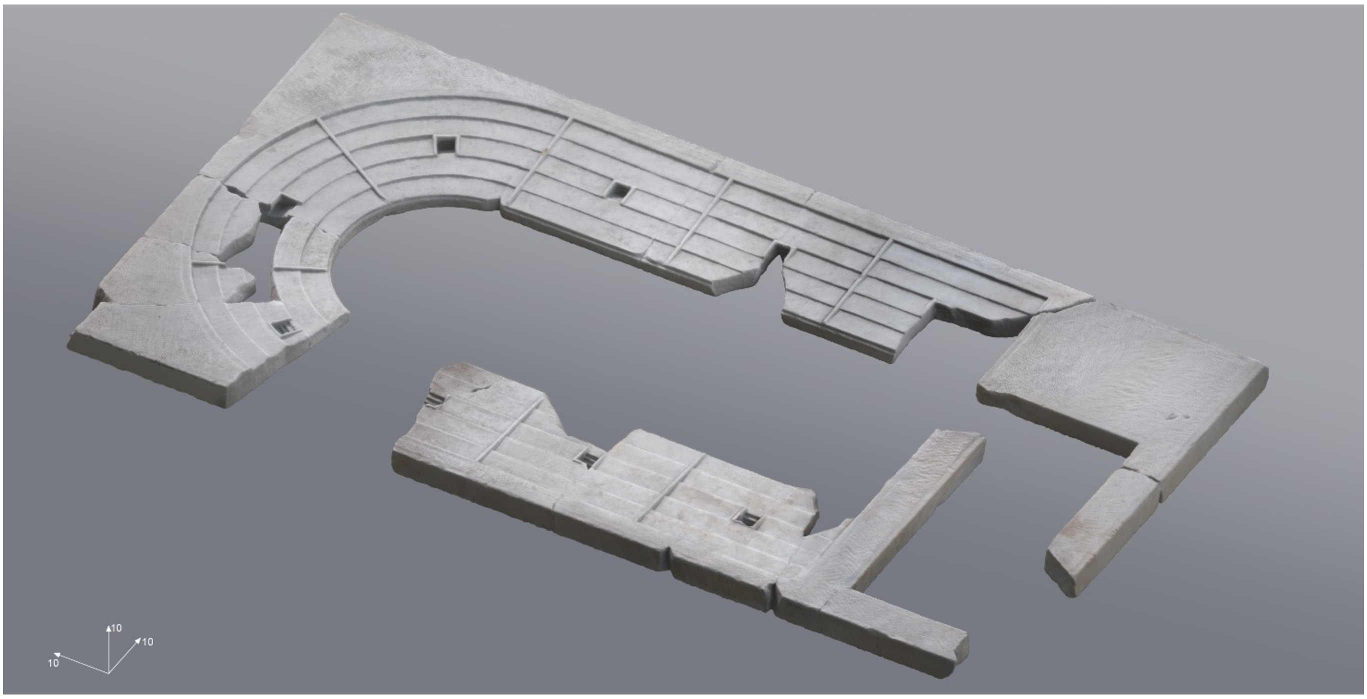

Figure 2. Axonometric view of the digital model. Illustration by F. Fantini.

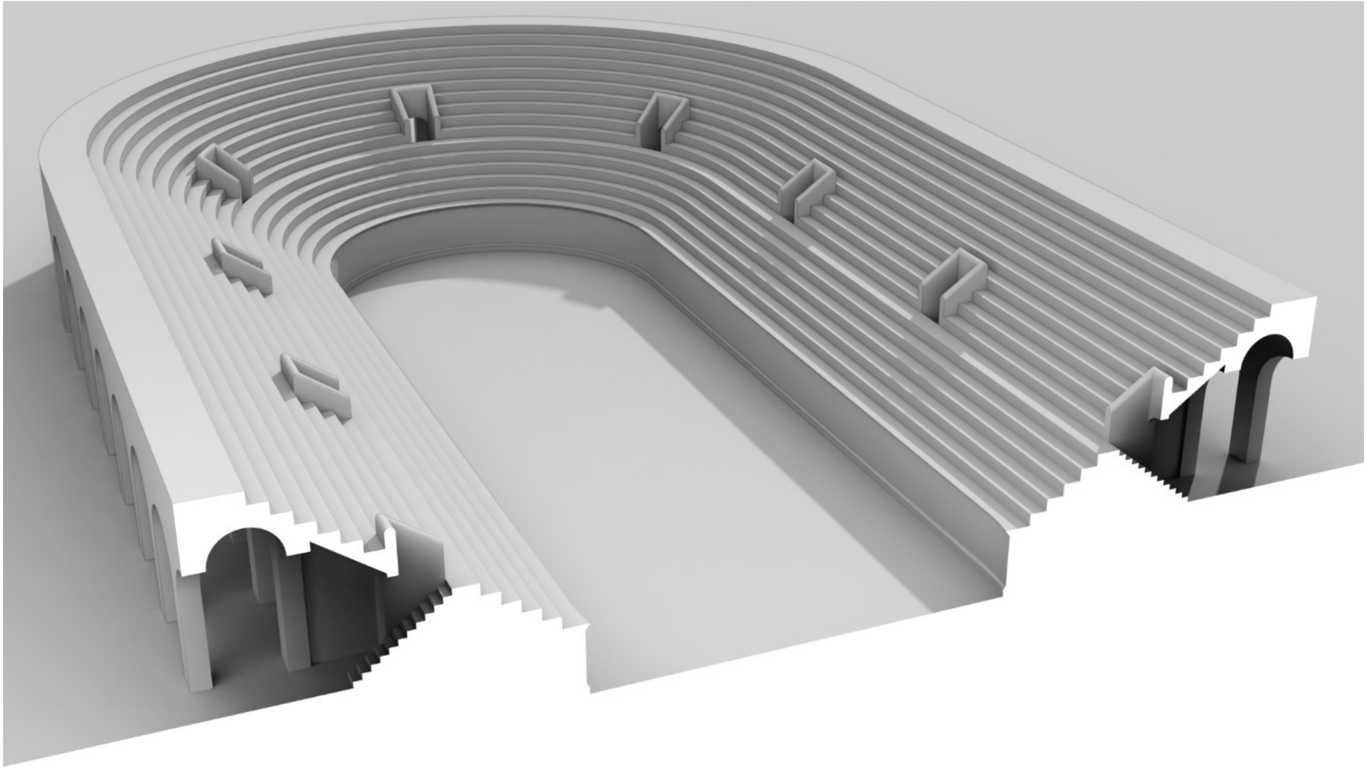

Figure 3. Reconstruction hypothesis. Illustration by S. Bertacchi.

1) according to Vitruvius' treatise, the architect should achieve *symmetria* in his project, by means of a series of calculations that have the aim of rationally organising and conceptually connecting each part of the building to a measure that is convenient from the point of view of calculating lengths, areas and volumes;
2) on the other hand, according to an earlier mathematician belonging to the Hellenistic scientific domain, the Syracusan Archimedes, the same term refers to the discovery of simple ratios that link areas and volumes of the sphere, cylinder, and cone. In fact, he wrote in the first book "On the Sphere and the Cylinder" that «these properties had always been intrinsic to the nature of the mentioned shapes and were ignored by those who before us dealt with geometry: none of them had realised that for these shapes there is a symmetry» [21].

In other words, for Archimedes, two geometric shapes are symmetrical if their areas and volumes are linked by simple proportionality expressed through a rational number, while for Vitruvius, the same word indicates the presence of the greatest common divisor, which is present in every part of the project, i.e. in every technical elaboration realised by the architect by applying the criteria of *dispositio* (*ichnographia*, *orthographia*, *scaenographia*). Whether this property is achieved through the use of modules is another matter that concerns the technique Vitruvius defines as *ordinatio* [4].

How the ancient architect obtained the measurement of a module is still quite complex; what emerges from the maquette and has been confirmed by recent studies on buildings for spectacle, such as theatres [22], is that the module was derived from a subdivision of the standard areas used by the Romans and that, if appropriately apportioned, allowed for quicker calculations and memorisation.

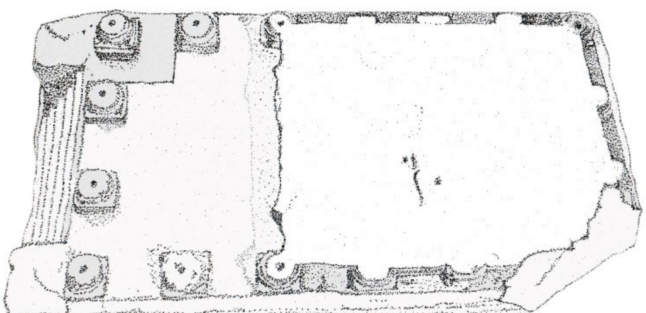

Figure 4. Model of a temple from Ostia Antica. Illustration by S. Bertacchi after [18], p. 125.

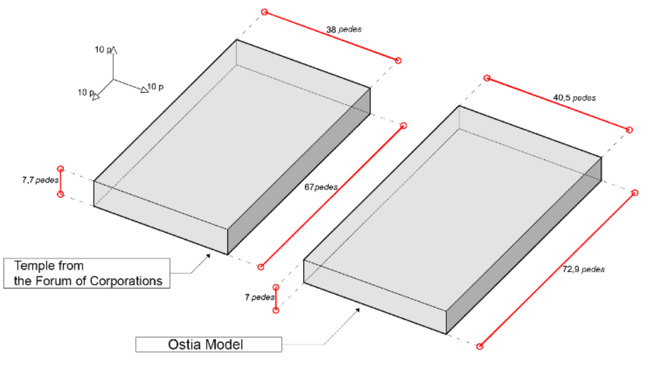

Figure 5. Dimensional comparison between the marble model (left) and the Temple in the Forum of Corporations in Ostia Antica (right) which is of a similar type (assuming the model scale 1:32). Illustration by S. Bertacchi.

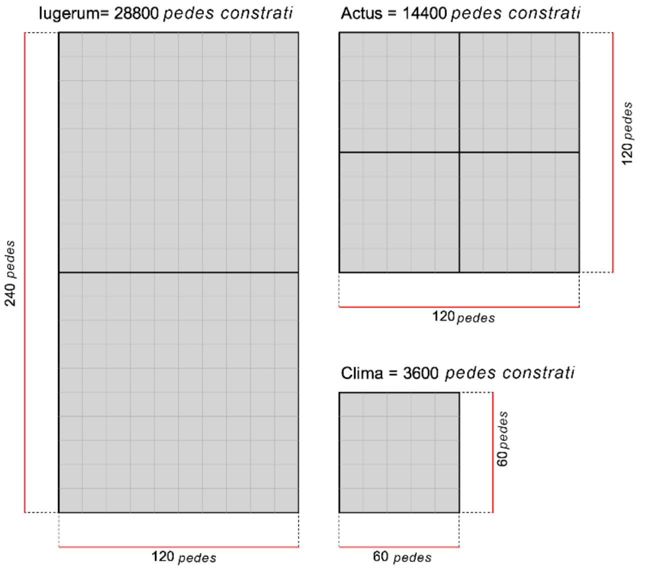

Figure 6. Different Roman units of measurement of areas. Illustration by S. Bertacchi.

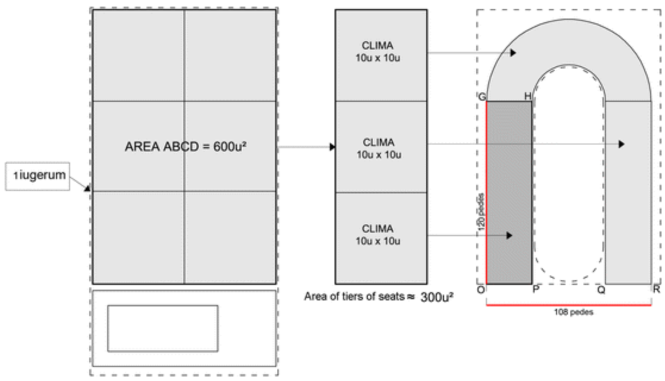

Figure 7. Planimetric analysis of the so-called stadium model from Villa Hadrian s Villa. Illustration by F. Fantini.

The best insight to this issue in the written sources is provided by Columella [23], who lists the widest range of standard surfaces compared to other authors [22] (Figure 6).

The interest towards Columella's writing derives from a consideration about a sub-module of the ancient unit of measurement, i.e. the *actus cuadratus* (120 × 120 *pedes constrati*) and the *clima* (60 × 60 *pedes constrati*): this area is very close to a measurement that we can define as architectural, in the sense that a square with 17.7 metres-side generates an area of about 300 m^2, which presumably was used as a sort of pre-dimensioning area for a building or a part of it.

The analysis carried out on the model of the building for spectacles led to the determination of a scale of representation [24], in particular it was assumed that the short side of the model, 12 *palmi* in length, could correspond to a side of 120 *pedes* in reality as for the *actus*: on the basis of this assumption, given that the present module of the model measures 3/5 of a *palmus*, it was concluded that the module in real scale would correspond to 6 *pedes* and therefore the scale of representation of the model corresponds to 1:40.

Once the reduction scale was obtained, new calculations could be carried out with regard to the areas that the architect assigned to each sector of the tiers of seats (two are rectangular in shape and one is a semi-circular crown) and it was realised that the three areas were equivalent, and each were identical to the measure called *clima* (3600 *pedes constrati*) (Figure 7).

But the *symmetria* pursued by the architect also concerns the other built volume, which is not easy to identify functionally, and which extends on the opposite side of the semicircular tiers (perhaps a gymnasium, a place for training, separate from the tiers) [25]. The area of such a rectangular space is in fact equal to a *clima*: thus, every single tier of seats (curved or straight) as well as this area are equivalent and equal to 3600 *pedes constrati*. In short, we can state that the entire construction area was equal to one *iugerum*, while the "covered" area was equal to half of it, i.e. one *actus*. Even the two non-built areas (dromos/arena and rectangular court), which are represented in the model as actual holes, are in fact connected by a simple proportionality: the arena is 3 times the area of what we can identify with the peristyle carved out of the rectangular annexed volume.

Buildings such as the theatre, stadium and circus have evident formal similarities with this maquette, and for this reason some tests were carried out on buildings for which up-to-date surveys were available.

Results confirmed the use of *clima* for the dimensioning of the tiers of seats not only at the design stage, but also at the modification stage. Theatrical buildings, in fact, always had at least two construction phases closely linked to the demographic development of the city: an initial design corresponding, in general, to the early imperial period was always followed by a second one in the later imperial period, and finally, but not always, by a third and final one of later expansion.

The verifications carried out, particularly in the case of the Roman theatre of Sagunto – for which a complete survey and archaeological study campaign was available –, demonstrated a clear intention of the architect to double the number of spectators in the second construction phase. The issue is not trivial from a mathematical point of view, as it requires designing a semi-circular crown with an area doubling the one of the first construction phase [26]. Vitruvius is of no help in this type of problem as he makes no mention of formulae or geometric constructions that can solve this problem.

In this respect, the clarifying reading of Heron's writings in Heiberg's critical edition [19], [27], provides extremely important keys to understanding how the architect could calculate the *cavea* (a semi-circular annular ring) for the purposes of determining the number of spectators to be seated. The problem is also posed in very similar terms for calculating the construction volume of arches (Figure 8 a,b).

Heron, in *Stereometrica* (I, 42 and 43) and *De mensuris* (I, 24), deals with the problem of the "measure of the theatres", which is in fact solved by squaring semi-circular crowns: the formula is very simple, but precise – see equation (1) –, since it is based on the sum of the low perimeter of the *cavea* and the one in summa *cavea*. The result is divided by 2 to find an average perimeter, which is then multiplied by the number of steps. In the formulae the measure of the foot is assimilated to that of the space occupied by each spectator:

$$N_s = \frac{E_s + I_s}{2} R, \tag{1}$$

where N_s is the number of spectators, E_s the semi-circle of the summa *cavea*, I_s the semi-circle of the orchestra, and R the number of steps [28].

Subsequent research on second-century A.D. architecture then focused on the theme of centric-layout spaces, particularly characterized by the presence of domes in *opus camenticium* [29],

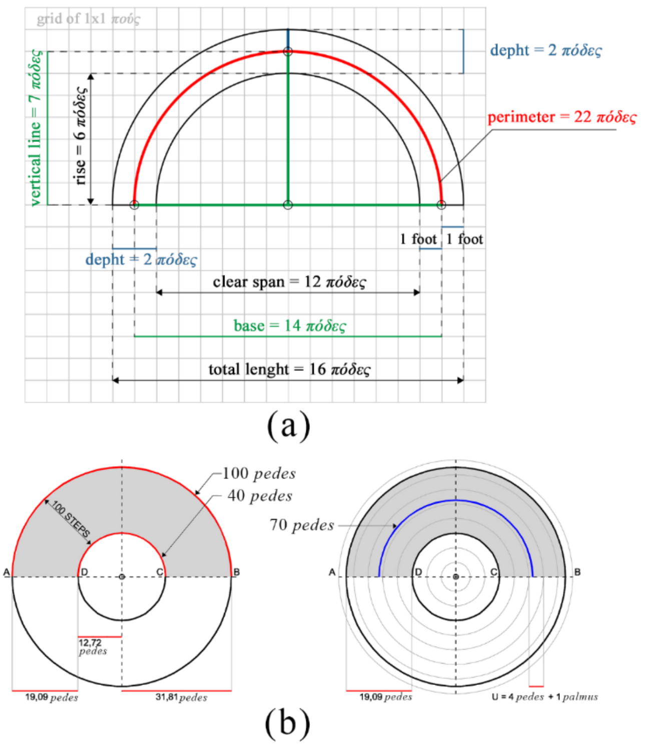

Figure 8. (a) The interpretation of Heron's formula for the calculation of the constructive volume of an arch (*Stereometrica* II, V, 29). (b) Heron's formula for the calculation of the number of spectators (*De mensuris*, 24). Illustration by S. Bertacchi.

[30]. As with the determination of the volume of the arch and the number of spectators in the theatre, domes can also be approached with similar procedures and especially if we know the radii of the intrados and extrados of a dome, the construction volume can easily be determined.

It is important to point out that research carried out on the basis of the most recent surveys allowed us to find dimensions that systematically recur in Roman buildings with a central plan covered by domes: studies by Fuchs [31]-[32] and Svenshon [33] on Roman and Byzantine buildings have shown that the number 7 (7 modules or multiples of 7 *pedes*) is a constant presence in vaulted spaces, because the approximation of π to 22/7 made it possible to simplify the measurement of the diameter with that of the denominator of the fraction without any irrational quantities, pointless from the perspective of the Roman builder. The formalisation of calculations certainly cannot be directly compared to ours based on Arabic numerals, but even from reading Heron it is clear that the basic concept, i.e. multiplying by 7 and dividing by 7, had several advantages for the ancient architect. In *Stereometrica* (I, 2), in order to calculate the volume and area of a sphere, Heron uses a diameter equal to 7 and then immediately afterwards states that the perimeter of the great circle is exactly 22 *pedes* (Figure 9). We are clearly far from the refinement of Euclid's or Apollonius of Perga's demonstrations, but when considered from a professional point of view and for a fast solution, the formulation is more than acceptable.

It seems no coincidence that the greatest development of complex vaulted systems occurred during the principate of Nero, who, a contemporary of Heron, had directed the engineering and creative efforts of his architects towards the realisation of the extremely thin dome of the octagonal hall of the Domus Aurea. The mastery of the structural limits of the *opus caementicium* and the availability of formulas aimed at the design of domes allowed

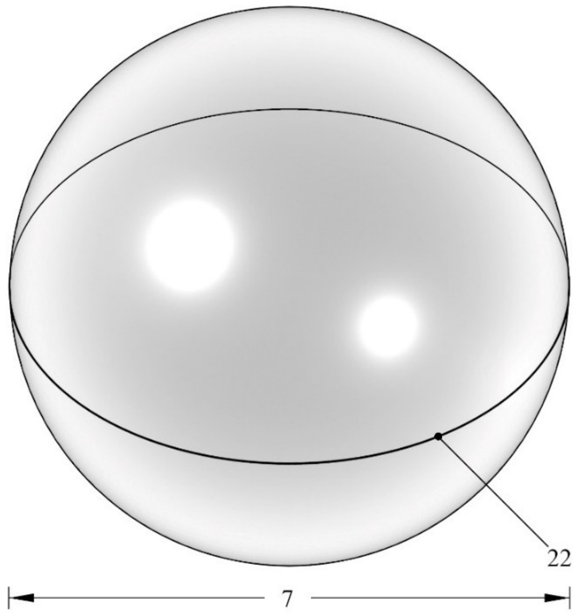

Figure 9. Calculation of the volume and area of a sphere, knowing the great circle (*Stereometrica* I, 2). Illustration by F. Fantini.

the later emperors with a passion for architecture, in particular Domitian and Hadrian, to enjoy the results of Alexandrian experiments on complex geometric forms and related calculations [30].

4. APPLICATIONS

Analysis of the model from Hadrian's Villa shows that there was a close relationship between the standard areas used by the Romans and the dimensioning of the rows of seats. Standard areas most probably corresponded to a specific number of spectators. An obvious analogy with the model can be found in an example of a construction with a very similar *ichnographia* from the 1st century B.C.: the amphitheatre of Caesarea of Mauretania built by Juba II [24].

During the initial phases of the designing process, a pre-dimensioning criterion was applied. The architect used numerically "convenient" areas, such as the *clima* (60 × 60 *pedes constrati*): as a matter of fact, the number 60 admits many dividers, easily adaptable with the determination of a flexible module, that could be certainly associated with the number of spectators (Figure 10). The use of standard areas from which to start the *ichnographia*, all the more so, evident even in buildings with a simpler layout. A residential building, called *Mouseia* (formerly the *Antiquarium*), located on the west side of the Canopus pool of

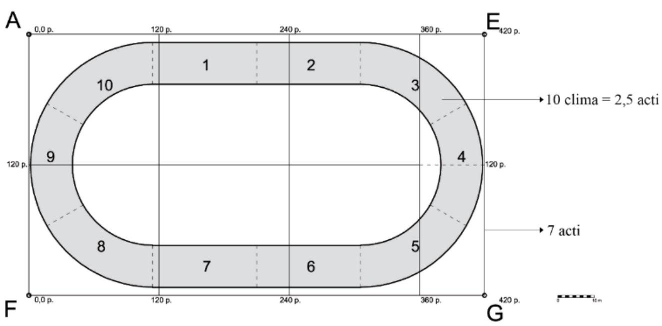

Figure 10. Planimetric analysis of the amphitheatre of Caesarea of Mauritania Illustration by F. Fantini.

Hadrian's Villa (Figure 11), seem to confirm the same approach. Standard areas are subdivided according to a modular grid, completely in line with the design method of buildings for entertainment, although with the simplified condition of not presenting curvilinear elements [34].

When analysing the simple planimetric design of the complex, it emerges clearly the application of design method based on the concept of a progressive subdivision of the area used as construction site. Also, the property called *symmetria* is present and evident when confronting covered and uncovered spaces with the standard areas (*actus*, *clima*, *scripulum*).

A third example is related to the design of complex domes based on the geometry of the sphere, the cylinder intersecting spheroidal triangular surfaces (sail vaults) which – in the writings attributed to Heron – are referred to as "*trikentron*" [30].

The Vestibule at the Golden Court is a centrally-plan building, structurally and functionally connected to a peristyle, that would serve as an archetype for many other later examples of octagonal halls covered with "compound domes": a semi-sphere intersects with an octagonal drum and other surfaces radially aligned. Semi-circular lunettes on the flat sides of the octagon (Figure 12) are indeed connected to the sphere by means of *trikentrons*. In *Stereometrica* II, the formula for calculating the surface area of a *trikentron* is expressed in an extremely simple manner even though it conceals an interesting numerical feature, that places the numerical solution somewhere between the surface area of a spherical triangle and a sector of a groined vault.

It should be noted that the volume of the sphere can be achieved by taking advantage of the so-called "rule of 7" [32]. Actually, the intermediate semispheric profile of the dome, made of *opus caementicium*, tuff, and brick, has a diameter of 35 *pedes*: a number easily divisible by 7 and thus able to simplify the calculation of the dome's volume. One question, still open in

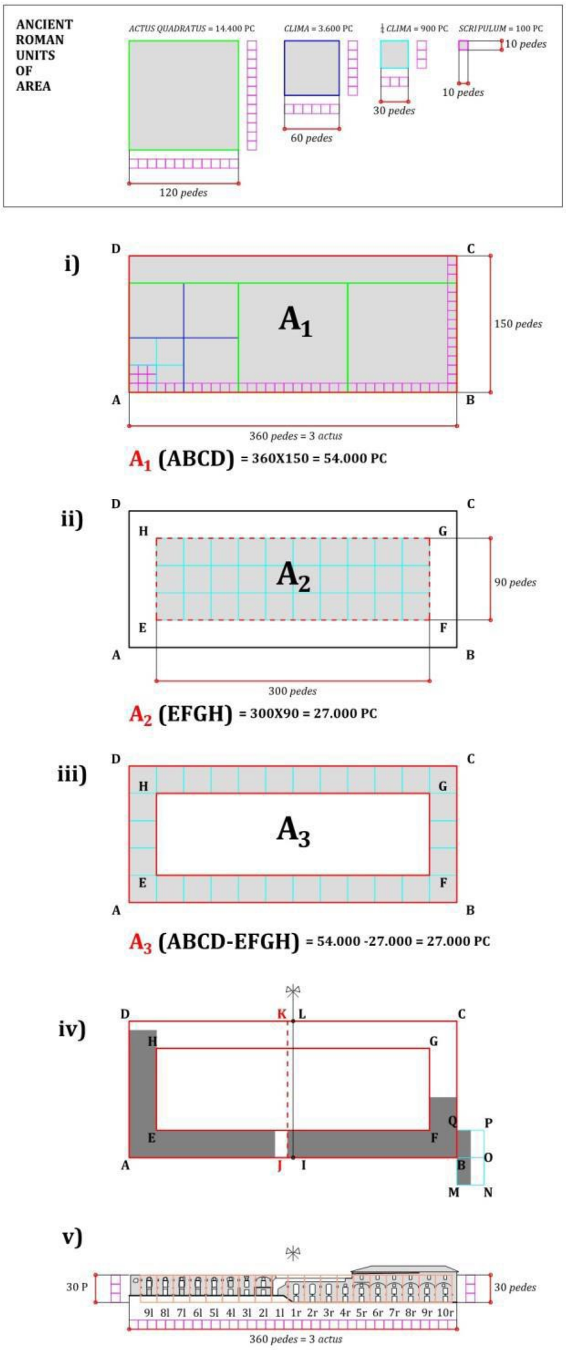

Figure 11. Planimetric analysis of the residential building located in the west area of the Serapeum-Canopus complex at Hadrian's Villa. Preliminary hypothesis for the plan of the original building (Roman units of measurement for surfaces: *pedes*, *actus*, PC=*pedes constrati*). i) Estimate of the global area of the upper building considering the plausible design of the plan (A1); ii) estimate area of the courtyard according to preserved remains and excavations (A2); iii) possible covered area of the building (A3); iv) current existing upper structures (dark grey hatching); v) proportions, height difference and modules of the eastern front. Illustration by S. Bertacchi.

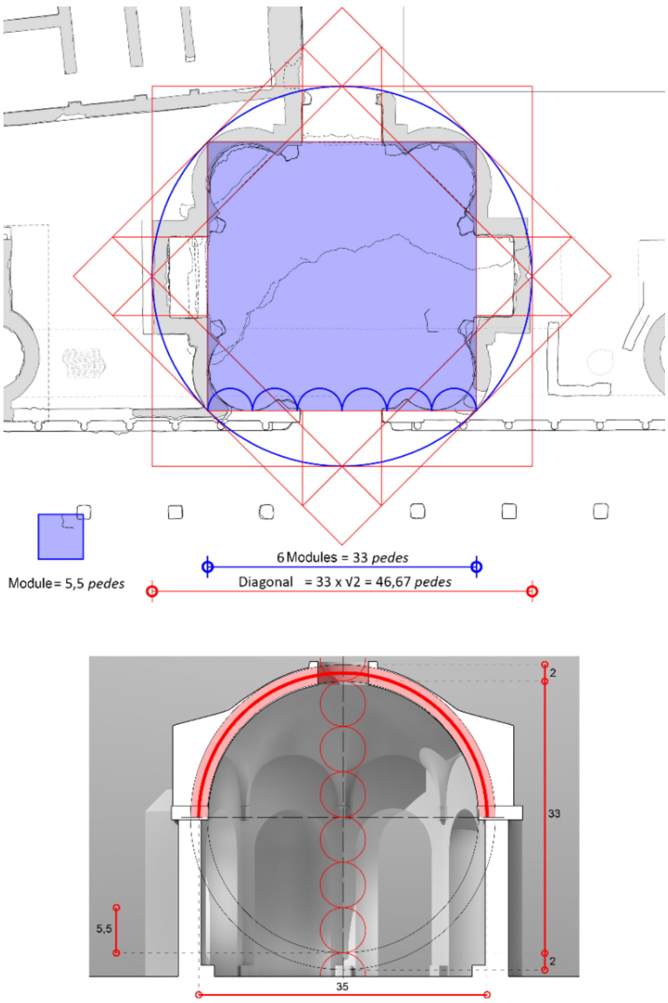

Figure 12. Design analysis of the access Vestibule at the Golden Court: *ichnographia* (*ad quadratum*) and *orthographia* (Heron's formulas). Illustration by F. Juan-Vidal.

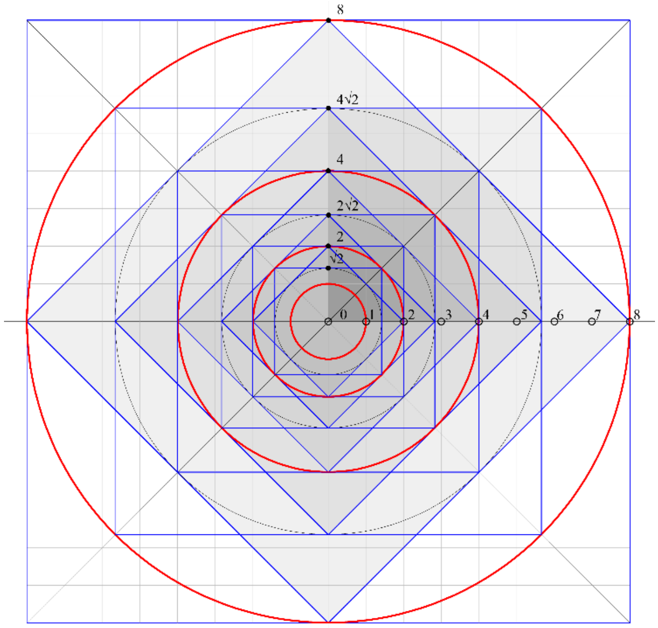

Figure 13. *Ad quadratum* pattern matching with the modular grid. Illustration by F. Fantini.

scholarly debate, is how the ancient architect would reconcile this set of formulas with the graphic tools habitually employed for the planimetric layout of the building. The construction known as "*ad quadratum*" is evidently used extensively in circular and octagonal buildings to create a pattern to be followed in the planimetric layout (*ichnographia*), which, in turn, will have to be harmonised with the absolute must of using modules (*symmetria*). By inscribing successive squares and circles, a geometric progression of ratio √2 generates alternate terms (2n) that double areas, it is also verified that the successive figures of the layout are again in conformity with the grid (Figure 13).

5. CONCLUSIONS

Vitruvius repeatedly expresses in his treatise the need to refer to rational quantities and possibly whole numbers of modules obtained through calculations and geometric constructions performed with the compass, i.e., forcing "*ad abundantiam*", the result of the calculations performed through similar handbooks to those ascribed to Heron. The method proposed here is based on this premise and can be divided into five priority aspects to be taken into account when performing the design analysis of ancient buildings:

1) research of sequential elements to derive the module present in the whole building;
2) exam of the existing relationship between the module and the standard area measurements used in the imperial age (*jugerum*, *actus*, *clima*);
3) check for simple numerical relationships linking parts of the building layout (covered areas/uncovered areas);
4) for curved layouts, with special regards to centric plans, research of a pattern based on inscribed and circumscribed circles with simple geometric figures such as triangles and squares (*ad quadratum* scheme), to be adjusted to previously obtained modules. In general, the final building measurements are the product of an overall pure mathematical calculations or geometric constructions;
5) verification of the presence of modules or measures that are multiples of 7 to understand which quantities (volumes, areas) the ancient architect had to calculate to make the project executive.

This kind of study has a twofold objective, on the one hand to find a basis for a fruitful dialogue with archaeologists through a philological investigation of technical texts. On the other, to regulate, through a clear codification, a type of research focusing on ancient design that is often the subject of empirical graphic analyses, lacking a real causal link between forms and functions/requirements.

Hadrian's Villa is an archaeological site known for the great formal variety of buildings dedicated to conviviality and well-being (*triclinia*, baths, buildings for spectacles), but also for the presence of more functional constructions with simple layouts such as the residential ones and those for administration [35]. The coexistence of complex monumental buildings, especially the domed ones with a mixtilinear layout, with the more elementary non-aulic buildings, allows us to clearly see the application of a rigorous design method, which in the more eclectic buildings becomes complicated, leading to solutions that are not always optimal. It is precisely this co-presence that allows us to recognise in a single ensemble the rule and its exception with great clarity, provided, however, that we have up-to-date surveys and multidisciplinary collaborations available.

Extending the surveys carried out with advanced tools also to other buildings belonging to the same cultural scene, such as those of Baiae (Temple of Venus near Naples in Campania, Italy) and of Rome (Horti Sallustiani), would allow an even more in-depth understanding of the design methods in vogue during the Hadrianic reign, which, despite their high originality, do not abandon the core of the classic design methods enunciated by Vitruvius. In an era in which research starting from digital surveys, then turned into 3D digital models, seems to be mainly directed towards the "transparent" representation of the data collected in order to obtain valid reconstructions [36], we find it important to underline the relevance of integrating knowledge to advance in the process of understanding the design methods adopted in antiquity: an indispensable basis for any philologically reliable reconstruction [37]-[38].

ACKNOWLEDGEMENT

The authors would like to thank the Istituto Villa Adriana Villa d'Este, Andrea Bruciati and Benedetta Adembri.

REFERENCES

[1] M. T. Bartoli, *Orthographia*, *Ichnographia*, *Scaenographia*, in: 2000 Anni di Vitruvio (Studi e documenti di architettura, no. 8). L. Vagnetti, L. Marcucci, M. T. Bartoli (editors). Edizione della Cattedra di Composizione Architettonica I e II della Facoltà di Architettura di Firenze, Firenze, 1978, pp. 197-208. [In Italian]

[2] S. Ferri (editor), Marco Vitruvio Pollione, Architettura (dai libri I-VII), Testo latino a fronte, Rizzoli BUR Classici greci e latini, Milano, 2002, ISBN 9788817127684. [In Italian/Latin]

[3] A. Corso, I disegni che corredavano il *De Architectura* nel contesto delle rappresentazioni antiche di temi architettonici, in: Vitruvio e il Disegno di Architettura. P. Clini (editor). Marsilio Editori, Venezia, 2012, ISBN 9788831714402, pp. 47-59. [In Italian]

[4] P. Gros (editor), Vitruvio. De Architectura, Einaudi, Torino, 1997, ISBN 9788806122393, p. 1405. [In Italian]

[5] J. B. Ward-Perkins, Architettura Romana, Electa, Milano, 1974, ISBN 9788843524730, p. 63. [In Italian]

[6] C. F. Giuliani, L'Edilizia nell'Antichità, Carocci Editore, Roma, 2018, ISBN 9788843092307, p. 119. [In Italian]

[7] F. O. Hultsch, Heronis Alexandrini geometricorum et stereometricorum reliquiae: accedunt Didymi Alexandrini Mensurae marmorum et anonymi variae collectiones ex Herone, Euclide, Gemino, Proclo, Anatolio aliisque, Weidmann, Berlin, 1864. [In Latin/Greek]

[8] W. Schmidt (editor), Heronis Alexandrini Opera quae svpersvnt omnia, Volumen I: Pneumatica et automata, B. G. Teubner, Berlin, 1976. [In Latin] DOI: 10.1515/9783110261431

[9] B. Adembri, S. Di Tondo, F. Fantini, New advancing of the research on the architecture with concave and convex rhythms at Hadrian's Villa: reconstruction hypothesis on the southern nymphaeum of the Piazza d'Oro, in: Proc. of the 16th CHNT 2011. Museen der Stadt Wien-Stadtarchäologie, Wien, Austria, 2012, ISBN 9783200027404, pp. 125-139.

[10] A. Ippolito, F. Borgogni, I modelli 3D nei rilievi di architettura, in: Metodologie Integrate per il Rilievo, il Disegno, la Modellazione dell'Architettura e della Città. E. Chiavoni, M. Filippa, M. Docci (editors). Gangemi, Roma, 2011, ISBN 9788849222081, pp. 71-77. [In Italian]

[11] P. Azara, M. Palà (editors), Las Casas del Alma, Maquetas Arquitectónicas de la Antigüedad (5500 a.C. - 300 d.C.), Edición Centre de Cultura Contemporània de Barcelona CCCB - Institut d'Edicions de la Diputació de Barcelona, Barcelona, 1997, ISBN 9788477945017. [In Spanish]

[12] B. Adembri, L. Cipriani, F. Fantini, Illustrare, pianificare e costruire nel mondo antico: casi di studio da Villa Adriana, in: I Tracciati di Cantiere: Disegni esecutivi per la Trasmissione e Diffusione delle Conoscenze Tecniche. C. Inglese, A. Pizzo (editors). Gangemi Editore, Roma, 2016, ISBN 9788849232660, pp. 90-107. [In Italian]

[13] D. Rathbone, Lives of Hadrian. From Cassius Dio and the Historia Augusta, Pallas Athene, London, 2008, ISBN 9781843680475.

[14] A. Viscogliosi, Traiano e gli imperatori adottivi, in: L'architettura del Mondo Antico. C. Bozzoni, V. Franchetti Pardo, G. Ortolani, A. Viscogliosi (editors). Laterza, Bari, 2006, ISBN 9788842080558, pp. 298-338. [In Italian]

[15] A. Viscogliosi, L'architettura Adrianea: di Adriano, per Adriano, sotto Adriano, dopo Adriano, in: Adventus Hadriani, Investigaciones sobre Arquitectura Adrianea. R. Hidalgo Prieto, G. E. Cinque, A. Viscogliosi, A. Pizzo (editors). L'Erma di Bretschneider, Roma, 2020, ISBN 9788891320131, pp. 11-36. [In Spanish]

[16] E. Salza Prina Ricotti, Villa Adriana. Il Sogno di un Imperatore, L'Erma di Bretschneider, Roma, 2000, ISBN 9788891313263. [In Italian]

[17] P. Pensabene, Modello templare ostiense in marmo lunense dal Collegio degli Augustali, Rivista dell'Istituto Nazionale d'Archeologia e Storia dell'Arte S. III XIX-XX 1996-1997 (1998), pp. 148-151. [In Italian]

[18] C. Caprino, Plastico marmoreo di uno stadio nella Villa Adriana a Tivoli, Rivista dell'Istituto Nazionale d'Archeologia e Storia dell'Arte S. III XIX-XX 1996-1997 (1998), pp. 113-151. [In Italian]

[19] J. L. Heiberg (editor), Heronis Alexandrini Opera quae supersunt Omnia. Volumen IV: Heronis Definitiones cum variis Collectionibus. Heronis quae feruntur Geometrica, B. G. Teubner, Berlin, 1976. [In Latin] DOI: 10.1515/9783110954227

[20] W. L. MacDonald, J. A. Pinto, Villa Adriana. La Costruzione e il Mito da Adriano a Louis I. Kahn, Mondadori Electa, Milano, 2006, ISBN 9788837045005, p. 16. [In Italian]

[21] A. Frajese (editor), Opere di Archimede, UTET, Torino, 1974, ISBN 8802018200, p. 70. [In Italian]

[22] C. Bianchini, F. Fantini, Dimensioning of ancient buildings for spectacles through *Stereometrica* and *De mensuris* by Heron of Alexandria, Nexus Netw. J. 17 (2015), pp. 23-54. DOI: 10.1007/s00004-014-0230-8

[23] C. Carena (editor), Lucio Giunio Moderato Columella. L'Arte dell'Agricoltura e Libro sugli Alberi, Einaudi, Torino, 1977, ISBN 8806098942. [In Italian]

[24] Y. Chennaoui, F. Juan Vidal, F. Fantini, Architectural models and urban planning, in: Proc. of the 17th CHNT 2012. Museen der Stadt Wien-Stadtarchäologie, Wien, Austria, 2013, ISBN 9783200032811, pp. 1-19.

[25] F. Fantini, Il modello di stadio da Villa Adriana, indagine su un progetto incompiuto, Università degli Studi di Firenze, Firenze, 2008, p. 98. Ph.D. thesis [In Italian]

[26] S. Lara Ortega, El trazado Vitruviano como mecanismo abierto de implantación y ampliación de los Teatros Romanos, Archivo español de arqueología 65(165-166) (1992), pp. 151-179. [In Spanish] DOI: 10.3989/aespa.1992.v65.475

[27] J. L. Heiberg (editor), Heronis Alexandrini Opera quae supersunt Omnia. Volumen V: Heronis quae feruntur Stereometrica et de Mensuris, B. G. Teubner, Berlin, 1976. [In Latin] DOI: 10.1515/9783110957051

[28] L. Cipriani, F. Fantini, S. Bertacchi, Composition and Shape of Hadrianic Domes, Nexus Netw. J. 22 (2020), pp. 1041-1061. DOI: 10.1007/s00004-020-00514-z

[29] L. Cipriani, F. Fantini, S. Bertacchi, The geometric enigma of small baths at Hadrian's Villa: mixtilinear plan design and complex roofing conception, Nexus Netw. J. 19 (2017), pp. 427-453. DOI: 10.1007/s00004-017-0344-x

[30] A. Roca, F. Juan-Vidal, L. Cipriani, F. Fantini, Heron's legacy: an example of ancient calculations applied to Roman imperial architecture, Nexus Netw. J. 25(1) (2023), pp. 185-192. DOI: 10.1007/s00004-023-00704-5

[31] W. Fuchs, Studio geometrico della forma architettonica del tempio di Vesta ricostruito nel Foro Romano, Aedes Vestae. Archeologia, Architettura e Restauro, Atti della Giornata di Studio, Parco archeologico del Colosseo, 9 giugno 2021, Roma, Italia, 2022, pp. 77-86. [In Italian]

[32] W. Fuchs, The new theory of the metrological framework of the Pantheon, Nexus Netw. J. 25(1) (2023), pp. 103-110. DOI: 10.1007/s00004-023-00679-3

[33] H. O. Svenshon, Heron of Alexandria and the Dome of Hagia Sophia in Istanbul, Proc. of the 3rd International Congress on Construction History, Brandenburg University of Technology Cottbus, Germany, 20th - 24th May 2009, pp. 1387-1394.

[34] A. Ottati, S. Bertacchi, B. Adembri, Integrated methods for documentation and analysis of archaeological heritage: the residential building along the western side of the Canopus at Hadrian's Villa. Initial results and research perspectives, SCIRES-it 8(2) (2018), pp. 85-106. DOI: 10.2423/i22394303v3n2p85

[35] E. Eramo, Gli edifici residenziali "non aulici" nella Villa Adriana di Tivoli: caserma dei Vigili, Hospitalia, Triclinio Imperiale, novità e indirizzi di analisi, in: Adventus Hadriani, Investigaciones sobre Arquitectura Adrianea. R. Hidalgo Prieto, G. E. Cinque, A. Viscogliosi, A. Pizzo (editors). L'Erma di Bretschneider, Roma, 2020, ISBN 9788891320131, pp. 607-632. [In Italian]

[36] F. I. Apollonio, F. Fallavollita, R. Foschi, The critical digital model for the study of unbuilt architecture, in: Research and Education in Urban History in the Age of Digital Libraries. UHDL 2019. Communications in Computer and Information Science. F. Niebling, S. Münster, H. Messemer (editors). Springer, Cham, 2021, 1501, pp. 3-24. DOI: 10.1007/978-3-030-93186-5_1

[37] I. Bassoli, F. Fallavollita, W. Fuchs, Hypothesis of reconstruction of the Roman theater of Urbs Salvia, SCIRES-it 12(1) (2022), pp. 151-164. DOI: 10.2423/i22394303v12n1p151

[38] F. Gabellone, Digital Twin: a new perspective for cultural heritage management and fruition, Acta IMEKO 11(1) (2022), pp. 1-7. DOI: 10.21014/acta_imeko.v11i1.1085