Acoustic determination of *g* by tracking a free-falling body using a smartphone as a "sonar"

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Abstract

The gravitational constant is determined by tracking the movement of a free-falling body in an aluminium pipe. For this purpose, a smartphone is used to generate sound waves of a specific frequency and to simultaneously detect the sound wave resonances in the tube. The ability of smartphones to generate and receive the sound waves to track a moving body, like a "submarine SONAR" does, is an essential point in this work. Concepts of kinematics and acoustics are combined to determine the gravitational acceleration within 1% when the result is compared with the reported value.

Keywords: smartphone, gravitational acceleration, Acoustics.

1. Introduction

The use of smartphone sensors in introductory and first-year university physics courses have become very popular in the last decade [1-3]. A wide range of topics within general physics courses has been covered by the published work, such as linear and circular motions, oscillations [2-4], beats [5,6], optics [7,8], acoustics [9-13], among others. The calculation of the gravitational constant using different method has always been of broad interest in teaching [14-16]. Even so, this topic continues to call the attention of Physics instructors as new interesting and creative works on smartphone sensors are still coming up. In particular, the microphone and the speakers of the smartphone appear in many works to characterize acoustic phenomena, e.g. for the determination of the speed of sound [11-13], the study of acoustic beats [5], or the Doppler effect [9].

In this work, we will use sound wave resonance in an air column to track the free-falling of a body inside an aluminium pipe. The smartphone is used with a double purpose, that is, to generate sound waves of a given frequency into the inside of the pipe and to listen the result of the sound wave resonance. From the information collected in the recorded sound file, the movement of the body is reconstructed, and the gravitational constant determined by means of a non-linear fitting.

2. Experimental setup and physics model

Figure 1 shows the experimental setup used for the experiment. On the right-hand side of the figure the mobile phone, the body and the aluminium pipe are shown. The body diameter closely fits the inner diameter of the pipe but such that a fine air layer still exists between the pipe and the body. The length of the pipe is 1.524 m and its inner diameter 0.0210 m. The diameter of the body is 0.0197 m and its height 0.058 m. A schematic representation of the pipe with a standing wave produced as the body falls is represented on the right-hand side of the figure.

The smartphone is used here with a double purpose, that is, to generate the single-frequency sine sound wave and to record the standing sound wave produced between the open end and a closed end, consisting of the upper end of the moving body through the pipe. For the single-tone sound wave generation, the free Android app. Physics Toolbox Suit [17] has been used (shown on the screen of the mobile in figure 1). When the body falls in the vertical pipe, sound amplitude variations are produced. These variations are registered by recording the sound with the Voice Recorder app., commonly incorporated in the smartphones bearing Android.

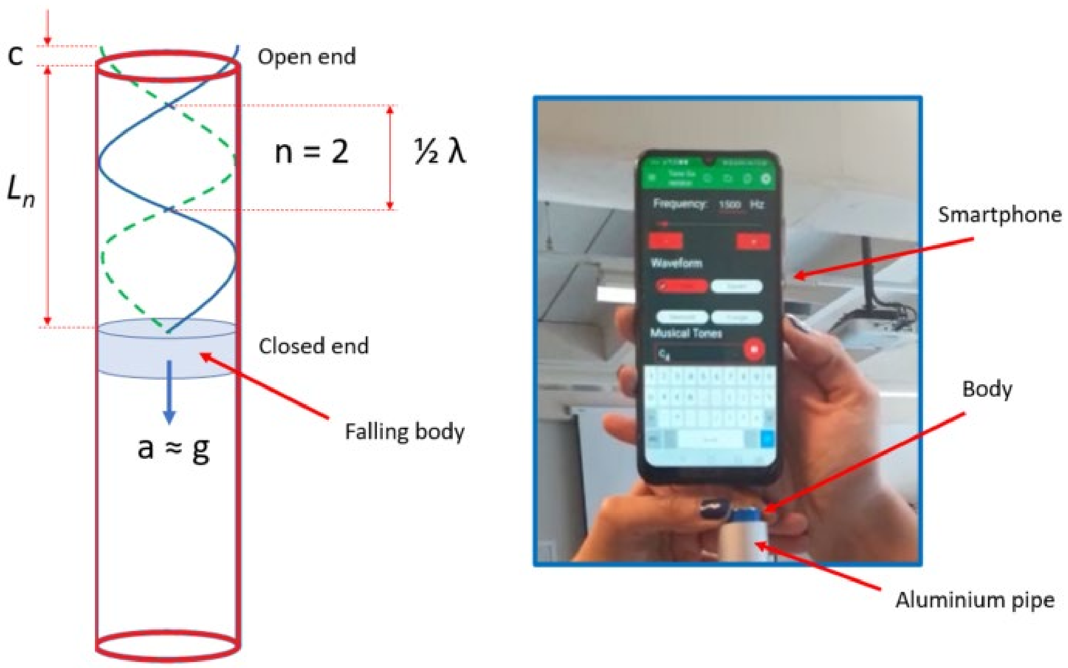

Figure 1. Schematic representation of the aluminium pipe, the body, and the standing wave on the left-hand side and a photo of the experimental setup on the right-hand side.

The described system can be modelled by an open-end air column for which resonances take place according to the following expression:

$$L_n + c = \frac{\lambda_n(2n+1)}{4};\ \text{with}\ n = 0, 1, 2, 3, \cdots \tag{1}$$

Every time the falling body reaches a resonant length, L_n, that is, a length that equals an odd multiple of $\frac{\lambda_n}{4}$, a resonance is achieved (i.e. a standing wave is produced and the sound intensity reaches a maximum). Here c is an end correction and λ is the wavelength of the sound wave.

3. Results and discussion

The resonances are detected in our experiment by the microphone and registered with the Voice Recorder application. For the best results it is recommended that the room remains silent during the experiments. Figure 2 shows the amplitude of the sound wave as plotted *versus* time. This information is contained in the recorded ".wav" file. It can be noticed that the resonances are well-defined along the curve.

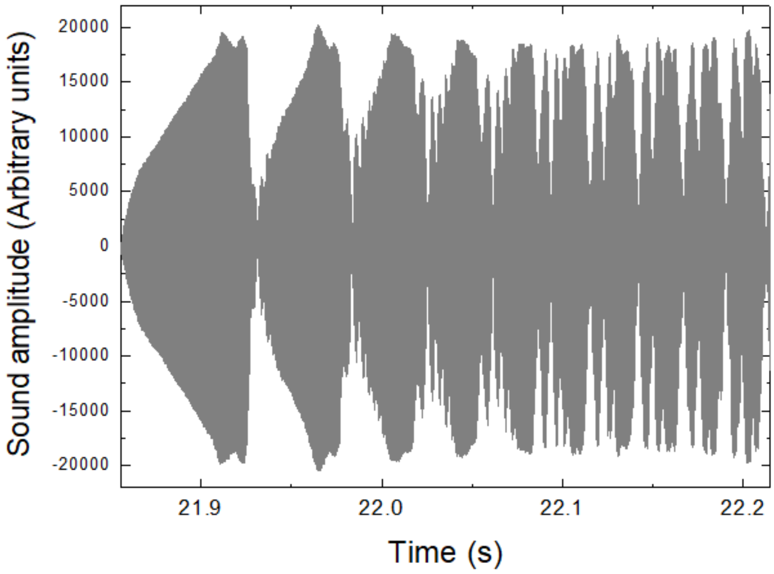

Figure 2. Audio recorded with the smartphone as the body was falling inside the pipe. The graph represents the amplitude of the sound versus time.

Taking $n = 0$ as the reference position, the following expression is obtained,

$$L_n - L_0 = \frac{\lambda(2n+1)}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}n \quad (2)$$

The falling body in the aluminium pipe follows a uniformly accelerated linear motion. Thus, the following expression $L_n - L_0 = v_0(t_n - t_0) + g(t_n - t_0)^2/2 = v_0\Delta t_n + g(\Delta t_n)^2/2$ can be substituted in Eq. (2) to obtain,

$$\frac{\lambda}{2}n = v_0\Delta t_n + g(\Delta t_n)^2/2 \quad (3)$$

Taking n apart in Eq. (3),

$$n = \frac{2v_0}{\lambda}\Delta t_n + \frac{g}{\lambda}(\Delta t_n)^2 \,, \quad (4)$$

where g is the gravitational acceleration.

Eq. (4) represents a second order polynomial of n *versus* Δt_n which can be expressed as follows,

$$n = a + b\Delta t_n + c(\Delta t_n)^2 \,, \quad (5)$$

where $a = 0$, $b = \frac{2v_0}{\lambda}$ and $c = \frac{g}{\lambda}$ are constant and can be obtained from a non-linear fitting to the experimental data.

Table I. The first column shows the consecutive values of n in Eq. 1, the second the corresponding time elapsed and, the third, the time elapsed relative to the time for $n = 0$ taken as reference.

n	t_n (s)	$\Delta t_n = (t_n - t_0)$ (s)
0	21.8567	0.0000
1	21.9315	0.0748
2	21.9837	0.1270
3	22.0254	0.1687
4	22.0615	0.2048
5	22.0937	0.2370
6	22.1179	0.2612

The values of t_n and $\Delta t_n = (t_n - t_0)$ are included in Table I as a function of the consecutive values of n in Eq. 1. Figure 3 shows an Excel plot of the time elapsed from the moment a node taken as reference ($n = 0$) is produced to the moments when each of the of the consecutive nodes are produced (Δt_n) *versus* $n = 1, 2, 3, 4, \cdots$. The last 6 nodes out of 16 nodes originated in the pipe have been use. The parameters resulting from the non-linear fitting of equation 5 to the experimental data are $a = 0.020$, $b = 8.640\ s^{-1}$ and $c = 53.495\ s^{-2}$. The coefficient of determination of $R^2 = 0.9995$ indicates the good quality of the fitting. The fitting has been carried out using the "Add Trendline" option implemented in Excel.

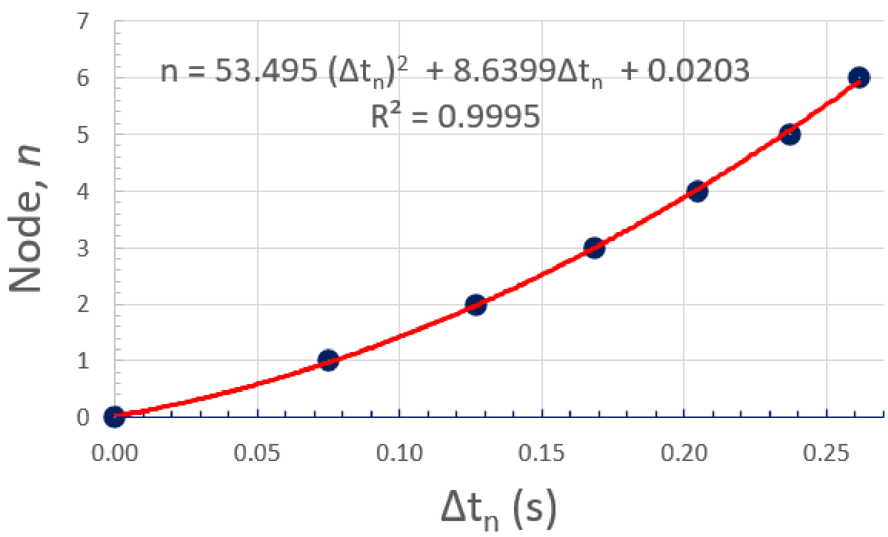

Figure 3. Time elapsed from a node taken as reference to each of the of the consecutive nodes. The blue dots represent the experimental data and the red line the non-linear fit.

From the parameter $c = \frac{g}{\lambda}$ and substituting $\lambda = v/f$, where v is the speed of sound in air and f its frequency, the gravitational acceleration can be determined as follows,

$$g = cv/f \tag{6}$$

The frequency chosen for this experiment was $f = 1900$ Hz. The speed of sound at the temperature of the experiments (24 °C) was 345.48 m/s [18]. The gravitational acceleration, calculated according to Eq. (6) is $g = 9.7\ \mathrm{m/s^2}$. The comparison with the reported standard value for the gravitational constant [19], $9.801\ \mathrm{m/s^2}$ yields a percentage difference lower than 1% which is a good indicator of the quality of the experiment.

4. Conclusions

This paper shows a simple experimental setup to determine the gravitational constant by combining concepts of kinematics and acoustics. Our value $g = 9.7\ \mathrm{m/s^2}$ underestimates the reported value as neither the air resistance nor the correction to the speed of sound inside the pipe were considered. The air resistance depends on the size of the gap between the falling body and the inner side of the tube, and the speed of sound is known to change when the sound wave becomes a guided wave in a tube. Another influencing factor is the bandwidth of the frequency as generated by the device. The experiments also show that the microphone, and the speaker can be effectively used simultaneously to describe an acoustic phenomenon.

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