25th

MATHEMATICAL MODELLING IN ENGINEERING & HUMAN BEHAVIOUR

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This event is partially financed by Grant CIAORG/2022/23.

UNIVERSITAT POLITÈCNICA DE VALÈNCIA

Modelling for Engineering & Human Behaviour 2023

València, July 11th-14th, 2023

This book includes the extended abstracts of papers presented at XXV Edition of the Mathematical Modelling Conference Series at the Institute for Multidisciplinary Mathematics *Mathematical Modelling in Engineering & Human Behaviour 2023*.

ISBN: 978-84-09-57681-4
Edited by:
I.U. de Matemàtica Multidisciplinar, Universitat Politècnica de València.
J.R. Torregrosa, J-C. Cortés and A. Vidal-Ferràndiz.

Cover Page Image by Jaume Hervás Raga, http://www.hagamosalgointeligente.com

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Preface

This book includes the extended abstracts presented at XXV Edition of the Mathematical Modelling Conference Series at the Institute for Multidisciplinary Mathematics Mathematical Modelling in Engineering & Human Behaviour.

December 2023

Juan Ramón Torregrosa
MME&HB 2023
Universitat Politècnica de València

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Competitiveness of Formula 1 championship from 2012 to 2022 as measured by Kendall corrected evolutive coefficient

Francisco Pedroche[1]

Institut Universitari de Matemàtica Multidisciplinària,
Universitat Politècnica de València
Camí de Vera s/n, 46022, València, Spain
pedroche@imm.upv.es,
WWW home page: http://personales.upv.es/pedroche/

Abstract. In this paper we analyze the FIA formula one world championships from 2012 to 2022 taking into account the drivers classifications and the constructors (*teams*) classifications of each Grand Prix. The needed data consisted of 22 matrices of sizes ranging from 25×20 to 10×19 that have been elaborated from the GP classifications extracted from the official FIA site. We have used the Kendall corrected evolutive coefficient, recently introduced, as a measure of Competitive Balance (CB) to study the evolution of the competitiveness along the years in both drivers and teams championships. In addition, we have compared the CB of F1 championships and two major European football leagues from the seasons 2012-2013 to 2022-2023.

Keywords: Kendall's tau, Formula One, Football, Competitive balance, sport rankings, contest

1 Introduction

A *ranking* naturally appears when we sort elements, being this a key action in more activities such as analysis of sport competitions [2], economic time series [14], comparison of algorithms performance [25], etc. Series of rankings can be studied from different perspectives. For example, to analyse sorting algorithms [15], to define measures of disarray [7], to use rank transformation to develop nonparametric methods in Statistics [5], to *learn to rank* in machine learning applications [4], etc. In this paper we are interesting in characterising a series of rankings by giving a coefficient that measures the disarray along the series in the classic manner of [11]. Specifically, we follow the definitions of [21], [20] and [6].

2 Kendall corrected evolutive coefficient

The Kendall corrected evolutive coefficient, denoted by $\widehat{\tau}^{\bullet}_{ev}$, was introduced in [21]. It takes as input a series of m rankings (with at most n elements) that can be *complete* (that is, the n elements are ranked in all the rankings) or incomplete. In addition, we consider the existence or not of ties between the ranked elements. Kendall corrected evolutive coefficient can be considered as an extension of a correlation coefficient of two rankings applied to m rankings and therefore, as output, $\widehat{\tau}^{\bullet}_{ev}$ gives a real number in $[-1, 1]$.

The coefficient $\widehat{\tau}^{\bullet}_{ev}$ reduces to some particular coefficients that are well documented and can be found in the literature. For example, when $m = 2$ and the rankings are complete and with no ties, then $\widehat{\tau}^{\bullet}_{ev}$ reduces to the classical Kendall's τ coefficient of disagreement (see [11], [12], [13]) that can be written as

$$\tau = \frac{2(P - Q)}{n(n-1)} \tag{1}$$

where P is the number of pair of elements that keep its relative order from the first ranking to the second one and Q is the number of pairs of elements that change its order. For example, taking $n = 3$, the rankings $\mathbf{a} = [1, 2, 3]$ and $\mathbf{b} = [3, 2, 1]$ have an associated $\tau = -1$ and the rankings $\mathbf{a} = [1, 2, 3]$ and $\mathbf{b} = [1, 2, 3]$ have an associated $\tau = 1$. When $m = 2$ and the rankings are complete and with ties, then $\widehat{\tau}^{\bullet}_{ev}$ is related to the *Kendall distance with penalty parameter* $p \in [0, \frac{1}{2}]$ defined in [8]. When $m > 2$ and the rankings are complete and with ties, then $\widehat{\tau}^{\bullet}_{ev}$ reduces the *corrected evolutive Kendall distance with penalty parameter* p introduced in [20].

In sport competitions it is most used the term *Competitive Balance* (CB) to measure the balance between the teams [27], [19]. A high measure of CB means that the competition is highly interesting since it is very difficult to predict the result of a match (or a race, in our case), while a low measure of CB means that the competition is very predictable, and therefore *boring* (see. [18], [17], [9], [10], [2], [3]). In this regard it is more convenient to use the measure called *Normalized Strength* (borrowed from complex networks terminology, see [6], [1]), and that we define here by

$$NS = \frac{1 - \widehat{\tau}^{\bullet}_{ev}}{2} \tag{2}$$

Note that NS is a normalized index, $NS \in [0, 1]$, and its value can be considered as a measure of CB. We will use this index in our analysis. The interested reader may find the precise definition of $\widehat{\tau}^{\bullet}_{ev}$ in [21] but we omit the details for the sake of brevity.

3 Formula One World Championships

Formula One (also known as Formula 1 or F1) organised by the Fédération Internationale de l'Automobile (FIA) is a well-known international racing for cars [23]. The drivers championship began in the season of 1950, while the constructors championship began in 1958. Along the years, there has been some modifications both in the format and in the rules that the participants must accomplish.

A Formula One season consists of a series of races, each of them known as Grand Prix (denoted as GP), that take place in several countries. For example, the F1 2022 season consisted of 22 GP and participated 10 teams and 22 drivers. A GP is held on a weekend. On friday and saturday some qualifying sessions fix the starting order (*the grid*) for the GP race that occurs on Sunday. In this paper we are interested only in the ranking corresponding to this GP races. This ranking is decided based on the timing of each driver and he receives a quantity of points depending on his ranking. From 2010 to 2018 the sharing of the points was given as shown in Table 1. The points assigned to the constructors in a GP is the sum of the points of the two drivers of the team that participated in that GP.

Table 1: Points scoring sharing since 2010

1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
25	18	15	12	10	8	6	4	2	1

From 2019 one additional point is given to the pilot that occupied a position in the top ten and furthermore has the fastest lap in the race. FIA has some rules to break ties between the pilots and therefore the ranking of the drivers can be considered as ranking with no ties. Note, therefore that each GP has its own classification. The final ranking (that is, the F1 Championship) of the season is made by accumulating the points of each GP, and, again, some rules are applied to break the ties, if any. Our collection of rankings are precisely the rankings of each GP in a season, both for drivers and constructors. We use these series of rankings to compute the corresponding $\widehat{\tau}_{ev}^{\bullet}$ of that season, and then the corresponding NS. We precisely describe the used rankings in the next section.

4 Description of the rankings

We have selected the F1 classifications from 2012 to 2022. Our criterium to select our dataset is based on taking the GP classifications of championships in where 1) the regulations does not vary too much, 2) the distribution of points (e.g. as given by Table 1) is quite stable, 3) the number of GP does not vary too much and 4) that the standings can be easily retrieved from the official FIA site [23]. For example, the 2012 season can be retrieved from the FIA site [24]. In Table 2 we show the number of drivers in each championship jointly with the number of GP in that year.

To describe our rankings we use the following notation (see [26], [21]). Let $V = \{v_1, v_2, \cdots, v_n\}$ be the objects to be ranked, with $n > 1$. The ranking is

Table 2: Number of drivers, teams and GP in each analyzed F1 Championship

Year	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
Drivers	25	23	24	22	24	25	20	20	23	21	22
Teams	12	11	11	10	11	10	11	10	10	10	10
GP's	20	19	19	19	21	20	21	21	17	22	22

given by

$$\mathbf{a} = [a_1, a_2, \cdots, a_n] \tag{3}$$

where a_i is the position of v_i in the ranking. Note that if $a_i = a_j$, then v_i and v_j are tied. If v_i is not ranked, then it is denoted as $a_i = \bullet$.

4.1 Drivers ranking

From the FIA site, we can retrieve the drivers classification for each GP of the considered championship. In these classifications we can see the ranking, the points obtained by each driver, and a note indicating whether the driver has finished the race or not. To construct our drivers ranking we consider that a driver that has not finished the race (or has not even start it) is an absent element in our ranking, and therefore it is indicated by $\bullet$. For example, in Table 3 we show our notation to describe the first three rankings of the 2012 championship.

4.2 Constructors ranking

From the FIA site we can retrieve the constructors classification for each GP of the considered championship. The points given to a constructor consist of the sum of the points of the two drivers of the corresponding team in each GP. In this case the FIA site offers the points obtained by each constructor. This gives us the opportunity to create two types of rankings, being the interest to see how our measure NS is affected by these types. The two considered methods are the following:

Method 1: We consider that the constructors that have 0 points are tied in the last position.

Method 2: We consider that the constructors that have 0 points are absent elements.

As an example, in Table 4 we show the constructors name, scoring and $\mathbf{a}_i$ vectors (by using Method 1 and Method 2) for the first three GP of FIA 2012 World Championship.

Table 3: Drivers' name, nationality, and $\mathbf{a}_i$ vector for three of the first GP of FIA 2012 World Championship. Elaborated from [23]. Note that • means that the driver did not start or did not finish the race. The rankings are *incomplete rankings with no ties*. The order of the drivers in the first column follows the (final) classification of the constructors championship. The drivers Raikkonen, Grosjean and D'Ambrosio belong to the same team (Lotus F1) while the rest of teams contributed with two drivers in the whole GP rankings of this championship.

Driver	Nat	GP1	GP2	GP3
Sebastien Vettel	DEU	2	11	5
Fernando Alonso	ESP	5	1	9
Kimi Raikkonen	FIN	7	5	14
Lewis Hamilton	GBR	3	3	3
Jenson Button	GBR	1	14	2
Mark Webber	AUS	4	4	4
Felipe Massa	BRA	•	15	13
Romain Grosjean	FRA	•	•	6
Nico Rosberg	DEU	12	13	1
Sergio Perez	MEX	8	2	11
Nico Hulkenberg	DEU	•	9	15
Kamui Kobayashi	JPN	6	•	10
Michael Schumacher	DEU	•	10	•
Paul Di Resta	GBR	10	7	12
Pastor Maldonado	VEN	13	19	8
Bruno Senna	BRA	16	6	7
Jean-Eric Vergne	FRA	11	8	16
Daniel Ricciardo	AUS	9	12	17
Vitaly Petrov	RUS	•	16	18
Timo Glock	DEU	14	17	19
Charles Pic	FRA	15	20	20
Heikki Kovalainen	FIN	•	18	23
Jérôme D'Ambrosio	BEL	•	•	•
Narain Karthikeyan	IND	•	22	22
Pedro De la Rosa	ESP	•	21	21

5 Results

5.1 Comparison of constructors and drivers championships

In order to compare the *competitivity balance* of the GP of drivers and constructors we have computed NS, given by (2) for the GP standings from 2012 to 2022 for drivers and for constructors (with Method 1 and Method 2). The results are shown in Table 5.

Table 4: Constructor's name, scoring and $\mathbf{a}_i$ vectors (by using Method 1 and Method 2) for the first three GP of FIA 2012 World Championship. The order of the teams in the first column follows the (final) classification of the championship.

	Score			Method 1			Method 2		
Constructors	GP1	GP2	GP3	GP1	GP2	GP3	GP1	GP2	GP3
Red Bull Racing	30	12	22	2	4	3	2	4	3
Scuderia Ferrari	10	25	2	4	1	6	4	1	6
Vodafone McLaren Mercedes	40	15	33	1	3	1	1	3	1
Lotus F1 Team	6	10	8	5	5	5	5	5	5
Mercedes AMG Petronas F1 Team	0	1	25	8	8	2	•	8	2
Sauber F1 Team	12	18	1	3	2	7	3	2	7
Sahara Force India F1 Team	1	8	0	7	6	8	7	6	•
Williams F1 Team	0	8	10	8	6	4	•	6	4
Scuderia Toro Rosso	2	4	0	6	7	8	6	7	•
Caterham F1 Team	0	0	0	8	8	8	•	•	•
Marussia F1 Team	0	0	0	8	8	8	•	•	•
HRT F1 Team	0	0	0	8	8	8	•	•	•

The data on Table 5 can be resumed on the box-and-whiskers plot shown on Figure 1. In more detail, the mean values of NS on the period 2012-2022, and the corresponding sample standard deviation, s, are as follows:
Mean value of NS for drivers: 0.2203, ($s = 0.018$).
Mean value of NS for constructors (Method 1): 0.2394, ($s = 0.035$).
Mean value of NS for constructors (Method 2): 0.2771, ($s = 0.070$).

Let us consider that NS is a random variable. By computing the Shapiro-Wilk test for normality [22] we obtain the p-values 0.61, 0.08 and 0.44 for the corresponding NS series for drivers, and constructors (Method 1 and Method 2) respectively. Therefore we cannot reject the normality of the distribution of NS of the corresponding samples. Regarding the mean values of NS for constructors by using Method 1 and Method 2, since they come from the same data (as an example, the scores in Table 4 give us the corresponding values for Method 1 and Method 2) we can use a comparison method for means coming from paired data. By using a t-test we obtain a p-value of 0.18 and therefore we cannot reject that the means are equal with a confidence interval of 95%. Since the value of the variances does not have a ratio major than 4 we can use the t-test for comparing the mean of NS by using Method 1, and the corresponding NS for drivers. We obtain that the p-value is 0.12 and therefore we cannot reject the null hypothesis that the means are equal. All in all we have the statistically the three values of NS are not different, with a confidence interval of 95%.

Table 5: NS for the series of GP of the Championships from 2012 to 2022 for drivers and constructors.

Year	NS Drivers	NS Constructors Method 1	NS Constructors Method 2
2012	0.2561	0.2456	0.4052
2013	0.2136	0.1924	0.3421
2014	0.1913	0.1616	0.3106
2015	0.2270	0.2722	0.2350
2016	0.2065	0.2218	0.2143
2017	0.2140	0.2632	0.2179
2018	0.2188	0.2559	0.1886
2019	0.2157	0.2772	0.2596
2020	0.2436	0.2562	0.3652
2021	0.2270	0.2413	0.2715
2022	0.2092	0.2455	0.2376

5.2 Comparison of competitiveness between F1 championships and two major European football leagues

A competitive balance measure like NS, based on sport ranking series, can be used to compare the CB of two different sports. For example, by computing the coefficient NS for two major European football leagues (Spanish League - commercially known as Laliga Santander in the season 2022/23-, and the English Premier league) we obtain the results shown in Table 6. We have used the series of standings from the season 2012-2013 to the season 2022-2023 for both the Spanish League (retrieving the data from the links on [28]) and Premier League (retrieving the data from [29]). The summary for the football leagues in the studied period is the following:

Mean value of NS for Spanish league: 0.059, ($s = 0.0094$).

Mean value of NS for Premier league: 0.056, ($s = 0.0062$).

As a consequence, by using the results on section 5.1 for NS of drivers and NS of constructors by using Method 1, we obtain that the mean value of NS for the F1 championships is about four times greater than the values of NS corresponding to the analyzed football leagues.

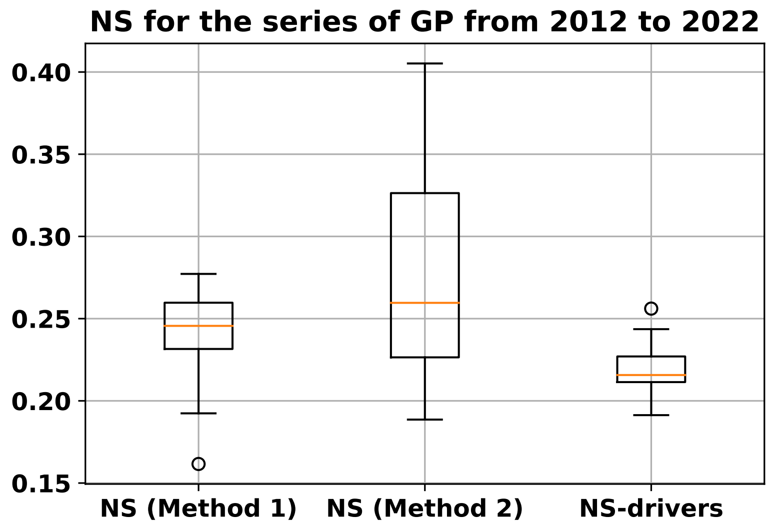

Fig. 1: Box-and-whiskers diagram for NS for the series of GP of the championships from 2012 to 2022 for drivers and constructors (by using the two methods explained on the text).

6 Conclusions

In this communication we have shown how to apply a recently introduced metric to calculate a measure of the competitive balance (CB) associated to Formula 1 championships, by taking into account the standings of the Grand Prix that compose each championship. We have introduced to methods (called Method 1 and Method 2) to compute the CB values of the F1 Constructors Championship in the period 2012-2022. We have obtained that these two methods do not offer mean values that can be considered statistically different. We think that this shows a good behaviour of our metric since both Method 1 and Method 2 are obtained by computing a linear combination from the same set of data (the F1 Drivers Championship) but with different treatment of the constructors that finish with zero points in a Grand Prix. We also have obtained that the CB of the F1 Drivers Championship and F1 Constructors Championship show similar values on the studied period, but with a slightly higher mean value for the Constructors Championship. As an example of the power of our metric, we have compared the CB of two different sports: the Formula 1 championships from 2012 to 2022 and the Spanish football league and Premier football league on the seasons 2012-2013 to 2022-2023. Our results show that the mean value of CB for the F1 championships is about four times greater than the values of CB corresponding to the analyzed football leagues.

Table 6: NS values for two European football leagues from season 2012/2013 to season 2022/2023.

Year	NS Spanish league	NS Premier league
2012	0.0615	0.0514
2013	0.0593	0.0656
2014	0.0546	0.0597
2015	0.0613	0.0563
2016	0.0435	0.0589
2017	0.0589	0.0550
2018	0.0688	0.0489
2019	0.0600	0.0643
2020	0.0757	0.0583
2021	0.0440	0.0461
2022	0.0595	0.0515

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