## Contents

1 Introduction
   1.1 $C_0$-semigroups ................................................. 1
      1.1.1 Linear dynamics for operators and $C_0$-semigroups ........ 3
   1.2 Criteria for hypercyclicity and Devaney Chaos .................. 4
      1.2.1 Dynamics of autonomous discretizations .................... 5
      1.2.2 Spectral conditions ....................................... 6
   1.3 Operators ...................................................... 8

2 Dynamic behaviour of a $C_0$-semigroup .......................... 11
   2.1 Definitions ..................................................... 11
   2.2 Distributionally chaotic dynamics of autonomous discretizations ............................................ 13

3 Sufficient conditions for distributional chaos .................... 31

4 Distributional chaos for translation $C_0$-semigroups ............ 39
   4.1 Existing results on the dynamics of translation $C_0$-semigroups .... 40
   4.2 Distributional chaos for translation $C_0$-semigroups .......... 41
   4.3 Backward shifts and translation $C_0$-semigroups .............. 46

5 Completely distributionally chaotic $C_0$-semigroups ............. 51

6 Solution semigroups of partial differential equations ............ 61
   6.1 Distributionally chaotic $C_0$-semigroups ....................... 61
   6.2 The Desch-Schappacher-Web Criterion implies distributional chaos .... 65
   6.3 Distributional chaos for birth-and-death processes with proliferations .... 68
   6.4 Further research lines ....................................... 71