Abstract

The analysis and vibration control is particularly important in many branches of engineering, especially mechanical, civil, aeronautics and automotive. This is so up to the point that almost this analysis is identified as a separate area within the dynamic analysis of structures. Since the beginning, dissipative or damping forces have been one of the most difficult to be represented. The viscous model, due to its simplicity and versatility, has been and still is the great paradigm of damping models. However, due to the introduction of memory materials in practical applications, it was necessary to improve the model to consider the viscoelasticity phenomenon; the viscoelasticity, although closely related to the speed of response, required the introduction of the so-called hereditary functions. These functions allow us to express the dissipative forces not only as functions of the instantaneous velocity, but also of the velocity history, from the beginning of the movement —hence the term *memory*—. Naturally, such theoretical advance into the model is also accompanied by computational disadvantages. In fact, before the response was obtained as solution of a system of linear differential equations and now a a system of integro-differential equations must be solved.

Analysis of the free vibrations of the viscoelastic damping systems leads to a nonlinear eigenvalue problem, the main feature of which is a frequency-dependent damping matrix. The study of the solution of eigenvalues and eigenvectors of this problem becomes of special importance in order to know the structural modes of vibration or if the frequency-domain response is sought. The eigenvalue set is formed by complex-conjugate pairs and a real eigenvalues, the number of which depends on the mathematical form of the damping function.

The main objectives of this Thesis are two: first, to deepen the knowledge of the eigenvalue problem of viscoelastic systems through the proposal of new numerical methods. Second, to develop a new viscous model that, under certain conditions, can reproduce the response of the viscoelastic model accurately enough.

The document is divided into eight chapters, among them, the main body is constituted by Chapters 2 to 7. All of them are research articles that have either

been published or are under review in sources included in the Journal Citation Reports (JCR). For this reason, all chapters retain the intrinsic structure of an article, including introduction and bibliography.

The first four chapters (2 to 5) are focused on the study of the aforementioned non-linear eigenvalue problem with two numerical methodologies proposed. The first is an iterative procedure based on the fixed-point scheme, specially developed for proportional or lightly non-proportional systems (those with decoupled or almost-uncoupled modes). The second methodology, presented in two separate chapters, is called parametric since the fundamentals are based on the perturbation of certain damping parameter and allows us to construct almost-analytical solutions of the eigenvalues, both for single and multiple degree-of-freedom systems. The study of the eigenvalue problem is completed by a chapter on real eigenvalues, also called non-viscous eigenvalues. To this end, a new mathematical characterization of these eigenvalues is presented and a new concept is introduced: the non-viscous set.

In the last two chapters (6 and 7) an equivalent viscous model is developed. The approach has been proposed as an approximated substitute of the response of viscoelastic systems. The analysis is performed with the study of the transfer function in the frequency domain. In a first stage (Chap 6), the related mathematical nature is analyzed. It is shown that the exact transfer function of a viscoelastic model can be expanded as a sum of a transfer function, characteristic of certain viscous model, plus a residual term. The latter is directly dependent on the induced damping level and on the modal coupling (non-proportionality of the damping matrix). In a second stage (Chap. 7), an real structural application formed by plane frame structures with free damping layers of viscoelastic material is developed. In this chapter the equivalent viscous model and an improved parametric method to compute the eigenvalues are applied, achieving the conjunction of the two objectives of the present Thesis.