DUALITY THEORY FOR P-TH POWER FACTORABLE OPERATORS AND KERNEL OPERATORS

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ABSTRACT

The present work is devoted to the analysis of a particular class of (linear and continuous) operators between Banach function spaces. The aim is to advance in the theory of the so-called p-th power factorable operators by analyzing all the aspects of the duality. This class of operators has proved to be useful both in the factorization theory of operators on Banach function spaces (Maurey-Rosenthal theory) and in Harmonic Analysis (optimal domains for the Fourier transform and convolution operators). In order to develop the corresponding duality theory and some applications, a new class of operators with extension properties involving both the operator and its adjoint is defined and studied. This is the family of the (p,q)-th power factorable operators, for $1 \leq p,q < \infty$, that can be characterized by means of a canonical factorization scheme through the p-th power space of the domain space and the dual of the q-th power space of the dual of the codomain space. An equivalent diagram factoring such an operator through $L^p(m)$ and $L^q(n)'$ for suitable vector measures m and n is also obtained, and this becomes the main tool (Chapter 3 and Chapter 4). Some other preliminary results concerning p-th powers of Banach function spaces are also necessary for constructing the above mentioned ones (Chapter 2).

Using these tools, some results characterizing the optimal range —the smallest Banach function space in which the operator can take values— for operators from a Banach space into a Banach function space are given (Chapter 3). Also, the idea of optimal factorization of an operator optimizing a previous one is developed and formally presented in terms of the diagram that a (p,q)-th power factorable operator must satisfy (Chapter 4). All these results extend the nowadays well known computation of the optimal domain for operators on Banach function spaces by means of vector measures. These computations have provided relevant results in several fields of the mathematical analysis by means of a description of the biggest Banach function spaces to which some special relevant operators —for instance, the Fourier transform and the Hardy operator— can be extended.

The theory is applied for finding new results in some concrete fields: as interpolation theory for operators between Banach function spaces, kernel operators (Chapter 5) and in particular, to the Laplace transform (Chapter 6).