Summary

A great variety of phenomena in science and engineering are modelled by using ordinary matrix differential equations of first order. The fundamental objective of this Ph.D. dissertation is the development of novel methods for the approximate solution of these equations, including equations of the linear type, differential matrix Sylvester equations, and differential matrix Riccati equations–all of them with variable coefficients. Matrix differential equations pose a higher level of difficulty for their resolution, compared to the conventional scalar equations due, to the obvious intricacy of the matrix structure: the dimensional increase of the problem, which comes hand in hand with an elevated number of necessary operations to obtain the solution, the absence of commutativity and other salient properties of the usual scalar case.

One of the proposed methods employs approximations based on cubic matrix splines. These methods have been implemented in the form of packages in the scientific programming language MATLAB. A selection of this code is made available in the appendix of this dissertation. The aim in mind was to present algorithms readily exportable to other programming environments with great ease of use.

The study continues with an extension of the method for ordinary matrix differential equations of second order, but by avoiding the traditional approach, which consists of the transformation of the problem to a system of first order, since this would increase the dimension of the new problem and come with additional computational overhead.

Regarding the usage of splines of higher than third order, we have obtained good numerical approximations, for both, first- and second-order problems.

The results comprised in the present dissertation have been published in several scientific journals of high impact. Furthermore, they have been communicated at various ECMI conferences (European Conference on Mathematics for Industry) to accomplish the highest possible degree of dissemination.

The topical classification of this thesis, following the standard *Mathematics Subject Classification* (MCS 2010) of the *American Mathematical Society* (AMS) is given by: **41A15**, **65D07**, **65F30**, **65L05**.