Contents

Acknowledgements **I**

Abstract **III**

Resumen **V**

Resum **VII**

Index **VIII**

List of algorithms **XIII**

List of tables **XV**

List of figures **XVII**

1 Introduction **1**
1.1 Constraint Programming 1
1.2 Dynamism and Uncertainty 3
1.3 Reactive Approaches 5
1.4 Proactive Approaches 7
1.4.1 Stability, Flexibility and Robustness 7
1.4.2 Searching for Robust Solutions 9
1.4.3 Searching for Stable Solutions 11
1.5 Dynamism Knowledge 12
1.6 Contributions and Outline 14

2 Technical Background **19**
2.1 Constraint Satisfaction Problems (CSPs) 19
2.1.1 Domains 20
2.1.2 Assignments 21
2.1.3 Constraints 22
2.1.4 Solutions 24
2.1.5 Spaces associated with CSPs 24
2.2 Topological and Metric Spaces 25

2.3 Dynamic Constraint Satisfaction Problems 29
2.4 Constraint Satisfaction and Optimization Problems 29
2.4.1 Flexible CSPs and Valued CSPs 30
2.4.2 Weighted CSPs 31
2.5 Inference 33
2.5.1 Levels of Consistency 33
2.5.2 Generalized Arc Consistency 35
2.6 Hybrid Methods and Efficiency Improvements 36
2.6.1 Maintaining Arc Consistency 37
2.6.2 Variable Ordering Heuristics 38
2.6.3 Restarting Techniques 39
2.6.4 Branch & Bound 40

3 New Dynamism Framework for CSPs with Ordered Domains 43
3.1 Dynamism in CSPs with Ordered Domains 43
3.2 Related Dynamism Assumptions in the Literature 46
3.3 Extending Robustness and Stability Concepts 48
3.3.1 Robustness 48
3.3.2 Stability 49
3.4 New Theoretical Approaches for this Environment 51
3.4.1 The Centroid of the Solution Spaces of CSPs 51
3.4.2 Neighbourhood and 'Onion topology' 54
3.5 New Techniques for this Environment 59
3.6 Summary and Limitations 61

4 Probabilistic Enumeration-based Technique 63
4.1 Additional Information about the Dynamism 64
4.2 New Constraints Generation 65
4.2.1 Properties of the New Generated Constraints 67
4.3 Cost Assignment 68
4.4 Algorithm 70
4.4.1 Algorithm Description 70
4.4.2 Computational Complexity 72
4.5 Example 72
4.6 Summary and Limitations 75

5 Enumeration-based Technique for Finite Linear CSPs 77
5.1 Dynamism over Linear CSP with Ordered Domains 78
5.2 Distances Computation and Cost Assignment 80
5.3 Algorithm 81
5.3.1 Algorithm Description 81
5.3.2 Computational Complexity 82
5.4 Example 83
5.5 Summary and Limitations 86

6 Enumeration-based Technique by Coverings **89**
6.1 The Concept of Coverings 90
6.2 Algorithm for Calculating Coverings 93
6.2.1 Algorithm Description 94
6.2.2 Computational Complexity 97
6.3 Modeling CSPs as WCSPs by Coverings 98
6.3.1 Cost Assignment 98
6.3.2 Modeling Algorithm 99
6.4 Examples 100
6.4.1 2-dimensional Example 100
6.4.2 3-dimensional Example 103
6.5 Summary and Limitations 105

7 Search Algorithm for Finding Robust and Stable Solutions **107**
7.1 Neighbourhood in the Solution Space 108
7.2 Objective Function 111
7.2.1 Formalization 113
7.3 Search Algorithm 114
7.4 Summary and Limitations 118

8 A Case Study: Scheduling Problems **121**
8.1 Modeling Scheduling Problems as CSPs 122
8.2 Dynamism in Scheduling 123
8.3 Robustness Measurement in Scheduling 124
8.4 Neighbourhood in Scheduling 125
8.5 Adaptation of the Search Algorithm to Scheduling 127
8.6 Adaptation of the Coverigns Technique to Scheduling 131
8.7 Alternative Proactive Approaches in Scheduling 132
8.8 Example 133
8.8.1 Obtaining Robust Schedules 134
8.8.2 Trade-off between Robustness and Makespan 138
8.9 Summary and Limitations 139

9 Evaluation **141**
9.1 Problems Setting 142
9.1.1 Structured and Random CSPs 142
9.1.2 Uninformed Approaches Comparison 143
9.1.3 Solvers and Parameters Setting 144
9.2 Dynamism Simulation in Uninformed Environments 144
9.2.1 Dynamism in the Search Space 145
9.2.2 Dynamism in the Solution Space 146
9.3 Probabilistic Technique Evaluation 147
9.3.1 Experiments based on the Number of Constraints Additions 148
9.3.2 Experiments based on the Dynamism Functions 151

9.4 Technique for Finite Linear CSPs Evaluation 151
9.5 Coverings Evaluation based on the CSP Parameters 155
9.6 Non-convex Random CSPs Evaluation 161
9.7 Scheduling Benchmarks Evaluation 166
9.8 Summary and Limitations 178

10 Conclusions **183**
10.1 Contributions 184
10.1.1 New Dynamism Framework for Ordered Domains 184
10.1.2 Proposed Techniques and Designed Algorithms 185
10.2 Conclusions of our Approaches 186
10.2.1 Performance 187
10.2.2 Strengths and Limitations 188
10.3 Future Work 190
10.3.1 Considering Certain Dynamism Information 190
10.3.2 Combining Optimality Criteria 191
10.4 Final Conclusions 192
10.5 Associated Publications 193

A Weighted CSPs File Format Specification **197**
A.1 File Format Description 197
A.2 Example 198