Acoustic Doppler effect applied to the study of linear motions

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Abstract. In this work, the change of frequency of a sound wave due to Doppler effect has been measured using a smartphone. For this purpose, a speaker at rest and a smartphone placed on a cart on an air track were used. The change in frequency was measured by using an application for Android™ "Frequency Analyzer" developed by us specifically for this work. This fact made possible the analysis of four types of mechanical motions: uniform linear motion, uniform accelerated linear motion, harmonic oscillations and damped harmonic oscillations. These experiments are suitable for undergraduate students. The main novelty of this work was the possibility of measuring the instantaneous frequency as a function of time with high precision. The results were compared with alternative measurements yielding a good agreement.

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1. Introduction

Doppler effect is a very important physical phenomenon with a great variety of applications nowadays. For example, some radars use this effect to determine the velocity of moving objects, and in Astronomical sciences, the frequency change in the electromagnetic waves is used to measure the speed of stars and galaxies. In Medicine, the Doppler effect is used for the construction of images and for the measurement of the blood flow. Other applications to Satellite Communication and in Acoustics have been reported.

The Doppler effect consists of a change in the frequency received by the receptor when the source moves relative to it, that is, the frequency increases when the receptor approaches the source and decreases when the receptor moves away from the source [1, 2, 3, 4, 5, 6, 7]. In our case, we have used a source at rest, and the receptor was moving. Then, the frequency registered by a moving observer, being the source at rest, is expressed by,

$$f^{'} = f\left(1 \pm \frac{\nu_0}{\nu}\right), \tag{1}$$

where f is the frequency of the source, ν_0 is the velocity of the observer, and ν is the velocity of the wave, that is, the sound in the air. The "plus" sign indicates that the observer approaches to the source (the frequency increases), and the "minus" sign indicates that the observer moves away (the frequency decreases).

There are various articles on acoustic Doppler effect experiments reported in the literature [8, 9, 10, 11, 12, 13]. Different lay-outs have been used. For instance, in reference [8] ultrasonic emitters placed at the end of a rotating arm were the moving sources and a fixed emitter provided a frequency for comparison. The signal was directed to a dual channel oscilloscope in order to detect a frequency shift or the sum of both signals on the oscilloscope display.

In reference [9] one speaker was carried by a student moving at a walking speed of about 1 m/s. The Doppler shifts was measured with a microphone and a digital frequency meter. They also measured Doppler shifts using the microphone and an oscilloscope to observe and measure the beat frequency produced between the moving speakers shifted signal and the sound from a second stationary speaker. In reference [10] authors used a computer Windows program and a sound card to record the Doppler shifted sound from a moving speaker at constant velocity on an air track. They used a fast Fourier transform (FFT) analysis of the sound data to determine the peak positions visually, and subsequently, the frequency shift. The precise determination of the peak centers was somewhat subjective. The authors also performed experiments on accelerated motions, measuring a frequency shift in the FFT representation for which a good qualitative agreement between the theory and data was shown. In reference [11] a microphone was connected directly to the computer sound card and a small loudspeaker was attached to the end of a vibrating metal rod. The effect of the speaker speed on the frequency spectrum of the sound received by the microphone was observed. A Matlab®

program was used to acquire the sound data and perform the Fast Fourier Transform. The estimated value of the rod vibration frequency was consistent with the visual observation of the vibrating rod. In reference [12] the authors used a speaker attached to a string moving in uniform circular motion. The sound waves were recorded and Fourier analyzed. The calculated power spectrum of the measured sounds was represented over some periods of time (sonograms). The theoretical curves of the measured sonogram were superposed yielding a good agreement. In reference [13] the authors measured the instantaneous Doppler-shifted frequency of a free-falling sound source. The acceleration of gravity was calculated from the slope of the frequency *versus* time.

The main novelty of our work lies in being able to measure the frequency as a function of time with very high precision. For this purpose, a very simple measurement instrument is used, that is, a smartphone as receptor which moves with respect to a speaker (source). No additional calculations are need. The smartphone shows the measured frequency as a function of time directly on the device display. This is an advantage with respect to previous experiments where a microphone, a speaker and a computer program to analyze the results were independent components. Here, the computer program and the receptor are integrated in a portable device.

In the present work, the experimental setup included a speaker connected to a signal generator and a smartphone attached to the cart of an air track. The single frequency signal emitted by the speaker is captured by the microphone of the smartphone. The speaker is fixed while the smartphone may move on the air track with different relative velocities. The analysis of the signal was performed with the application for Android™ "Frequency Analyzer"[14]. Although there is a variety of applications available in Google Play™ repository which can be used in different Physics experiments [15, 16, 17, 18, 19, 20], we have not found any application able to determine the fundamental frequency of the sound wave with sufficiently high precision for our purposes. For this reason, we have developed our own free application for Android [21].

The Doppler effect was used to characterize four mechanical motions, that is, the uniform linear motion, uniformly accelerated linear motion, simple harmonic oscillations, and damped harmonic oscillations, respectively. The main idea is that the frequency change measured from the Doppler effect is related to the speed of the smartphone which is placed on the cart of the air track. These experiments can be easily perform in a physical laboratory, in which an air track is available, and with the students' smartphone. These experiments allow the students to better understand the Doppler effect, and in addition, to study the different linear motions by an alternative method.

The outline of the paper is the following. In section 2, the methodology for the experiments is described. In section 3, results for each of the four studied motions are discussed. Finally, in section 4, some conclusions are drawn.

2. Experiments

By using an air track, linear motions with speeds less than 1 m/s can be studied. This means that the change in frequency due to Doppler effect associated to theses speeds should be less than 0.3 %, so that, to obtain good results, the frequency has to be measured with very high precision. For this purpose, an application for Android™ capable of measuring the fundamental frequency of sound with a precision less than 0.04 % and to sample the frequency every 0.1 s, was developed. We decided to implement our own application since no application for this purpose was available in Google Play™ store on the internet. However, we had to take into account that the sound produced by the air track when it is on can cause interferences and consequently additional errors in the frequency measurement.

In order to quantify such errors, the frequency of a sound wave with 3000 Hz has been measured with the smartphone at rest on the air track with the air track switched on. The value measured by the smartphone was (3000.1 ± 0.5) Hz, which corresponds to a relative error in the measurement of the frequency of 0.017 % and a discrepancy with the expected value of 0.003 %.

From the arguments given above, we can concluded that this application can be used for measuring the Doppler effect since it is capable to measure frequencies within a precision of 0.5 Hz for a signal of 3000 Hz, so that we will be able to measure frequency changes for smartphone speeds starting from 0.06 m/s.

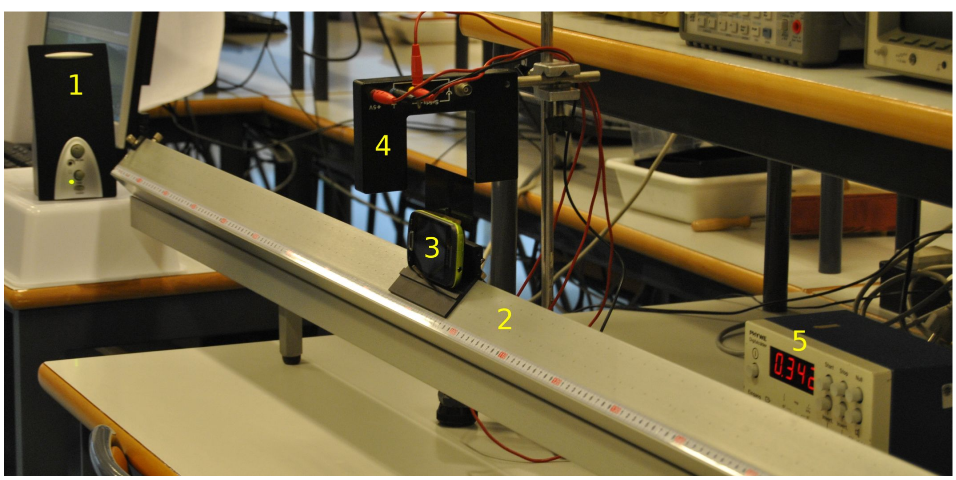

Figure 1. Photograph of the experimental set-up used to measure the frequency of a linear motion. In the figure, (1) is the speaker, (2) the air track, (3) the smartphone, (4) the photodetector, and (5) the time counter.

Figure 1 shows the experimental setup for the measurement of the frequency with the smartphone on the air track. The smartphone was moved with four different linear motions: uniform linear motion, uniformly accelerated linear motion, harmonic oscillations and damped harmonic oscillations. The figure includes the speaker (1) which emits the signal, the smartphone (2) attached to a cart on the air track (3), and the photometer which detects the pass of the cart through it (4) and activates the time counter (5) to measure the speed of the cart for uniform motion and the period of the oscillations for harmonic motion. For the harmonic motions, two springs were attached to the the cart. In all experiments, a smartphone model Samsung Galaxy Mini S5570 was used. The signals of unique frequency are generated by connecting the speaker to an AC frequency generator.

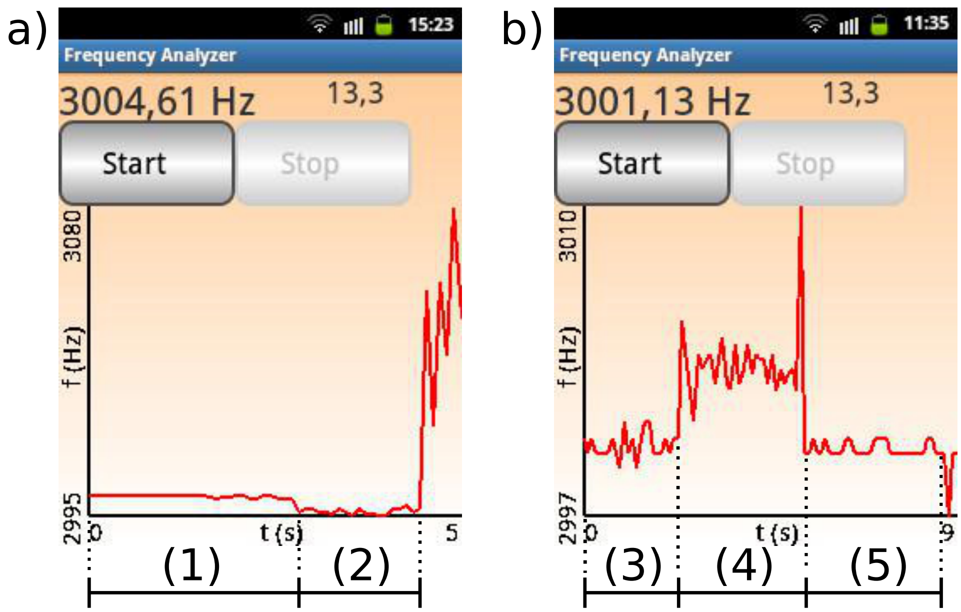

Figure 2. Snapshots of the Frequency Analyzer application for two measurements of the frequency when the smartphone moves with uniform motion on the air track: a) moving away from the speaker and b) approaching to the speaker.

In Figure 2, two examples of the frequency measurement with the mobile phone are shown. In order to assure that the application measures properly the frequency change due to the relative motion, the smartphone was left quiet for a few seconds, and afterwards, it was moved at approximately constant velocity. In both cases, the frequency of the source was 3000 Hz. In Figures 2a and 2b, the zone from which the experiment starts can be clearly observed (labeled as (1) and (3) respectively), that is, the region where the frequency is constant. Once the motion is started (figure 2a), a decrease in the frequency is observed (2) since for this case, the receptor moved away from the source. Finally, when the smartphone got too far from the speaker (source), the amplitude of the signal received by the microphone decreased, and the application was not able to determine the frequency with precision. Those values should not be taken into account in the calculations. In Figure 2b, an increase in frequency is observed (4) since the mobile phone approached the source. In this case, the mobile phone was stopped when it reached the end of the air track, and the initial constant frequency can be observed again (5).

In the first experiment, for the uniform linear motion, the cart was moved with approximately constant velocity. By means of a photometer, the pass of the cart (with the smartphone) moving at constant velocity was detected, and by means of a time counter the elapsed time was measured. The speed of the cart was then calculated by dividing the distance traveled over the time elapsed. This value was later compared with the result derived from the Doppler effect.

In the second experiment, for the uniformly accelerated linear motion, the air track was set at the inclination of (0.030±0.001) rad which allowed a component of the gravity acceleration to act along the air track direction,

$$a = g \sin \alpha = (0.294 \pm 0.013)\ \mathrm{m/s^2}. \tag{2}$$

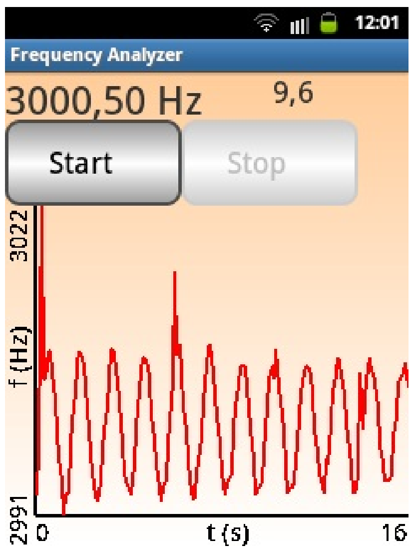

Figure 3. Snapshot of the Application window for a measurement of the frequency in a simple harmonic motion.

For the third and fourth experiments, the cart was attached to two fix points by means of two equal springs. In Figure 3, an example of the measurement of the frequency in the smartphone moving with simple harmonic oscillations is shown. The frequency of the source was 3000 Hz. In the figure, the sinusoidal variation of the signal is clearly observed. From the data obtained in this measurement, the amplitude is calculated, 7.5 Hz, which corresponds to the maximum speed in the harmonic motion, 0.85 m/s.

Finally, the case of the damped simple harmonic oscillations was presented. The air supply of the air track was decreased to let some friction act between the air track and the cart with the smartphone attached.

3. Results and discussions

3.1. Uniform linear motion

In Figure 4, the frequency as a function of time in a uniform linear motion is shown. During the first 1.6 s, the smartphone was at rest (red circle points) and registered

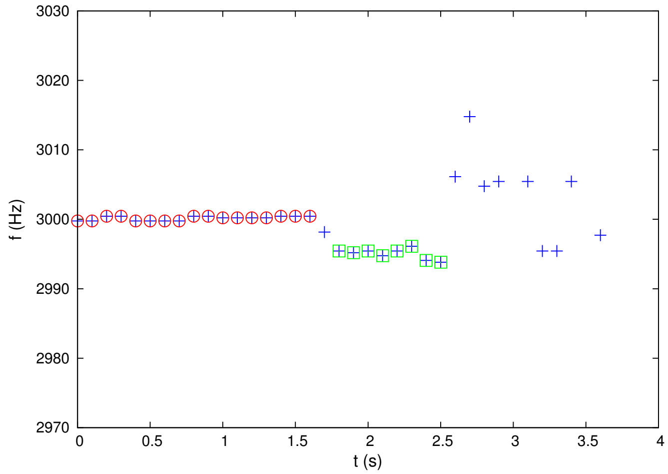

Figure 4. Frequency *versus* time in a uniform linear motion. Red circles represent the mobile phone at rest, and green boxes when it moved away from the source.

a frequency of (3000.1±0.3) Hz. From this moment on, the cart moved at constant velocity (green box points), for a frequency of (2995.0±0.7) Hz registered by the mobile phone. Once the smartphone moved far away from the speaker, the intensity of the sound signal decreased and the application was not able to determine the frequency with precision (after 2.6 s). The measurements for times longer than 2.6 s was not included in the calculations.

With these data and taking into account that the speed of sound at the temperature of the experiments, 24°C, is 347 m/s [22], the speed of the cart (with the smartphone) was calculated. The resulting value was (0.592±0.002) m/s, which was compared with the value obtained from using the photo-detector and the chronometer, (0.585±0.009) m/s. A great agreement between both results for a percentage discrepancy of 1.2 % was obtained.

3.2. Uniformly accelerated linear motion

Figure 5 shows the results for the speed as obtained from the frequency shift due to the Doppler effect for the mobile phone moving with uniformly accelerated linear motion. A small corrugation in the signal with a duration of 0.08 m/s approximately can be observed. This is related to small variations of the frequency measured around 0.7 Hz in this case. During the first 3 s, the smartphone remained at rest and from this moment on, it started moving under the action of the component of the gravity force that appeared due to the inclination of the air track. For a uniformly accelerated linear motion the acceleration is constant and equals the slope of the curve of the velocity *versus* time. The% calculation yielded (0.303±0.021) m/s^2. This represents a discrepancy of 3 % with the value obtained from Equation 2.

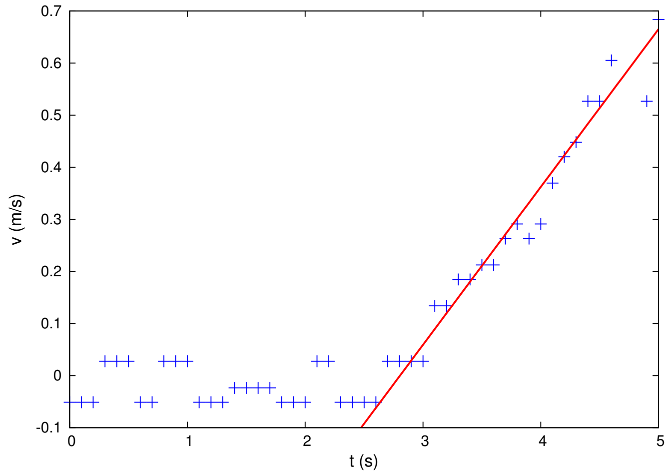

Figure 5. Absolute value of the velocity versus time for a uniformly accelerated linear motion. Blue crosses represent the experimental data, and the red line the linear fit.

3.3. Free simple harmonic motion

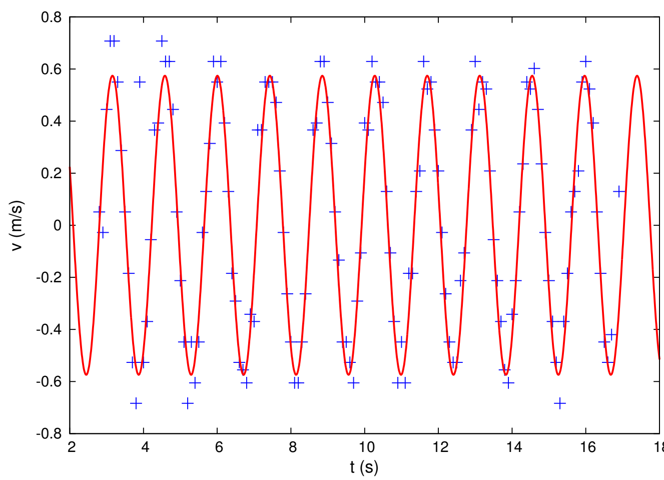

Figure 6. Velocity of the smartphone versus time for the data obtained in the simple harmonic motion. Blue crosses represent the experimental data, and the red curve the fit.

The values of the velocity *versus* time as calculated from the frequencies measured with the smartphone moving with simple harmonic oscillations are shown in Figure 6. The experimental data was fitted to the Equation of the velocity of the simple harmonic motion,

$$v(t) = v_0 \cos(\omega_0 t + \phi_0). \tag{3}$$

% The values of the parameters resulting from the fitting were $v_0 = (0.57\pm0.02)$ m/s (amplitude), $\omega_0 = (4.414\pm0.007)$ rad/s (natural frequency of the system) and $\phi_0 = (4.91\pm0.07)$ rad (initial phase). From the value of the frequency ω_0, the period of the oscillatory motion can be calculated, T=(1.423±0.002) s. This value was compared with the measurement of the time counter, T_{tc}=(1.428±0.001) s. A very low discrepancy of 0.3 % between both measurements was obtained.

3.4. Damped simple harmonic motion

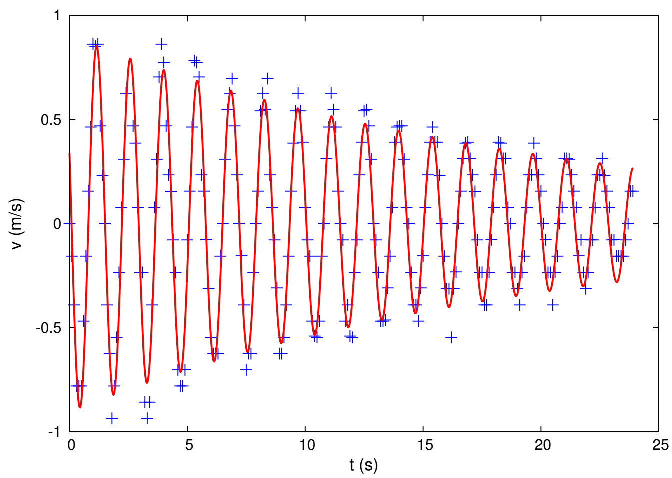

Figure 7. Velocity of the smartphone calculated from the data obtained in the damped simple harmonic motion. Blue crosses represent the experimental data, and the red curve represents the fit.

In Figure 7, the results of the speed as calculated from the frequency measured with the smartphone moving with damped simple harmonic oscillations are shown. The graph shows clearly the under-damped behavior of the system. It can be observed a decrease in the the amplitude with time which is due to the friction force between the cart and the air track. The time dependence of the position as a function of time in an under-damped harmonic motion is given by the following expression,

$$x(t) = Ae^{-\gamma t}\sin(\omega t + \phi_0), \quad (4)$$

where A is the amplitude of the motion, γ is the damping constant, ω is the damped angular frequency and ϕ_0 the initial phase. The damped frequency is a function of the natural frequency calculated in the previous experiment and of the damping constant, that is, $\omega^2 = \omega_0^2 - \gamma^2$. By taking the first derivative with respect to time, the expression for the velocity is obtained,

$$v(t) = Ae^{-\gamma t}\left(-\gamma \sin(\omega t + \phi_0) + \omega \cos(\omega t + \phi_0)\right). \quad (5)$$

The fitting of the experimental results to this equation yielded the following values for the parameters: A = (0.206±0.006) m, γ = (0.050±0.003) s^{-1}, and ϕ_0 = (1.17±0.02) rad.

From the value of the damping constant, and the natural angular frequency calculated in the previous experiment, the damped frequency was obtained,

ω=(4.414±0.007) rad/s. The system was weakly damped, that is $\gamma << 1$, the damped frequency was nearly equal to the natural frequency, so that the differences between them were less than the error obtained in the fit.

4. Conclusions

The Doppler effect was studied by using a speaker and a smartphone as useful laboratory measurement instruments. The single frequency sound signal emitted by a speaker was captured by the microphone of the smartphone. The speaker was fixed while the smartphone moved on an air track with different types of mechanical motions. The frequency change as a function of time was measured with high precision by using our own application for Android™ "Frequency Analyzer" developed specifically for this work. Four experiment were performed. In the first case, the constant velocity of the cart measured by the Doppler effect, and by using a time counter were compared for a discrepancy of 1.2 %. In the second experiment, a uniformly accelerated motion was studied. The acceleration calculated from the speed derived from the Doppler effect was compared to the value calculated from the Newton's Second Law taking into account the inclination angle of the air track. A discrepancy of 3% between the results indicated a very good agreement. Free and damped harmonic motions were also studied. A good agreement between the experimental data and the theoretical expressions was also obtained.

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