A note on mean ergodic composition operators on spaces of holomorphic functions

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Abstract   We characterize those composition operators defined on spaces of holomorphic functions of several variables which are power bounded, i.e. the orbits of all the elements are bounded. This condition is equivalent to the composition operator being mean ergodic. We also describe the form of the symbol when the composition operator is mean ergodic.

Keywords   Composition operators · Mean ergodic operators · Power bounded operators · Spaces of holomorphic functions

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1 Introduction and notation

The purpose of this note was to study the behaviour of orbits of composition operators $C_\varphi(f) := f \circ \varphi$, φ a holomorphic self map, on spaces $H(U)$ of holomorphic functions defined on an open connected subset (=*domain*) U of $\mathbb{C}^d$ or, more generally, of a Stein manifold Ω. The space $H(U)$ is endowed with the compact open topology and it is a nuclear Fréchet space, hence Montel. We are interested here in the case when the orbits of all the

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elements under C_φ are bounded. If this occurs, the operator C_φ is called *power bounded*. A related interesting question is when C_φ is *mean ergodic* [3,9,17,22]. We complement important results on universality which were obtained in [5,6,12]. These papers do not discuss when C_φ is power bounded or mean ergodic on $H(U)$.

There is a huge literature about the dynamical behavior of various linear continuous operators on Banach, Fréchet and more general locally convex spaces; see the survey papers by Grosse-Erdmann [10,11] and the recent books by Bayart and Matheron [4] and by Grosse-Erdmann and Peris [13]. For the general theory of composition operators on Banach spaces of holomorphic functions see [8,20], and for composition operators on $H(U)$ see [21].

Let $T : E \to E$ be a continuous linear operator on a Hausdorff locally convex space E. The iterates of T are denoted by $T^n := T \circ \cdots \circ T, n \in \mathbb{N}$. If the sequence $(T^n)_{n \in \mathbb{N}}$ is equicontinuous in the space $L(E)$ of linear operators from E to E, T is called *power bounded*. In case $E = H(U)$, it is a Fréchet space and the uniform boundedness principle can be applied to conclude that T is power bounded if and only if the orbit $\{T^n(x) : n \in \mathbb{N}\}$ is bounded for every $x \in E$.

A continuous linear operator T on E is called *mean ergodic* if the limits

$$Px := \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} T^n x, \quad x \in E, \tag{1}$$

exist in E. A power bounded operator T is mean ergodic precisely when $X = \ker(I - T) \oplus \overline{\mathrm{im}(I - T)}$. Moreover, $\mathrm{im} P = \ker(I - T)$ and $\ker P = \overline{\mathrm{im}(I - T)}$. If the space E is barrelled and T is mean ergodic, the sequence $\frac{1}{N} \sum_{n=1}^{N} T^n$ converges pointwisely to a continuous linear projection P; see [22, Ch. VIII, Sect. 3]. If the convergence is uniform on bounded sets we call T *uniformly mean ergodic*. There is a classical theory of mean ergodic operators which goes back to fundamental papers of Yosida and Hille especially in the Banach case; cf. [17]. For more details on the locally convex theory see [2,3,22] and the references therein.

We characterize those composition operators C_φ, $C_\varphi(f) := f \circ \varphi$, on the space of holomorphic functions $H(U)$ on a domain U in a Stein manifold Ω, $\varphi : U \to U$ holomorphic, such that C_φ is power bounded; see Proposition 1. We prove that power boundedness is equivalent to (uniform) mean ergodicity. Finally in Theorem 1 we describe precisely the form of φ such that C_φ is power bounded. The evaluation of these equivalent conditions and the precise form of the projection which appears as limit of the Cesaro sums is determined in concrete cases in Corollaries 2 and 3. Our results here are utilized in [7] to investigate power bounded composition operators on spaces of real analytic functions.

We will use Kobayashi semi-distance $k_V(\cdot, \cdot)$ on a complex manifold $V \subseteq \mathbb{C}^d$. Here the beautiful book [16] is a standard reference. Every holomorphic map $\varphi : V \to V$ is always non-expansive with respect to k_V. The manifold V is called *hyperbolic* if k_V is a distance (and then it induces the standard topology of V). Every domain biholomorphic to a bounded set is automatically hyperbolic [16, Cor. 4.1.10, Prop. 3.2.2]. The manifold V is called *Kobayashi complete* if (V, k_V) is a complete metric space, or equivalently, if every ball in this space is relatively compact [16, Prop. 1.1.9]. Every bounded open set $V \subseteq \mathbb{C}^d$ such that every boundary point admits a weak peak function is Kobayashi complete [16, Cor. 4.1.11]. Both hyperbolicity and Kobayashi completeness are biholomorphic invariants. Recall that every Stein manifold embeds via a proper map into $\mathbb{C}^d$ for suitable $d \in \mathbb{N}$ [15, 5.3.9].

By $B(x, r)$ and $B_{k_V}(x, r)$ we denote, respectively, Euclidean and Kobayashi balls of center x and radius r. The notation $K \Subset U$ means that K is a compact subset of the open set U. For non-explained notions from functional analysis we refer to [18]. For complex analysis of several variables see [14]. For dynamics of holomorphic maps see [1,19].

2 Power bounded composition operators on spaces of holomorphic functions

Let $\varphi : U \to U$ be a continuous self-map on a topological space U, we say that φ has *stable orbits on U* if for every compact subset K of U there is a compact subset $L \Subset U$ such that $\varphi^n(K) \subseteq L$ for $n \in \mathbb{N}$. This concept plays an important role later on.

Proposition 1 *Let U be a connected domain of holomorphy in $\mathbb{C}^d$ (or even in a Stein manifold) and let $\varphi : U \to U$ be a holomorphic mapping. The following assertions are equivalent:*

(a) $C_\varphi : H(U) \to H(U)$ is power bounded.
(b) $C_\varphi : H(U) \to H(U)$ is uniformly mean ergodic.
(c) $C_\varphi : H(U) \to H(U)$ is mean ergodic.
(d) The map φ has stable orbits on U.
(e) There is a fundamental family of connected compact sets (L_j) in U such that $\varphi(L_j) \subseteq L_j$ for every $j \in \mathbb{N}$.

Proof (a)$\Rightarrow$(b): Follows from [2, Prop. 2.4] since $H(U)$ is a Fréchet Montel space.

(b)$\Rightarrow$(c): Obvious.

(c)$\Rightarrow$(d): By [2, Prop. 2.2], the sequence $\left(\frac{1}{n}C_{\varphi^n}\right)$ tends to zero, so it is bounded and equicontinuous on $H(U)$. For every K compact in U we find L compact in U such that $(1/n)C_{\varphi^n}$ extends to $(1/n)\widehat{C_{\varphi^n}} : A(L) \to A(K)$ as a continuous map, where K and L have non-empty interior and $A(K)$ denotes the completion of $H(U)$ with respect to the sup-norm on K. Since U is a domain of holomorphy it is holomorphically convex and we can assume that L is equal to its U-holomorphic hull, i.e.,

$$L = \left\{ w \in U : |f(w)| \leq \sup_{z \in L} |f(z)| \quad \forall\ f \in H(U) \right\}.$$

Suppose that there are $n \in \mathbb{N}$ and $a \in \varphi^n(K)\setminus L$. Then there is a function $f \in H(U)$ such that $f(a) = 1$ but $\sup_{z \in L}|f(z)| \leq \varepsilon$, $\varepsilon \in (0,1)$. Since f^k tends uniformly to zero on L as $k \to \infty$ but does not tend to zero at a, the map C_{φ^n} is not continuous from $A(L)$ to $A(K)$; a contradiction.

(d)$\Rightarrow$(e): Let K be connected compact set in U. Select a connected compact set $L \supseteq K$ such that $\varphi^n(K) \subseteq L$ for $n \in \mathbb{N}$. Apply condition (d) again to find a compact set M such that $\varphi^n(L) \subseteq M$ for every $n \in \mathbb{N}$. Then

$$\tilde{L} := \overline{L \cup \bigcup_{n=1}^{\infty} \varphi^n(L)}$$

is a compact connected set such that $\varphi(\tilde{L}) \subseteq \tilde{L}$ and $\tilde{L} \supseteq K$. It is connected since $\varphi^n(L)$ is connected and $\varphi^n(L) \cap L$ contains $\varphi^n(K)$. By induction we can construct (L_j) as required.

(e)$\Rightarrow$(a): Obvious. $\square$

Remark 1. The conditions (d) and (e) in Proposition 1 are always equivalent. The implication (c)$\Rightarrow$(d) requires in the proof that U is a domain of holomorphy, the converse implication does not—it suffices that U is a domain in a Stein manifold.

Remark 2. Every domain U in a Stein manifold has a unique maximal extension V such that V is a Stein manifold and $H(U)$ and $H(V)$ are isomorphic as Fréchet algebras (points of V are exactly multiplicative functionals on $H(U)$, see [14, Sect. I.H, Th. III.J.1, III.R.5]). Since $C_\varphi : H(U) \to H(U)$ is an algebra homomorphism, φ extends uniquely to $\psi : V \to V$

such that C_φ corresponds to C_ψ. These remarks permit us to conclude that the equivalence (a)⇔(b)⇔(c) in Proposition 1 holds for arbitrary domain U in a Stein manifold.

We analyze the holomorphic functions $\varphi : U \to U$ for which condition (d) in Theorem 1 is satisfied.

Proposition 2 *Let $\varphi : U \to U$ be a holomorphic map and let U be a domain in a Stein manifold. Assume that there is a fundamental family of compact subsets (L_j) of U such that $\varphi(L_j) \subseteq L_j$ for every $j \in \mathbb{N}$. Then for every $j \in \mathbb{N}$ there is a fundamental family of (open!) neighbourhoods V of L_j in U such that $\varphi(V) \subseteq V$. In particular, every compact set $K \subseteq U$ is contained in a hyperbolic open subset $V \subseteq U$ such that $\varphi(V) \subseteq V$.*

Proof Clearly U is a manifold embedded into $\mathbb{C}^d$. Without loss of generality we may assume that $L_j \Subset \overset{\circ}{L}_{j+1}$. Fix $j \in \mathbb{N}$. There is $\varepsilon > 0$ such that $\big(L_j + B(0, \varepsilon)\big) \cap U \subseteq L_{j+1}$. Here $L + B(0, \varepsilon)$ means in fact the algebraic sum of L and $B(0, \varepsilon)$ in $\mathbb{C}^d$. By the Cauchy estimates, the sequence $(\varphi^n)_n$ is uniformly equicontinuous on L_{j+2}. Then

$$\forall \, \delta > 0 \; \exists \, 0 < \eta(\delta) < \delta \; \forall \, n \in \mathbb{N} : \quad x, y \in L_{j+1},$$
$$\|x - y\| < \eta(\delta) \Rightarrow \|\varphi^n(x) - \varphi^n(y)\| < \delta.$$

Let us define $W_0 := L_j$, $W_k := \big(\varphi(W_{k-1}) + B\big(0, \eta\big(\frac{\varepsilon}{2^k}\big)\big)\big) \cap U$ and prove inductively that

$$W_k \subseteq L_{j+1} \quad \text{and} \quad \varphi^n(W_k) \subseteq \left(L_j + B\left(0, \sum_{j=1}^{k} \frac{\varepsilon}{2^j}\right)\right) \cap U. \tag{2}$$

The condition is true for $k = 0$. Now, assuming (2) for $k - 1$ we get

$$W_k = \left(\varphi(W_{k-1}) + B\left(0, \eta\left(\frac{\varepsilon}{2^k}\right)\right)\right) \cap U$$
$$\subseteq \left(L_j + B\left(0, \sum_{j=1}^{k-1} \frac{\varepsilon}{2^j}\right) + B\left(0, \eta\left(\frac{\varepsilon}{2^k}\right)\right)\right) \cap U \subseteq L_{j+1}.$$

Moreover, we have

$$\varphi^n(W_k) = \varphi^n\left(\underbrace{\left(\varphi(W_{k-1}) + B\left(0, \eta\left(\frac{\varepsilon}{2^k}\right)\right)\right) \cap U}_{\subseteq L_{j+1}}\right) \subseteq \left(\varphi^{n+1}(W_{k-1}) + B\left(0, \frac{\varepsilon}{2^k}\right)\right) \cap U$$
$$\subseteq \left(L_j + B\left(0, \sum_{j=1}^{k-1} \frac{\varepsilon}{2^j}\right) + B\left(0, \frac{\varepsilon}{2^k}\right)\right) \cap U \subseteq \left(L_j + B\left(0, \sum_{j=1}^{k} \frac{\varepsilon}{2^j}\right)\right) \cap U.$$

Since W_k are open subsets of U and $\varphi(W_k) \subseteq W_{k+1}$ thus $\tilde{W} := \bigcup_{k=0}^{\infty} W_k$ is an open set in U such that $\varphi(\tilde{W}) \subseteq \tilde{W} \subseteq L_j + B(0, \varepsilon)$. Taking $\varepsilon > 0$ arbitrarily small we get the conclusion. Since every submanifold of a bounded open set in $\mathbb{C}^d$ is hyperbolic [16, Cor. 4.1.10, Prop. 3.2.2], the last sentence of the statement follows. $\qquad\square$

Proposition 3 *Let $\varphi : U \to U$ be a holomorphic map and let U be a domain in a Stein manifold.*

(a) If U is hyperbolic and every orbit of φ in U is relatively compact then φ has stable orbits in U and, hence, $C_\varphi : H(U) \to H(U)$ is power bounded.

(b) If U is Kobayashi complete hyperbolic and there is a relatively compact orbit of φ then φ has stable orbits in U and, hence, $C_\varphi : H(U) \to H(U)$ is power bounded.

Proof (a) Fix a compact set K. For every $x \in K$ its φ-orbit $O_x := \{\varphi^n(x) : n \in \mathbb{N}\}$ is relatively compact. Hence there is $\varepsilon(x) > 0$ such that $U_x := \{z \in U : k_U(z, O_x) < \varepsilon(x)\}$ is relatively compact in U. As $(B_{k_U}(x, \varepsilon(x))_{x \in K}$ is a covering of K, there is a finite subcovering $(B_{k_U}(x_i, \varepsilon(x_i)))_{i=1,\ldots,m}$. Since φ is non-expansive with respect to the Kobayashi metric of U, for each $z \in K$, $\varphi^n(z) \in \bigcup_{i=1}^m U_{x_i}$ for each $z \in K$, and the set $\bigcup_{i=1}^m U_{x_i}$ is relatively compact.

(b) Let the φ-orbit O_x of x be relatively compact. Let K be an arbitrary compact subset of U containing O_x, thus $K \subseteq B_{k_U}(x, r)$ for some $r > 0$. Since φ is non-expansive with respect to the Kobayashi metric of U, orbits of all points of K are contained in $B_{k_U}(x, 2r)$. This set is relatively compact since in a complete hyperbolic set all Kobayashi balls are relatively compact. □

Remark For $U = \mathbb{C}$, part (b) of Proposition 3 does not hold: it is enough to take $\varphi(z) = 2z$.

Corollary 1 *Let U be a topologically contractible bounded strongly pseudoconvex domain in $\mathbb{C}^d$ with $\mathcal{C}^3$ boundary. Let $\varphi : U \to U$ be a holomorphic map. Then $C_\varphi : H(U) \to H(U)$ is power bounded if and only if φ has a fixed point.*

Proof By Proposition 1, if C_φ is power bounded φ^n cannot converge to a boundary point. By [16, Th. 5.5.9], if φ^n not converge to a boundary point then it has a fixed point. By [16, Cor. 4.1.12], U is complete hyperbolic. Sufficiency follows from Proposition 3 (b). □

A precise description of maps φ such that $C_\varphi : H(U) \to H(U)$ is power bounded follows from an argument of Abate originally used for taut manifolds, see [16, Th. 5.5.4].

Theorem 1 *Let U be a domain in a Stein manifold and let $\varphi : U \to U$ be a holomorphic map. If φ has stable orbits on U, then there is a holomorphic submanifold M of U and a holomorphic surjective retraction $\rho : U \to M$ such that $\psi := \varphi|_M$ is an automorphism of M. Moreover,*

$$G := \overline{\{\psi^n : n \in \mathbb{N}\}}^{H(M,M)}$$

is a compact abelian group of automorphisms on M such that every cluster point of (φ^n) in $H(U, U)$ is of the form $\gamma \circ \rho$ where $\gamma \in G$. The φ-orbit of every element $z \in U$ tends to some G-orbit of elements of M, in particular, it tends to M and this convergence is uniform with respect to z belonging to a fixed compact set.

Finally,

$$P(f)(z) := \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^N C_{\varphi^n}(f)(z) = \int_G f(\gamma \circ \rho(z)) dH(\gamma),$$

where H is the Haar measure on G, and the image of the projection P satisfies

$$\mathrm{im}\, P = \left\{ f : f \text{ is constant on } \rho^{-1}(\{\gamma \circ \rho(z) : \gamma \in G\}) \quad \forall z \in U \right\}.$$

Proof Most of the statement follows immediately form the proof of [16, Th. 5.5.4] except the uniform convergence of orbits and description of the projection P. We prove this part below.

Since every sequence (φ^{n_j}) contains a subsequence convergent to $\gamma \circ \rho$ for some $\gamma \in G$ it follows that the set of all cluster points of the orbit:

$$O_x := \{\varphi^n(x) : n \in \mathbb{N}\}$$

is exactly equal to

$$G_{\rho(x)} := \{\gamma \circ \rho(x) : \gamma \in G\}.$$

Let $K \Subset U$ be an arbitrary compact set, then there is a compact set $L \Subset U$, $L \supseteq K$ such that $\varphi^n(K) \subseteq L$ for every $n \in \mathbb{N}$. Clearly, for every $x \in K$ we have $G_{\rho(x)} \subseteq L$ and, by Proposition 2, L is contained in a hyperbolic open set $V \subseteq U$, $\varphi(V) \subseteq V$.

The family of sets

$$G_{\rho(x)}(\varepsilon) := \left\{z \in V : k_V\left(z, G_{\rho(x)}\right) < \varepsilon\right\}$$

is a neighbourhood basis of $G_{\rho(x)}$ and if the orbits falls into $G_{\rho(x)}(\varepsilon)$, it cannot escape because φ is a non-expansive with respect to k_V. For every $\varepsilon > 0$ there is a finite set $\{x_1, \ldots, x_m\} \subseteq K$ such that

$$K \subseteq \bigcup_{j=1}^{m} B_{k_V}\left(x_j, \frac{\varepsilon}{2}\right).$$

Moreover, there is n_0 such that for every $n > n_0$ and every $j = 1, \ldots, m$ we have

$$k_V\left(\varphi^n(x_j), G_{\rho(x_j)}\right) < \frac{\varepsilon}{2}.$$

Clearly, for some j, $G_{\rho(x)} \subseteq G_{\rho(x_j)}\left(\frac{\varepsilon}{2}\right)$. Thus for every $x \in K$ and $n > n_0$ for some $j = 1, \ldots, m$ we have

$$k_V\left(\varphi^n(x), G_{\rho(x)}\right) < \frac{\varepsilon}{2} + k_V\left(\varphi^n(x), G_{\rho(x_j)}\right)$$

$$\leq \frac{\varepsilon}{2} + k_V\left(\varphi^n(x), \varphi^n(x_j)\right) + k_V\left(\varphi^n(x_j), G_{\rho(x_j)}\right) < \frac{3}{2}\varepsilon,$$

and orbits of $x \in K$ tend uniformly to $G_{\rho(x)}$.

Since im $P = \ker(C_\varphi - \mathrm{id})$ [22, Ch. VIII, Sect. 3] it is easily seen that im P is the set of functions constant on closures of all orbits and for $x \in U$ we have

$$\overline{O_x} = \rho^{-1}\left(\{\gamma \circ \rho(x) : \gamma \in G\}\right).$$

On the other hand, $\ker P = \overline{\mathrm{im}\,(C_\varphi - \mathrm{id})}$ [22, Ch. VIII, Sect. 3] and for

$$S : H(U) \to H(U), \quad S(f)(z) := \int_G f(\gamma \circ \rho(z)) dH(\gamma)$$

we have $\ker S \supseteq \mathrm{im}\,(C_\varphi - \mathrm{id})$ and im $S = \mathrm{im}\,P$. Therefore $S = P$ on $H(U)$. □

Remark 1. The converse of Theorem 1 is also true. Indeed, for every compact set $K \Subset U$ the set

$$L_K := \bigcup_{z \in K} \{\gamma \circ \rho(z) : \gamma \in G\}$$

is compact since it is the image of the continuous map

$$G \times K \to U, \quad (\gamma, z) \mapsto \gamma(\rho(z)).$$

Let V be a relatively compact neighbourhood of L_K. Since orbits of $z \in K$ tend uniformly to L_K there is n_0 such that $\varphi^n(K) \subseteq V$ for every $n > n_0$. Thus

$$\bigcup_{n \in \mathbb{N}} \varphi^n(K) \subseteq \bigcup_{n=1}^{n_0} \varphi^n(K) \cup V$$

which is relatively compact.

Remark 2. If U is the unit ball then M must be an affine slice of U, see [1, Cor. 2.2.16].

The following consequences follow from Theorem 1 in special cases.

Corollary 2 *Let U be a connected open subset of $\mathbb{C}$ and let $\varphi : U \to U$ be a holomorphic self-map. If C_φ is power bounded then either φ is an automorphism of U or all orbits of φ tend to a constant $u \in U$ such that u is a fixed point of φ.*

If additionally $U = \mathbb{D}$, then in the automorphism case φ has also a fixed point u and

$$\varphi = \varphi_u^{-1} \circ r_\theta \circ \varphi_u, \quad \theta \in [0, 2),$$

where

$$\varphi_u(z) := \frac{u - z}{1 - \bar{u}z}, \quad r_\theta(z) = e^{i\theta\pi} z.$$

Moreover, we have

(i) *If φ is not an automorphism then the projection P associated to C_φ is given: $P(f)(z) = f(u)$.*

(ii) *If φ is an automorphism and $\theta = \frac{p}{q}$ is rational then $P(f)(z) = \frac{1}{q}\sum_{j=0}^{q-1} f(\varphi_u^{-1}(r_{\frac{jp}{q}}(\varphi_u(z))))$.*

(iii) *If φ is an automorphism and θ is irrational then $P(f)(z) = \frac{1}{2}\int_0^2 f(\varphi_u^{-1}(r_\theta(\varphi_u(z))))d\theta$.*

Proof It follows from Theorem 1 in the non-automorphism case (since M has to be zero-dimensional and connected). In the automorphism case we apply Denjoy-Wolff theory (see [19, Sec. 5]) and the well-known form of automorphisms on $\mathbb{D}$. □

Clearly the result above gives a description of mean ergodic composition operators in the case of one-connected one dimensional Riemann manifolds.

Corollary 3 *Let B_d be the unit ball in a d dimensional complex space $\mathbb{C}^d$. Let $\varphi : B_d \to B_d$ induce a power bounded map C_φ. Then φ satisfies one of the following three properties:*

(i) *φ has a fixed point u and all orbits of φ tend to it. In this case $P(f)(z) = f(u)$.*

(ii) *There is an m dimensional affine submanifold M of B_d, $1 \leq m < d$, (clearly, biholomorphically equivalent to the ball B_m in $\mathbb{C}^m$) with a holomorphic retraction ρ onto M, φ restricted to M is an automorphism, it has a fixed point u in M and the projection associated to C_φ has the form*

$$P(f) = \int_\Gamma f(\zeta^{-1} \circ A \circ \zeta \circ \rho(z))dH(A),$$

where Γ is a closed subgroup of the group of unitary transformations generated by $\zeta \circ \varphi \circ \zeta^{-1}$, H is the Haar measure on this group and $\zeta : M \to B_m$ is a biholomorphism such that $\zeta(u) = 0$.

(iii) φ is an automorphism of B_d with a fixed point and P looks as above for $m = d$.

Proof We apply Theorem 1, and consider separately the cases when M has dimension 0, m for $0 < m < d$ and d. In the second case since M is retractive it must be an affine one-connected submanifold (see [1, Cor. 2.2.16]), thus it is biholomorphically equivalent to the corresponding m dimensional ball. The rest follows as in the proof of Corollary 2. □

References

1. Abate, M.: Iteration Theory of Holomorphic Maps on Taut Manifolds. Mediterranean Press, Commenda di Rende (1989)
2. Albanese, A.A., Bonet, J., Ricker, W.J.: On mean ergodic operators. In: Curbera, G.P. et al. (eds.) Vector Measures, Integration and Related Topics, vol. 201, pp. 1–20. Operator Theory: Advances and Applications. Birkhäuser Verlag, Basel (2009)
3. Albanese, A.A., Bonet, J., Ricker, W.J.: Mean ergodic operators in Fréchet spaces. Anal. Acad. Sci. Fenn. Math. **34**, 401–436 (2009)
4. Bayart, F., Matheron, E.: Dynamics of linear operators. In: Cambridge Tracts in Mathematics, vol. 179. Cambridge University Press, Cambridge (2009)
5. Bernal-González, L., Montes-Rodríguez, A.: Universal functions for composition operators. Complex Var. **27**, 47–56 (1995)
6. Bernal-González, L.: Universal entire functions for affine endomorphisms of $\mathbb{C}^{\mathbb{N}}$. J. Math. Anal. Appl. **305**, 690–697 (2005)
7. Bonet, J., Domański, P.: Power bounded composition operators on spaces of analytic functions. Collect. Math. doi:10.1007/s13348-010-0005-9 (2010)
8. Cowen, C.C., MacCluer, B.D.: Composition operators on spaces of analytic functions. In: Studies in Advanced Mathematics. CRC Press, Boca Raton (1995)
9. Fonf, V.P., Lin, M., Wojtaszczyk, P.: Ergodic characterizations of reflexivity in Banach spaces. J. Funct. Anal. **187**, 146–162 (2001)
10. Grosse-Erdmann, K.G.: Universal families and hypercyclic operators. Bull. Am. Math. Soc. **36**, 345–381 (2001)
11. Grosse-Erdmann, K.G.: Recent developments in hypercyclicity. Rev. R. Acad. Cien. Ser. A Math. **97**, 273–286 (2003)
12. Grosse-Erdmann, K.G., Mortini, R.: Universal functions for composition operators with non-automorphic symbol. J. Anal. Math. **107**, 355–376 (2009)
13. Grosse-Erdmann, K.G, Peris, A.: Linear Chaos. Springer, Berlin (2011)
14. Gunning, R.C.: Introduction to Holomorphic Functions of Several Variables. Wadsworth & Brooks, Belmont (1990)
15. Hörmander, L.: An Introduction to Complex Analysis in Several Variables, 3rd edn. North-Holland, Amsterdam (1990)
16. Kobayashi, S.: Hyperbolic Complex Spaces. Springer, Berlin (1998)
17. Krengel, U.: Ergodic Theorems. de Gruyter, Berlin (1985)
18. Meise, R., Vogt, D.: Introduction to Functional Analysis. Clarendon Press, Oxford (1997)
19. Milnor, J.: Dynamics in One Complex Variable. Vieweg Verlag, Wiesbaden (2000)
20. Shapiro, J.H.: Composition operators and classical function theory. In: Universitext: Tracts in Mathematics. Springer, New York (1993)
21. Shapiro, J.H.: Notes on the Dynamics of Linear Operators. Lecture notes. http://www.math.msu.edu/~shapiro/Pubvit/Downloads/LinDynamics/lindynamics.pdf (1993)
22. Yosida, K.: Functional Analysis. Springer, Berlin (2001)