Pricing reverse mortgages in Spain.

Abstract

In Spain, as in other European countries, the continuous ageing of the population creates a need for long-term care services and their financing. However, in Spain the development of this kind of services is still embryonic. The aim of this article is to obtain a calculation method for reverse mortgages in Spain based on the fit and projection of dynamic tables for Spanish mortality, using the Lee and Carter model. Mortality and life expectancy for the next 20 years are predicted using the fitted model, and confidence intervals are obtained from the prediction errors of parameters for the mortality index of the model. The last part of the article illustrates an application of the results to calculate the reverse mortgage model promoted by the Spanish Instituto de Crédito Oficial (ICO) (Spanish State Financial Agency), for which the authors have developed a computer application.

Keywords: Reverse Mortgage, Lee-Carter model, Remaining Lifetime.

1 Introduction

A reverse mortgage is a real estate guaranteed credit, a product that provides a person who owns real estate with a monthly income, determined by several factors. Upon the death of the debtor, the heirs are required to pay off the loan, or the creditor, as a last resort, proceeds to execute the guarantee, which can mean selling the real estate to liquidate the debt, paying the inheritors the balance from the sale of the property, if any.

A reverse mortgage is not the only financial product that transforms real estate assets into income. Other formulas and products exist that can provide seniors with additional income, as is the case of the so-called housing pension, mortgage pension or the cession for rent of the principal residence to a third party. Details of these alternatives can be found in Herranz-Gonzalez (2006), who also considers similar products that exist in other countries, such as home-equity reversion in the United Kingdom or the Home Equity Conversion Mortgage (HECM) program in the United States, commented on in more detail in Taffin (2006). For an in depth review of principal formulas and variables used in the HECM program, interested readers can consult Skarr (2008), who also compares several possibilities for their calculation. Although they may be less popular, similar products also exist in other developed countries such as Australia, Canada, Denmark, Finland, Ireland, Japan, Netherlands, Norway and Sweden. The expected remaining lifetime at the age of contracting and the periodic incomes are determining factors when it comes to evaluating the income to be received (Herranz-Gonzalez, 2006). This is why the reverse mortgage can be considered a double application of life tables. On the one hand, the expected remaining lifetime estimated for each age limits the theoretical term of the financial income. On the other hand, the calculation of a life annuity to complement the financial income (annuity-certain) requires obtaining the necessary commutation symbols, which depend on life tables to determine the present actuarial value of the income. Life annuities are contingent on the death of the annuitant, while an annuity-certain is independent of any life event and is therefore a purely financial operation. If the life tables are not well calibrated, we can predict higher mortality probabilities than is actually the case among policyholders, the latter will have been undercharged and the insurance company will make a loss.

The correct evaluation of all these quantities requires well *calibrated* life tables, meaning that they adequately reflect the evolution of mortality with over the course of time, which returns us again to our interest in dynamic life tables.

The aim of the present article is to obtain a calculation method for reverse mortgages in Spain based on the fit and projection of dynamic tables for Spanish mortality, using the model proposed by Lee and Carter (Lee and Carter, 1992). For these reasons, the paper has three steps:

1. To model the behavior of Spanish mortality during the period 1980-2005 using Lee-Carter's model

2. To predict the probability of death and residual life expectancy for future years for a range of ages that includes the most advanced ones

3. To price reverse mortgages in Spain.

Section 2 consists of a review of different international studies on reverse mortgages. Sections 3 and 4 present the methodology for the construction of dynamic life tables for the Spanish population, commenting on the advantages, disadvantages and adaptation of the Lee-Carter model. Section 5 is dedicated to the procedure used for the calculation of reverse mortgages according to the conditions established by the Instituto de Crédito Oficial (ICO[1]). Section 6 is devoted to a computer application allowing the simulation of incomes for this type of mortgage. The last Section, 7, presents the conclusions that are reached.

2 A general perspective of reverse mortgages

Reverse mortgages are the object of study of Costa-Font (2009), who consider them to be an interesting option for the Spanish population. The authors study the preferences of the population in relation to complementary financial instruments for personal care associated with advanced ages, such as housing pensions, reverse mortgages and life annuities. They conclude that a reverse mortgage is useful essentially for those that wish for "ageing at home" (the option preferred by Spanish people), alone or with the assistance of hired caregivers, and the need to complement a modest pension to increase their quality of life. Another study (Blay-Berrueta, 2007) analyses the dependent Spanish population and proposes the reverse mortgage as an alternative to long-term case insurance. In the context of Spain, Costa-Font (2012) concludes that private pension schemes are comparatively undeveloped.

In general, this instrument combines two types of traditional risks: the risk of the homeowner's longevity, managed by the insurance companies, and the interest rates risk, very familiar to credit firms. There is also a third risk, probably the least identified and understood (Taffin, 2006), associated with the value of the property once the loan is paid off. The borrowed capital depends, principally, on the borrower's age, interest rates and assumptions about the increase in housing prices. Given the lifelong nature of reverse mortgages, the lower the borrower's age, the lower the capital granted, the

[1]Instituto de Crédito Oficial is a State-owned corporate entity attached to the Ministry of Economy and Finance through the Secretariat of State for the Economy. It has the status of the State's Financial Agency in Spain. www.ico.es

consequence being that this type of loan is granted preferentially to older people. The average borrower's age is estimated to be around 75. There is a theoretical minimum threshold which in the USA is set at 62, while in the United Kingdom it is set at 60, sometimes even at 55, below which reverse mortgages should not be granted.

Wang et al. (2008) propose a method to transfer and finance the wide range of risks inherent to reverse mortgages from a lender's point of view. The article deals with what the literature calls *crossover* risk, which occurs when the outstanding balance on the loan exceeds the home equity value before the loan is settled. According to the authors, this risk is a combination of other risks, which include the above-mentioned ones of occupation or longevity, interest rates and house prices. An occupation time that surpasses the foreseen time will increase the value of the loan and could surpass the value of the property. The increase in life expectancy in recent decades makes this risk increasingly plausible. On the other hand, and given that the payment of the loan must be covered by the value of the property, an environment with high interest rates and a real estate market in crisis increases the risk of *crossover*. As a consequence of these risks, securitization emerged in the U.S. financial market in the 1970s.

To these risks the authors add other less important ones, but worth taking into account. In particular they mention *maintenance* and *expenses*. The first occurs when the borrower does not carry out the necessary repairs for the correct maintenance of the property, resulting in depreciation. Articles by Miceli and Sirmans (1994), Shiller and Weiss (2000) and Davidoff and Welke (2007) deal with this in detail. The second is a consequence of an increase in expenses associated with the management of reverse mortgages in situations of inflation. An in depth study of the risk of crossover can be found in Chinloy and Megbolugbe (1994).

An important contribution to the literature on reverse mortgages is that of Kutty (1998), who focuses on this product as a way of alleviating poverty, given that the elderly population is poor in earnings but own their homes and, therefore, are "House rich and cash poor". In the Spanish case we have already quoted the work of Blay-Berrueta (2007), which presents the reverse mortgage as an alternative to private insurance for the joint financing of long-term care in Spain. After a detailed survey of long-term care, including a calculation of its cost, the author makes a comparative review of reverse mortgages in the USA, the UK and Spain. With respect to reverse mortgages in Spain he describes and calculates the reverse mortgages already offered by some financial entities and compares their results. Sánchez-Álvarez et al. (2007) also deal with the design of reverse mortgages in the Spanish market. In this paper, we describe in detail the characteristics and calculation method of reverse mortgages designed by the Instituto de Crédito Oficial in Section 5.

3 Construction of projected life tables

In 1992, Lee and Carter proposed their famous model for obtaining dynamic life tables, and studied project mortality rates in the USA. Since then, many authors have used the model to study mortality in other developed countries such as: Canada (Lee and Nault, 1993), Chile (Lee and Rofman, 1994), Japan (Wilmoth, 1996), Belgium (Brouhns et al., 2002), Austria (Carter and Prkawetz, 2001), England and Wales (Renshaw and Haberman, 2003a), Australia (Booth and Tickle, 2003) and Spain (Guillen and Vidiella-i-Anguera, 2005; Debón et al., 2008b).

Lee and Carter (1992) accepted that their model would be better if it were used for projecting aggregate measures like life expectancy rather than fundamental measures like individual age-specific mortality rates. In addition, demographers have traditionally centered their work on the study of life expectancy but, as Booth et al. (2006a) indicate, it is difficult to establish a direct relationship between the precision with which this measure of mortality is predicted or estimated and the relative precision of mortality rates, the measurement that is being modeled. Hence, that author puts special emphasis on indicating that while it is important to achieve precise predictions of life expectancy, the evaluation of error in estimated mortality rates and predictions is essential.

On the other hand, the ever increasing longevity of populations in more developed countries requires research that pays special attention to mortality at advanced ages. Poor fit and predictions for these ages suppose a serious risk for the insurer due to the possibility of not being able to pay the annuities, as well as for the policyholder, who can see the danger of not receiving payment.

The analysis pays special attention to the study of errors in the predictions of both measurements of mortality, by obtaining confidence intervals. In order to simplify the procedure and reduce the computational time as much as possible, the confidence intervals were obtained following a suggestion by

Lee and Carter (1992), who suggest that the main source of uncertainty induced in predicting comes from the temporal component of the model, k_t.

3.1 Lee-Carter's Model

Unlike other models (Heligman and Pollard, 1980; Forfar et al., 1988), designed to graduate static life tables, Lee-Carter's model was developed exclusively for the graduation of dynamic life tables. Since its publication it has enjoyed wide acceptance in the actuarial and demographic world due to its simplicity and good results. The model expresses the measure of mortality as an exponential function that depends on age and time,

$$m_{xt} = \exp(a_x + b_x k_t + \epsilon_{xt}), \tag{1}$$

or equally

$$\ln(m_{xt}) = a_x + b_x k_t + \epsilon_{xt}. \tag{2}$$

With respect to the interpretation of the parameters, it should be noted that:

1. the a_x coefficients describe the average shape of the age profile,
2. the b_x coefficients describe the pattern of deviations from this age profile when the parameter k_t varies $\left(\frac{d \ln m_{xt}}{dt} = b_x \frac{dk_t}{dt}\right)$,
3. the values of k_t represent the trend of mortality throughout the period studied.

The errors ϵ_{xt}, with average zero and variance σ_ϵ^2, reflect historic fluctuations that are not captured by the model.

Expressions (1) and (2) are in fact reduced versions of Lee-Carter's model. The more general form applied to the probability of death q_{xt}, is

$$\ln(q_{xt}) = a_x + \sum_{i=1}^{r} b_x^i k_t^i + \epsilon_{xt}, \tag{3}$$

where r is the range of the matrix $\ln(q_{xt}) - a_x$. Some authors, see Debón et al. (2008b), prefer to model the $logit(q_{xt})$ instead of its logarithm,

$$\text{logit}(q_{xt}) = \ln\left(\frac{q_{xt}}{1 - q_{xt}}\right) = a_x + \sum_{i=1}^{r} b_x^i k_t^i + \epsilon_{xt}. \tag{4}$$

The reasons for this change are two-fold: first is the remark in Lee (2000), where the author points out that nothing ensures that estimations obtained from (3) will not exceed 1, and this problem can be avoided by modelling the *logit* death rates. On the other hand, by adding terms to (1) or (2), interactions between age and time can be better captured, as Booth et al. (2002) and Renshaw and Haberman (2003b) indicate.

The structure of the model is invariable under any of the following transformations of the parameters, $(a_x, b_x^i/c, ck_t^i)$ or $(a_x + cb_x^i, b_x, k_t^i - c)$, $\forall c$, which requires their normalization, $\sum_x b_x^i = 1$ and $\sum_t k_t^i = 0$, in order to get one single solution. The model cannot be fitted by normal regression techniques as the values of the index k_t are not observable. The estimation of parameters in (4) can be carried out by singular value decomposition (SVD) of the matrix $\ln\left(\frac{q_{xt}}{1-q_{xt}}\right) - \hat{a}_x$ (Lee and Carter, 1992), conditional generalized linear models (GLM) (Currie et al., 2004), or the method of maximum likelihood (Brouhns et al., 2002). Details on the estimation of parameters according to these three methods can be found in Debón et al. (2008b). The SVD method normally includes a second stage adjustment to the estimated values of k_t that ensures an equality of observed deaths and predicted deaths within the fitting period. A further method that was proposed by Wilmoth (1993) is weighted least squares.

The prediction of future mortality values requires a last step, the fitting of a time series to the values of the estimated mortality index, $\hat{k}_t$. Substituting the prediction $\hat{k}_{t_n+s}$ in Lee-Carter's model, the $\hat{q}_{x,t_n+s}$, $s = 1, 2, \ldots$, is obtained, by means of

$$\ln\left(\frac{\hat{q}_{x,t_n+s}}{1-\hat{q}_{x,t_n+s}}\right) = a_x + \hat{b}_x \hat{k}_{t_n+s}, \;\; s > 0. \tag{5}$$

An alternative prediction method is proposed by Lee (2000) and Renshaw and Haberman (2003b) using the final data as a starting point. They suggest obtaining projected mortality rates by aligning them with the final crude rates of mortality. To this end, (Renshaw and Haberman, 2006) propose the expression,

$$\ln\left(\frac{\hat{q}_{x,t_n+s}}{1-\hat{q}_{x,t_n+s}}\right) = \ln\left(\frac{\dot{q}_{xt_n}}{1-\dot{q}_{xt_n}}\right) + \hat{b}_x(\hat{k}_{t_n+s} - \hat{k}_{t_n}), \;\; s > 0.$$

Some authors have proposed modifications to this method, including Carter and Lee (1992), Wilmoth (1993) and Lee (2000) himself in an article where he compares his method to other alternatives such as that of McNown and Rogers (1989, 1992). Modifications are also proposed in Booth et al. (2002), Li and Lee (2005) and Debón et al. (2011), these proposing modifications to the Lee-Carter method to forecast mortality for a group of countries, taking into account their membership of a group instead of considering them individually. More recently, Czado et al. (2005) and Pedroza (2006) introduced a Bayesian estimation of the parameters, the latter by means of state-space models.

The principal criticism of the Lee-Carter model is that the parameters a_x and b_x depend only on age and that the prediction of future values of mortality are only based on k_t, which supposes that no interaction exists between age and time. Its advantages are, among others, the easy interpretation of its parameters and its parsimony (Lee, 2000; Booth et al., 2002). The model at present enjoys a considerable popularity due to its good results and to its simplicity, which is why there is a growing literature devoted to it.

3.2 Treatment of ages above 85 years

The crude rates of mortality for advanced ages yield unreliable results due to the low number of subjects exposed to risk. The pattern for advanced and very advanced ages is highly influenced by random fluctuations due to the scarcity of data. Cossette et al. (2007) consider a large number of studies in which demographers and actuaries suggest diverse techniques to calculate and complete the measurement of mortality for advanced ages. Studies by Coale and Guo (1989), Coale and Kisker (1990), Thatcher et al. (1998), Lindbergson (2001) and Thatcher et al. (2002) deserve mentioning due to their great influence. An extensive and complete list of studies referring to this subject can be consulted in Booth (2006). An adaptation of the model proposed by Denuit and Goderniaux (2004) is used here, varying only in the fitting method. The starting point in the original method is a constrained log-quadratic regression model of the form

$$\ln(q_{xt}) = a_t + b_t x + c_t x^2 + \epsilon_{xt}, \tag{6}$$

with ϵ_{xt} i.i.d., $N(0, \sigma^2)$. The model is fitted separately to each calendar year t and for ages $x \geq 75$. We fix two constraints, the first one is $q_{130t} = 1, \forall t$, and the second $\frac{\partial q_x(t)}{\partial x} |_{x=130} = 0$. These two restrictions lead to the following relation between the coefficients,

$$a_t + b_t x + c_t x^2 = c_t (130 - x)^2, \quad \forall t. \tag{7}$$

Substituting (7) in (6) produces the equation

$$\ln(q_{xt}) = c_t(130 - x)^2. \tag{8}$$

The proposed adaptation consists of fitting the previous equation using a *GLM* with *log* link, as we have considered a Binomial distribution for the number of deaths, $D_{xt} \sim Bi(E_{xt}, q_{xt})$. Therefore, we propose a small improvement to the fit of this model that consists of obtaining the estimators of c_t by maximum likelihood. This considerably improves the fit.

4 Application to Spanish mortality data

4.1 Description of data and preliminary analysis

The mortality data for men and women in Spain, corresponding to the period 1980 to 2005 and a range of ages from 0 to 125, are analyzed in this chapter. As indicated in the previous section, the special interest in ages over 99 justifies such a wide range of ages, even at the risk of appearing excessive.

The crude estimations of the mortality rates, q_{xt}, that are the necessary input for dynamic models, were obtained with the procedure used by the Spanish Instituto Nacional de Estadística (INE[2]),

$$\dot{q}_{xt} = \frac{1/2(d_{xt} + d_{x(t+1)})}{P_{xt} + 1/2 d_{xt}}, \tag{9}$$

where d_{xt} is the number of deaths in year t at age x, $d_{x(t+1)}$ is the number of deaths in year $t+1$ at age x, and P_{xt} the population that on December 31^{st} of year t is x years old. Formula (9) can be applied to all ages between 1 and 99 years, but age 0, due to the concentration of deaths in the first months of life, requires an alternative expression,

$$\dot{q}_{0t} = \frac{0.85 d_{0t} + 0.15 d_{0(t+1)}}{P_{0t} + 0.85 d_{0t}}. \tag{10}$$

It can be seen that in both (9) and (10), the denominator is an estimation of E_{xt}, those initially exposed to risk.

The data set with which the subsequent analyses will be carried out is formed by the original values of $\dot{q}_{xt}$ for $x = \{0, 1, \ldots, 85\}$, and for $x =$

[2]National Institute of Statistics, www.ine.es.

$\{86, \ldots, 125\}$, $\dot{q}_{xt}$ from the fit of a quadratic regression in (8). The comparison of both groups of values can be seen in Figures 1 and 2, in which the effect of smoothing in the advanced ages is evident.

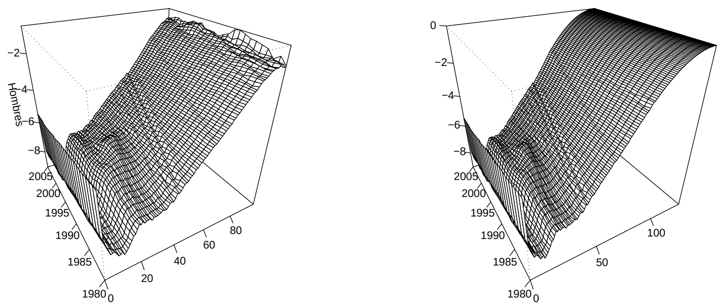


Figure 1: Probabilities of death for men for ages from 0 to 99 (left) and for ages from 0 to 130 (right).

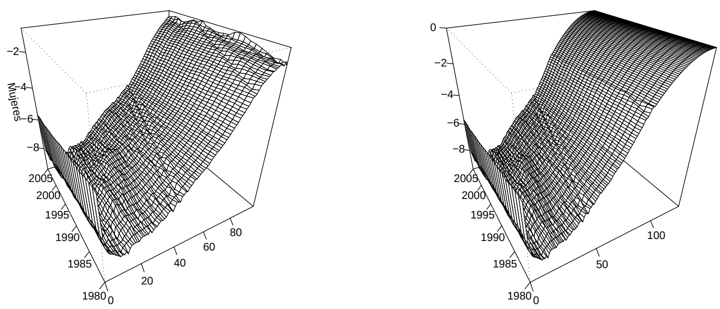


Figure 2: Probabilities of death for women for ages from 0 to 99 (left) and for ages from 0 to 130 (right).

Once the data was completed, a model allowing a sufficiently smooth mortality surface to be obtained, which could also be used to project the long-term probability of death, was designed. Starting from the fitted model, projections of mortality rates for the following 20 years were obtained, that is, from 2006 to 2025.

The data for population and deaths were obtained from the official data produced by the INE. For the period 1991-2005, population figures are published on the INE's web page (`www.ine.es`) for ages from 0 to 100 and beyond. For those pertaining to the previous period, 1980-1991, the range ends at age 85 or more, the remaining ages (from 85 to 100 and beyond) were requested from the INE.

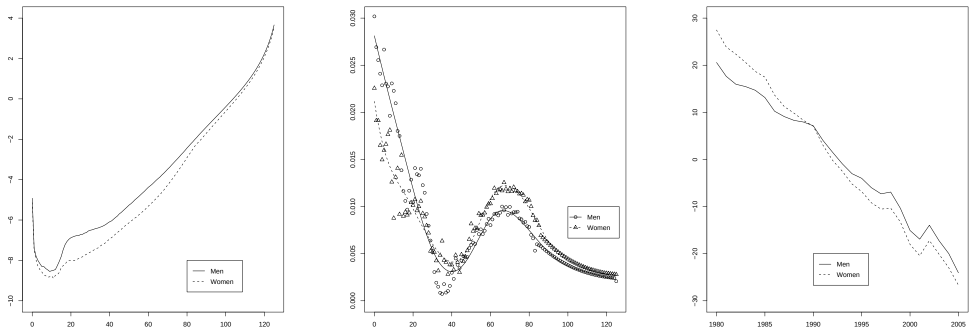
4.2 Application of the Lee-Carter model

Several authors have studied Spanish mortality with dynamic models. Felipe et al. (2002), use Heligman-Pollard's law (Heligman and Pollard, 1980) to evaluate the way in which calendar year (1975-1993) affects mortality patterns in the Spanish population for a range of ages from 0 to 90. Guillen and Vidiella-i-Anguera (2005), use a Poisson log-bilinear version of the Lee-Carter model, proposed by Wilmoth (1993) and Brouhns et al. (2002), to analyze Spanish mortality data corresponding to the period 1975 -1998 and ages from 0 to 105. In Debón et al. (2008b), the authors apply Lee-Carter's model with one or two terms to data for the period 1980-1999 and ages from 0 to 96, improving on the results obtained by the previous authors due to the better adaptability of the model to mortality observed at intermediate ages. Finally, in Debón et al. (2008a), mortality is analyzed from a new perspective by introducing geostatistical techniques, designed for the analysis of spatio-temporal data, which improve on all of the previously obtained results, although increasing the model complexity and computation time. It appears to be a promising new approximation but very complex for use in the context of work of a practical nature. For these same reasons the simplest Lee-Carter model, the one with only one term, was chosen to develop the analysis whose results are summarized below. The R code used for the fit can be found in Debón et al. (2009).

4.2.1 Fitting results

Due to the large number of parameters estimated in this model, $126\times2+26 = 278$ for each sex, the presentation of numerical results would require long tedious tables, which is why it is preferable to present it in the form of a graph in Figure 3, which also has the advantage of making its evolution with age or time easier to understand.

A first general comment regards the different behavior of a_x according



(a) a_x (b) b_x (c) k_t

Figure 3: Estimated Values for Lee-Carter's model.

to gender, Figure 3(a) showing that mortality for women is lower than for men. On the other hand, the increase in mortality that the graph shows for men between 15 and 40 could be explained by deaths due to accidents, which some authors call the *accident hump*. The positive values of b_x at all the ages, see Figure 3(b), indicate that mortality diminishes over time. Nevertheless, some authors (Debón et al., 2008b) have found negative values of b_x for intermediate and advanced ages, which would indicate an increase in mortality with each calendar year. The ages range from 24 to 40, showing a relative increment in mortality that can be explained (Felipe et al., 2002; Guillen and Vidiella-i-Anguera, 2005) by the effect of the AIDS epidemic during the period of time studied. Finally, due to the structure of the data, for yearly series for a range of ages without grouping, the values of b_x show an irregular pattern that makes smoothing necessary to avoid certain anomalies localized in specific elderly groups (Renshaw and Haberman, 2003c,b), an undesirable effect from an actuarial point of view. Cubic splines were used for the smoothing.

For the values for the rate of death k_t, Figure 3(c) shows an obviously decreasing tendency, which is more pronounced for women than men. Tuljapurkar et al. (2000), investigating the G7 countries (Canada, France, Germany, Italy, Japan, UK, and US) find that mortality at each age has declined exponentially at a roughly constant rate in every country over this period. For a more recent multi-country comparison of various stochastic mortality models, see Booth et al. (2006b). Our results are similar to those obtained by these studies.

4.2.2 Goodness-of-fit

In order to carry out a diagnosis of the fitted model, Figures 4 and 5 show the deviance residuals for both sexes against age, calendar year and cohorts.

The residuals would have a true satisfactory behavior if they varied between $[-2, 2]$, which does not happen between ages 20 to 40 for the period 1990-1995 or for the cohorts corresponding to those born between 1950 and 1975. In these intervals the residuals are much greater than in the rest of the graphs. In addition, the better quality of fit for women is evident, which is because the logit of their mortality rates has less variability and a more linear behavior.

4.2.3 Prediction and confidence intervals

For the fit of the time series to the mortality indexes and their forecasts, the application of (5) is required. We used the auto.arima and forecast functions from the R forecast package (Hyndman, 2008), which returns the best ARIMA model according to Akaike information criterion (AIC). AIC is a measure of the relative goodness-of-fit of a statistical model. Hence, AIC not only rewards goodness-of-fit, but also includes a penalty that is an increasing function of the number of estimated parameters. The best-suited models turned out to be $ARIMA(2, 1, 1)$ for men and $ARIMA(0, 1, 0)$ for women. The general expression for an $ARIMA(p, d, q)$ is,

$$\phi(B)(1 - B)^d \hat{k}_t = p + \theta(B)u_t,$$

where $\phi(B)$ and $\theta(B)$ are polynomials in B with p and q degrees, respectively, B is the backward shift operator and u_t is a white noise. So, for $ARIMA(2, 1, 1)$ we have

$$\hat{k}_t + (\phi_1 - 1)\hat{k}_{t-1} + (\phi_1 - \phi_2)\hat{k}_{t-2} + \phi_3\hat{k}_{t-3} = p + u_t - \theta_1 u_{t-1},$$

and for $ARIMA(0, 1, 0)$

$$\hat{k}_t - \hat{k}_{t-1} = p + u_t.$$

The values of k_t were predicted for $t = 2006, \ldots, 2025$ and once substituted in (5) they provide the logit(q_{xt}) projections. The projection surfaces obtained for men and women are shown in Figure 6.

Confidence intervals should be obtained to measure the uncertainty of the mortality forecasts (Pedroza, 2006). Koissi et al. (2006) and Brouhns et al. (2005) describe two possible methods to obtain confidence intervals, the first by means of parametric bootstrap techniques and the second by nonparametric techniques. Later, Renshaw and Haberman (2008), who point out the problems with the Monte Carlo approach, reviewed the three approaches suggested in the literature to boot-strapping. The narrow amplitude of the confidence intervals obtained attracts the attention of the aforementioned authors who provide some explanations for this fact. Li et al. (2009) attribute the phenomenon to the rigidity of the structure of the Lee-Carter model, and to avoid it they relax the structure by incorporating the heterogeneity of each age-period cell. Therefore, as the confidence intervals obtained by means of bootstrap techniques are computationally more costly, we resorted to the method originally proposed in Lee and Carter (1992). Those authors proposed obtaining the intervals from the prediction errors for the k_t parameters projected by the $ARIMA$ models. This approximation assumes that the principal source of uncertainty is k_t.

Mortality predictions need to be accompanied by prediction intervals for the estimations obtained. One way to combine all sources of uncertainty is to use bootstrapping procedures as Brouhns et al. (2005) and Koissi et al. (2006) do. In the case of Spain, this method was used by Debón et al. (2008b), who obtained prediction intervals for the predictions provided by the Lee-Carter model with one or two terms. Parametric and non-parametric bootstrap techniques are used, in both cases based on the binomial distribution assumption, as distinct from the work by Brouhns et al. (2005), Koissi et al. (2006) and Renshaw and Haberman (2008), who employ the Poisson distribution. Another difference to point out is the residuals sampled in the non-parametric case. While Debón et al. (2008b) sample over the logit residuals given, Koissi et al. (2006) and Renshaw and Haberman (2008) sample over the deviance residuals. The narrow prediction intervals obtained by classical bootstrap techniques have attracted the attention of other researchers in this field (Lee and Carter, 1992; Lee, 2000; Booth et al., 2002; Koissi et al., 2006), who provide different explanations. In the case of spatial (two-dimensions: age, time) dependence of residuals, ordinary bootstrap is not valid (Liu and Braun, 2010). Therefore, Liu and Braun (2010) propose prediction intervals by using a residual-based block-bootstrap. More recently, D'Amato et al. (2012) have introduced a tailor-made bootstrap instead of an ordinary bootstrap to improve the methodology for forecasting mortality in order to enhance model performance and increase forecasting power by capturing the dependence structure of neighboring observations in the population.

4.2.4 Prediction of q_{xt} for the period 2006-2025

The fit of the model for 1980 to 2005 and the forecasts obtained for the probabilities of death from 2006 to 2025 are shown in Figure 7. To facilitate interpretation only some of the ages, the five ten-year periods from 10 to 50, are shown. The figure shows that the model presents reasonable future tendencies without inconsistencies for the ages shown and also for the rest of the ages, meaning that the decreasing tendency predicted for some ages does not cross the predictions for higher ages. In part, this good behavior is due to the smoothing of the estimated values of b_x, which was referred to before. Figure 7 shows the confidence intervals of $\text{logit}(q_{xt})$ for predictions beyond 2005.

In the case of advanced ages, which for the reasons mentioned before are of greater interest, five year periods between 70 and 95 years, both inclusive, are shown. Figure 8 shows that all of them present unremarkable future tendencies. The deceleration in the probability of death as the years pass can be explained by the selective survival of healthier individuals to older ages (Horiuchi and Wilmoth, 1998).

The narrow amplitude of the confidence intervals should be emphasized, even if the ones shown here have been obtained following the method proposed by Lee and Carter. This fact is more evident in the case of men, as the fluctuations of male mortality for the ages in the accident hump in the period of time considered are difficult to perceive, such that the parameters b_x reflect little decrease in mortality rates, thus reducing the amplitude of intervals given by $b_x(k_t^{sup} - k_t^{inf})$. In the case of women the evolution of mortality is more linear and homogeneous, which implies greater values for b_x (see Figure 3(b)) and wider intervals. This, which seems contrary to logic, is a consequence of not having incorporated the uncertainty of the estimations for b_x into the calculation of confidence intervals, which constitutes a weakness of the method, and calls for the search for alternatives. An additional reason to explain the narrow amplitude of confidence intervals should be sought in the process of smoothing used for ages over 85 years old, which eliminates fluctuations that result in low values for b_x. Therefore, the use of bootstrapping is recommended to capture parameter error better.

It should be noted that Delwarde et al. (2007) propose a version of Lee-Carter's model whose parameters are estimated by means of a penalized log-likelihood, directly obtaining smoothed estimations for b_x.

4.2.5 Prediction of e_{xt} for the period 2006-2025

An indicator of the evolution of mortality over time, widely used by actuaries, is the residual life expectancy at age x in year t, e_{xt}, whose expression is given by

$$e_{xt} = \frac{T_{xt}}{l_{xt}},$$

where l_{xt} and T_{xt} represent, respectively, the total number of people that have reached x years in the calendar year t and the sum of years that all of them expect to live. Its interest is evident in financial and actuarial contingent claims that depend on the remaining age of the party contracting them. Reverse mortgages are a paradigmatic example of such contracts.

Figure 9 shows the life expectancy for ages 70, 75, 80, 85 and 90, estimated in the period 1980-2005 and projected for the period 2006-2025, as well as their confidence intervals.

The increase in life expectancy over time, clear for both sexes, is greater for women. The narrow amplitude of the confidence intervals is also clear here, and is more acute for women for the reasons outlined in the previous section. Debón et al. (2008b) calculated confidence intervals by applying parametric and nonparametric bootstrap techniques, obtaining very similar results.

Life expectancy at age x, e_{xt}, can be obtained for a specific period t or for the corresponding cohort $t - x$, definitions being available in Denuit (2007). Computation of life expectancy and annuities by cohort generates greater risk than computation by period, manifested in the respective wider confidence and prediction intervals (Renshaw and Haberman, 2008). In this paper we use period-based definitions and calculations.

5 ICO program for reverse mortgages

A reverse mortgage is a loan used to release the home equity of the main residence in order to generate an income for the beneficiary over an established period of time. In Spain it is regulated in a generic form by Law 41/2007.

The reverse mortgage model proposed by the ICO is more restrictive than the generic model regulated by this law, as it is a model designed for the disabled or people over 70 years of age who receive a low monthly income. This mode of reverse mortgage is a complement to Law 39/2006, known as the Ley de Dependencia (Long-Term Care Law). Its principal characteristics are described below:

The *borrowers* must have resided in Spain for at least the previous 5 years, with an age equal to or above 70 and who are owners of their principal residence, which can be mortgaged.

The *principal limit* of the loan is 70% of the present appraisal value of the property plus the premium corresponding to life insurance.

The *expected duration* of the operation is $e_{xt} + 5$, e_{xt} being the residual life expectancy of the borrower.

The *maximum monthly income* for the borrower is calculated by equaling the present net value of the monthly income during the residual life expectancy, e_{xt}, with the financing granted, with a monthly limit of 2,000 Euros. This monthly income is increased every year by a percentage of approximately $r = 3\%$.

The borrower is obliged to subscribe to:

- An *insurance* policy, as a single premium, that covers the monthly income in case of survival beyond the expected duration of the operation, as well as the interest in favor of the financial entity for the outstanding of the account.
- A *multi-risk home insurance* policy for the property under guarantee.

The mathematical analysis of the financial operation requires the calculation of all the terms involved by application of the equivalence principal. This supposes that the total loan has to be equal to all the earned income, both being evaluated at the same time, in this case at the beginning of the contract. That is,

$$V_A(1+i)^n = V_T + G,$$

where

V_A is the present value of the property,

V_T is the appraised value of the property,

G is the total sum of initial expenses which, given the conditions of the program, are estimated as $1,06\%$ of V_T,

i is the annual interest,

n is the duration of the contract in years, whose expected value is $n = e_{xt} + 5$.

The total sum of the actual loan granted, P_R, from which the value of the monthly income to be received is determined for the financial part of the operation, whose duration is n years, as well as the actuarial part. The lifelong income if n years are exceeded, is

$$P_R = V_A - G - U_I, \tag{11}$$

where U_I represents the single premium of interests in favor of the borrowing entity in the case of survival of the beneficiary beyond the foreseen n years. It is a constant deferred monthly income whose value is obtained with the expression

$$U_I = 12 V_T i_m \left(\frac{N_{x+n+1}}{D_x} + \frac{11}{24} \frac{D_{x+n}}{D_x} \right),$$

where i_m is the monthly financial interest rate,

$$i_m = (1+i)^{1/12} - 1.$$

The usual mathematics in this type of financial-actuarial operation lead to the following expression for y, the monthly sum to be received in the first year by the borrower and updated annually with interest r

$$y = \frac{P_R}{a_{\overline{12}|i_m} \frac{(1+i)[1-r^n(1+i)^{-n}]}{1+i-r} + 12 r^{n-1} \left(\frac{N^*_{x+n+1}}{D^*_x} + \frac{11}{24} \frac{D^*_{x+n}}{D^*_x} \right)},$$

where the symbols with an * are calculated with the technical interest $i_2 = (1+i-r)/r$, and

$$a_{\overline{12}|i_m} = \frac{1 - (1+i_m)^{-12}}{i_m}.$$

6 A reverse mortgage simulator

As a complement to what has been set out in the previous sections, the authors have developed a computer tool. It permits the simulation of the most relevant data of a reverse mortgage in accordance with the method developed by the ICO, starting from the personal characteristics of a theoretical borrower. The package also provides the fitted dynamic tables for Spanish mortality using the data corresponding to the period 1980-2005, the most recent published by the Instituto Nacional de Estadística, for ages 0 to 125, as well as its projection over the next 20 years.

The data to be introduced for the simulation of the monthly sums for the reverse mortgage are:

the *year* of inception of the reverse mortgage in a range between 2007 and 2026,

the *age* and *sex* of the borrower,

the *financial* and *actuarial interest*,

the appraised *value of the property* and, optionally,

the *initial expenses*, as a percentage, and *the yearly growth rate*, fixed by default as $1,06\%$ and 3%, respectively.

The result of the application is a table, which includes, for all the years of the mortgage's duration, the value of the monthly income to be received every month, the multi-risk insurance premium and the resulting net income. Incomes and premiums are updated year by year according to the rate of growth that has been fixed. Table 1 shows the effect of changes in financial and interest rates on the net income by means of a simulation of a mortgage contracted in 2010 for a 70 year old man, and a property with an appraised valued of 100,000 Euros, for yearly financial rates of 8, 6 and 4% and actuarial interest rates of 3.5, 2.5 and 1.5%.

7 Conclusions

As a final comment, it is worth highlighting three distinctive features of the methodology presented here in relation to the work of other authors.

	financial interest 8%			financial interest 6%			financial interest 4%		
year	3.5	2.5	1.5	3.5	2.5	1.5	3.5	2.5	1.5
1	98.68	90.41	80.57	139.63	132.68	124.32	185.31	179.47	172.41
2	101.64	93.13	82.98	143.82	136.66	128.05	190.87	184.86	177.58
3	104.69	95.92	85.47	148.14	140.76	131.89	196.59	190.40	182.91
4	107.83	98.80	88.04	152.58	144.98	135.85	202.49	196.12	188.39
5	111.06	101.76	90.68	157.16	149.33	139.92	208.57	202.00	194.04
6	114.39	104.81	93.40	161.87	153.81	144.12	214.82	208.06	199.87
7	117.83	107.96	96.20	166.73	158.42	148.44	221.27	214.30	205.86
8	121.36	111.20	99.09	171.73	163.18	152.90	227.91	220.73	212.04
9	125.00	114.53	102.06	176.88	168.07	157.48	234.74	227.35	218.40
10	128.75	117.97	105.12	182.19	173.11	162.21	241.79	234.17	224.95
11	132.61	121.51	108.28	187.66	178.31	167.07	249.04	241.20	231.70
12	136.59	125.15	111.52	193.29	183.66	172.09	256.51	248.44	238.65
13	140.69	128.91	114.87	199.08	189.17	177.25	264.21	255.89	245.81
14	144.91	132.77	118.32	205.06	194.84	182.57	272.13	263.56	253.18
15	149.26	136.76	121.87	211.21	200.69	188.04	280.30	271.47	260.78
16	153.74	140.86	125.52	217.54	206.71	193.68	288.71	279.62	268.60
17	158.35	145.09	129.29	224.07	212.91	199.49	297.37	288.00	276.66
18	163.10	149.44	133.17	230.79	219.29	205.48	306.29	296.64	284.96
19	167.99	153.92	137.16	237.72	225.87	211.64	315.48	305.54	293.51
20	173.03	158.54	141.28	244.85	232.65	217.99	324.94	314.71	302.32

Table 1: Simulation of a reverse mortgage under different financial and actuarial interest rates

1. Our methodology allows us to obtain estimations of q_{xt} for the range of ages 0-130 by maximum likelihood.

2. As far as we know, the Lee-Carter model has not been used in the graduation of Spanish mortality data to the full range of ages 0-125.

3. The models used in this paper have been fitted for the full range of ages. Many authors achieve better fits by eliminating the early ages, which they justify by arguing that actuarial operations begin at a more advanced age. Contrary to this criterion, we have decided to include them as we consider that their influence on the fit should not be underestimated.

Our model for pricing reverse mortgages is of additional interest for being applicable to mortality data for a wide range of ages from other countries.

A future line of work would be a comparative study of simulation strategies for assessing risk in mortality rate predictions and associated estimates of life expectancy and annuity values in both period and cohort frameworks. The previous methods for the calculation of a reverse mortgage (Table 1) are sensitive to the particular choice of parameters. The work by Renshaw and Haberman (2008) go in this direction, quantifying the effect of mortality projections on life expectancy and annuity values through the computation of prediction intervals. Another possibility is to answer, in the context of Spain, the Davidoff (2012) question: *Can "High Costs" Justify Weak Demand for the Home Equity Conversion Mortgage?*

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References

Blay-Berrueta, D. (2007). *Sistemas de cofinaciaciación de la dependencia: seguro privado frente a hipoteca inversa*. Cuadernos de la Fundación, Fundación Mapfre Estudios, Madrid.

Booth, H. (2006). Demographic forecasting: 1980 to 2005 in review. *International Journal of Forecasting*, 22(3):547–582.

Booth, H., Hyndman, R., Tickle, L., and de Jong, P. (2006a). Lee-Carter mortality forecasting: a multi-country comparison of variants and extensions. *Demographic Research*, 15(9):289–310.

Booth, H., Hyndman, R., Tickle, L., and de Jong, P. (2006b). Lee-Carter mortality forecasting: a multi-country comparison of variants and extensions. *Demographic Research*, 15(9):289–310.

Booth, H., Maindonald, J., and Smith, L. (2002). Applying Lee-Carter under conditions of variable mortality decline. *Population Studies*, 56(3):325–336.

Booth, H. and Tickle, L. (2003). The future aged: new projections of Australia's ederly population. *Population Studies*, 22(4):38–44.

Brouhns, N., Denuit, M., and Keilegom, I. V. (2005). Bootstrapping Poisson log-bilinear model for mortality forecasting. *Scandinavian Actuarial Journal*, 2005(3):212–224.

Brouhns, N., Denuit, M., and Vermunt, J. (2002). A Poisson log-bilinear regression approach to the construction of projected lifetables. *Insurance: Mathematics & Economics*, 31(3):373–393.

Carter, L. and Lee, R. (1992). Modeling and forecasting U. S. sex differentials in mortality. *International Journal of Forecasting*, 8(3):393–411.

Carter, L. and Prkawetz, A. (2001). Examining structural shifs in mortality using the Lee-Carter method. Mpidr wp 2001-007, Center for Demography and Ecology Information, University of Wisconsin-Madison.

Chinloy, P. and Megbolugbe, I. (1994). Reverse mortgages: contracting and crossover. *Journal of the American Real Estate and Urban Economics Association*, 22(2):367–386.

Coale, A. and Guo, G. (1989). Revisited regional model life tables at very low levels of mortality. *Population Index*, 55:613–643.

Coale, A. and Kisker, E. (1990). Defects in data old age mortality in the United States: New procedures for calculating approximately accurate mortality schedules and lifes tables at the highest ages. *Asian and Pacific Population Forum*, 4:1–31.

Cossette, H., Delwarde, A., Denuit, M., Guillot, F., and Étienne Marceau (2007). Pension plan valuation and mortality projection: A case study with mortality data. *North American Actuarial Journal*, 11(2):1–34.

Costa-Font, J. (2009). Ageing in place? exploring elderly people's housing preferences in Spain. *Urban studies*, 46(2):295–316.

Costa-Font, J. (2012). Housing-related well-being in older people: the impact of environmental and financial influences. *Urban studies*, -(-):1–17.

Currie, I., Kirkby, J., Durban, M., and Eilers, P. (2004). Smooth Lee-Carter models and beyond. In *Workshop on Lee-Carter Methods*, www.ma.hw.ac.uk iain/workshop/workshop.html (4th March 2005).

Czado, C., Delwarde, A., and Denuit, M. (2005). Bayesian Poisson log-bilinear mortality projections. *Insurance: Mathematics & Economics*, 36(3):260–284.

D'Amato, V., Haberman, S., Piscopo, G., and Russolillo, M. (2012). Modelling dependent data for longevity projections. *Insurance: Mathematics and Economics*, 51(3):694 – 701.

Davidoff, T. (2012). Can 'high costs' justify weak demand for the home equity conversion mortgage? Technical report, Available at SSRN.

Davidoff, T. and Welke, G. (2007). Selection and moral hazard in the reverse mortgage market. Technical report, Haas School of Business, UC Berkeley.

Debón, A., Montes, F., and Martínez-Ruiz, F. (2011). Statistical methods to compare mortality for a group with non-divergent populations: an application to spanish regions. *European Actuarial Journal*, 1:291–308.

Debón, A., Montes, F., and Sala, R. (2009). *Tablas de mortalidad dinámicas. Una aplicación a la hipoteca inversa en España*. Fundación ICO. Publicaciones de la Universitat de València, valència.

Debón, A., Montes, F., Mateu, J., Porcu, E., and Bevilacqua, M. (2008a). Modelling residuals dependence in dymanic life tables. *Computational Statistics and Data Analysis*, 52(3):3128–3147.

Debón, A., Montes, F., and Puig, F. (2008b). Modelling and forecasting mortality in Spain. *European Journal of Operation Research*, 189(3):624–637.

Delwarde, A., Denuit, M., and Eilers, P. (2007). Smoothing the Lee-Carter and poisson log-bilinear models for mortality forecasting: a penalized log-likelihood approach. *Statistical Modelling*, 7(1):29–48.

Denuit, M. (2007). Distribution of the random future life expectancies in log-bilinear mortality projections models. *Lifetime Data Analysis*, 13(3):381–397.

Denuit, M. and Goderniaux, A. (2004). Closing and projecting lifetables using log-linear models. *Mitteilungen. der Schweizerischen Aktuarvereingung*, 1:29–49.

Felipe, A., Guillén, M., and Pérez-Marín, A. (2002). Recent mortality trends in the Spanish population. *British Actuarial Journal*, 8(4):757–786.

Forfar, D., McCutcheon, J., and Wilkie, A. (1988). On graduation by mathematical formula. *Journal of the Institute of Actuaries*, 115 part I(459):1–149.

Guillen, M. and Vidiella-i-Anguera, A. (2005). Forecasting Spanish natural life expectancy. *Risk Analysis*, 25(5):1161–1170.

Heligman, L. and Pollard, J. (1980). The age pattern of mortality. *Journal of the Institute of Actuaries*, 107:49–80.

Herranz-Gonzalez, R. (2006). Hipoteca inversa y figuras afines. Informes Portal Mayores 49, IMSERSO, Madrid, http://www.imsersomayores.csic.es/documentos/documentos/herranz-hipoteca-01.pdf.

Horiuchi, S. and Wilmoth, J. (1998). Decelaration in the age pattern of mortality at older ages. *Demography*, 35:391–412.

Hyndman, R. J. (2008). *forecast: Forecasting functions for time series*. R package version 1.11.

Koissi, M., Shapiro, A., and Högnäs, G. (2006). Evaluating and extending the Lee-Carter model for mortality forecasting confidence interval. *Insurance: Mathematics & Economics*, 38(1):1–20.

Kutty, N. (1998). The scope for poverty alleviation among elderly homeowners in the United States through reverse mortgages. *Urban studies*, 35(1):113–129.

Lee, R. (2000). The Lee-Carter method for forecasting mortality, with various extensions and applications. *North American Actuarial Journal*, 4(1):80–91.

Lee, R. and Carter, L. (1992). Modelling and forecasting U. S. mortality. *Journal of the American Statistical Association*, 87(419):659–671.

Lee, R. and Nault, F. (1993). Modeling and forecasting provincial mortality in Canada. Montreal. World Congress of the International Union for Scientific Study of Population.

Lee, R. and Rofman, R. (1994). Modelación y proyección de la mortalidad en Chile. *Notas Población*, 22(59):182–213.

Li, N. and Lee, R. (2005). Coherent mortality forecast for a group of populations: an extension of the Lee-Carter method. *Demography*, 42(3):575–593.

Li, S.-H., Hardy, M., and Tan, K. (2009). Uncertainty in mortality forecasting: an extensión to the classical Lee-Carter approach. *Astin Bulletin*, 31:137–164.

Lindbergson, M. (2001). Mortality among the elderly in Sweden. *Scandinavian Actuarial Journal*, (1):79–94.

Liu, X. and Braun, W. J. (2010). Investigating mortality uncertainty using the block bootstrap. *Journal of Probability and Statistics*, 2010:385–399.

McNown, R. and Rogers, A. (1989). Forecasting mortality: A parametrized time series aproach. *Demography*, 26(4):645–660.

McNown, R. and Rogers, A. (1992). Forecasting cause-specific mortality using time series methods. *International Journal of Forecasting*, 8(3):413–432.

Miceli, T. and Sirmans, C. (1994). Reverse mortgages and borrower maintenance risk. *Journal of the American Real Estate and Urban Economics Association*, 22(2):433–450.

Pedroza, C. (2006). A bayesian forecasting model: predicting U.S. male mortality. *Biostatistics*, 7(4):530–550.

Renshaw, A. and Haberman, S. (2003a). Lee-Carter mortality forecasting: a parallel generalized linear modelling aproach for England and Wales mortality projections. *Journal of the Royal Statistical Society C*, 52(1):119–137.

Renshaw, A. and Haberman, S. (2003b). Lee-Carter mortality forecasting with age specific enhancement. *Insurance: Mathematics & Economics*, 33(2):255–272.

Renshaw, A. and Haberman, S. (2003c). On the forecasting of mortality reduction factors. *Insurance: Mathematics & Economics*, 32(3):379–401.

Renshaw, A. and Haberman, S. (2006). A cohort-based extension to the Lee-Carter model for mortality reduction factors. *Insurance: Mathematics & Economics*, 38(3):556–570.

Renshaw, A. and Haberman, S. (2008). On simulation-based approaches to risk measurement in mortality with specific reference to poisson Lee-Carter modelling. *Insurance: Mathematics & Economics*, 42(2):797–816.

Shiller, R. and Weiss, A. (2000). Moral hazard in home equity conversion. *Real Estate Economics*, 28(1):1–31.

Skarr, D. (2008). Financial planner's guide to the FHA insured home equity conversion mortgage. *Journal of Financial Planning*, 21(5):68–75.

Sánchez-Álvarez, I., López-Ares, S., and Quiroga-García, R. (2007). Diseño de hipotecas inversas en el mercado español. Proyecto 205/05 3, Instituto de Mayores y Servicios Sociales.

Taffin, C. (2006). La hipoteca inversa o vitalicia. Informes externos, Asociación Hipotecaria Española.

Thatcher, A., Kannisto, V., and Andreev, K. (2002). The survivor ratio method for estimating numbers at high ages. *Demographic Research*, 6(1):1–18.

Thatcher, A., Kannisto, V., and Vaupel, J. (1998). *The force of mortality at ages 80 to 120*. Odense, Denmark: Odense University Press.

Tuljapurkar, S., Li, N., and Boe, C. (2000). A universal pattern of mortality decline in the G7 countries. *Nature*, 405(6788):789–792.

Wang, L., Valdez, E., and Piggott, J. (2008). Securization of longevity risk in reverse mortgages. *North American Actuarial Journal*, 12(4):345–370.

Wilmoth, J. (1993). Computational methods for fitting and extrapolating the Lee-Carter model of mortality change. Technical report, Departament of Demography, University of California, Berkeley.

Wilmoth, J. (1996). *Health and Mortality among Elderly Populations*, chapter Mortality projections for Japan: A comparison of four methods, pages 266–287. Oxford University Press.

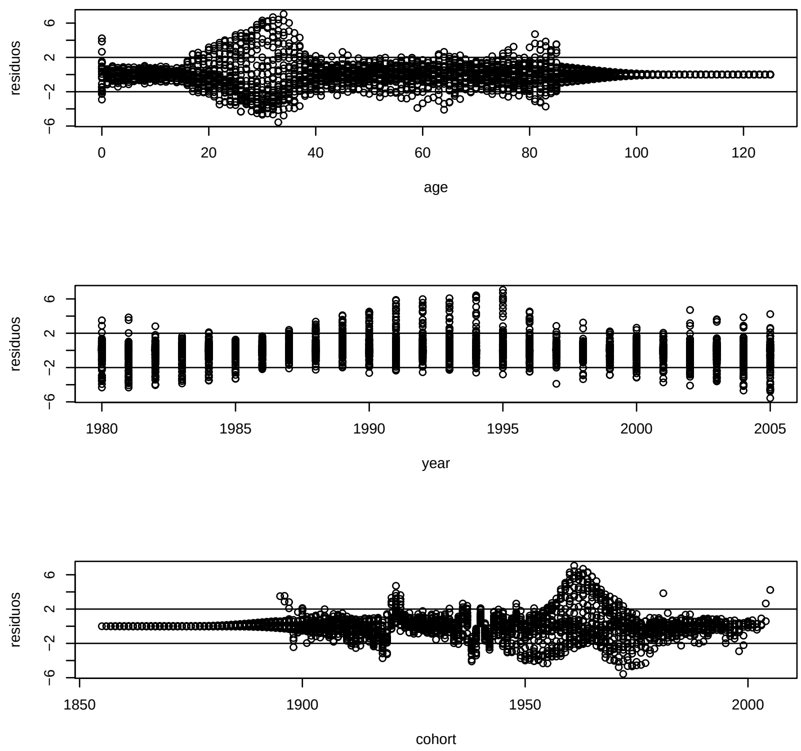

Figure 4: Deviance residuals for men.

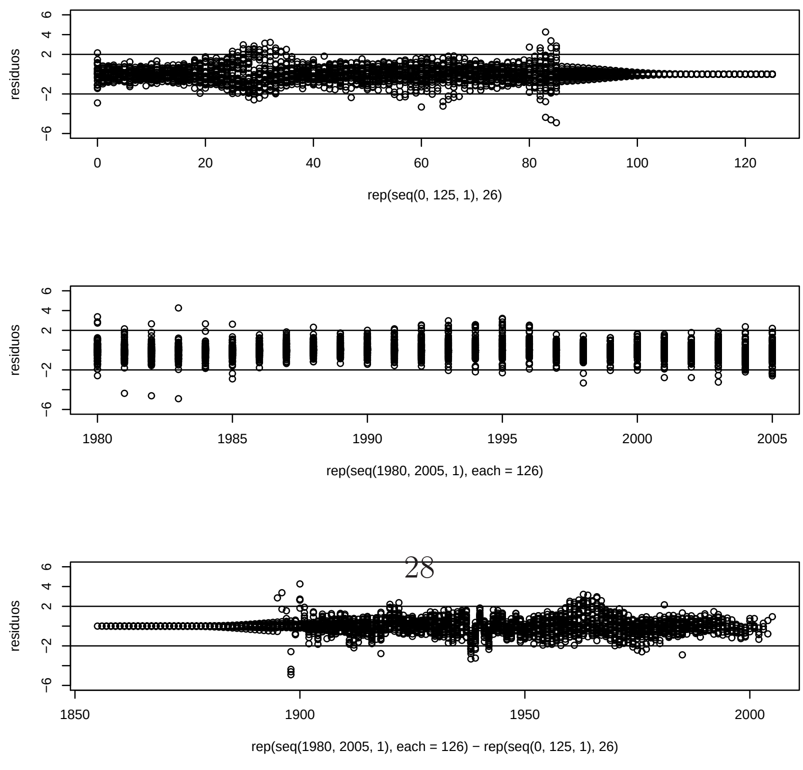

Figure 5: Deviance residuals for women.

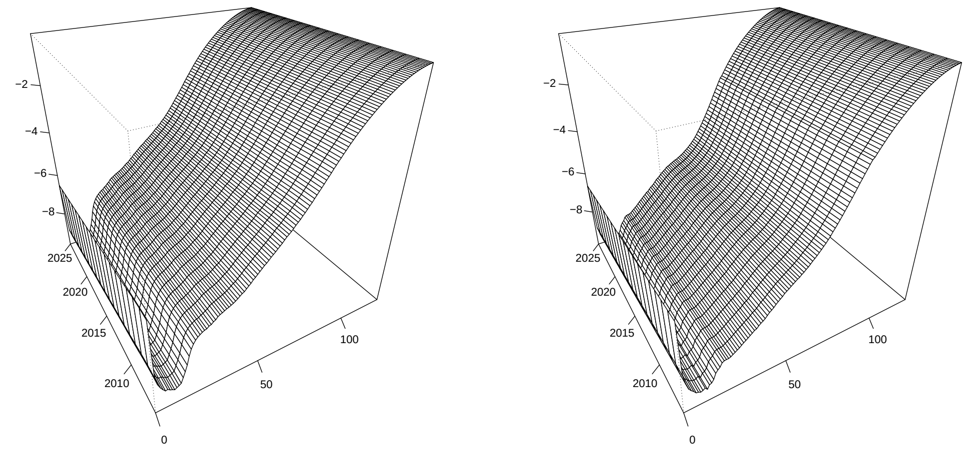

Figure 6: Projections for the period 2006-2025.

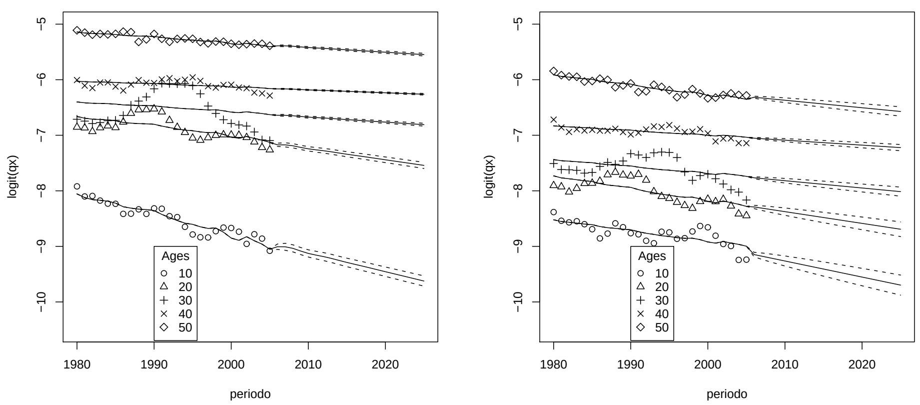

Figure 7: Predictions for certain age groups for men (left) and women (right).

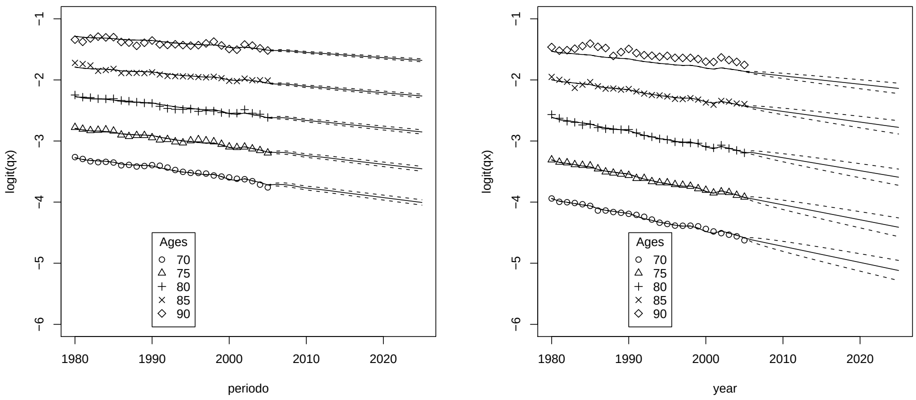

Figure 8: Predictions for advanced ages for men (left) and women (right).

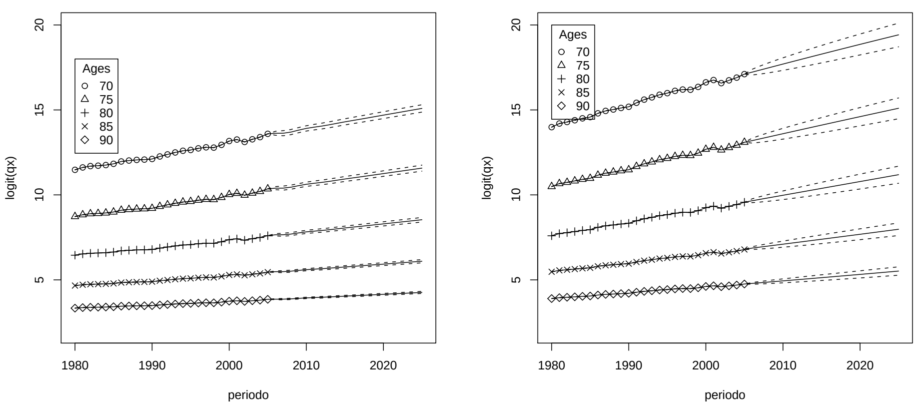

Figure 9: Residual life expectancy for advanced ages for men (left) and women (right).