Convex Combination Filtered-X Algorithm for Multichannel Active Noise Control

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***Abstract*—Adaptive filtering schemes exhibit a compromise between convergence speed and steady-state mean square error. Convex combination approaches that provide meaningful performance have been recently developed for system identification. The purpose of this work is to apply the convex combination strategy to multichannel active noise control systems, taking into account the secondary path between the adaptive filter output and the error sensor and the eventual unavailability of the disturbance signal, which depends on the filtering scheme considered. Even though this strategy involves a computational burden higher than the classic adaptive filters, it exhibits optimum performance in term of convergence speed and steady-state mean square error.**

I. Introduction

The existing diversity of developed adaptive algorithms for sound control applications is justified by the different performances that they can offer. It is well known that adaptive filter performance is mainly a compromise between convergence speed, steady-state behavior, and computational complexity. Generally speaking, algorithms with high convergence speed give large mean square error (MSE) at steady-state. In contrast, algorithms with good properties at steady-state show slow convergence speed. Therefore, it seems interesting to combine the good performance of two algorithms that offer complementary capabilities (high convergence speed and good steady-state MSE) in order to obtain a unique algorithm that would provide optimum performance in the above aspects. These algorithms would not have to be different, but simply set to different configuration parameters. This idea has motivated the development of strategies for combining two or more algorithms like the convex combination strategy introduced in [1].

The aim of the convex combination approach is to combine two algorithms with complementary capabilities so that the overall performance of the global adaptive filter is better than the performance of each algorithm working separately [1]. This complementarity requires the parallel combination of two algorithms so this approach involves a computational burden at least double that of the classic adaptive filters. The convex combination of simple adaptive algorithms, such as the considered least mean squares (LMS) algorithms, would require a computational cost similar to more complex adaptive filters. However, even in real-time applications, they are easily implemented in common hardware platforms. Although computational cost is an important issue in real-time applications, our work mainly focuses on the improvement of adaptive filters performance for multichannel active control. So, this work describes how to combine two LMS algorithms, one with a high convergence parameter μ and the other with a low one, both constrained to the range of μ values to avoid possible instabilities. Any other algorithm can be used instead of LMS, but it is robust and the simplest. Convex combination approaches can be applied in real-time systems that require good performance such as multichannel sound systems and more specifically to active noise control (ANC) [2] systems. ANC remains a topic of interest and a considerable number of publications deal with real-time systems based on this noise reduction technique [3]. ANC applications that require a control system that follow changes are common [4], [5]. Hence, the parallel combination scheme can be extrapolated to an ANC system by using the different existing filtering structures [6] and providing an algorithm that either quickly converges or easily readjusts with the plant or the input signal, and reaches the steady-state with a small MSE. It should also be noted that other strategies such as the variable step size algorithms [7] achieve a good final error in steady-state without penalizing the convergence speed of the algorithm. Moreover their computational cost is moderate in contrast with the convex combination strategy studied where two algorithms have to be running simultaneously. { However, this kind of algorithms do no improve the performance of convex combination [8], [9] and normally introduce several parameters, and some a priori knowledge about the statistics of the filtering scenario (as the SNR, in example) is needed for appropriately tuning them. } In any case, the main goal of this work is to extend the convex combination approach already proposed and successfully evaluated for adaptive identification systems, to ANC systems. {Si bien en [10] ya se abordó parte del problema presentando un sistema de ANC monocanal basado en la estructura modificada de filtrado-x, en este artículo se describe como usar la combinación convex para un sistema generico de ANC multicanal usando cualquiera de las estructuras habituales de ANC. { Although the algorithms proposed in the present paper have a significant computational cost, an optimum real-time implementation based on new hardware facilities such as multicore processors [11] would allow to exploit the inherent parallelism of the convex strategy proposed.

This paper is organized as follows. Section II briefly describes the convex combination of adaptive filters and con-

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cretely the convex combination of two LMS algorithms. In Section III, a detailed description of the application of convex strategies to the main types of filtering schemes for single ANC is provided. This study is extended to a multichannel ANC system in Section IV. Finally, Section V presents some experimental results that validate the convex approaches introduced. Conclusions are given in Section VI.

II. Convex combination of adaptive filters

In a classic adaptive filter, the target is to minimize a cost function dependent on the desired signal $d(n)$ and on the input signal $x(n)$ that feeds the adaptive filter (see Fig. 1).

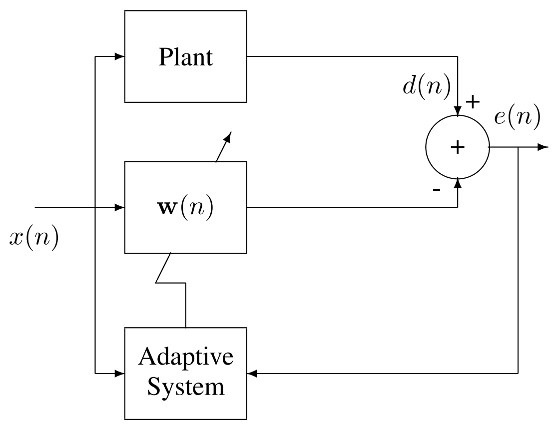

Fig. 1. Block diagram of an adaptive system for system identification.

In a convex combination scheme, we use two adaptive filters whose single outputs (see $y_1(n)$ and $y_2(n)$ in Fig. 2) are suitably combined in order to obtain the output of the parallel filter as the weighted sum of the single outputs,

$$y(n) = \lambda(n)y_1(n) + [1-\lambda(n)]y_2(n), \tag{1}$$

where $\lambda(n)$ is a mixing parameter in the range $(0,1)$. This parameter controls the combination of the two filters at each iteration, and comes from

$$\lambda(n) = \frac{1}{1+e^{-a(n)}}, \tag{2}$$

where $a(n)$ is updated in order to minimize the instantaneous square error of the overall filter, $J(n) = e(n)^2 = [d(n) - y(n)]^2$, by using the gradient descent method [1]. Thus $a(n)$ is given by the following adaptation rule,

$$a(n+1) = a(n) + \mu_a e(n)[e_2(n) - e_1(n)]\lambda(n)[1-\lambda(n)], \tag{3}$$

where $e_1(n)$ and $e_2(n)$ are the output error signals of the component filters, and μ_a is a step-size parameter that controls changes in $a(n)$ from one iteration to the next. { Puesto que la elección del parámetro μ_a puede resultar crítico, podemos normalizar dicha constante de paso para independizar el comportamiento del algoritmo de la SNR, [9]. Así, el algoritmo resulta más robusto cuando normalizamos esta contante de paso de igual forma que se realiza en el algoritmo NLMS. En este caso, la señal $e_2(n) - e_1(n)$ haría las veces de la señal de entrada y la Eq. (3) quedaría, según [9] como:}

$$a(n+1) = a(n) + \frac{\mu_a}{p(n)}\mu_a e(n)[e_2(n) - e_1(n)]\lambda(n)[1-\lambda(n)], \tag{4}$$

{ siendo $p(n)$ una estimación de la potencia de la señal $e_2(n) - e_1(n)$, calculada según: }

$$p(n) = \beta p(n-1) + (1-\beta)[e_2(n) - e_1(n)]^2. \tag{5}$$

{ Además, en los casos en el que el algoritmo convex haya llegado a un estado donde uno de los algoritmo que interviene en la combinación ofrece prestaciones mucho mejores sobre el otro, combiene que en la Eq. (1) la salida sólo dependa del mejor de los algoritmos. Para esto, se suele saturar el valor de $\lambda(n)$ a 0 o 1 cuando la constante $a[n]$ alcance unos umbrales determinados. Según [8], los umbrales óptimos para saturar el valor de $\lambda(n)$ se corresponde con $a = \pm 4$ }

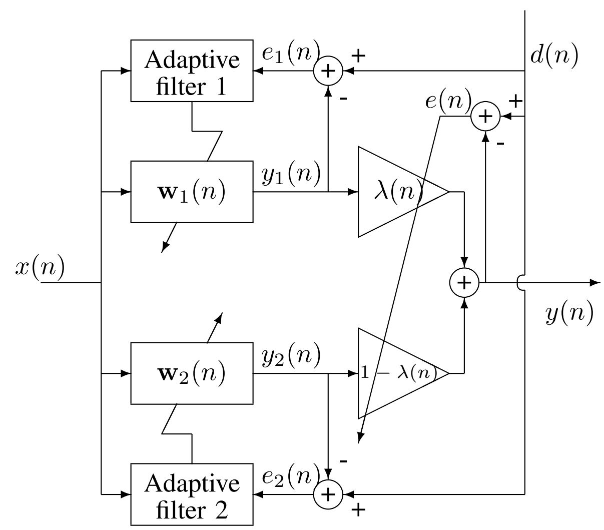

Fig. 2. Block diagram of the convex adaptive filtering structure.

Every single algorithm follows its own coefficient update equation, which are given by

$$\mathbf{w}_1(n) = \mathbf{w}_1(n-1) + \mu_1\mathbf{x}_1(n)e_1(n), \tag{6}$$

and

$$\mathbf{w}_2(n) = \mathbf{w}_2(n-1) + \mu_2\mathbf{x}_2(n)e_2(n), \tag{7}$$

where $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ are vectors with the most recent L_1 and L_2 samples of the input signal $x(n)$, respectively. L_1 and L_2 denote the dimensions of each adaptive filter. The convex LMS algorithm chooses a different step-size parameter suitable for each single algorithm (μ_1 and μ_2).

Although in [1] the convex combination of adaptive filters for system identification is discussed in detail, this strategy is summarized in this section in order to better understand similarities and differences of this approach with its application to an ANC system.

III. Convex filtered-x algorithms for active noise control

The convex combination of adaptive filters { previously proposed in [1] for system identification, in [12] for variable tap-length filters, and discussed in [8] } is introduced to ANC in this section. Two LMS algorithms will be considered, but other adaptive algorithms could be used.

First, the essential differences between an adaptive system for system identification (see Fig. 1) and its application to ANC (see Fig. 3) should be noted.

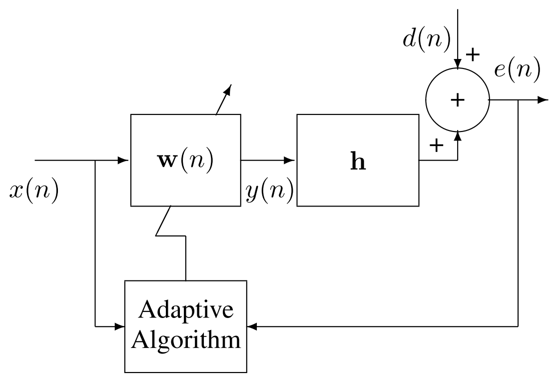

Fig. 3. Active noise control system.

Application of adaptive filtering to ANC requires some modifications because an ANC configuration is a particular adaptive scheme. One of the main differences introduced by ANC systems is due to the presence of an unavoidable system response between the adaptive filter output and the error sensor. This response $\mathbf{h}$ (called secondary path) is comprised of the transducer and the channel responses. The usual way to take into account this response and avoid negative effects on the adaptive algorithm performance consists in filtering the reference signal $x(n)$ through a copy of this response. Moreover an acoustical combination of the disturbance signal $d(n)$ and the adaptive filter output filtered by $\mathbf{h}$ is produced. Then, in general, the adaptive algorithm for ANC does not provide the disturbance signal $d(n)$ but the error signal $e(n) = d(n) + y(n) * \mathbf{h}$, and so it is not straightforward to obtain the error signals $e_1(n)$ and $e_2(n)$, since $e_p(n)$, for $p = 1, 2$ would be given by $d(n) + y_p(n) * \mathbf{h}$. Therefore, both the existence of an acoustic path and the acoustical combination of the disturbance signal and the adaptive filter output are the main differences between an adaptive system for channel identification and its application to ANC. The required signals can be estimated in different ways, depending on the filtering structure used for the ANC system. The filtering structures most frequently used in ANC are: the conventional filtered-x structure, the modified filtered-x structure and the adjoint filtered-x structure.

A. *Conventional filtered-x structure*

The conventional filtered-x structure [13] offers a good trade-off between computational complexity and convergence speed. The way to take into account the secondary path $\mathbf{h}$ and avoid negative effects on adaptive algorithm performance consists in filtering the reference signal $x(n)$ through a copy of this response, see Fig. 4.

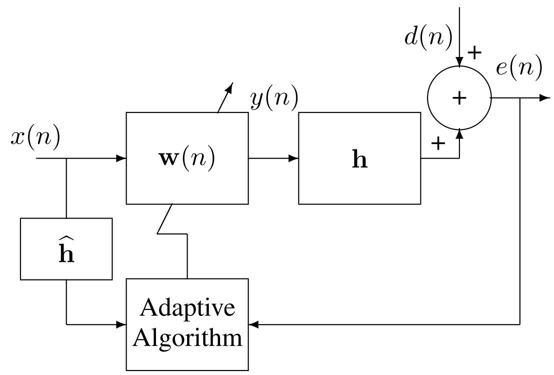

Fig. 4. Conventional filtered-x structure for ANC.

Therefore, the following considerations should be made by using the commented structure. The error signal is given by,

$$\begin{aligned} e(n) &= d(n) + y(n) * \mathbf{h} \\ &= d(n) + \mathbf{h} * \{\lambda(n) y_1(n) + [1-\lambda(n)] y_2(n)\}. \end{aligned} \tag{8}$$

If an estimate of the secondary path is known, $\widehat{\mathbf{h}}$, the disturbance signal could be calculated by,

$$\begin{aligned} \widehat{d}(n) &= e(n) - y(n) * \ \widehat{\mathbf{h}} \\ &= e(n) - \widehat{\mathbf{h}} * \{\lambda(n) y_1(n) + [1-\lambda(n)] y_2(n)\}, \end{aligned} \tag{9}$$

and the output error signals of the component adaptive filters are

$$e_1(n) = \widehat{d}(n) + \widehat{\mathbf{h}} * y_1(n) = e(n) + [1-\lambda(n)][y_1(n) - y_2(n)] * \widehat{\mathbf{h}}, \tag{10}$$

and

$$e_2(n) = \widehat{d}(n) + \widehat{\mathbf{h}} * y_2(n) = e(n) + \lambda(n)[y_2(n) - y_1(n)] * \widehat{\mathbf{h}}. \tag{11}$$

Notation in Table I and expressions (10) and (11), will be used to describe the convex conventional filtered-x LMS (Convex-FXLMS) algorithm. According to this notation, the steps of the Convex-FXLMS algorithm and its computational cost are detailed in **Algorithm 1**. It should be noted that, as expected, the complexity of the Convex-FXLMS ($2L_1+2L_2+3M+13$ multiplications per iteration) is higher than that of the conventional filtered-x LMS (FXLMS) algorithm with a single filter of L coefficients ($2L+M+1$ multiplications per iteration). Nevertheless, the aim of this approach is to improve performance regarding convergence speed and final residual error at the expense of an increase in computational cost. By using this strategy, the computational cost increases about four times regarding the FXLMS in the case where $L = L_1 = L_2$,

TABLE I
NOTATION OF THE CONVEX FILTERED-X ALGORITHMS.

L_1	Length of the adaptive filter $\mathbf{w}_1(n)$
L_2	Length of the adaptive filter $\mathbf{w}_2(n)$
M	Length of FIR filter that models $\mathbf{h}$
$x(n)$	Reference signal at time n
$y_1(n)$	Output signal of the adaptive filter 1 at time n
$y_2(n)$	Output signal of the adaptive filter 2 at time n
$e(n)$	Error signal at time n
$w_{1l}(n)$	lth coefficient in the adaptive filter 1
$w_{2l}(n)$	lth coefficient in the adaptive filter $w_2(n)$
$\widehat{\mathbf{h}}$	Estimated impulse response of the FIR filter modelling the secondary path $\mathbf{h}$ $\widehat{\mathbf{h}} = [\widehat{h}_1\ \widehat{h}_2, \cdots, \widehat{h}_M]^T$
$\mathbf{x}(n)$	$[x(n)\ x(n-1), \cdots, x(n-M+1)]^T$
$v(n)$	Reference signal $x(n)$ filtered by the plant model $\widehat{\mathbf{h}}$ at time n
$\mathbf{v}_1(n)$	$[v(n)\ v(n-1), \cdots, v(n-L_1+1)]^T$
$\mathbf{v}_2(n)$	$[v(n)\ v(n-1), \cdots, v(n-L_2+1)]^T$
$\mathbf{x}_1(n)$	$[x(n)\ x(n-1), \cdots, x(n-L_1+1)]^T$
$\mathbf{x}_2(n)$	$[x(n)\ x(n-1), \cdots, x(n-L_2+1)]^T$
$\mathbf{y}_1(n)$	$[y_1(n)\ y_1(n-1), \cdots, y_1(n-M+1)]^T$
$\mathbf{y}_2(n)$	$[y_2(n)\ y_2(n-1), \cdots, y_2(n-M+1)]^T$

although as discussed below, other strategies can provide a lower relative increase.

Algorithm 1 Convex-FXLMS algorithm.

Input: Reference signal $x(n)$ and error signal $e(n)$
Output: Output of the parallel filter $y(n)$

Update the vectors $\mathbf{x}(n)$, $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$
$y_1(n) = \mathbf{w}_1^T(n)\mathbf{x}_1(n)$, (Multipl.: L_1)
$y_2(n) = \mathbf{w}_2^T(n)\mathbf{x}_2(n)$, (Multipl.: L_2)
$y(n) = \lambda(n)y_1(n) + [1-\lambda(n)]y_2(n)$, (Multipl.: 2)
Update the vectors $\mathbf{y}_1(n)$ and $\mathbf{y}_2(n)$
$y_{f1}(n) = \mathbf{y}_1^T(n)\widehat{\mathbf{h}}$, (Multipl.: M
$y_{f2}(n) = \mathbf{y}_2^T(n)\widehat{\mathbf{h}}$, (Multipl.: M)
$p(n) = \beta p(n-1) + (1-\beta)[y_{f2}(n) - y_{f1}(n)]^2$, (Multipl.: 3
$a(n) = a(n-1) + \frac{\mu_a}{p(n)} e(n)[y_{f2}(n) - y_{f1}(n)]\lambda(n)[1-\lambda(n)]$, (Multipl.: 4)
$\lambda(n) = \dfrac{1}{1+e^{-a(n)}}$
$v(n) = \mathbf{x}^T(n)\widehat{\mathbf{h}}$, (Multipl.: M)
Update the vectors $\mathbf{v}_1(n)$ and $\mathbf{v}_2(n)$
$\mathbf{w}_1(n) = \mathbf{w}_1(n-1) - \mu_1\mathbf{v}_1(n)\{e(n) + [1-\lambda(n)][y_{f1}(n) - y_{f2}(n)]\}$ (Multipl.: L_1+2)
$\mathbf{w}_2(n) = \mathbf{w}_2(n-1) - \mu_2\mathbf{v}_2(n)\{e(n) + \lambda(n)[y_{f2}(n) - y_{f1}(n)]\}$, (Multipl.: L_2+2)

B. Modified filtered-x structure

Although the modified filtered-x structure [14] provides a slightly better convergence speed than the conventional filtering scheme, it is more demanding from a computational cost point of view, as can be seen in Fig. 5. However, this complexity simplifies application of the convex strategy. Specifically, this scheme allows the recovery of an estimate of the disturbance signal $d(n)$ ($d'(n)$ in Fig. 5) which is needed to calculate the error signals $e_1(n)$ and $e_2(n)$.

The convex algorithm based on the modified filtered-x structure (Convex-MFXLMS) is described in **Algorithm 2** according to notation in Table I.

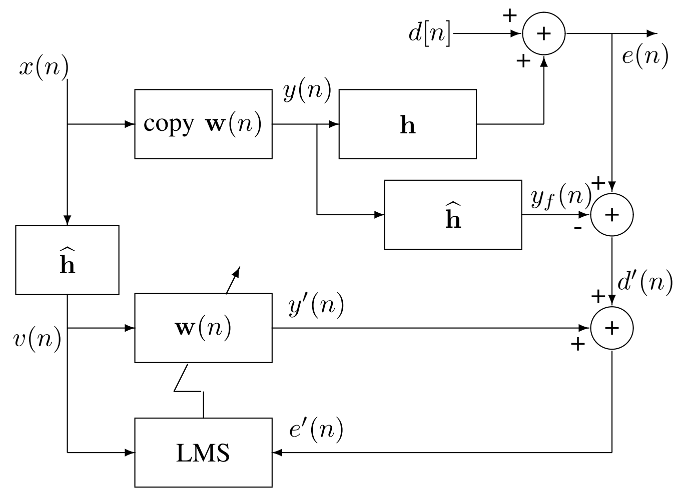

Fig. 5. Modified filtered-x structure for ANC.

Algorithm 2 Convex-MFXLMS algorithm.

Input: Reference signal $x(n)$ and error signal $e(n)$
Output: Output of the parallel filter $y(n)$

Update the vectors $\mathbf{x}(n)$, $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$
$v(n) = \mathbf{x}^T(n)\widehat{\mathbf{h}}$, (Multipl.: M)
$y_1(n) = \mathbf{w}_1^T(n)\mathbf{x}_1(n)$, (Multipl.: L_1)
$y_2(n) = \mathbf{w}_2^T(n)\mathbf{x}_2(n)$, (Multipl.: L_2)
Update the vectors $\mathbf{v}_1(n)$, $\mathbf{v}_2(n)$, $\mathbf{y}_1(n)$ and $\mathbf{y}_2(n)$
$y_1'(n) = \mathbf{v}_1^T(n)\mathbf{w}_1(n)$, (Multipl.: L_1)
$y_2'(n) = \mathbf{v}_2^T(n)\mathbf{w}_2(n)$, (Multipl.: L_2)
$y_{f1}(n) = \mathbf{y}_1^T(n)\widehat{\mathbf{h}}$, (Multipl.: M)
$y_{f2}(n) = \mathbf{y}_2^T(n)\widehat{\mathbf{h}}$, (Multipl.: M)
$d_1'(n) = e(n) - y_{f1}(n)$
$d_2'(n) = e(n) - y_{f2}(n)$
$e_1'(n) = d_1'(n) + y_1'(n)$
$e_2'(n) = d_2'(n) + y_2'(n)$
$p(n) = \beta p(n-1) + (1-\beta)[e_2'(n) - e_1'(n)]^2$, (Multipl.: 3
$a(n) = a(n-1) + \frac{\mu_a}{p(n)} e(n)[e_2'(n) - e_1'(n)]\lambda(n)[1-\lambda(n)]$, (Multipl.: 4)
$\lambda(n) = \dfrac{1}{1+e^{-a(n)}}$
$y(n) = \lambda(n)y_1(n) + [1-\lambda(n)]y_2(n)$, (Multipl.: 2)
$\mathbf{w}_1(n+1) = \mathbf{w}_1(n) - \mu_1\mathbf{v}_1(n)e_1(n)$, (Multipl.: L_1+1)
$\mathbf{w}_2(n+1) = \mathbf{w}_2(n) - \mu_2\mathbf{v}_2(n)e_2(n)$, (Multipl.: L_2+1)

The total number of multiplications per iteration required by the Convex-MFXLMS algorithm reaches $3(M+L_1+L_2)+8$. However, if a modified filtered-x LMS (MFXLMS) algorithm with a single adaptive filter of L length is considered, only $3M+2L+11$ multiplications are required. Thus, a relative roughly three-fold increase of the MFXLMS, when $L = L_1 = L_2$, happens due to the use of convex structure.

C. Adjoint filtered-x structure

The adjoint scheme for adaptive filtering [15] is the simplest of the structures considered from a computational point of view, but it offers the worst convergence performance, see Fig. 6. Usually, this structure is not applied, apart from cases where the computational cost is severely constrained. Therefore, it does not seem logical to apply the convex strategy to the adjoint filtering scheme, since its computational load will significantly increase providing a performance similar to those of the other two schemes. Furthermore, the computational saving is not achieved in single channel systems, see Table III-C. Nevertheless, this algorithm has been included in the experiments carried out in order to obtain a whole understanding of the convex combination approach. Therefore, the convex algorithm based on the adjoint filtered-x structure (Convex-AFXLMS) is described in **Algorithm 3** according to the notation in Table I.

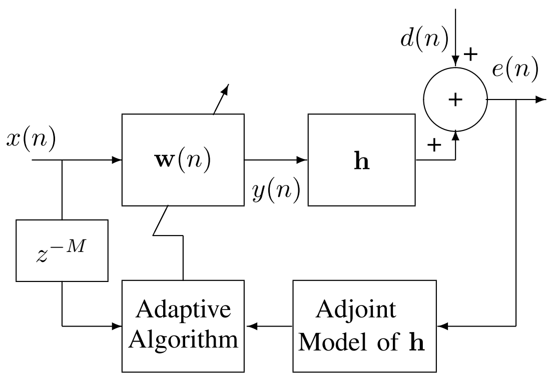

Fig. 6. Adjoint filtered-x structure for ANC.

The convex approach based on the adjoint filtered-x scheme presents a problem similar to the Convex-FXLMS algorithm, since only one error signal is provided and two error signals ($e_1(n)$ and $e_2(n)$) are required in order to independently control each algorithm. As was shown above, both error signals can be obtained from an estimate of the secondary acoustic path. Therefore, proceeding similarly to Section III-A, and considering the following vectors

- $\mathbf{h}' = [\widehat{h}_M, \widehat{h}_{M-1}, \ldots, \widehat{h}_1]^T$
- $\mathbf{e}(n) = [e(n), e(n-1), \ldots, e(n-M+1)]^T$,

where $\mathbf{h}'$ is the adjoint model of $\widehat{\mathbf{h}}$. The convex approach algorithm based on the adjoint filtered-x structure (Convex-AFXLMS) can be developed as shown in **Algorithm 3**.

TABLE II
NUMBER OF MULTIPLICATIONS PER ITERATION OF THE FILTERED-X LMS ALGORITHM BASED ON DIFFERENT FILTERING SCHEMES, AND THEIR CONVEX APPROACHES (CONVEX-FXLMS, CONVEX-MFXLMS AND CONVEX-AFXLMS). TYPICAL CASE: $L = L_1 = L_2 = 30$, AND $M = 10$.

Algorithm	Multiplications per iterat.	Typ. case
FXLMS	$2L + M + 1$	71
Convex-FXLMS	$2(L_1 + L_2) + 3M + 13$	163
MFXLMS	$3M + 2L + 1$	91
Convex-MFXLMS	$3(M + L_1 + L_2) + 11$	221
AFXLMS	$2L + M + 1$	71
Convex-AFXLMS	$2(L_1 + L_2) + 3M + 13)$	163

IV. CONVEX FILTERED-X LMS ALGORITHMS FOR MULTICHANNEL ANC

To extend the convex filtered-x LMS algorithms described in the previous section, to the multichannel case, a generic multichannel ANC system (I reference signals, J secondary sources, and K error sensors) is considered and illustrated in Fig. 7.

Algorithm 3 Convex-AFXLMS algorithm.

Input: Reference signal $x(n)$ and error signal $e(n)$
Output: Output of the parallel filter $y(n)$

Update the vectors $\mathbf{e}(n)$, $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$
$y_1(n) = \mathbf{w}_1^T(n)\mathbf{x}_1(n)$, (Multipl.: L_1)
$y_2(n) = \mathbf{w}_2^T(n)\mathbf{x}_2(n)$, (Multipl.: L_2)
$y(n) = \lambda(n)y_1(n) + [1-\lambda(n)]y_2(n)$, (Multipl.: 2)
Update the vectors $\mathbf{y}_1(n)$ and $\mathbf{y}_2(n)$
$y_{f1}(n) = \mathbf{y}_1^T(n)\widehat{\mathbf{h}}$ (Multipl.: M)
$y_{f2}(n) = \mathbf{y}_2^T(n)\widehat{\mathbf{h}}$, (Multipl.: M)
$p(n) = \beta p(n-1) + (1-\beta)[y_{f2}(n) - y_{f1}(n)]^2$, (Multipl.: 3)
$a(n) = a(n-1) + \frac{\mu_a}{p(n)} e(n)[y_{f2}(n) - y_{f1}(n)]\lambda(n)[1-\lambda(n)]$, (Multipl.: 4)
$\lambda(n) = \dfrac{1}{1 + e^{-a(n)}}$
$e'(n) = \mathbf{h}'^T\mathbf{e}(n)$, (Multipl.: M)
$\mathbf{w}_1(n) = \mathbf{w}_1(n-1) - \mu_1\mathbf{x}_1(n-M+1)\{e'(n) + [1-\lambda(n)][y_{f1}(n-M+1) - y_{f2}(n-M+1)]\}$, (Multipl.: $L_1 + 2$)
$\mathbf{w}_2(n) = \mathbf{w}_1(n-1) - \mu_2\mathbf{x}_2(n-M+1)\{e'(n) + \lambda(n)[y_{f2}(n-M+1) - y_{f1}(n-M+1)]\}$, (Multipl.: $L_2 + 2$)

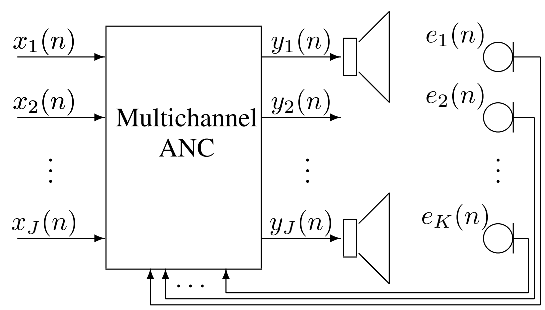

Fig. 7. Block diagram of a multichannel active noise control system.

It should be noted that this system presents JK secondary paths and IJ adaptive filters, and when the convex strategy is applied, the number of single adaptive filters doubles and the computational load increases compared with the single channel case. Notation in Table III will be used to describe the multichannel algorithms based on the convex strategy.

The management of multiple error signals and the simultaneous updating of multiple adaptive filters become the main difficulties in extending the convex filtered-x algorithms described in Section 3 to the multichannel case. Regarding the update rule of the adaptive filter coefficients, the target is to minimize a cost function dependent on the sum of the mean square errors [16], given by

$$J(n) = E\left\{\sum_{k=1}^{K} e_k^2(n)\right\}. \quad (12)$$

Using the gradient descent method and approximating the mean square errors by their instantaneous values, thus, every

TABLE III
NOTATION OF THE CONVEX MULTICHANNEL ANC SYSTEMS.

I	Number of reference sensors
J	Number of actuators
K	Number of error sensors
L_1 and L_2	Length of the adaptive filters
$x_i(n)$	ith reference signal at time n
$y_{1j}(n)$	jth actuator signal at time n from the adaptive filter 1
$y_{2j}(n)$	jth actuator signal at time n from the adaptive filter 2
$e_k(n)$	kth error sensor signal at time n
$w_{1(i,j,l)}(n)$	lth coefficient in the adaptive filter 1 linking $x_i(n)$ and $y_{1j}(n)$ at time n
$w_{2(i,j,l)}(n)$	lth coefficient in the adaptive filter 1 linking $x_i(n)$ and $y_{2j}(n)$ at time n
$\widehat{\mathbf{h}}_{j,k}$	Estimated FIR filter modelling the acoustic plant $\mathbf{h}_{j,k}$ that links the kth error sensor and the jth actuator $\widehat{\mathbf{h}}_{j,k} = [\widehat{h}_{j,k,1}\ \widehat{h}_{j,k,2}, \cdots, \widehat{h}_{j,k,M}]^T$
$v_{i,j,k}(n)$	Reference signal $x_i(n)$ filtered by the estimated plant model $\widehat{\mathbf{h}}_{j,k}$ at time n
$\mathbf{w}_{1(i,j)}(n)$	$[w_{1(i,j,1)}(n)\ w_{1(i,j,2)}(n), \cdots, w_{1(i,j,L_1)}(n)]^T$
$\mathbf{w}_{2(i,j)}(n)$	$[w_{2(i,j,1)}(n)\ w_{2(i,j,2)}(n), \cdots, w_{2(i,j,L_2)}(n)]^T$
$\mathbf{x}_i(n)$	$[x_i(n)\ x_i(n-1), \cdots, x_i(n-M+1)]^T$
$\mathbf{v}_{1(i,j,k)}(n)$	$[v_{i,j,k}(n)\ v_{i,j,k}(n-1), \cdots, v_{i,j,k}(n-L_1+1)]^T$
$\mathbf{v}_{2(i,j,k)}(n)$	$[v_{i,j,k}(n)\ v_{i,j,k}(n-1), \cdots, v_{i,j,k}(n-L_2+1)]^T$
$\mathbf{x}_{1i}(n)$	$[x_i(n)\ x_i(n-1), \cdots, x_i(n-L_1+1)]^T$
$\mathbf{x}_{2i}(n)$	$[x_i(n)\ x_i(n-1), \cdots, x_i(n-L_2+1)]^T$
$\mathbf{y}_{1j}(n)$	$[y_{1j}(n)\ y_{1j}(n-1), \cdots, y_{1j}(n-M+1)]^T$
$\mathbf{y}_{2j}(n)$	$[y_{2j}(n)\ y_{2j}(n-1), \cdots, y_{2j}(n-M+1)]^T$

single algorithm follows its own update equation,

$$\mathbf{w}_{1(i,j)}(n) = \mathbf{w}_{1(i,j)}(n-1) - \mu_1 \sum_{k=1}^{K} \mathbf{v}_{i,j,k}(n) e_k(n)$$
$$\text{and} \qquad (13)$$
$$\mathbf{w}_{2(i,j)}(n) = \mathbf{w}_{2(i,j)}(n-1) - \mu_2 \sum_{k=1}^{K} \mathbf{v}_{i,j,k}(n) e_k(n).$$

This update rule was first proposed for the MELMS algorithm in [17]. Moreover, $a(n)$ is updated to minimize the same cost function $J(n)$ described in (12) and approximated by its instantaneous value, yielding

$$\begin{aligned} &a(n+1) \\ &= a(n) - \frac{1}{2}\mu_a \nabla \sum_{k=1}^{K} e_k^2(n) \\ &= a(n) + \mu_a \lambda(n)[1-\lambda(n)] \sum_{k=1}^{K} e_k(n)[e_{2k}(n) - e_{1k}(n)] \\ &= a(n) + \mu_a \lambda(n)[1-\lambda(n)] \sum_{k=1}^{K} e_k(n)[y_{f2k}(n) - y_{f1k}(n)] \end{aligned} \qquad (14)$$

where $y_{flk}(n)$ are the output signals of the adaptive filters $l = 1, 2$ and filtered by their corresponding estimated acoustic paths.

{ Al igual que en el caso monocanal Eq. (4), la constante que controla la actualización de $a(n)$ se puede normalizar para mejorar el funcionamiento del algoritmo en el caso en que la SNR varíe. De esta forma Eq. (14) quedaría:

$$\begin{aligned} &a(n+1) \\ &= a(n) + \mu_a \lambda(n)[1-\lambda(n)] \sum_{k=1}^{K} \frac{e_k(n)}{p_k(n)} [y_{f2k}(n) - y_{f1k}(n)] \end{aligned} \qquad (15)$$

siendo:

$$p_k(n) = \beta p_k(n-1) + (1-\beta)[e_{2k}(n) - e_{1k}(n)]^2. \qquad (16)$$

}

With the considerations derived from (13) and (15), the convex multichannel LMS algorithms are developed by further applying the three filtering schemes of Section 3 for the single channel case. Thus, Algorithm 4 presents the convex multichannel FXLMS (multichannel Convex-FXLMS) algorithm based on the conventional filtered-x scheme. Algorithm 5 illustrates the convex multichannel MFXLMS (multichannel Convex-MFXLMS) algorithm based on the modified filtered-x scheme, and finally, the convex multichannel AFXLMS (multichannel Convex-AFXLMS) algorithm with the adjoint structure embedded is reported in Algorithm 6. The total number of multiplications per iteration required for the different multichannel algorithms is given in Table IV. It can be seen that the multichannel Convex-MFXLMS algorithm needs more multiplications per iteration than the corresponding MFXLMS algorithm and the other convex approaches. Moreover, for the typical case analyzed, the three multichannel convex approaches proposed require approximately twice the multiplications of their single channel counterpart.

V. Simulation Results

In this section we want to compare the performance of each parallel algorithm of section III and IV with their respective single version. { También se han comparado con el algoritmo de paso variable descrito en [7] y se ha estudiado la desviación que se produce en los resultados cuando los algoritmos no disponen de estimación perfecta de los caminos acústicos. For these purposes, different experiments have been carried out. Specifically, a single channel and a 1:2:2 ($J = 2$ actuators and $K = 2$ error sensors) multichannel ANC systems have been considered. A gaussian random signal of zero mean and unit variance and a periodic noise composed of four harmonics added to gaussian noise of 0.001 variance have been used as reference signal ($x(n)$ in Fig. 3). The acoustic paths (both primaries and secondaries) have been modelled by FIR filters with a length of $M = 20$ coefficients, and the adaptive filters have been designed to $L = L_1 = L_2 = 40$ coefficients. Perfect secondary path estimate and SNR of 20,10 and 5 dB in secondary path estimation are assumed.} In the different cases analyzed the convex approaches combine two algorithms of complementary capabilities trying to extract the best properties of the component filters. Thus, for single channel ANC, two LMS algorithms based on the different filtering schemes work with the same number of filter coefficients ($L = L_1 = L_2$) and with different step sizes chosen by trial and error (one of them for a high μ_2 with a high convergence speed, and the other for a low μ_1 that exhibits the better steady-state MSE.{ El valor de β para la estimacíón de la potencia en las Eq. (5) y (16) fue fijado a 0.9, y se usó un valor de $\mu_a = 1$.} The same considerations are assumed in the multichannel experiments.

In order to evaluate the different approaches, the learning curves were obtained for the simulations. These curves are calculated here from the ratio between the instantaneous estimated power at the error sensor with and without active noise control, expressed in decibels, that is,

$$10 \cdot \log_{10} \frac{e^2(n)}{d^2(n)}. \tag{17}$$

For the multichannel case, which provides in general, more than one error signal, we will add the instantaneous power of the different contributions. Therefore, the corresponding learning curves are given from,

$$10 \cdot \log_{10} \left(\frac{\sum_{k=1}^{K} e_k^2(n)}{\sum_{k=1}^{K} d_k^2(n)} \right). \tag{18}$$

In order to reduce the variance of the learning curves and have a better idea of performance in both the steady and the transient state, 3000 simulations were averaged to obtain each learning curve.

With the previously described configuration, different kinds of experiments have been carried out. { Primeramente se comprobó la respuesta del algoritmo convex para las diferentes configuraciones descritas anteriormente para los sistemas de ANC simulados cuando tenemos estiamación perfecta de los canales acústicos. Tanto en el caso monocanal como en el multicanal, el comportamiento de estos algoritmos es el esperado y las curvas de aprendizaje obtenidas se aproximan al mejor de los dos casos individuales. Además, si comparamos las prestaciones de la configuración convex con las de los algoritmos de paso variable, se observa también una clara mejoría. En las figuras 8 (a), 9 (a) y 10 (a), se representan estas curvas para el caso monocanal y las tres estrucutras descritas. Aunque en la mayoría de las aplicciones de ANC podemos considerar que los caminos acústicos se pueden estimar con suficiente precisión como para considerarlos estimación perfecta, se ha querido estudiar los efectos de los errores de estimación introduciendo cierto error en las estimaciones y comprobando cómo de significativa es la degradación de los resultados. Al igual que en [6], para contrastar la robustez de estos algoritmos frente a la estimación imperfecta de los caminos acústicos, se ha introducido en los caminos estimados $\widehat{\mathbf{h}}$ un cierto. Los resultados de simulación confirman que mientras que dicho error no es suficientemente signifcativo (SNR$<$20 dB) la degradación de los resultados es despreciable. También se pueden concluir que la estructura de filtrado-x convencional es la más robusta frente a la estimación imperfecta y la adjunta la más sensible, tal y como se aprecia en las figuras 8 (b), 9 (b) y 10 (b). Además, la degradación que sufren estos algoritmos no es mayor que la que puedan sufrir los algoritmos individuales que forman la combinación, por lo que se puede concluir que la combinación convex es robusta frente a errores en la estimación de los caminos acústicos. Sólo SNR muy pequeñas degradan significativamente las curvas de aprendizaje (sobre todo en la estructura modificadda), pero ha de tenerse en cuenta que por lo general podemos obtener estimaciones de los caminos acústicos con SNR$>$20 dB. Por tanto, dada la robustez de estos algoritmos, podemos considerar la asunción de estimación perfecta, como bastante realista.

Para el caso de los sistemas de ANC más complicados como sistemas multicanal o vaiantes con el tiempo, se obtienen conclusiones similares a las derivadas del caso monocanal. Como ejemplo se muestran las curvas de aprendizaje obtenidas para el caso de la estructura adjunta y un sistema de ANC 1:2:2 en la figura 11 (a). Para el resto de estructuras, los resultados son similares. En la figura 11 (b) se muestran las curvas de aprendizaje para entornos de ANC variables con el tiempo. En este caso the algorithms have been studied when the multichannel ANC system configuration is time variant and the statistic of the input signal were varied during the experiment. Then, the filters that model the acoustic paths were changed every $25,000$ iterations and the algorithms were run for $75,000$ iterations. That means the algorithms should converge three times. Moreover, the reference signal variance was also changed in the last $25,000$ iterations. These non-stationary conditions allow to evaluate the ability of the algorithms in tracking changes in both the acoustic system and the input signal.

Algorithm 4 Multichannel Convex-FXLMS algorithm.

Input: Reference signals $x_i(n)$ and error signals $e_k(n)$

Output: Output of the parallel filter $y_j(n)$

Update the vectors $\mathbf{x}_i(n)$, $\mathbf{x}_{1i}(n)$ and $\mathbf{x}_{2i}(n)$

$y_{1j}(n) = \sum_{i=1}^{I} \mathbf{w}_{1(i,j)}^T(n)\mathbf{x}_{1i}(n)$, (Multipl.: L_1IJ)

$y_{2j}(n) = \sum_{i=1}^{I} \mathbf{w}_{2(i,j)}^T(n)\mathbf{x}_{2i}(n)$, (Multipl.: L_2IJ)

$y_j(n) = \lambda(n)y_{1j}(n) + [1-\lambda(n)]y_{2j}(n)$, (Multipl.: $2J$)

Update the vectors $\mathbf{y}_{1j}(n)$ and $\mathbf{y}_{2j}(n)$

$y_{f1k}(n) = \sum_{j=1}^{J} \mathbf{y}_{1j}^T(n)\widehat{\mathbf{h}}_{j,k}$, (Multipl.: MJK)

$y_{f2k}(n) = \sum_{j=1}^{J} \mathbf{y}_{2j}^T(n)\widehat{\mathbf{h}}_{j,k}$, (Multipl.: MJK)

$p_k(n) = \beta p_k(n-1) + (1-\beta)[y_{f2k}(n) - y_{f1k}(n)]^2$, (Multipl.: $3K$)

$a(n) = a(n-1) + \mu_a\lambda(n)[1-\lambda(n)]\sum_{k=1}^{K} \frac{e_k(n)}{p_k(n)}[y_{f2k}(n) - y_{f1k}(n)]$, (Multipl.: $3+K$)

$\lambda(n) = \frac{1}{1+e^{-a(n)}}$

$v_{i,j,k}(n) = \widehat{\mathbf{h}}_{j,k}^T\mathbf{x}_i(n)$, (Multipl.: $MIJK$)

Update the vectors $\mathbf{v}_{1(i,j,k)}(n)$ and $\mathbf{v}_{2(i,j,k)}(n)$

$\mathbf{w}_{1(i,j)}(n) = \mathbf{w}_{1(i,j)}(n-1) - \mu_1\sum_{k=1}^{K} \mathbf{v}_{1(i,j,k)}(n)\{e_k(n) + [1-\lambda(n)][y_{f1k}(n) - y_{f2k}(n)]\}$, (Multipl.: $[(L_1+2)K+1]IJ$)

$\mathbf{w}_{2(i,j)}(n) = \mathbf{w}_{2(i,j)}(n-1) - \mu_2\sum_{k=1}^{K} \mathbf{v}_{2(i,j,k)}(n)\{e_k(n) + \lambda(n)[y_{f2k}(n) - y_{f1k}(n)]\}$, (Multipl.: $[(L_2+2)K+1]IJ$)

TABLE IV

NUMBER OF MULTIPLICATIONS PER ITERATION OF THE MULTICHANNEL FILTERED-X LMS ALGORITHM BASED ON DIFFERENT FILTERING SCHEMES, AND ITS CONVEX APPROACHES (MULTICHANNEL CONVEX-FXLMS, MULTICHANNEL CONVEX-MFXLMS AND MULTICHANNEL CONVEX-AFXLMS). TYPICAL CASE: $L = L_1 = L_2 = 30$, $M = 10$, $I = 1$ AND $J = K = 2$.

Algorithm	Multiplications per iteration	Typical case
Multichannel FXLMS	$LIJ + MIJK + IJ(LK+1)$	222
Multichannel Convex-FXLMS	$(L_1+L_2+2)IJ + 2J + 3 + 4K + (L_1+L_2+4+M)IJK + 2MJK$	515
Multichannel MFXLMS	$IJK(2L+M) + IJ(L+1) + JKM$	382
Multichannel Convex-MFXLMS	$(L_1+L_2)(2K+1)IJ + 4K + 3 + (I+2)MJK + 2J + 2IJ$	739
Multichannel AFXLMS	$IJ(2L+1) + KM$	142
Multichannel Convex-AFXLMS	$(L_1+L_2)IJ + 2J + 2MJ + 3 + 4K + MJK + [(L_1+L_2+4)K+1]IJ$	473

Además, puesto que el objetivo de estos algoritmos es el ANC, al margen del ruido Gaussiano se han empleado otro tipo de señales diferentes como señales de ruido a cancelar, como por ejemplo el ruido periódico que simula el clásico ruido de motor. Este ruido se ha simulado añadiendo a una base de ruido gaussiano de baja potencia, señales periódicas de baja frecuencia. En particular, se ha considerado un tono de frecuencia discreta 0.0023 y sus primeros tres armónicos. Las fases iniciales de estos tonos tomaban valores aleaotorios. Los algoritmos han funcionado de forma similar en todos los casos independientemente del tipo de ruido empleado. En la figura 12 se muestran las curvas de aprendizaje de los experimentos realizados con ruido periódico usando la estructura modificada tanto para el caso monocanal como para el multicanal. }

VI. CONCLUSION

The application of the convex combination strategy for single channel and multichannel practical ANC systems has been proposed throughout the present paper. The convex approach has been applied to an ANC system by using three different filtering structures and the LMS algorithm: the conventional filtered-x structure that provides the Convex-FXLMS algorithm, the modified filtered-x structure that yields the Convex-MFXLMS algorithm, and the Convex-AFXLMS algorithm based on the adjoint filtered-x LMS structure. Both the adaptive filter coefficients and the parameter that controls the combination of filter outputs follow the update rule based on a steepest descent method. Although the computational cost of the convex approaches is higher (as always happens with parallel combination strategies that involve two algorithms working independently), they are a suitable solution to develop meaningful and robust algorithms with high convergence speed and good steady-state MSE performance. Such algorithms would be especially appropriate when the control system is time variant or the acoustical system may suffer unexpected changes, for their ability to follow changes on system conditions without worsening their steady-state performance.

{ Además, estos algoritmos han demostrado ser robusto

Algorithm 5 Multichannel Convex-MFXLMS algorithm.

Input: Reference signals $x_i(n)$ and error signals $e_k(n)$
Output: Output of the parallel filter $y_j(n)$

Update the vectors $\mathbf{x}_i(n)$, $\mathbf{x}_{1i}(n)$ and $\mathbf{x}_{2i}(n)$

$v_{i,j,k}(n) = \widehat{\mathbf{h}}_{j,k}^T \mathbf{x}_i(n)$, (Multipl.: $MIJK$)

$y_{1j}(n) = \sum_{i=1}^{I} \mathbf{w}_{1(i,j)}^T(n)\mathbf{x}_{1i}(n)$, (Multipl.: L_1IJ)

$y_{2j}(n) = \sum_{i=1}^{I} \mathbf{w}_{2(i,j)}^T(n)\mathbf{x}_{2i}(n)$, (Multipl.: L_2IJ)

Update the vectors $\mathbf{v}_{1(i,j,k)}(n)$, $\mathbf{v}_{2(i,j,k)}(n)$, $\mathbf{y}_{1j}(n)$ e $\mathbf{y}_{2j}(n)$

$y'_{1_j}(n) = \sum_{i=1}^{I}\sum_{k=1}^{K} \mathbf{v}_{1(i,j,k)}^T(n)\mathbf{w}_{1(i,j)}(n)$, (Multipl.: L_1IJK)

$y'_{2j}(n) = \sum_{i=1}^{I}\sum_{k=1}^{K} \mathbf{v}_{2(i,j,k)}^T(n)\mathbf{w}_{1(i,j)}(n)$, (Multipl.: L_2IJK)

$y_{f1k}(n) = \sum_{j=1}^{J} \mathbf{y}_{1j}^T(n)\widehat{\mathbf{h}}_{1(j,k)}$, (Multipl.: MJK)

$y_{f2k}(n) = \sum_{j=1}^{J} \mathbf{y}_{2j}^T(n)\widehat{\mathbf{h}}_{j,k}$, (Multipl.: MJK)

$d'_{1k}(n) = e_k(n) - y_{f1k}(n)$

$d'_{2k}(n) = e_k(n) - y_{f2k}(n)$

$e'_{1k}(n) = d'_{1k}(n) + \sum_{j=1}^{J} y'_{1j}(n)$

$e'_{2k}(n) = d'_{2k}(n) + \sum_{j=1}^{J} y'_{2j}(n)$

$p_k(n) = \beta p_k(n-1) + (1-\beta)[e'_{2k}(n) - e'_{1k}(n)]^2$, (Multipl.: $3K$)

$a(n) = a(n-1) + \mu_a\lambda(n)[1-\lambda(n)]\sum_{k=1}^{K}\frac{e_k(n)}{p_k(n)}[e'_{2k}(n) - e'_{1k}(n)]$, (Multipl.: $K+3$)

$\lambda(n) = \frac{1}{1+e^{-a(n)}}$

$y_j(n) = \lambda(n)y_{1j}(n) + [1-\lambda(n)]y_{2j}(n)$, (Multipl.:$2J$)

$\mathbf{w}_{1(i,j)}(n) = \mathbf{w}_{1(i,j)}(n-1) - \mu_1\sum_{k=1}^{K}\mathbf{v}_{1(i,j,k)}(n)e'_{1k}(n)$, (Multipl.: $[L_1K+1]IJ$)

$\mathbf{w}_{2(i,j)}(n) = \mathbf{w}_{2(i,j)}(n-1) - \mu_2\sum_{k=1}^{K}\mathbf{v}_{2(i,j,k)}(n)e'_{2k}(n)$, (Multipl.: $[L_2K+1]IJ$)

frente a errores en la estimación de los caminos acústicos.
}

Simulation results in stationary and non-stationary conditions have validated the expected performance of the convex approaches for single channel and multichannel ANC systems.

Further research suggests the implementation of convex schemes in real-time applications that require good convergence performance such as sound reproduction or control.

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Algorithm 6 Multichannel Convex-AFXLMS algorithm.

Input: Reference signals $x_i(n)$ and error signals $e_k(n)$
Output: Output of the parallel filter $y_j(n)$

Update the vectors $\mathbf{x}_{1i}(n)$, $\mathbf{x}_{2i}(n)$ and $\mathbf{e}_k(n)$

$y_{1j}(n) = \sum_{i=1}^{I} \mathbf{w}_{1(i,j)}^T(n)\mathbf{x}_{1i}(n), \quad$ (Multipl.: $L_1 IJ$)

$y_{2j}(n) = \sum_{i=1}^{I} \mathbf{w}_{2(i,j)}^T(n)\mathbf{x}_{2i}(n), \quad$ (Multipl.: $L_2 IJ$)

$y_j(n) = \lambda(n)y_{1j}(n) + [1-\lambda(n)]y_{2j}(n), \quad$ (Multipl.: $2J$)

$y_{f1k}(n) = \sum_{j=1}^{J} \mathbf{y}_{1j}^T(n)\widehat{\mathbf{h}}_{j,k}\ , \quad$ (Multipl.: MJ)

$y_{f2k}(n) = \sum_{j=1}^{J} \mathbf{y}_{2j}^T(n)\widehat{\mathbf{h}}_{j,k}\ , \quad$ (Multipl.: MJ)

$p_k(n) = \beta p_k(n-1) + (1-\beta)[y_{f2k}(n) - y_{f1k}(n)]^2$, (Multipl.: $3K$)

$a(n) = a(n-1) + \mu_a \lambda(n)[1-\lambda(n)]\sum_{k=1}^{K} \frac{e_k(n)}{p_k(n)}[y_{f2k}(n) - y_{f1k}(n)]\ , \quad$ (Multipl.: $3+K$)

$\lambda(n) = \frac{1}{1+e^{-a(n)}}$

$e'_k(n) = \sum_{j=1}^{J} \mathbf{h}'^T_{j,k}\mathbf{e}_k(n), \quad$ (Multipl.: MJK)

$\mathbf{w}_{1(i,j)}(n) = \mathbf{w}_{1(i,j)}(n-1) - \mu_1 \sum_{k=1}^{K} \mathbf{x}_{1i}(n-M+1)\{e'_k(n) + [1-\lambda(n)][y_{f1k}(n-M+1) - y_{f2k}(n-M+1)]\}\ , \quad$ (Multipl.: $[(L_1+2)K+1]IJ$)

$\mathbf{w}_{2(i,j)}(n) = \mathbf{w}_{2(i,j)}(n-1) - \mu_2 \sum_{k=1}^{K} \mathbf{x}_{2i}(n-M+1)\{e'_k(n) + \lambda(n)[y_{f2k}(n-M+1) - y_{f1k}(n-M+1)]\}\ , \quad$ (Multipl.: $[(L_2+2)K+1]IJ$)

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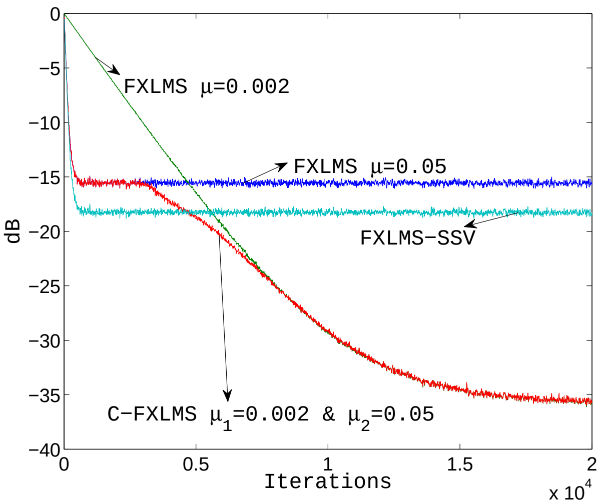
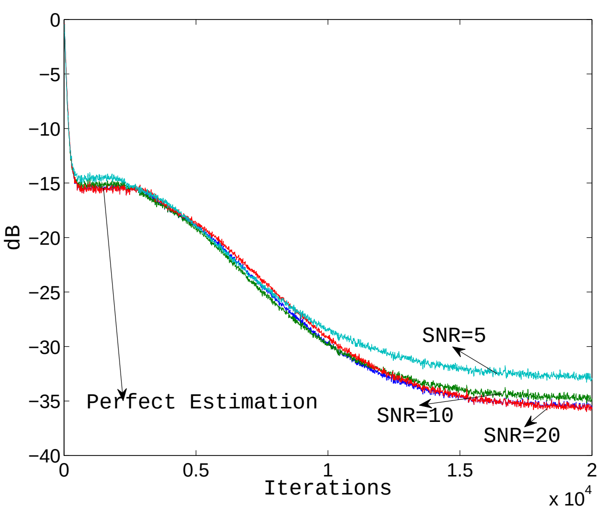
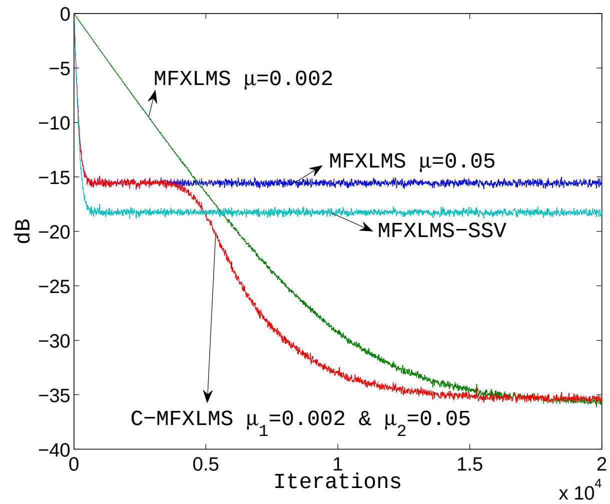


(a)

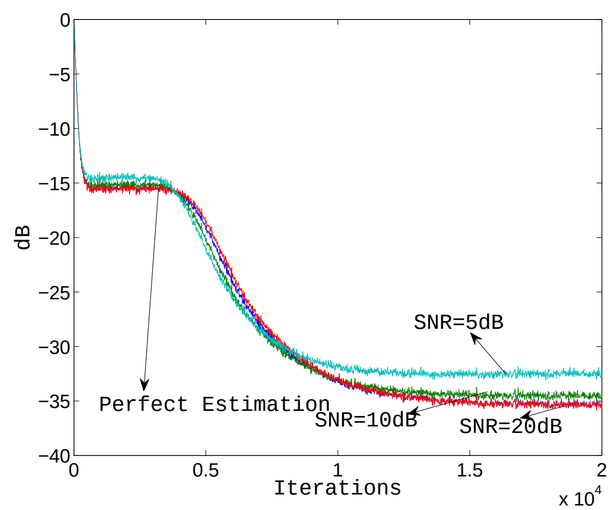


(b)

Fig. 8. Learning curves of the FXLMS algorithms, FXLMS-SSV algorithm and of their convex combination approach (Convex-FXLMS algorithm) for the single channel ANC systems conditions for time invariant and perfect estimation a) and with error in path estimate b)

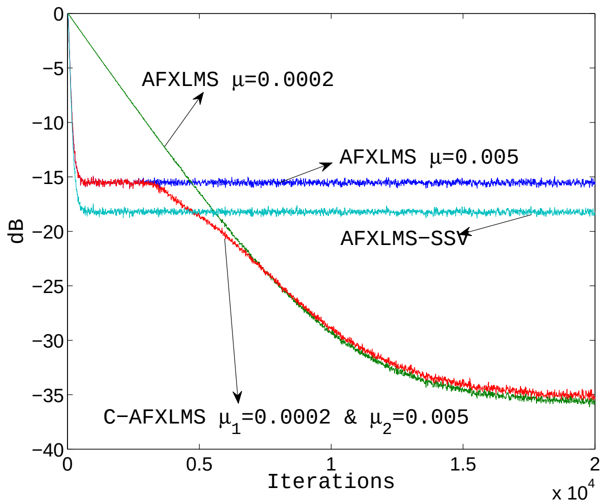


(a)

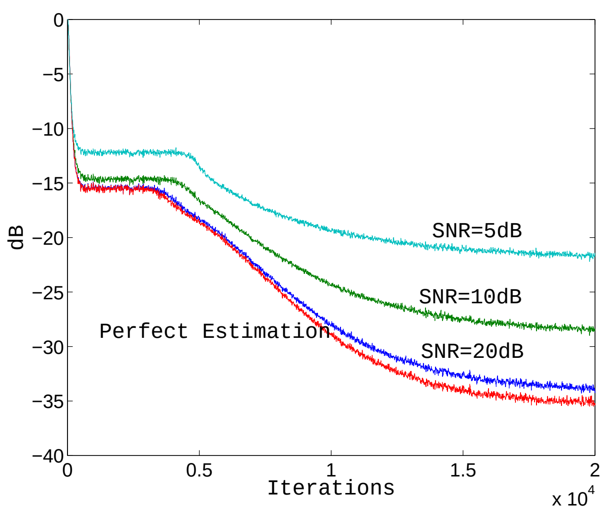


(b)

Fig. 9. Learning curves of the MFXLMS algorithms, MFXLMS-SSV algorithm and of their convex combination approach (Convex-MFXLMS algorithm) for the single channel ANC systems conditions for time invariant and perfect estimation a) and with error in path estimate b).

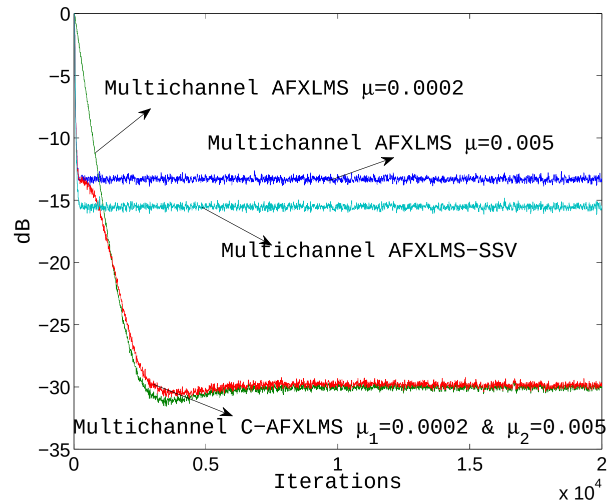


(a)

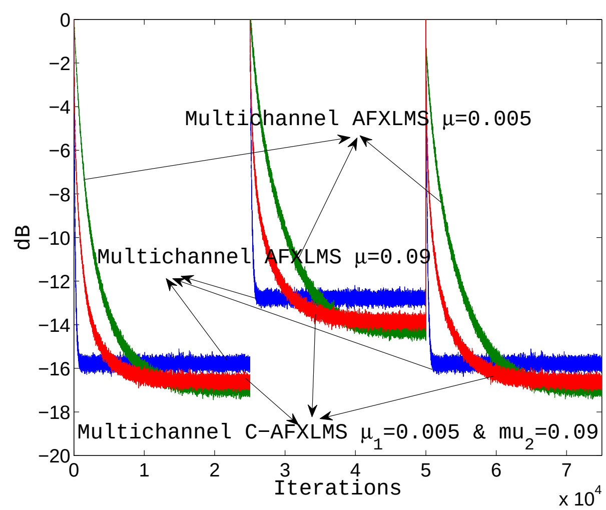


(b)

Fig. 10. Learning curves of the AFXLMS algorithms, AFXLMS-SSV algorithm and of their convex combination approach (Convex-AFXLMS algorithm) for the single channel ANC systems conditions for time invariant and perfect estimation a) and with error in path estimate b).

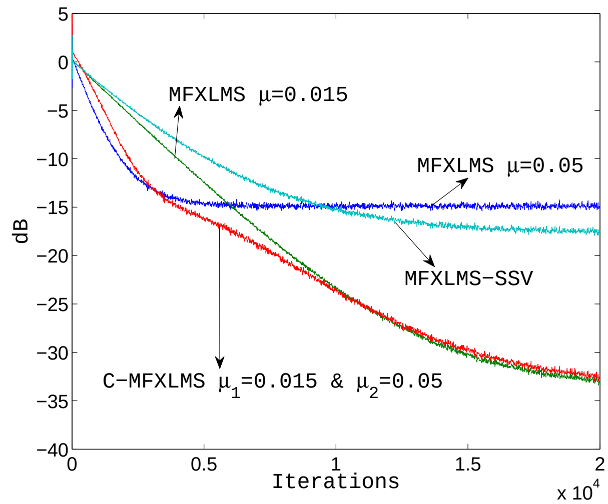


(a)

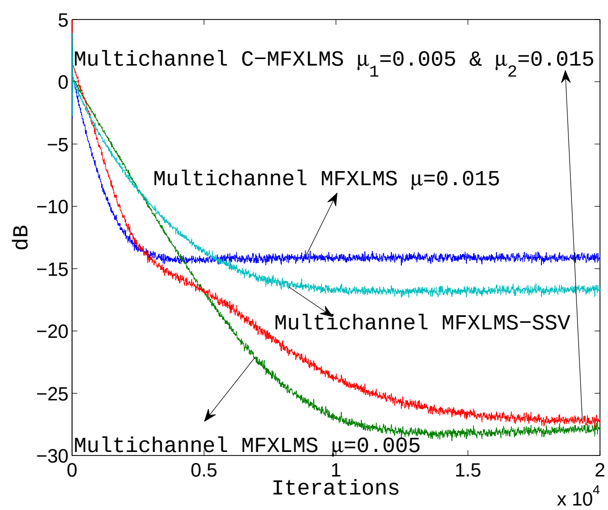


(b)

Fig. 11. Learning curves of the AFXLMS algorithms, AFXLMS-SSV algorithm and of their convex combination approach (Convex-AFXLMS algorithm) for: a) the multichannel ANC time invariant system and and b) the multichannel ANC systems for non-stationary conditions.



(a)



(b)

Fig. 12. Learning curves of the MFXLMS algorithms, MFXLMS-SSV algorithm and of their convex combination approach (Convex-MFXLMS algorithm) with periodic noise for: a) the single channel b) the multichannel ANC systems.