Abstract

Solving nonlinear equations and systems is one of the most important problems of applied mathematics, from both a theoretical and practical point of view, as well as many branches of science, engineering, physics, computing , astronomy, finance, ... a look at the bibliography and the list of great mathematicians who have worked on this issue reveals a high level of contemporary interest in it. Although the rapid development of digital computers led to the effective implementation of many numerical methods, in the practice is necessary to analyze different problems such as computational efficiency based on the time spent by the processor, the design of iterative methods that have a rapid convergence to the desired solution, the rounding error control, information about the error bounds of the approximate solution obtained, the initial conditions that guarantee safe convergence, etc.. These problems are the starting point for this work.

The overall objective of this memory is to design efficient iterative methods to solve an equation or a system of nonlinear equations. The best known scheme for solving nonlinear equations is Newton's method, whose generalization to systems of equations was proposed by Ostrowski. In recent years, as shown in extensive literature, the construction of iterative methods has increased considerably, both one point as multipoint, in order to achieve optimal order of convergence and better computational efficiency. In general, in this memory we have used the technique of weight functions to design methods for solving equations and systems, both derivative-free as with derivatives in their iterative expression.

In Chapter 2 we introduce the previous concepts underpinning the development of the different topics. Among them we have included those related to iterative methods for solving nonlinear problems in one and several variables; the concept of optimal method (based on Kung-Traub's conjecture); demonstration techniques used to test the local order of convergence as well as the divided differences operator [x, y; F], and the basics of the complex dynamics of rational functions that we will use to analyze the dynamical behavior of the operator associated with any iterative method.

In Chapters 3 and 4 we have developed optimal iterative methods of orders 4 and 8, with and without derivatives for solving nonlinear equations. In both chapters we start referring to the state of the art, to show then designed new methods, which include known families but also new iterative schemes, then continue with the analysis of the convergence of these kinds of methods, establishing some particular cases, which are analyzed in detail and we finish with the numerical evidence relating to the proposed iterative schemes. Specifically, in Chapter 3, the results obtained by modifying the classical Gauss method for determining preliminary orbits, to include in the process iterative schemes of high order of convergence are presented. Meanwhile, in Chapter 4 the dynamic properties of some eighth-order iterative schemes as well as their stability properties which are verified on different test functions.

In Chapter 5, we present optimal higher-order iterative methods with derivatives to solve nonlinear equations. After the development of these methods and the analysis of their convergence, that class of iterative schemes becomes another derivative-free, maintaining its optimality. Finally, results of some numerical tests, including the determination of preliminary orbits of satellites are showed.

The dynamical behavior of the operator associated with an iterative method when applied on a nonlinear function provides important information about its stability and reliability. Dynamical analysis of an iterative method focuses on the study of the asymptotic behavior of the fixed points (roots, or not, of the equation) operator and in the basins of attraction associated with them. For parametric families of iterative methods, the analysis of free critical points allows to select the most stable of these family members. The analysis of the complex dynamics of methods designed for nonlinear equations is carried out in Chapter 6, where we focus on one family of optimal methods presented in previous chapters. Thus, once established scaling theorem, we analyze the behavior of rational operator associated with the method acting on quadratic polynomials, calculating its fixed

and critical points and analyzing their stability. We show the parameter planes associated to the different free critical points and study some particular cases by means of the dynamical planes which include some basins of attraction that do not correspond to the roots.

Then, in Chapter 7 we extend to systems the iterative techniques defined in the scalar case, but using matrix weight functions. So we construct methods of any order by adding successive steps with the same structure. Finally, we use the divided difference operator to extend to the multivariable case some iterative schemes that, a priori, cannot be extended directly. All these methods are part of the numerical study presented at the end of the chapter, in which the theoretical results are confirmed.

This memory concludes with a chapter on open problems and future work lines. Some of these problems have arisen as a result of the progress made.