

Document downloaded from:

<http://hdl.handle.net/10251/44615>

This paper must be cited as:

González Sorribes, A.; Sala, A.; García Gil, P.J.; Albertos Pérez, P. (2013). Robustness analysis of discrete predictor-based controllers for input-delay systems. *International Journal of Systems Science*. 44(2):232-239. doi:10.1080/00207721.2011.600469.



The final publication is available at

<http://dx.doi.org/10.1080/00207721.2011.600469>

Copyright Taylor & Francis: STM, Behavioural Science and Public Health Titles

RESEARCH ARTICLE

Robustness analysis of discrete predictor-based controllers for input-delay systems

A. Gonzalez^{a*}, A. Sala^a, P. Garcia^a, and P. Albertos^a

*Instituto de Automática e Informática Industrial, Universitat Politècnica de València, Cno. Vera s/n,
46022 Valencia, Spain;*

(Received 00 Month 200x; final version received 00 Month 200x)

In this paper, robustness to model uncertainties are analyzed in the context of discrete predictor-based state-feedback controllers for discrete-time input-delay systems with time-varying delay, in an LMI framework. The goal is comparing robustness of predictor-based strategies with respect to other (sub)optimal state feedback ones. A numerical example illustrates that improvements in tolerance to modelling errors can be achieved by using the predictor framework.

Keywords: Time-varying delay-dependent stability; Discrete h-step ahead predictor; Linear matrix inequality (LMI).

1. Introduction

Time delays are often encountered in practical control systems, such as aircraft, chemical or process control systems Richard (2003), Normey-Rico and Camacho (2007). In many cases, such delays are time-varying Yue. (2005), Pan et al. (2006), Gao et al. (2008), especially in networked control systems Nilsson (1998).

The stability analysis and stabilization of delayed systems has been widely explored in the literature under two main approaches:

- (a) use of conventional controller schemes: static state feedback Pan et al. (2006), Valter et al. (2008), Du et al. (2008), Yong et al. (2008), Guangdeng et al. (2009), and static output feedback or dynamic controllers Gao et al. (2004), Liu et al. (2006).
- (b) Dead-time compensation techniques (DTC), with the aim to eliminate the delay from the characteristic equation by incorporating some sort of “prediction”. Two classical approaches are worth mentioning: Smith Predictor Smith (1959), Palmor (1996), Normey-Rico and Camacho (2007) (for time-constant delays) and the so-called finite spectrum assignment (FSA), Manitius and Olbrot (1979), Wang et al. (1998), Yue. (2005), Zhong (2006). Experimental applications of DTC techniques can be found in, for instance Hagglund (1996), Normey-Rico and Camacho (2007).

This paper will compare static versus predictor-based controllers for discrete delay systems. The more appealing characteristic of predictor-based control is that, without modelling error, its performance approaches that of a delay-free system when delay is known Yue. (2005). However, one of the widest criticisms to the predictor-control schemes is the high sensitivity to model uncertainties and delay mismatches Michiels and Niculescu (2003). This fact can be explained taking into account that modelling error tends to accumulate as model equations are integrated

*Corresponding author. Email: angonsor@upvnet.upv.es

(or iterated in discrete-time) in order to obtain future state predictions. This motivates the practical interest of analyzing the performance-robustness tradeoff of such predictor implementations. As most control applications are computer based, this paper will focus in studying the differences in performance-robustness tradeoff of the two above paradigms, for a discrete-time case.

Robustness of discrete non-predictor controllers have been addressed in, for instance, Boukas (2006), Du et al. (2008), Yong et al. (2008), Zhang et al. (2008).

Some results concerning robustness analysis with FSA predictor-based controllers have been proposed in the literature (see, for instance a time-varying delay case for continuous-time Yue. (2005)). For the discrete version of FSA, known as predictor scheme Goodwin and Sin (1984) some results (existence of a small-enough neighborhood) regarding model and delay variations Lozano et al. (2004), Garcia et al. (2006) have been reported.

The objective of this paper is extending the ideas in the just cited references in order to analyze robust stability of discrete predictor-based state-feedback controllers under time-varying delay, only assuming knowledge of delay bounds. As a result, the paper will illustrate that a suitably designed predictor scheme can improve the robustness margins with respect to a (sub)optimal memoryless robust controller design in some cases.

The paper is organized as follows: next section discusses preliminary results, section 3 provides the problem statement and auxiliary lemmas. Section 4 provides a robust stability analysis theorem. Section 5 provides a numerical example comparing robustness of various predictor and non-predictor designs and a conclusion section closes the paper.

2. Preliminaries

Let us consider the following discrete-time linear-time-variant input delayed system:

$$x_{k+1} = Ax_k + Bu_{k-d_k}, \quad k = 0, 1, \dots \quad (1)$$

$$u_l = \phi_u(l), \quad -d_M \leq l < 0 \quad (2)$$

where $\phi_u(l)$ represents some initial conditions for the input control action u_{-d_M}, \dots, u_{-1} and $A \in \mathbb{R}^{m \times m}$, $B \in \mathbb{R}^{m \times n}$ are the nominal plant parameter matrices. This kind of models arise in, for instance, computer-controlled industrial reactors or vessels where some components flow around feed pipelines (mass and energy transport): the delay would be a function of the pipe length and fluid speed, so if the latter is not constant, the delay would be varying.

The delay $d_k > 0$ is assumed not measurable, but known to vary randomly in an interval $d_m \leq d_k \leq d_M$. The lower and upper delay bounds (d_m and d_M , respectively), are assumed known.

To simplify the further developments and without loss of generality, $\phi_u(k) = 0$ will be assumed on the sequel.

As previously discussed, most approaches to control of input-delay systems are based on either direct static state-feedback or a predictor-based feedback.

Static state feedback. If a static state-feedback control law is proposed

$$u_k = Kx_k \quad (3)$$

where $K \in \mathbb{R}^{n \times m}$ is the control gain matrix. The closed loop realization yields

$$x_{k+1} = Ax_k + BKx_{k-d_k} \quad (4)$$

The memoryless feedback has as an advantage the fact that stability analysis and controller synthesis have been explored in the context of discrete Lyapunov-Krasovskii functionals Gao and Chen (2007), Zhang et al. (2008). However, it seems clear that there is an inherent conservativeness because: (1) the feedback considers only partial information about the true state of the delayed process (in fact, x can no longer be regarded as the “state” vector as more past input information is needed for future predictions), (2) The Lyapunov-Krasovskii functionals do not usually represent with full generality the class of Lyapunov functions possibly needed.

Prediction-based control. As an alternative, consider the state-feedback prediction-based control law

$$u_k = K\bar{x}_{k+h} \quad (5)$$

where \bar{x}_{k+h} is the future state prediction h -step ahead, being h an user-defined parameter (the expected delay). The basic intuitive argumentation is the fact that $u_{k-h} = K\bar{x}_{k+h-h}$ so, if \bar{x} correctly approximates the true state x , then the term Bu_{k-h} would equal BKx_k which is the same than that of a delay-free state feedback law. Obviously, the idea works, in principle, with constant and known delay. The objective of this paper is exploring the performance/robustness tradeoff of these ideas in a varying delay case, as discussed in next section.

Assuming $x_{k+1} = Ax_k + Bu_{k-h}$, then $x_{k+2} = A^2x_k + ABu_{k-h} + Bu_{k-h+1}$ and similar expressions can be obtained for x_{k+3} and so on. Hence, the predictor \bar{x} in (5) can be computed as Goodwin and Sin (1984), Lozano et al. (2004):

$$\bar{x}_{k+h} = A^h x_k + \sum_{i=0}^{h-1} A^{h-i-1} B u_{k+i-h} \quad (6)$$

Note that the control actions used in the above expressions are u_{k-h}, \dots, u_{k-1} , so indeed this prediction is a *causal* expression.

For constant delays $d_k = d$, the resulting closed-loop expression is Garcia et al. (2006):

$$x_{k+1} = (A + BK)x_k + BKA^h(x_{k-d} - x_{k-h}) \quad (7)$$

which, indeed, recovers a delay-free closed loop if $d = h$, as intuitively expected.

Uncertainty plays a significant role in the practical applications of predictor schemes. Indeed, as mentioned in the introduction, prediction accumulates such model uncertainty over time.

How to characterize such uncertainty and the robustness of the designs against it has been preliminarily considered in literature. When the actual delay d is close to the designed predictor delay h , the closed loop behavior has been shown to be robustly stable: in Lozano et al. (2004) sampling period jitter and small delay variations (significantly less than one nominal sample period), as well as small modelling errors, are approximately transformed to uncertainty in the discrete-time input delay model (1); the authors show that stability is preserved in a small enough neighborhood around the nominal system matrices. In Garcia et al. (2006) the framework is extended allowing for a predictor delay h different to d , but considering both h and d to be constant. In summary, in these references the size of the robustness neighborhood is not computed and the results are only valid for constant delay in the discretized model.

As discussed in the introduction, this paper has two goals: first, extending the ideas in the just cited references in order to analyze robust stability of discrete predictor-based state-feedback controllers under time-varying delay and, second, to show that in some cases there is a performance/robustness improvement with respect to a non-predictor static feedback control scheme.

3. Problem formulation

This paper will consider a robustness analysis for the above-considered delay systems and controllers. In particular, let us consider the discrete-time linear-time-variant input delayed system depicted in (1) with norm-bounded uncertainties:

$$x_{k+1} = (A + \Delta_{A_k})x_k + (B + \Delta_{B_k})u_{k-d_k} \tag{8}$$

$$u_l = \phi_u(l), \quad -d_M \leq l < 0 \tag{9}$$

The system uncertainties $\Delta_{A_k}, \Delta_{B_k}$ are assumed to satisfy the following constraints

$$[\Delta_{A_k}, \Delta_{B_k}] = \bar{\alpha}G\Delta_k[H_a, H_b] \tag{10}$$

$$\Delta_k^T \Delta_k \leq I \tag{11}$$

where G, H_a, H_b are known matrices of appropriate dimensions.

The objective of the following sections is to present a set of LMI constraints able to quantify the trade-offs involved in predictor-based schemes: nominal performance versus robustness to time-varying delay and model uncertainty. In this setting, $\bar{\alpha}$ will be considered an overall size parameter to be later optimized.

Notation and auxiliary lemmas. The following well-known lemma is given here to develop some of the main results presented

Lemma 3.1: *Given appropriate matrices X and Y and a symmetric matrix Z ,*

$$Z + X\Delta Y + Y^T \Delta^T X^T < 0 \tag{12}$$

holds for all Δ satisfying $\Delta^T \Delta \leq I$ if and only if there exists a scalar $\epsilon > 0$ such that

$$Z + \epsilon X X^T + \epsilon^{-1} Y^T Y < 0 \tag{13}$$

In the example section, the following result, obtained from minor modifications to (Gao and Chen 2007, lemma 2) will be used for comparison purposes.

Lemma 3.2: *The closed-loop system (4) is robustly asymptotically stable if there exists matrices $P, Q, R, Z_1 > 0$, a scalar value $\rho > 0$ and some matrices M, N, S, W of suitable dimensions satisfying the convex optimization problem depicted below.*

minimize ρ s.t

$$\begin{pmatrix} -P & 0 & \phi_4 & 0 & \sqrt{d_M}G & 0 \\ (*) & -P & \phi_3 & 0 & G & 0 \\ (*) & (*) & \phi_1 + \Psi_2 + \Psi_2^T & \Psi_4 & 0 & \phi_2 \\ (*) & (*) & (*) & \Psi_5 & 0 & 0 \\ (*) & (*) & (*) & (*) & -\rho I & 0 \\ (*) & (*) & (*) & (*) & (*) & -I \end{pmatrix} < 0 \tag{14}$$

$$\Psi_2 = (M + N, S - M, -S - N) \tag{15}$$

$$\Psi_4 = (M\sqrt{d_M}, S\sqrt{d_M - d_m}, N\sqrt{d_M}) \quad (16)$$

$$\Psi_5 = \text{diag}(-Z_1, -Z_1, -P + Z_1) \quad (17)$$

$$\phi_1 = \begin{pmatrix} -P + \tau Q + R & 0 & 0 \\ (*) & -Q & 0 \\ (*) & (*) & -R \end{pmatrix} \quad (18)$$

$$\phi_2 = (H_a P, H_b W, 0)^T \quad (19)$$

$$\phi_3 = (A_0 P, B_0 W, 0) \quad (20)$$

$$\phi_4 = (\sqrt{d_M}(A_0 - I)P, \sqrt{d_M}B_0 W, 0) \quad (21)$$

Furthermore, the suboptimal controller gain can be obtained directly as $K = WP^{-1}$ guaranteeing stability for model uncertainties Δ_{A_k} and Δ_{B_k} fulfilling (10) with $\bar{\alpha} = \sqrt{1/\rho}$. This is the reason of searching for the minimum ρ .

Proof The result is obtained in a straightforward way by setting $P = Z_1 + Z_2$ in Theorem 2, Gao and Chen (2007) and applying a congruence transformation by pre-and post-multiplying by $\text{diag}(P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I)$. Note that P in (14)–(21) actually denotes the inverse of the decision variable P in Gao and Chen (2007). \square

More recent results improve over Gao and Chen (2007) in stability analysis Zhang et al. (2008), and also including model uncertainty Guo and Li (2009). However, regarding controller design, the first work poses the problem as a BMI to be solved by CCL El Ghaoui et al. (1997) and the second one does not improve upon Gao and Chen (2007) when adaptations in order to get LMI synthesis conditions analogue to Lemma 3.2 are implemented (memoryless controller). In this paper, only LMI approaches will be considered.

4. Robust Stability Analysis of predictor-based state feedback

In this section LMI constraints are provided to check stability in a discrete-time predictor control law on a time-varying input delay system in presence of norm-bounded uncertainties.

In order to establish robust stability results in delay systems, some state transformations are needed, as pointed out in Yue. (2005). The following lemma proposes the needed transformation for the discrete-time case.

Lemma 4.1:

If A is invertible¹, the closed-loop realization of model (8) with the control law (5) can be expressed as

¹Note that in the discretization of continuous-time systems with state matrix A_c , the resulting $A = e^{A_c T}$ is always non-singular.

$$z_{k+1} = A_1 z_k + B_d z_{k-d_k} + B_h z_{k-h} + \sum_{i=1}^{h-1} B_i z_{k-i} \quad (22)$$

$$A_1 = A + BK + A^h \Delta_{A_k} A^{-h} \quad (23)$$

$$B_d = A^h (B + \Delta_{B_k}) K \quad (24)$$

$$B_h = -A^h (I + \Delta_{A_k} A^{-1}) BK \quad (25)$$

$$B_i = -A^h (\Delta_{A_k} A^{-h+i-1}) BK \quad i = 1, \dots, h-1 \quad (26)$$

Proof Let us define the new state z_k

$$z_k = A^h x_k + \sum_{i=0}^{h-1} A^{h-i-1} B u_{k-h+i} \quad (27)$$

being z_k the prediction of x_{k+h} , i.e., $z_k = \bar{x}_{k+h}$ using the nominal model with no uncertainties. The one-step ahead z_{k+1} is

$$z_{k+1} = A^h x_{k+1} + \sum_{i=0}^{h-1} A^{h-i} B u_{k-h+i+1} \quad (28)$$

Substituting x_{k+1} from the system model (1) we obtain

$$z_{k+1} = A^h [(A + \Delta_{A_k}) x_k + (B + \Delta_{B_k}) u_{k-d_k}] + \quad (29)$$

$$+ \sum_{i=0}^{h-1} A^{h-i} B u_{k-h+i+1} \quad (30)$$

From (27) the state x_k can be obtained depending on z_k and the last h control actions as

$$x_k = A^{-h} z_k + \sum_{i=0}^{h-1} A^{-1-i} B u_{k-h+i} \quad (31)$$

By replacing x_k in (29) and making the suitable operations the following is obtained

$$z_{k+1} = (A + A^h \Delta_{A_k} A^{-h}) z_k + B u_k - A^h (I + \Delta_{A_k} A^{-1}) B u_{k-h} + \quad (32)$$

$$+ A^h (B + \Delta_{B_k}) u_{k-d_k} - \sum_{i=1}^{h-1} A^h (\Delta_{A_k} A^{-h+i-1}) B u_{k-i}$$

Finally, taking into account the control law $u_k = K z_k$ the proof is completed. □

Once the above realization is available, the following theorem presents the main robust stability analysis result, using the well known augmented delay-free realization of discrete delay systems.

For convenience, the notation $\delta = d_M - d_m + 1$ will be introduced. Also, $d_i = d_m + i - 1$ for $i \in \{1, \dots, \delta\}$.

Theorem 4.2:

The closed-loop system (22) is asymptotically stable for some previously-designed K and h if there exists δ matrices \bar{P}_i and δ^2 scalars $\epsilon_{ij} > 0$ for $i, \in \{1, \dots, \delta\}$ and $j \in \{1, \dots, \delta\}$ such that

$$\bar{P}_i > 0 \quad \forall i \in \{1, \dots, \delta\} \quad (33)$$

and, for all $i, j \in \{1, \dots, \delta\} \times \{1, \dots, \delta\}$

$$\begin{pmatrix} \bar{P}_i + \epsilon_{ij} \bar{H}(d_i)^T \bar{H}(d_i) & (*) & (*) \\ \bar{P}_j \bar{A}(d_i) & -\bar{P}_j & (*) \\ 0 & \bar{G}^T \bar{P}_j & -\rho \epsilon_{ij} I \end{pmatrix} < 0 \quad (34)$$

where

$$\bar{A}(d_i) = \begin{pmatrix} A + BK \Gamma_1(d_i) & \Gamma_2(d_i) & \cdots & \cdots & \Gamma_{d_M}(d_i) \\ I & 0 & 0 \cdots & \cdots & 0 \\ 0 & I & 0 \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 \cdots & I & 0 \end{pmatrix}$$

being

$$\Gamma_l(d_i) = \gamma(d_i, l) A^h B K - \gamma(h, l) A^h B K \quad (35)$$

for $1 \leq l \leq d_M$ and

$$\bar{G} = (A^h G \ 0 \ \cdots \ 0)^T \quad (36)$$

$$\bar{H}(d_i) = \begin{pmatrix} H_a A^{-h} \\ -H_a A^{-h} B K + \gamma(d_i, 1) H_b K \\ -H_a A^{-h+1} B K + \gamma(d_i, 2) H_b K \\ \cdots \\ -H_a A^{-1} B K + \gamma(d_i, h-1) H_b K \\ \gamma(d_i, h) H_b K \\ \cdots \\ \gamma(d_i, d_M) H_b K \end{pmatrix}^T \quad (37)$$

where the scalar function $\gamma(x, y)$ is defined here as

- $\gamma(x, y) = 1$, if $x = y$
- $\gamma(x, y) = 0$, otherwise

Moreover, a bound for model uncertainties in (10), $\bar{\alpha}$, can be found by setting an objective function over ρ in such way that the proposed robust analysis task can be solved as:

minimize ρ subject to constraints (33), (34)

If feasibility is achieved, the proved size of model uncertainties keeping stability can be obtained as $\bar{\alpha} = \sqrt{1/\rho}$.

Proof

Consider a generic quadratic parameter-dependent Lyapunov functional involving the full-augmented process state

$$V_k = \sum_{p=0}^{d_M} \sum_{r=0}^{d_M} x_{k-p}^T P_{d_k}^{p,r} x_{k-r} \tag{38}$$

The matrices $P_{d_k}^{p,r}$ are defined such as the Lyapunov functional (38) is always positive definite, $V_k > 0$, that is,

$$\bar{P}_{d_i} := \bar{P}_i = \begin{pmatrix} P_i^{0,0} & P_i^{0,1} & \dots & P_i^{0,d_M} \\ P_i^{0,1} & P_i^{1,1} & \dots & P_i^{1,d_M} \\ \dots & \dots & \dots & \dots \\ P_i^{d_M,0} & P_i^{d_M,1} & \dots & P_i^{d_M,d_M} \end{pmatrix} > 0$$

for every $i \in \{1, \delta\}$. Without loss of generality we may consider that $P_i^{p,r} = (P_i^{r,p})^T$. In such a way (38) can be rewritten as

$$V(k, d_k) = \Phi_k^T \bar{P}_{d_k} \Phi_k \tag{39}$$

where Φ_k is defined as the augmented process state vector $\Phi_k = [x_k, \dots, x_{k-d_M}]$. By suitable algebraic manipulations the augmented system (22) with Φ_k can be expressed as

$$\Phi_{k+1} = [\bar{A}(d_k) + \bar{G}\Delta_k\bar{H}(d_k)]\Phi_k \tag{40}$$

where $\bar{A}(d_k)$, \bar{G} and $\bar{H}(d_k)$ are defined in (35), (36) and (37), respectively.

By imposing $V(d_{k+1}, k + 1) - V(d_k, k) < 0$ and making the usual Schur complement for discrete-system stability analysis, it is easy to obtain the well-known LMI constraints for every $d_i = d_k$ and $d_j = d_{k+1}$

$$\begin{pmatrix} -\bar{P}_i & (*) \\ \bar{P}_j[\bar{A}(d_i) + \bar{\alpha}\bar{G}\Delta_k\bar{H}(d_i)] - \bar{P}_j \end{pmatrix} < 0 \tag{41}$$

this last expression can be written as

$$\bar{A}_u(d_i, d_j) + \bar{\alpha}\bar{G}_u(d_j)\Delta_k\bar{H}_u(d_i) + \bar{H}_u^T(d_i)\Delta_k^T\bar{G}_u^T(d_j) < 0 \tag{42}$$

where

$$\bar{A}_u(d_i, d_j) = \begin{pmatrix} -\bar{P}_i & (*) \\ \bar{P}_j\bar{A}(d_i) - \bar{P}_j \end{pmatrix} \tag{43}$$

$$\bar{G}_u(d_j) = (0 \ \bar{P}_j\bar{G})^T \tag{44}$$

$$\bar{H}_u(d_i) = (\bar{H}(d_i) \ 0) \tag{45}$$

According to lemma 3.1, the inequality (42) holds for all d_k satisfying $\Delta_k^T\Delta_k \leq I$ if and only if there exists a scalar $\epsilon_{ij} > 0$ such that

$$\bar{A}(d_i, d_j) + \epsilon_{ij}^{-1} \bar{\alpha}^2 \bar{G}_u(d_j) \bar{G}_u^T(d_j) + \epsilon_{ij} \bar{H}_u^T(d_i) \bar{H}_u(d_i) < 0 \quad (46)$$

Substituting (43), (44) and (45) into (46) we have

$$\begin{pmatrix} -\bar{P}_i + \epsilon_{ij} \bar{H}^T(d_i) \bar{H}(d_i) & & & \\ & \bar{P}_j \bar{A}(d_i) & & \\ & & -\bar{P}_j + \epsilon_{ij}^{-1} \bar{\alpha}^2 \bar{P}_j \bar{G} \bar{G}^T \bar{P}_j & \\ & & & (*) \end{pmatrix} < 0 \quad (47)$$

By applying again the Schur complement, the above inequality is equivalent to

$$\begin{pmatrix} \bar{P}_i + (\epsilon_{ij}) \bar{H}^T(d_i) \bar{H}(d_i) & (*) & (*) \\ & \bar{P}_j \bar{A}(d_i) & (*) \\ & 0 & \bar{G}^T \bar{P}_j - (\epsilon_{ij}) \bar{\alpha}^{-2} I \end{pmatrix} < 0 \quad (48)$$

which completes the proof. \square

Remark 1: Note that if we make $\epsilon_{ij} = \epsilon$ theorem 4.2 is an LMI problem with the change of variable $P' = P/\epsilon$.

Remark 2: For large values of the delay, the number of decision variables in (38) may exhaust computational resources. Another option may be eliminating the dependence on the delay or replacing the functional by:

$$V_k = \sum_{p=0}^{d_M} \sum_{r=\max(0, p-q)}^{\min(p+q, d_M)} x_{k-p}^T P_{d_k}^{p,r} x_{k-r}$$

with a complexity parameter, q , which transforms matrix (39) from a full matrix ($q = d_M$) to a band one or even to a diagonal matrix ($q = 0$). In fact, many discrete Lyapunov-Krasovskii proposals can be considered a particular case of the generic augmented-state Lyapunov function discussed above.

5. Numerical Example

Consider the following system:

$$x_{k+1} = (A + \Delta A_k)x_k + (B + \Delta B_k)u_{k-d_k} \quad (49)$$

with

$$A = \begin{pmatrix} 1.01 & 0 \\ 0 & 0.7 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (50)$$

and the model uncertainties matrices defined at (10)

$$G_d = (1 \ 1)^T, H_a = (0.1 \ 0.1), H_b = 1 \quad (51)$$

The following two design approaches will be compared with respect to robustness bounds:

- Static state-feedback: maximizing robustness via lemma 3.2, taking the delay bounds d_m and d_M .
- Predictor scheme: dynamic feedback based on predictor scheme taking the predictor delay $h = d_m$. The controller gain K is designed via lemma 3.2 but replacing the values for d_m and d_M with the new values $d'_m = 0$ and $d'_M = d_M - d_m$, respectively.

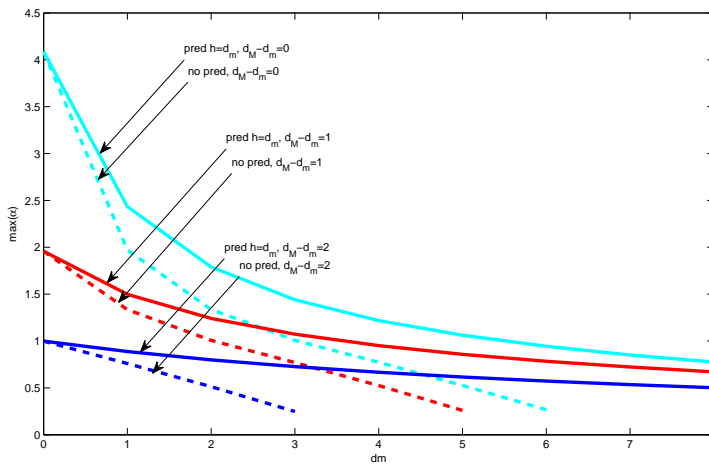


Figure 1. Achieved robustness $\bar{\alpha}$ obtained with the proposed predictor scheme vs controller designed via lemma 3.2.

Figure 1 compares the maximum tolerance to model errors in $\bar{\alpha}$ obtained in both cases. In the legend, the predictor-based design results are labeled as 'pred' and the static state feedback are labeled as 'no pred', respectively.

The lower bound delay d_m takes the value depicted on the abscissae-axis for every case. The cyan lines (top) plot the proved uncertainty bound in case of constant delay $d_m = d_M$. The other lines plot two different time-varying delay cases: (a) the red ones (middle) plot the case of $d_M = d_m + 1$, and (b) the blue ones (bottom) present the case $d_M = d_m + 3$. When no value $\bar{\alpha}$ is provided for some d_m it means that no feasible solution was found. The figure shows that, at least for this example, the predictor-based feedback design strategy achieves a better tolerance to modelling errors than the (sub)-optimal static design. As intuitively expected, the provable model error bound decreases as both minimum delay and delay range increase.

Simulation results. As the LMI stability analysis and controller design techniques may be conservative, in order to test whether the larger robustness bounds for the predictor approach are actually confirmed in the time response, a simulation has been carried out.

The previous system has been simulated by considering time-varying input delay bounds of $d_m = 4$ and $d_M = 5$, respectively, and a constant value of uncertainty $\Delta_k = -0.5$ at the system model in expression (10). The state response for $y = x_1$ is depicted in Figure 2, as follows:

- Delayed case without predictor (dash-dotted line). The static state feedback controller has been designed to maximize $\bar{\alpha}$ via lemma 3.2 with $d_m = 4$, $d_M = 5$ obtaining $K = [-0.0419, 0.0147]$, $\bar{\alpha} = 0.442$.
- Delayed case with predictor (solid line). The feedback gain has been designed by the aforementioned predictor-based design criterion ($h = 4$) obtaining $K = [-0.1504, -0.0448]$, $\bar{\alpha} = 0.952$.

The referred figure shows that the predictor scheme has a much better performance under modelling error and delay range settings than the non-predictor state-feedback approach: *both* performance and robustness are improved by the sensible use of the predictor.

In summary, we have shown that the predictor scheme may improve the performance/robustness trade-off for static state-feedback controllers with respect to a suboptimal design tackled with some proposed recent results found in the literature, also allowing to ensure closed-loop stability for a larger range of delay values.

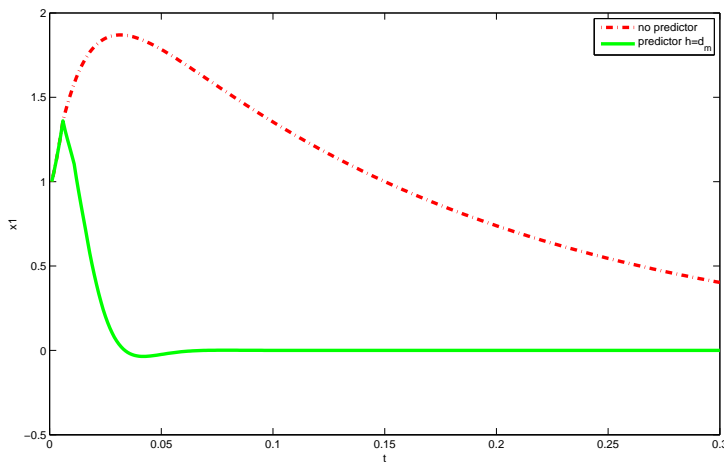


Figure 2. Comparison of the system response (state x_1) achieved between the proposed predictor scheme vs controller designed via lemma 3.2.

6. Conclusion

In this paper, a study of the robustness of discrete-time predictor-based state feedback control loops has been carried out. A Lyapunov function on the augmented delay-free plant has been used. If so wished, such Lyapunov function can be reduced to ordinary discrete Lyapunov-Krasovskii expressions.

The result allows to extract information on how model iterations diminish tolerance to modelling errors as prediction horizon h grows. It also enables comparative robustness analysis with other non-predictor alternatives.

A numerical example has shown that, in some cases, predictor-based approaches are, for the same performance levels, more robust than static feedback laws even if the latter consider the actual minimum and maximum delay bounds.

Acknowledgements

The authors are grateful to grants DPI2008-06737-C02-01, DPI2008-06731-C02-01, DPI2011-27845-C02-01 and PROMETEO/2008/088 from Spanish and Valencian governments.

References

- Boukas, E. (2006), "Discrete-time systems with time-varying time-delay: stability and stabilizability," *Mathematical Problems in Engineering*, 2006(ID42489), 1–10.
- Du, D., Jiang, B., and Zhou, S. (2008), "Delay-dependent robust stabilisation of uncertain discrete-time switched systems with time-varying state delay," *International Journal of Systems Science*, 39(3), 305–313.
- El Ghaoui, L., Oustry, F., and AitRami, M. (1997), "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE transactions on automatic control*, 42(8), 1171.
- Gao, H., and Chen, T. (2007), "New results on stability of discrete-time systems with time-varying state delay," *IEEE Transactions on Automatic Control*, 52(2), 328–334.
- Gao, H., Chen, T., and Lam, J. (2008), "A new delay system approach to network-based control," *Automatica*, 44(1), 39–52.
- Gao, H., Lam, J., Wang, C., and Wang, Y. (2004), "Delay-dependent output-feedback stabili-

- sation of discrete-time systems with time-varying state delay," *IEE Proc.-Control Theory Appl*, 151(6), 691.
- Garcia, P., Castillo, P., Lozano, R., and Albertos, P. (2006), "Robustness with respect to delay uncertainties of a predictor observer based discrete-time controller," *Proc. of the 45th IEEE Conference on Decision and Control*.
- Goodwin, G., and Sin, K., *Adaptive Filtering Prediction and Control*, Prentice-Hall, Englewood Cliffs, NJ (1984).
- Guangdeng, Z., Linlin, H., and Hongyong, Y. (2009), "Further results concerning delay-dependent H^{∞} control for uncertain discrete-time systems with time-varying delay," *Mathematical Problems in Engineering*, 2009.
- Guo, Y., and Li, S. (2009), "New Stability Criterion for Discrete-Time Systems with Interval Time-Varying State Delay," *Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*.
- Hagglund, T. (1996), "An industrial dead-time compensating PI controller," *Control Engineering Practice*, 4(6), 749–756.
- Liu, X., Martin, R., Wu, M., and Tang, M. (2006), "Delay-dependent robust stabilisation of discrete-time systems with time-varying delay," *IEE Proc.-Control Theory Appl*, 153(6), 689.
- Lozano, R., Castillo, P., Garcia, P., and Dzul, A. (2004), "Robust prediction-based control for unstable delay systems: Application to the yaw control of a mini-helicopter," *Automatica*, 40(4), 603–612.
- Manitius, A.Z., and Olbrot, A. (1979), "Finite spectrum Assignment problem for Systems with Delays," *tac*.
- Michiels, W., and Niculescu, S. (2003), "On the delay sensitivity of Smith predictors," *International Journal of Systems Science*, 34(8), 543–551.
- Nilsson, J. (1998), "Real-time control systems with delays," *Lund, Sweden: Lund Institute of Technology*.
- Normey-Rico, J., and Camacho, E., *Control of dead-time processes*, Springer Verlag (2007).
- Palmor, Z. (1996), "Time-delay compensation Smith predictor and its modifications," *The control handbook*, pp. 224–237.
- Pan, Y., Marquez, H., and Chen, T. (2006), "Stabilization of remote control systems with unknown time varying delays by LMI techniques," *International Journal of Control*, 79(7), 752–763.
- Richard, J. (2003), "Time-delay systems: an overview of some recent advances and open problems," *automatica*, 39(10), 1667–1694.
- Smith, O. (1959), "Closer Control of loops with dead time Chemical Engineering Progress," *Chemical Engineering Progress*, 53.
- Valter, J. et al. (2008), "Robust stabilization of discrete-time systems with time-varying delay: an LMI approach," *Mathematical Problems in Engineering*, 2008.
- Wang, Q., Lee, T., and Tan, K., *Finite spectrum assignment for time-delay systems*, Springer Verlag (1998).
- Yong, H., Min, W., Qinglong, H., and Jinhua, S. (2008), "Delay-dependent H^{∞} control of linear discrete-time systems with an interval-like time-varying delay," *International Journal of Systems Science*, 39(4), 427–436.
- Yue., D. (2005), "Delayed feedback control of uncertain systems with time-varying input delay," *aut*, 41, 233–240.
- Zhang, B., Xu, S., and Zou, Y. (2008), "Improved stability criterion and its applications in delayed controller design for discrete-time systems," *Automatica*, 44(11), 2963–2967.
- Zhong, Q., *Robust control of time-delay systems*, Springer Verlag (2006).