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A note on local bases and convergence in fuzzy metric spaces

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Abstract

In the context of fuzzy metrics in the sense of George and Veeramani, we study when certain families of open balls centered at a point are local bases at this point. This question is related to p-convergence and s-convergence.

Key words: p (s)-convergence; (principal) (stationary) fuzzy metrics.
MSC: 54A40, 54D35, 54E50

1 Introduction

George and Veeramani, [1,3], introduced and studied a notion of fuzzy metric with the help of continuous t-norms. If M is a fuzzy metric on a (non-empty)

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set X, a topology τ_M is deduced from M. In [2,8] the authors show that the class of topological spaces which are fuzzy metrizable agrees with the class of metrizable spaces. Later, several authors have contributed to the development of this theory, for instance [8,10,11,16–18].

An interesting aspect in this type of fuzzy metric is that it includes in its definition a parameter t. This feature has been successfully used in engineering applications such color image filtering [7,14,15] and perceptual color differences [5,13]. From the mathematical point of view it allows to introduce novel (fuzzy metric) concepts that only have natural sense in the fuzzy metric setting. For instance, the concept of p-convergence [4] and s-convergence [6] of sequences, which satisfy the implications

$$s - convergence \Rightarrow convergence \Rightarrow p - convergence$$

The convergence of a sequence to a point x_0 in a metric space (X, d) involves some local base constituted by balls centered at x_0. If ξ is any family of open balls centered at x_0 such that $\bigcap \xi = \{x_0\}$ and x_0 is not isolated in (X, d) then ξ is a local base at x_0. (In this paper $\bigcap \xi$ denotes the intersection of all members of ξ). The purpose of this note is to study this assertion in the fuzzy setting. We consider first, a general case, and later some families of balls that, in a natural way, appear when studying p-convergence and s-convergence. Notice that a centered ball at x_0 in a fuzzy metric space $(X, M, *)$ is denoted by $B(x_0, r, t)$ where $r \in]0, 1[$, $t > 0$.

We show in this paper that the above assertion is false, in general, for a fuzzy metric space (Example 7). Now, if ξ is constituted by balls of the form $\{B(x_0, r, r) : r \in J\}$, where $J \subset]0, 1[$, or M is stationary (Definition 2) then the above assertion holds.

In [4] it is proved that any sequence p-convergent to x_0 in $(X, M, *)$ is convergent if and only if $\{B(x_0, r, t) : r \in]0, 1[\}$ is a local base at x_0, for each $t > 0$. Fuzzy metric spaces in which all p-convergent sequences are convergent were called principal. So it seems natural to study families of open balls, centered at x_0, for a fixed $t > 0$. We show that if $\mathcal{B}$ is any of these families the above assertion is true in principal fuzzy metric spaces, but in general it is false.

In [6] it is proved that any sequence convergent to x_0 is s-convergent in $(X, M, *)$ if and only if $\bigcap_{t>0} B(x_0, r, t)$ is a local neighborhood of x_0 in (X, τ_M), for each $r \in]0, 1[$. Fuzzy metric spaces in which all convergent sequences are s-convergent were called s-fuzzy metric spaces. So, it is natural to study families of open balls centered at x_0 with a fixed radius $r \in]0, 1[$. If $\mathcal{D}$ is any of these families the above assertion is true in co-principal fuzzy metric spaces (Definition 19), and a similar result is obtained when (X, τ_M) is compact (Theorem

23). The answer in a more general context is an open problem (Problem 17). Some examples are provided, along the paper, that illustrate the theory.

The structure of the paper is as follows. In Section 2 we include the preliminaries about fuzzy metrics. In Section 3 we study the question of when a family ξ of open balls centered at x_0 in a (principal) fuzzy metric space $(X, M, *)$, is a local base at x_0 provided that $\bigcap \xi = \{x_0\}$. The same question related to s-fuzzy metrics is studied in Section 4.

2 Preliminaries

Definition 1 *(George and Veeramani [1]). A fuzzy metric space is an ordered triple $(X, M, *)$ such that X is a (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times]0, \infty[$ satisfying the following conditions, for all $x, y, z \in X$, $s, t > 0$:*

(GV1) $M(x, y, t) > 0$;
(GV2) $M(x, y, t) = 1$ *if and only if* $x = y$;
(GV3) $M(x, y, t) = M(y, x, t)$;
(GV4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(GV5) $M(x, y, \_) :]0, \infty[\rightarrow]0, 1]$ *is continuous.*

If $(X, M, *)$ is a fuzzy metric space, we will say that $(M, *)$, or simply M, is a *fuzzy metric* on X. This terminology will be also extended along the paper in other concepts, as usual, without explicit mention.

George and Veeramani proved in [1] that every fuzzy metric M on X generates a topology τ_M on X which has as a base the family of open sets of the form $\{B_M(x, \epsilon, t) : x \in X, 0 < \epsilon < 1, t > 0\}$, where $B_M(x, \epsilon, t) = \{y \in X : M(x, y, t) > 1 - \epsilon\}$ for all $x \in X$, $\epsilon \in]0, 1[$ and $t > 0$, and they proved that for each $x \in X$ the family $\{B(x, \frac{1}{n}, \frac{1}{n}) : n \in \mathbb{N}\}$ is a local base at x. A sequence $\{x_n\}$ in (X, τ_M) converges to $x \in X$ if and only if $\lim_n M(x_n, x, t) = 1$ for all $t > 0$. Also, in [1] the authors defined the closed ball $B_M[x, r, t] = \{y \in X : M(x, y, t) \geq 1\}$ and proved that it is a closed set in τ_M. If confusion is not possible we write simply B instead of B_M.

Let (X, d) be a metric space and let M_d be a function on $X \times X \times]0, \infty[$ defined by

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}$$

Then $(X, M_d, \cdot)$ is a fuzzy metric space, [1], and M_d is called the *standard fuzzy metric* induced by d. The topology τ_{M_d} coincides with the topology $\tau(d)$

on X deduced from d.

Definition 2 *A fuzzy metric M on X is said to be stationary, [9], if M does not depend on t, i.e. if for each $x, y \in X$, the function $M_{x,y}(t) = M(x, y, t)$ is constant. In this case we write $M(x, y)$ and $B(x, r)$ instead of $M(x, y, t)$ and $B(x, r, t)$, respectively.*

The next definitions and results are given for a fuzzy metric space $(X, M, *)$.

Definition 3 *(Mihet [12]). A sequence $\{x_n\}$ in X is called* p-convergent *to x_0 if $\lim_n M(x_n, x_0, t_0) = 1$ for some $t_0 > 0$.*

Definition 4 *(Gregori et al. [4]). $(X, M, *)$ is called* principal *(or simply, M is principal) if $\{B(x, r, t) : r \in]0, 1[\}$ is a local base at $x \in X$, for each $x \in X$ and each $t > 0$. M is principal if and only if all p-convergent sequences are convergent in (X, τ_M).*

Definition 5 *(Gregori et al. [6]) A sequence $\{x_n\}$ in X is called* s-convergent *to $x_0 \in X$ if $\lim_n M(x_n, x_0, \frac{1}{n}) = 1$. $(X, M, *)$ is called an* s-fuzzy metric space *(or simply, M is an s-fuzzy metric) if any convergent sequence is s-convergent.*

Theorem 6 *(Gregori et al. [6]) $\bigcap_{t>0} B(x_0, r, t)$ is a neighborhood of $x_0 \in X$ for each $r \in]0, 1[$ if and only if any convergent sequence to x_0 is s-convergent.*

Throughout the paper J will denote a non-empty subset of $]0, 1[$.

3 Local bases in (principal) fuzzy metric spaces

If ξ is a family of open sets in a metric space that constitutes a local base at x_0 then $\bigcap \xi = \{x_0\}$. Conversely, if we assume that x_0 is not isolated and ξ is constituted by a family of open balls centered at x_0 such that $\bigcap \xi = \{x_0\}$ then it can be asserted that ξ is a local base at x_0. We will see in the next example that this assertion is false, in general, in fuzzy metric spaces.

Example 7 *Consider the fuzzy metric space, [4], $(X, M, \cdot)$ where $X =]0, 1]$, $A = X \cap \mathbb{Q}$, $B = X \setminus A$ and M is given by*

$$M(x, y, t) = \begin{cases} \frac{\min\{x,y\}}{\max\{x,y\}} \cdot t, \; (x \in A, y \in B) \; \text{ or } \; (x \in B, y \in A), t \in]0, 1[, \\ \frac{\min\{x,y\}}{\max\{x,y\}}, \quad \text{elsewhere.} \end{cases}$$

It is easy to see that $\{1\}$ is not open, and that $B(1, r, t) =]1 - r, 1]$ for all $r \in]0, 1[$ and all $t > 1$. Consider for (some) $t > 1$ the family $\xi = \{B(1, r, t) : r \in]0, 1[\}$. We have that $\bigcap \xi = \{1\}$ but ξ is not a local base at 1, since $B(1, \frac{1}{2}, \frac{1}{2}) =$

$]\frac{1}{2}, 1] \cap \mathbb{Q}$ *and obviously* $B(1, r, t) \not\subseteq B(1, \frac{1}{2}, \frac{1}{2})$ *for all* $r \in]0, 1[$, *and all* $t > 1$.

The next proposition shows that the above assertion holds for stationary fuzzy metric spaces and at least for a particular case in fuzzy metric spaces.

Proposition 8 *Let* $(X, M, *)$ *be a (stationary) fuzzy metric space and suppose that* x_0 *is not isolated. Let* $\mathcal{B} = \{B(x_0, r, r) : r \in J\}$ *(or* $\mathcal{B} = \{B(x_0, r) : r \in J\}$ *if* M *is stationary). If* $\bigcap \mathcal{B} = \{x_0\}$ *then* $\mathcal{B}$ *is a local base at* x_0.

Proof. Since $\{x_0\}$ is not open then $\inf J = 0$ and the conclusion is obvious.

Denote by J_1 and J_2 two non-empty subsets of $]0, 1[$ where $\inf J_1 = \inf J_2 = 0$. The following is an immediate corollary.

Corollary 9 *Let* $(X, M, *)$ *be a fuzzy metric space and suppose that* x_0 *is not isolated. Let* $\mathcal{B} = \{B(x_0, r, t) : r \in J_1, t \in J_2\}$. *If* $\bigcap \mathcal{B} = \{x_0\}$ *then* $\mathcal{B}$ *is a local base at* x_0.

This last proposition is false, in general, if we remove the condition that $\{x_0\}$ is not open, even if M is stationary, as illustrate the following examples.

Example 10 *Consider the fuzzy metric space* $(]0, 1[, M, \cdot)$ *where* M *is given by*

$$M(x, y, t) = \begin{cases} 1, & x = y \\ xyt, & x \neq y, t \leq 1 \\ xy, & x \neq y, t > 1 \end{cases}$$

In [4], it is proved that τ_M *is the discrete topology.*

Let $x_0 \in]0, 1[$ *and consider the family* $\mathcal{B} = \{B(x_0, r, r) : r \in]\frac{1}{x_0+1}, 1[\}$.

It is easy to verify that $B(x_0, r, r) = \{x_0\} \cup]\frac{1-r}{r x_0}, 1[$. *We have that* $\bigcap \mathcal{B} = \{x_0\}$, *but* $\mathcal{B}$ *is not a local base at* x_0, *since* $\mathcal{B}$ *does not contain* $\{x_0\}$.

Example 11 *Consider the stationary fuzzy metric space* $([0, \infty[, M, \cdot)$, *[5], where* M *is given by*

$$M(y, x) = M(x, y) = \begin{cases} \frac{\min\{x,y\}}{\max\{x,y\}}, & x, y \in]0, \infty[\\ \frac{1}{2y}, & x = 0, y \geq 1 \\ \frac{y}{2}, & x = 0, y < 1 \\ 1, & x = y = 0 \end{cases}$$

It is easy to verify that $\{0\} \in \tau_M$.

For $r \in]\frac{1}{2}, 1[$ *we have that* $B(0,r) = \{0\} \cup]2(1-r), \frac{1}{2(1-r)}[$. *Consider the family* $\mathcal{B} = \{B(0,r) : r \in J\}$, *where* $J =]\frac{1}{2}, 1[$. *We have that* $\bigcap \mathcal{B} = \{0\}$ *but* $\mathcal{B}$ *is not a local base at* 0.

Remark 12 *(On principal fuzzy metric spaces) In any fuzzy metric space* $(X, M, *)$ *it is easy to verify that for a fixed* $t_0 > 0$ *it holds that* $\bigcap\{B(x_0, r, t_0) : r \in]0,1[\} = \{x_0\}$. *Then it makes sense to study families of open balls centered at* x_0 *with fixed* t_0. *Now, if* M *is not principal then we can find* $x_0 \in X$ *and* $t_0 > 0$ *such that* $\xi = \{B(x_0, r, t_0) : r \in]0,1[\}$ *is not a local base at* x_0. *So from* $\bigcap \xi = \{x_0\}$ *we cannot assert that* ξ *is a local base at* x_0, *even if* x_0 *is not isolated (indeed, this is the case of Example 7 since the family* ξ *is really* $\{B(1, r, 1) : r \in]0,1[\}$*). So, our aimed study only has sense in principal fuzzy metrics and the obtained results are the following.*

Proposition 13 *Let* $(X, M, *)$ *be a fuzzy metric space and suppose that* x_0 *is not isolated. For a fixed* $t_0 > 0$ *consider a family* $\zeta = \{B(x_0, r, t_0) : r \in J\}$ *such that* $\bigcap \zeta = \{x_0\}$. *They are equivalent:*

(i) ζ *is a local base at* x_0.
(ii) $\{B(x_0, r, t_0) : r \in]0,1[\}$ *is a local base at* x_0.
(iii) Any sequence $\{x_n\}$ *in* X *such that* $\lim_n M(x_n, x_0, t_0) = 1$ *is convergent (to* x_0*).*

Proof.

By [4] Theorem 11 we have that (*iii*) implies (*ii*), and with similar arguments to the ones used in the proof of this theorem it is proved that (*ii*) implies (*iii*). Then (*ii*) and (*iii*) are equivalents. Obviously, (*i*) implies (*ii*). We see that (*ii*) implies (*i*).

We claim that $\inf J = 0$ (in other case, $\{x_0\} = B(x_0, \alpha, t_0)$ for some $\alpha \in]0,1[$, a contradiction). Now, consider an open ball $B(x_0, r, t)$. We can find $\delta \in]0,1[$ such that $B(x_0, \delta, t_0) \subset B(x_0, r, t)$. Take $j \in J$ with $j < \delta$ and then $B(x_0, j, t_0) \subset B(x_0, \delta, t_0)$, so ζ is a local base at x_0.

Corollary 14 *Let* $(X, M, *)$ *be a fuzzy metric space without isolated points. For each* $x \in X$ *and each* $t > 0$ *put* $\zeta_x^t = \{B(x, r, t) : r \in J\}$. *Then* $(X, M, *)$ *is principal if and only if* ζ_x^t *is a local base at* x, *for each* $x \in X$ *and each* $t > 0$, *whenever* $\bigcap \zeta_x^t = \{x\}$.

Remark 15 *Notice that the converse of this corollary is true even if* X *has isolated points, since* $\{B(x_0, r, t) : r \in]0,1[\}$ *is a local base at* $x_0 \in X$, $t > 0$.

Now, the fuzzy metric M of Example 10 is principal, and the family $\mathcal{B}$ satisfies $\bigcap \mathcal{B} = \{x_0\}$, where $\{x_0\}$ is open, and $\mathcal{B}$ is not a local base at x_0.

4 Local bases in s-fuzzy metric spaces

The study of families of balls centered at x_0 with fixed radius turns interesting when studying s-fuzzy metrics (see Theorem 6). Hence, we are interested in this type of families. Consider a family $\mathcal{D} = \{B(x_0, r_0, t) : t \in J\}$. In the next example we will see that from $\bigcap \mathcal{D} = \{x_0\}$ we cannot assert that $\mathcal{D}$ is a local base at x_0.

Example 16 *Let $(X, M, \cdot)$ the fuzzy metric space of Example 10.*

Consider the family of open balls $\mathcal{D} = \{B(\frac{2}{3}, \frac{2}{3}, t) : t \in]\frac{1}{2}, 1]\}$ centered at $x_0 = \frac{2}{3}$ with radius $r_0 = \frac{2}{3}$. We have that $B(\frac{2}{3}, \frac{2}{3}, t) = \{\frac{2}{3}\} \cup]\frac{1}{2t}, 1]$ for $t \in]\frac{1}{2}, 1]$ and then $\bigcap \mathcal{D} = \{\frac{2}{3}\}$. Now, $\mathcal{D}$ is not a local base at $\frac{2}{3}$ since τ_M is the discrete topology.

The following is an open question.

Problem 17 *Let $(X, M, *)$ be a fuzzy metric space, and suppose that x_0 is not isolated. Consider for a fixed $r_0 \in]0, 1[$ the family $\mathcal{D} = \{B(x_0, r_0, t) : t \in J\}$. If $\bigcap \mathcal{D} = \{x_0\}$, is $\mathcal{D}$ a local base at x_0?*

Remark 18 *(With respect to Problem 17). If x_0 is not isolated in $(X, M, *)$ and $\bigcap\{B(x_0, r_0, t) : t \in J\} = \{x_0\}$ then $\bigcap\{B(x_0, r_0, t) : t > 0\}$ is not a neighborhood of x_0 and thus there exists a convergent sequence to x_0 which is not s-convergent. So, if $(M, *)$ is an s-fuzzy metric without isolated points then $\bigcap \mathcal{D} \neq \{x_0\}$, for any $r_0 \in]0, 1[$.*

For giving some partial answer to this problem we introduce a dual concept to principal fuzzy metrics, as follows.

Definition 19 *We will say that the fuzzy metric space $(X, M, *)$ (or simply, M) is* co-principal *if for each $x \in X$ and each $r \in]0, 1[$, the family $\mathcal{D}_x^r = \{B(x, r, t) : t > 0\}$ is a local base at x.*

Notice that if M is co-principal then M is an s-fuzzy metric space if and only if τ_M is the discrete topology. Clearly, stationary fuzzy metrics (excepting trivial cases) are not co-principal.

Proposition 20 *The standard fuzzy metric is co-principal.*

Proof. Let (X, d) be a metric space and consider the standard fuzzy metric space $(X, M_d, \cdot)$. As usual, $B_d(x; \delta)$ denotes the open ball in (X, d) with center x and radius δ.

Let $x \in X$ and $r \in]0, 1[$. It is easy to see that $B_{M_d}(x, r, t) = B_d(x; \frac{rt}{1-r})$ for each $t > 0$. Since the family $\{B_d(x; \frac{rt}{1-r}) : t > 0\}$ is a local base at x for $\tau(d)$ and $\tau(d) = \tau_{M_d}$, [1], we conclude that the family $\{B_{M_d}(x, r, t) : t > 0\}$ is a local base at x for τ_{M_d}.

The proof of the next proposition is obvious.

Proposition 21 *Let $(X, M, *)$ be a fuzzy metric space and suppose that x_0 is not isolated. For a fixed $r_0 \in]0, 1[$ consider a family $\mathcal{D} = \{B(x_0, r_0, t) : t \in J\}$ such that $\bigcap \mathcal{D} = \{x_0\}$. Then $\mathcal{D}$ is a local base at x_0 if and only if $\{B(x_0, r_0, t) : t > 0\}$ is a local base at x_0.*

Corollary 22 *Let $(X, M, *)$ be a co-principal fuzzy metric space without isolated points. Let $\mathcal{D} = \{B(x_0, r_0, t) : t \in J\}$. If $\bigcap \mathcal{D} = \{x_0\}$ then $\mathcal{D}$ is a local base at x_0.*

Notice that we cannot formulate last corollary as Corollary 14 because we cannot assert that $\bigcap\{B(x_0, r_0, t) : t > 0\}$ is $\{x_0\}$. A similar result to Corollary 22 can be obtained replacing co-principal by compactness, as shows the next theorem.

Theorem 23 *Let $(X, M, *)$ be a compact fuzzy metric space, let $\delta \in]0, 1[$ and suppose that x_0 is not isolated. Let $\mathcal{D} = \{B(x_0, \delta, t) : t \in J\}$. If $\bigcap \mathcal{D} = \{x_0\}$ then $\mathcal{D}_\epsilon$ is a local base at x_0, for each $\epsilon < \delta$ where $\mathcal{D}_\epsilon = \{B(x_0, \epsilon, t) : t \in J\}$.*

Proof. We have that $\inf J = 0$, since we suppose that $\{x_0\}$ is not open. Take $\epsilon \in]0, \delta[$ and consider a sequence $\{t_n\} \subset J$ convergent to 0. Clearly $\bigcap_n B(x_0, \delta, t_n) = \bigcap_n B(x_0, \epsilon, t_n) = \{x_0\}$.

Take $\epsilon_1 \in]0, 1[$ such that $\epsilon < \epsilon_1 < \delta$. Since $B(x_0, \epsilon, t) \subset B[x_0, \epsilon_1, t] \subset B(x_0, \delta, t)$ for all $t > 0$, then $\bigcap_n B[x_0, \epsilon_1, t_n] = \{x_0\}$.

Put $V_n = B[x_0, \epsilon_1, t_n]$ for $n = 1, 2, ...$ We will see that $\{V_n : n \geq 1\}$ is a local base at x_0. Consider an open ball $B(x_0, r, t)$ with $r \in]0, 1[$, $t > 0$. Suppose, contrarily, that for all $n \geq 1$, $V_n \not\subseteq B(x_0, r, t)$. Then put $E_n = V_n \cap (B(x_0, r, t))^c \neq \emptyset$, for all $n = 1, 2, ...$

Since $\{V_n : n \geq 1\}$ is a decreasing family then $\{E_n : n \geq 1\}$ is also a decreasing family of closed sets with $E_n \neq \emptyset$ for each $n \geq 1$. Further, the intersection of finite elements of that family, $E_{n_1}, ..., E_{n_k}$, is non-empty (indeed, if $i =$

$\max\{n_1, ..., n_k\}$, then $\bigcap_{j=1}^{k} E_{n_j} = E_i$). So, the family $\{E_n : n \geq 1\}$ has the finite intersection property. Since X is compact then $\bigcap E_n \neq \emptyset$, a contradiction (indeed, $y \in \bigcap_n E_n$ implies $y \in V_n$ for $n \geq 1$ with $y \neq x_0$).

So, there exists $m \in \mathbb{N}$ such that $V_m \subset B(x_0, r, t)$ and then $B(x_0, \epsilon, t_m) \subset B[x_0, \epsilon_1, t_m] \subset B(x_0, r, t)$. Hence $\{B(x_0, \epsilon, t) : t \in J\}$ is a local base at x_0.

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