TRANSPARENT FUZZY LOGIC BASED METHODS FOR SOME HUMAN RESOURCES PROBLEMS

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RESUMEN: Las políticas de selección y reasignación de recursos humanos deberían planearse y llevarse a cabo cuidadosamente a causa de su importancia en el futuro de las empresas. Cuando se contrata a una persona, la empresa está haciendo una inversión en capital humano. Por eso, los gerentes y directores deben usar diferentes herramientas que les permitan tomar una buena decisión. En este trabajo presentamos algunas herramientas basadas en matemáticas borrosas para procesos de selección y reasignación de personal y analizamos las ventajas e inconvenientes derivados de su aplicación. Además, mostramos algunos ejemplos, incluyendo el uso de un software específicamente diseñado.

Palabras claves: Selección de personal, recursos humanos, fuzzy, software

ABSTRACT: Personnel selection and reallocation are human resources policies that should be planned and implemented accurately because of their importance to the future of the company. When a person is hired, he or she represents an investment in human capital. This is the reason because managers have to use different tools in order to make good decisions. In this paper we present some fuzzy tools for personnel selection and reallocation processes and the advantages and disadvantages of their applying. Moreover, we show some examples, including the use of a specific software.

Keywords: Personnel selection, human resources, fuzzy, software.
1. Introduction

An accurate human resources management takes into account the company circumstances and allows managers to optimize production costs and to achieve corporative goals. Involved policies are complicated to quantify because of the human nature, and implies focusing in some concepts like validity, trust and criteria fixing.

Companies usually have excellent but limited human resources, which represent a significant advantage over competitors. Moreover, human resources are difficult to replace or to substitute due to social interactions, causal ambiguity and the unique evolution of each employee. In this case, recruitment and selection policies are a key factor for competitiveness, although standard indicators are not normally useful because of human nature [1].

It is well known that management implies such a quick system of interactions that deterministic mathematical techniques cannot keep pace. It would be useful to include all the information, in particular the subjective information provided by experts. In any decision-making process, the mathematical model can be affected by the numerical accuracy of the introduced quantities. Then, an appropriate approach to deal with uncertainty appearing in most real cases is the fuzzy set theory, which allows us to take decisions in uncertain scenarios. Fuzzy theory considers some elements that are essential to deal with economic, social and technological situations, as uncertainty in data, and the modeller or manager capacity to add any additional information.

When mathematical models are useful in decision-making processes, there are some advantages such as quick and clear solutions that are easy to understand. On the other hand, difficulties appear because, in a general way, mathematical models quantify magnitudes that can hardly be matched to human resources policies. Moreover, managers do not often understand (and they do not have to) complicated mathematical reasoning. To avoid this, we use models based in fuzzy logic (for a survey on fuzzy reasoning schemes, see [2]). Then, we not only can add uncertainty and subjectivity to the model, but we can also build more transparent and easy to understand models for managers. Fuzzy logic does not increase the difficulty of traditional mathematics and it is closer to human thinking. Also, it allows thinking on future policies to avoid rigidity requirements that makes the model non-sense and prevents us from ignoring other solutions that could be useful.

In this paper we focus on selection and reallocation policies, since they are essential for the survival of the company. For example, managers can face a merger, absorption or acquisition; in this case, managers try to design an optimal staff while respecting the existing jobs as much as possible in restructuring and reallocation policies.

Hiring policies have to be applied in a meticulous way in order to assign each employee the right job. This process requires extensive knowledge about tasks, personal competencies, characteristics of the organization and its needs. Staff restructuring is conditioned by economic, social and technological variables. Most of them are a source of uncertainty that affects decision making. Obviously, employees have to be assigned jobs according to quantity and quality requirements.

Employees are considered as a strategic resource. Then, human resources managers aim to create value for the company, rather than to reduce costs. This is the reason why knowledge and experience do not make a difference in competitive advantage by themselves in order to add value to the company. We have to take into account motivation, commitment, behavior, etc. The main purpose is to get a perfect match between the employee and the job. Thus, an excellent (and not merely successful) performance in tasks and activities of the job can be achieved to get an advantage over competitors because this performance is
difficult to copy or to imitate. Competence management enables us to achieve this dual objective, as it integrates not only expertise, but other human attributes, objective and subjective, broader and more complex.

Several authors define the term competence. For instance, Spencer and Spencer [3] consider a competence as an underlying characteristic of an individual that is causally related to effective or superior performance in a situation or work, defined in terms of a criterion. Boyatzis [4] defines competence as a set of behavior patterns that one person has to develop to efficiently perform his or her tasks and functions. Then, a competence is a set of patterns about underlying human characteristics (knowledge, skills, experiences, behaviors, etc.) that allow the individual to achieve an effective or excellent performance in a task or work. In short, competencies are knowledge, skills, attitudes, etc., which turn excellent the development of certain tasks and activities as well as the achievement of particular results.

In this paper we present the problem of personnel selection and restructuring staff, some fuzzy tools to solve them and the advantages and disadvantages of their applying.

2. Problems and notation

The two most common situations in the process of personnel selection in a company are: hiring an applicant to fill a job or hiring some applicants to cover different jobs. Note that the problem of hiring some applicants for some equal jobs is easily reducible to the first situation, while the second problem is analogous to an issue of reallocation or staff restructuring.

Our goal is to rank the candidates to a job so that the ordering process reflects the criteria of the problem to be solved. In the models presented in this paper, we use fuzzy numbers to order candidates. Different fuzzy ordering techniques can be found in [5, 6, 7, 8, 9, 10, 11].

Both hiring and staff restructuring policies require a correct evaluation of people to identify and develop their full potential. The assessment tools we use link individual subjectivity and scales of values to a formal and systematic process that managers use to create and retain judgments about subordinates. The aim is that, in the future, working conditions for employees and their promotion may improve. Measurement scales have a low cost and they are simple in design and in valuation methods. Then, decision makers can complete the process in a short time [12]. As the company must implement policies related to recruitment and staff restructuring on numerous occasions throughout its history, the choice of an appropriate method for this is important.

Suppose we have \( n \) candidates and let us call \( \text{Cand} = \{P_1, \ldots, P_n\} \) the set of candidates. In general, the personnel selection process has these steps:

**Step 1:** Design of the profiles of the jobs, i.e. selection of relevant competencies of the jobs to be occupied. Let us call \( \text{C} = \{c_1, \ldots, c_k\} \) the set of competencies.

**Step 2:** Evaluation of candidates. The internal or external experts who evaluate the candidates can provide information in different ways. Many times, expert systems use numbers to describe the experts' degrees of belief in their statements. However, sometimes it is difficult to assign an exact numerical value to the experts' degree of belief. In this case, we can consider an interval of possible values. Let us call these situations Case 1 and Case 2.

Case 1. Experts are able to assign a number between 0 and 1 that reflects the adequacy of a particular candidate to a particular competence, i.e. we have the following matrix:
\[
\begin{array}{cccc}
  c_1 & c_2 & \cdots & c_R \\
  \tilde{P}_1 & b^1_1 & b^1_2 & \cdots & b^1_j & \cdots & b^1_R \\
  \tilde{P}_2 & b^2_1 & b^2_2 & \cdots & b^2_j & \cdots & b^2_R \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
  \tilde{P}_n & b^n_1 & b^n_2 & \cdots & b^n_j & \cdots & b^n_R 
\end{array}
\]

In this case, each candidate could be seen as the fuzzy set (type 1)

\[
\tilde{P}_j = \left\{ \left( c_i, \mu_{c_i}(\tilde{P}_j) = b^j_i, 1 \right) \in [0,1] \right\}^R \text{ for all } j = 1, \ldots, n.
\]

Case 2. It can happen that experts prefer to evaluate each candidate in each competence by using a subinterval of [0,1]. In this case, the information would be as:

\[
\begin{align*}
\tilde{P}_1 & \Phi \left[ \begin{bmatrix} c_1^1 & c_2^1 & \cdots & c_R^1 \\ b_1^1 & b_2^1 & \cdots & b_R^1 \end{bmatrix} \right] \\
\tilde{P}_2 & \Phi \left[ \begin{bmatrix} c_1^2 & c_2^2 & \cdots & c_R^2 \\ b_1^2 & b_2^2 & \cdots & b_R^2 \end{bmatrix} \right] \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{P}_n & \Phi \left[ \begin{bmatrix} c_1^n & c_2^n & \cdots & c_R^n \\ b_1^n & b_2^n & \cdots & b_R^n \end{bmatrix} \right]
\end{align*}
\]

In this case, each candidate can be seen as the \_\_\_\_fuzzy set (type 2)

\[
\tilde{P}_j = \left\{ \left( c_i, \mu_{c_i}(\tilde{P}_j) = b^j_i, 1 \right) \in P([0,1]) \right\}^R \text{ for all } j = 1, \ldots, n.
\]

Remark: \( \Phi \)-fuzzy sets have other names in the literature, like interval-valued fuzzy sets [13], vague sets [14], grey fuzzy sets [15] or type-2 fuzzy sets [16].

Step 3: Design, if any, of the profiles of ideal candidates. It can happen that the manager of the company has no ideal candidates and simply wants to find top-level requirements. Otherwise, information on ideal candidates should be determined with the same structure as the information about candidates, i.e. either the ideal candidate is a fuzzy set as

\[
\tilde{I} = \left\{ \left( c_i, \mu_{c_i}(\tilde{I}) = a^i_j, 1 \right) \in [0,1] \right\}^R
\]

or it is a \( \Phi \)-fuzzy set as

\[
\tilde{I}\Phi = \left\{ \left( c_i, \mu_{c_i}(\tilde{I}\Phi) = a^i_j, 1 \right) \in P([0,1]) \right\}^R
\]

Step 4: Analysis of the matching degree of each candidate to each ideal profile. Depending on the available information and the goals of the company we propose in the next sections some methods to select or, if necessary, reassign, candidates to jobs.

Often we express judgments about different alternatives in fuzzy multicriteria decision making problems by using fuzzy preference relations [17]. We are also interested in pointing out the restrictions that should
preferably be satisfied before others. Because it is possible to partially satisfy a constraint in fuzzy optimization, we can indicate the level of trade off of some constraints by using weighing factors. Weights represent the relative importance of one or several constraints: the higher the weight of a particular restriction, the greater the importance of the result. Then, the result is highly conditioned by the constraints that the decision maker considers to be the most important one. That is, weighing factors represent the user's preferences regarding the optimal solution [18]. Also, membership functions can also be modified to study the influence of constraint’s flexibility on the results of optimization.

In the selection process, we can weigh competences and adequacy coefficients if we consider that some are more important than others. We cannot ignore this option since the result could be different depending on the weight to give [19]. To assign different weights \( w_i \), we use coefficients in the range \([0,1]\). The closer \( w_i \) is to 1, the more important it is. To facilitate calculations, we can normalize the weights.

### 3. Top-level requirement

If the company does not have an ideal candidate to compare real competencies with, then we can compare each candidate with the maximum permitted level. The candidate who has the shortest distance is the most competent according to this method.

In case each candidate was a fuzzy set type-1, we can calculate the ranking of the candidates by using:

\[
\text{DM}(P_j) = \frac{1}{R} \sum_{i=1}^{R} \left( 1 - \frac{b_{ij}}{c_i} \right) \quad \text{for all } j = 1, ..., n
\]

In case each candidate was a \( \Phi \)-fuzzy set, we should consider the complementary of each interval \([b_{ij}^1, b_{ij}^2]\), that is, \([1 - \frac{b_{ij}^2}{c_i}, 1 - \frac{b_{ij}^1}{c_i}]\) and decide if we defuzzify ex-ante or ex-post.

If defuzzification is done by using the mid-point of each interval, the three methods lead to the same result. Top-level requirement applying ex-ante defuzzification can be calculated as:

\[
\text{DM}(P_j) = \frac{1}{2R} \sum_{i=1}^{R} \left( 2 - \frac{b_{ij}^1}{b_{ij}^2} - \frac{b_{ij}^2}{b_{ij}^1} \right) \quad \text{for all } j = 1, ..., n
\]

If we decide to defuzzify ex post, top-level requirement is given by:

\[
\text{DM}(P_j) = \frac{2R - \sum_{i=1}^{R} b_{ij}^1 - \sum_{i=1}^{R} b_{ij}^2}{2R} \quad \text{for all } j = 1, ..., n
\]

Let us see. Consider 5 candidates and 4 competences. We have the matrix given by experts with fuzzy sets type-1. The top level requirement could be calculated as:

<table>
<thead>
<tr>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_4 )</th>
<th>( \frac{1}{R} \sum_{i=1}^{R} \left( 1 - \frac{b_{ij}}{c_i} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>0.2</td>
<td>0.8</td>
<td>0.4</td>
<td>0.95</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>0.65</td>
<td>0.15</td>
<td>0.55</td>
<td>0.4</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>0.9</td>
<td>0.45</td>
<td>0.25</td>
<td>0.85</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>0.15</td>
<td>0.75</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>( P_5 )</td>
<td>0.45</td>
<td>0.05</td>
<td>0.75</td>
<td>0.6</td>
</tr>
</tbody>
</table>
The most appropriate candidate is $P_3$.

If each candidate is a fuzzy set type-2 we have the following data

<table>
<thead>
<tr>
<th></th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>[0.1, 0.3]</td>
<td>[0.6, 1]</td>
<td>[0.3, 0.5]</td>
<td>[0.9, 1]</td>
</tr>
<tr>
<td>$P_2$</td>
<td>[0.5, 0.8]</td>
<td>[0.1, 0.2]</td>
<td>[0.3, 0.8]</td>
<td>[0.3, 0.5]</td>
</tr>
<tr>
<td>$P_3$</td>
<td>[0.8, 1]</td>
<td>[0.4, 0.5]</td>
<td>[0.2, 0.3]</td>
<td>[0.8, 0.9]</td>
</tr>
<tr>
<td>$P_4$</td>
<td>[0, 0.3]</td>
<td>[0.7, 0.8]</td>
<td>[0.4, 0.6]</td>
<td>[0, 0.2]</td>
</tr>
<tr>
<td>$P_5$</td>
<td>[0.3, 0.6]</td>
<td>[0, 0.1]</td>
<td>[0.5, 1]</td>
<td>[0.4, 0.8]</td>
</tr>
</tbody>
</table>

Notice that the previous matrix was composed by the midpoints of these intervals.

If we defuzzify ex-ante, we obtain

$$2 - b_{ij}^1 - b_{ij}^2 \quad \sqrt{\frac{1}{2R} \sum_{i=1}^{R} (c_i - b_i^j - b_i^j)^2}$$

<table>
<thead>
<tr>
<th></th>
<th>2 - $b_{ij}^1 - b_{ij}^2$</th>
<th>$\sqrt{\frac{1}{2R} \sum_{i=1}^{R} (c_i - b_i^j - b_i^j)^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>1.6 0.4 1.2 0.1</td>
<td>0.41</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.7 1.7 0.9 1.2</td>
<td>0.56</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.2 1.1 1.5 0.3</td>
<td>0.38</td>
</tr>
<tr>
<td>$P_4$</td>
<td>1.7 0.5 1.8</td>
<td>0.62</td>
</tr>
<tr>
<td>$P_5$</td>
<td>1.1 1.9 0.5 0.8</td>
<td>0.53</td>
</tr>
</tbody>
</table>

We can observe easily that the result is the same as the one from the previous method, as we know.

Finally, let’s see that the same happens if we defuzzify ex-post.

<table>
<thead>
<tr>
<th></th>
<th>$\sum_{i=1}^{R} b_{ij}^1$</th>
<th>$\sum_{i=1}^{R} b_{ij}^2$</th>
<th>$R - \sum_{i=1}^{R} b_{ij}^1$</th>
<th>$R - \sum_{i=1}^{R} b_{ij}^2$</th>
<th>$\sqrt{\frac{2R}{2}} - \sum_{i=1}^{R} b_{ij}^1 - \sum_{i=1}^{R} b_{ij}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>1.9 2.8</td>
<td>2.1 1.2</td>
<td>0.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_2$</td>
<td>1.2 2.3</td>
<td>2.8 1.7</td>
<td>0.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_3$</td>
<td>2.2 2.7</td>
<td>1.8 1.3</td>
<td>0.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_4$</td>
<td>1.1 1.9</td>
<td>2.9 2.1</td>
<td>0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_5$</td>
<td>1.2 2.5</td>
<td>2.8 1.5</td>
<td>0.53</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Measuring distances from an ideal

When the manager of a company has in mind an ideal candidate, a transparent and straight-forward selection method is to measure the distance from each candidate to the ideal and then rank candidates according to their closeness.

We can define Minkowski distance between the ideal and each candidate as:

$$d(I, P_j) = \left( \sum_{i=1}^{R} |c_i - b_i^j|^p \right)^{1/p}$$

The Minkowski distance is a generalization from a wide range of well-known distances such as Hamming distance ($p=1$), Euclidean distance ($p=2$), geometric distance ($p=0$) or harmonic distance ($p=-1$).

Hamming distance, proposed by Hamming in 1950 [20], and Euclidean distance are the most used in
fuzzy logic applications [21]. Also, depending on the characteristics of the problem, normalized or weighed distances can be used.

We consider the Hamming distance for the matrix of data from previous example, with the following ideal:

\[
I = \begin{bmatrix}
0.3 & 0.45 & 0.25 & 0.85
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th></th>
<th>0.1</th>
<th>0.35</th>
<th>0.15</th>
<th>0.1</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_1)</td>
<td>0.6</td>
<td>0.3</td>
<td>0.45</td>
<td>0.05</td>
<td>1.45</td>
</tr>
<tr>
<td>(P_2)</td>
<td>0.15</td>
<td>0.35</td>
<td>0.25</td>
<td>0.75</td>
<td>1.3</td>
</tr>
<tr>
<td>(P_3)</td>
<td>0.15</td>
<td>0.4</td>
<td>0.5</td>
<td>0.25</td>
<td>1.3</td>
</tr>
</tbody>
</table>

In the case of type-2 fuzzy sets, Minkowski distance is defined as:

\[
d(I, P_j) = \left( \sum_{i=1}^{n} \left( \left| a_{ci}^1 - b_{ci}^1 \right|^p + \left| a_{ci}^2 - b_{ci}^2 \right|^p \right)^{1/p} \right)^{1/p}
\]

With the same remarks as in type-1 fuzzy sets.

These kinds of distances work well when data are type-1 fuzzy sets but not when inputs are intervals. For example, if we consider the Euclidean distance, then \(d([-2,2,] [-1,1]) = d([-2,1], [-1,2])\). However, since we are dealing with fuzzy numbers, it would seem natural that the distance between the first pair was shorter than the distance between the second pair, because both intervals in the first pair represent a number close to zero more or less accurately, whereas the intervals in the second pair represent a number with negative trend and a number with positive trend respectively.

These apparent anomalies occur because these distances cannot be reduced to their usual equivalent distance, a necessary feature when type-1 and type-2 fuzzy sets are mixed. To face with this problem there are two alternatives: we can defuzzify \textit{ex ante} or use distances defined taking into account all the points of the interval and not just end points (to calculate distances between intuitionistic fuzzy sets, see [22]).

Another difficulty arises when data are mixed and we consider the type-1 fuzzy sets as degenerate intervals. For example, let us consider 0 = [0,0] and the intervals [-1,3] and [1,3]. Again it would seem natural that the distance between [0,0] and [-1,3] was shorter than the distance between [0,0] and [1,3], but they are equals. Then, we need to consider not so simple transformations as we will see in the next Section.

5. Adequacy coefficient

When candidates and the ideal candidate are represented by type-1 fuzzy sets, the adequacy coefficient of candidate \(P_j\) to competency \(c_i\) according to ideal \(I\) could be calculated as

\[
K_{a}(P_j \rightarrow I) = \min \left\{ 1, 1 - a_{ci} + b_{ci} \right\}
\]
Then, the adequacy coefficient of candidate $P_j$ according to ideal $I$ is:

$$K(P_j \rightarrow I) = \frac{1}{R} \sum_{i=1}^{R} K_{ci}(P_j \rightarrow I)$$

Considering the previous type-1 data we have:

$$\min\left\{1, 1 - a_{ci} + b_{ci} \right\} K(P_j \rightarrow I)$$

<table>
<thead>
<tr>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$K(P_j \rightarrow I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.97</td>
</tr>
<tr>
<td>1</td>
<td>0.7</td>
<td>0.55</td>
<td>1</td>
<td>0.81</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.85</td>
<td>0.7</td>
<td>0.25</td>
<td></td>
<td>0.7</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.75</td>
<td>0.93</td>
</tr>
</tbody>
</table>

The most appropriate candidate for the job is $P_3$.

When candidates and the ideal candidate are all of them $\Phi$-fuzzy sets, we could calculate the $\Phi$-adequacy coefficient by defuzzifying ex ante or ex post. If we decide to defuzzify ex ante we can apply the above mentioned method. On the contrary, if we decide to defuzzify at some point in the process we should calculate

$$K_{ci}^{\Phi}(P_j \rightarrow I^{\Phi}) = \frac{1}{R} \left[ \frac{l\left(\left[\begin{array}{c} b_{ci}^{1j} \cap a_{ci}^{1j} \\ a_{ci}^{2j} \end{array}\right]\right]}{l\left(\left[\begin{array}{c} b_{ci}^{1j} \cup a_{ci}^{1j} \\ a_{ci}^{2j} \end{array}\right]\right)} \right]$$

$$K^{\Phi}(P_j \rightarrow I^{\Phi}) = \frac{1}{R} \sum_{i=1}^{R} K_{ci}(P_j \rightarrow I^{\Phi})$$

Adequacy coefficients are ranked from highest to lowest so that candidate with the highest coefficient is the most appropriated for the job [23].

Remark 1: As the distances, some anomalies occur when type-1 and type-2 fuzzy sets are mixed. We could avoid these anomalies (also in the case of distances) making some previous transformations. If the candidate obtains $b_{ci}^{i}$ in the $i$-th competence, and the ideal value of the competence is $[a_{ci}^{1i}, a_{ci}^{2i}]$, then $b_{ci}^{i}$ must be transformed as follows:

$$T(b_{ci}^{i}) = \begin{cases} [a_{ci}^{i}, b_{ci}^{i}] & \text{if } a_{ci}^{i} < b_{ci}^{i} \\ b_{ci}^{i} & \text{if } a_{ci}^{i} \geq b_{ci}^{i} \end{cases}$$

But if the $i$-th competence of the candidate is $[b_{ci}^{1i}, b_{ci}^{2i}]$ and the ideal value for this competence is $a_{ci}^{i}$, then the transformation of $a_{ci}^{i}$ is the following:

$$T(a_{ci}^{i}) = \begin{cases} [b_{ci}^{1i}, a_{ci}^{i}] & \text{if } b_{ci}^{1i} < a_{ci}^{i} \\ a_{ci}^{i} & \text{if } b_{ci}^{1i} \geq a_{ci}^{i} \end{cases}$$
As an example, we can show the example in [24]. Suppose that the human resources department in a company has to choose two candidates to join the staff. For this, the department considers eight candidates to be evaluated in seven competences:

\[
\begin{bmatrix}
    \tilde{P}_1^\Phi \\
    \tilde{P}_2^\Phi \\
    \tilde{P}_3^\Phi \\
    \tilde{P}_4^\Phi \\
    \tilde{P}_5^\Phi \\
    \tilde{P}_6^\Phi \\
    \tilde{P}_7^\Phi \\
    \tilde{P}_8^\Phi \\
\end{bmatrix}
= \begin{bmatrix}
    0.4 & 0.5, 0.7 & 0.6, 0.7 & 0.3, 0.5 & 0.8 & 0.2, 0.4 & 0.6 \\
    0.2, 0.4 & 0.6 & 0.7, 0.9 & 0.5, 0.7 & 0.7, 0.8 & 0.2, 0.5 & 0.4, 0.6 \\
    0.4, 0.5 & 0.3, 0.5 & 0.8, 0.9 & 1 & 0.5, 0.6 & 1 & 0.3, 0.5 \\
    0.1 & 0.2, 0.4 & 0.2 & 0.5, 0.6 & 0.6, 0.8 & 0.3, 0.4 & 0.2, 0.5 \\
    0.2, 0.4 & 0.1, 0.3 & 0.5, 0.6 & 0.2, 0.3 & 0.2, 0.5 & 0.2 & 0.8, 0.9 \\
    0.3, 0.4 & 0.5, 0.6 & 1 & 0.3, 0.4 & 0.4, 0.7 & 0.5 & 1 \\
    0.1, 0.3 & 0.8 & 0.4, 0.5 & 0.4, 0.5 & 1 & 0.3, 0.5 & 0.5, 0.7 \\
    0.2 & 0.2, 0.5 & 0.3, 0.5 & 0.5, 0.6 & 0.8, 0.9 & 0.7, 0.9 & 0.3, 0.7 \\
\end{bmatrix}
\]

We suppose that the ideal candidate is

\[
\tilde{1}^\Phi = \begin{bmatrix} 0.3, 0.7 \\ 0.6, 0.8 \\ 0.7, 1 \\ 0.5, 1 \\ 0.8 \\ 0.4, 0.7 \\ 0.7, 1 \end{bmatrix}
\]

Applying the adequacy coefficient we have the result:

\[
\begin{align*}
\mu_I^{-}(\tilde{P}_1) &= 0.328 \\
\mu_I^{-}(\tilde{P}_2) &= 0.486 \\
\mu_I^{-}(\tilde{P}_3) &= 0.571 \\
\mu_I^{-}(\tilde{P}_4) &= 0.143 \\
\mu_I^{-}(\tilde{P}_5) &= 0.171 \\
\mu_I^{-}(\tilde{P}_6) &= 0.464 \\
\mu_I^{-}(\tilde{P}_7) &= 0.321 \\
\mu_I^{-}(\tilde{P}_8) &= 0.428
\end{align*}
\]

\[
\tilde{P}_4 < \tilde{P}_5 < \tilde{P}_1 < \tilde{P}_6 < \tilde{P}_7 < \tilde{P}_2 < \tilde{P}_3 \ \Rightarrow \text{We choose } \tilde{P}_3 \text{ and } \tilde{P}_2
\]

Remark 2: In some problems, it could be more useful to calculate the removal index, i.e. the complementary of the adequacy coefficient.

6. Staff restructuring and reallocation

Every day, managers are forced to take decisions that affect all the people in the company. This is the case of staff restructuring and reallocation policies, to which it is necessary to carry out a proper assessment of individuals, taking into account the concepts of justice and equity, in order to identify and develop their potential.

We have two cases: the first one is to decide which employees are going to stay in the company in a staff restructuring; the second one is to assign each employee the best job to obtain an optimal performance. In the first case, the number of jobs is lesser than the number of employees. This might not happen in the second case.

To solve both problems, we adapt the Hungarian algorithm or König algorithm which develops an optimization process based on two sets of elements, which are related through a matrix.
For this, we define the ideal profiles for $s$ jobs and evaluate the competencies of each of $n$ candidates for each job. After calculating the adequacy coefficients of the candidates, we apply the algorithm, briefly described because it is widely discussed in the literature (see [25, 26]):

- Construct a matrix that shows the adequacy coefficients of each candidate for each job, and its complementary matrix to reflect the distance between the competencies of each candidate and the ideal profile. Therefore, we had a maximization problem and now we deal with a minimization problem.
- Subtract from all the elements of each row the smallest value. Do the same in the columns. Assignment begins in rows with fewer zeros to follow in increasing order.
- Mark one of the zeros in each row and cross out the other zeros in this row and column.
- If the optimal solution has not been found, that is, there is a zero marked in each row or column, continue pointing with an arrow the rows in which there is not a zero marked and the columns in which there is a crossed out zero in a row marked with an arrow. This process is repeated until there is no row or column to point out.
- Finally, draw a line in the rows not pointed by arrows and the pointed columns themselves, to build a matrix of cells not crossed out. The smallest not crossed out data is taken and subtracted from the other elements of the rows with a line. With the new results, repeat the process from the second step to find the optimal solution.

The result shows the most appropriate candidate for each job taking into account overall competencies of all the employees together.

**Numerical example solved with StaffManager 1.0**

We have designed the software StaffManager. This tool can solve personnel selection problems, staff restructuring problems and reallocation problems. The results are presented in a report designed by StaffManager according to the preferences of decision makers. StaffManager is easy to use and performs all necessary calculations in a transparent and simply way.

The problem to solve is the assignment of 20 employees to 20 jobs evaluated in 10 competencies. The whole performance has to be optimal.

**JOBS VALUATION**

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**WEIGHTS**

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Figure 1 shows the optimal assignment in the final report provided by StaffManager 1.0. An adequacy index and the global adequacy index, that is, the fit between the set of candidates and the set of jobs.

Figure 1. Results.
7. Conclusions and further research

This paper shows how human resources policies are important for the strategic management of the company, particularly regarding selection and staff restructuring policies. Thus, the choice of an appropriate method according to external circumstances and internal organization is highly desirable and recruitment and hiring have to be carefully planned to achieve the best fit between an employee and a job. Consequently, managers can optimize costs and achieve corporate objectives. Nowadays, as it happens in most management problems, the personnel selection process is difficult but necessary.

Mathematical models are useful to take decisions. They present some advantages such as providing clear and fast solutions easy to understand. However, difficulties can arise because, in general, mathematical models quantify magnitudes that are not always objective. To avoid this problem, we develop models based in fuzzy set theory. These models are flexible and allow the incorporation of data uncertainty and the available information, either objective or subjective. On the other hand, fuzzy logic, despite being close to human thinking, does not increase the difficulty of traditional mathematics and allows managers to design flexible fuzzy future policies. Then, the model does not lose its sense and it can provide solutions that could be missed in a classical model (crisp).

In the personnel selection process, an inflexible treatment of valuations of the candidates can obstruct the ranking process due to the underestimation of all the requirements. The overall valuation of competencies neutralizes the positive ones, and this may be unfair. In this paper we present different complementary methods for personnel selection and staff restructuring to rank the candidates to a job. We include the use of intervals to allow more flexibility and better reflect the usual forms of valuation in companies, as well as considering the integration of all information from managers.

Since the theory of fuzzy subsets has already been applied to human resource management on several occasions with good results, we use fuzzy tools that include individual subjectivity and scales of values. Thus, the models fit more accurately to managers’ thinking when implementing human resources policies. Decisions regarding the personnel selection policy for one or more vacancies are determined by a ranking of candidates according to their competences; at the end of the process, managers choose the most appropriate one for the job.

For the development of selection and restructuring models presented in this paper we have applied competence management. The main idea of this approach is to obtain a perfect match between the employee and the job. An excellent or superior performance can be achieved to get an advantage over competitors, because it is difficult to copy or to imitate. Competence management includes not only knowledge and expertise, but also other human attributes, both objective and subjective, more extensive and complex. Inherent subjectivity in decision-making process fully justifies the used fuzzy techniques. The main advantages from competence management are the following: the use of a common language, the focus of all efforts to improve business results, it allows to know future behaviors and facilitates comparison between job profiles and competencies profiles from people, by taking the individual, and not the job, as the unit of analysis. As a consequence, decision-making has become more decentralized, organizational charts have become flatter and different tasks can be compared among themselves. Therefore, an employee can organize more efficiently his or her mobility opportunities, promotion, career, replacement, rotation, etc.

One of the main contributions to competencies approach to human resource management is that the use of a common language in the company is facilitated, because all the people are dealing with observable behaviors that allow a good performance at work. In consequence, it is easier that human resources managers and the rest of the organization agree [27]. In addition, all efforts are focused to improve results. Also, future behaviors can be known: when a person has a given behavior, under given
conditions, we can expect he or she repeats the same behavior in similar conditions. Moreover, this approach facilitates the comparison between the profile of a job and the profile of the people, by taking as unit of analysis the competencies of the individual and not the workplace.

**Acknowledgements**

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**References**


