A TWO-STAGE GROWTH MODEL FOR GILTHEAD SEA BREAM (*Sparus aurata*) BASED ON THE THERMAL GROWTH COEFFICIENT

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Abstract

Several authors have proposed models to describe fish growth taking the influence of temperature into account, and one of most interesting is the "Thermal unit Growth Coefficient" (*TGC*). Recent research has demonstrated that *TGC* varies throughout the growth cycle of fish, making it necessary to establish different stanzas. In this work, the original *TGC* model using 1/3 as exponent was compared with a new model considering 2/3. Likewise, two stages for the growth of gilthead sea bream under commercial conditions in marine farms were detected by means of *TGC* seasonal models using the continuous temperature curve. A critical value for weight around 117g was obtained, which could mark the transition between two growth dynamics. To describe the weight evolution during a complete production cycle, the two growth stages were described by two separate seasonal *TGC* models (1/3-*TGC* model and 2/3-*TGC* model), and with an integrated model named Mixed-*TGC* model, which presents interesting properties of continuity and differentiability and could be an important tool for fish farm management.

Keywords: Seasonal growth, temperature curve, marine cages production

1.- Introduction

The importance of growth models in aquaculture has been demonstrated by the publication of a large number of papers in recent years (Baer *et al.*, 2011; Dumas *et al.*, 2007, 2010; Dumas and France, 2008; Libralato and Solidoro, 2008; Mayer *et al*. 2008, 2009; Moses *et al*., 2008; Seginer and Halachmi, 2008), most of which are based on the metabolic growth model developed early last century (Pütter, 1920; Bertalanffy, 1938, 1957; Parker and Larking, 1959; Ursin, 1967) to describe fish growth. Most of the classic models were based on the assumption that growth depends on live weight affected by the exponent 2/3 (surface rule), and later models (Cho, 1992; Cho and Bureau, 1998) have also used this value. Nevertheless, some authors have questioned the general use of this exponent because Ursin (1968) estimated lower values than 2/3 in some fish, and Moses *et al*. (2008) cited values around 3/4 in some vertebrates.

An alternative is to use the "thermal unit growth coefficient (*TGC*)" model reported by Iwama and Tautz (1981) in hatcheries, and developed by Cho (1992), Cho and Bureau (1998) and Dumas *et al*. (2007) in growing trout, and Mayer *et al*. (2008, 2009) in gilthead sea bream. This model is a particular version of the von Bertalanffy equation that incorporates a cumulative water temperature, which allows an estimation of fish growth in several temperature conditions, constituting an interesting tool for aquaculture management. In the case of gilthead sea bream, growth patterns were considered as a function of cumulative effective temperature Σ (T_i - 12), because growth is zero, or negative, for water temperature below 12 ºC (Mayer *et al*. 2008, 2009). Other models have also considered the average temperature (Petridis and Rogdakis, 1996; Lupatsch and Kissil, 1998; Lupatsch *et al*., 2003) but their practical application was difficult.

Ursin (1963), Akamine (1993), Moreau (1987) and Fontoura and Agostinho (1996) studied the inclusion of sinusoidal temperature curve in the Bertalanffy growth model, and recently Leon *et al*. (2006) used a temperature function applied to growth model from Hernandez *et al*. (2003). Alternatively, Dumas and France (2008) proposed a model to illustrate the seasonal *TGC* growth of ectotherms using a one year temperature periodic function. Seginer and Halachmi (2008) also applied the sinusoidal temperature curve to the exponential growth model from Lupatsch and Kissil (1998) to study management aspects in intensive gilthead sea bream aquaculture.

Another advantage of the *TGC* model was the simplicity of application in aquaculture, as it was possible to estimate the weight throughout the production cycle using a single value of *TGC* (obtained in the same production conditions). However, Dumas *et al*. (2007) suggested the need to use different *TGC* values for different trout stages during the growth period (< 20 g, 20-500 g, > 500 g). This would indicate that new studies revising the *TGC* model in other species are necessary.

In a previous paper Mayer *et al.* (2008) studied various growth models for the gilthead sea bream considering the variability of water temperature. The evolution of a set of average weights calculated from different samples obtained in 20 batches where analysed. One of the key findings of the paper was that the best models (including *TGC* model) were those that considered the accumulated effective temperature as an independent variable, instead the time. In a second work, Mayer *et al*. (2009) explored full samples considering all the individual weights of sea bream from the batches studied in Mayer *et al.* (2008) using a discriminant analysis and quantile regression techniques, with reference to the classic *TGC* model. It was suggested that it was possible to differentiate two groups of gilthead sea bream with homogeneous and heterogeneous growth characterized by a different evolution of the weight dispersion. The factors that influenced the dynamics and the diversity of growth were the seasonal change of water temperature and the weight distribution of the fishes provided by the hatchery.

The aim of this paper was to develop a new approach to the growth of gilthead sea bream under commercial production conditions with great fluctuations in water temperature, including the sinusoidal temperature curve in the *TGC* model, and considering the different stages throughout the growth period, in order to improve the estimation of growth on aquaculture farms. Our initial goal was to detect the existence of significant changes in the dynamics of the evolution of the average weight of fish over a complete cycle of production considering two-step *TGC* model that established the existence of a "critical or transition" live weight, which indicates indicated a change point in the dynamics of growth of the gilthead sea bream.

2.-Material and Methods

2.1. Mathematical Models

Considering a general model of growth given by the initial value problem,

$$\begin{cases} \dfrac{dW}{dt} = g(W,t), \\ W(t_0) = W_0, \end{cases} \qquad (1)$$

where W is the weight and t is the time, a model can be achieved that takes into account seasonal fluctuations in temperatures by replacing in (1) the time variable t, by a function $ST(t_0, t)$ (ST was used for simplicity) which represents the accumulated temperature in the time interval $[t_0, t]$ (Akamine, 1993). Indeed, assuming that at the initial time t_0, the accumulated temperature is zero, $ST(t_0, t_0)$ =0 we have

$$\begin{cases} \dfrac{dW}{dST} = g(W, ST), \\ W(0) = W_0, \end{cases} \qquad (2)$$

Models (1) and (2) describe different temporal dynamics. The model (2) takes the sum of temperature as an independent variable to describe the evolution of time and the growth is described from the instantaneous rate of weight gain per unit of accumulated temperature.

Taking into account the chain rule

$$\frac{dW}{dt} = \frac{dW}{dST} \cdot \frac{dST}{dt} \qquad (3)$$

and that

$$\frac{dST}{dt} = T(t) \qquad \text{i.e.} \qquad ST(t_0, t) = \int_{t_0}^{t} T(x)\, dx \qquad (4)$$

where $T(t)$ is the continuous function that provides the temperature at any moment t, from the model (2) we obtain immediately a seasonal time-dependent model

$$\begin{cases} \dfrac{dW}{dt} = g(W, ST)T(t), \\ W(t_0) = W_0, \end{cases} \qquad (5)$$

In the case of indeterminate allometric growth the basic model (Parker and Larkin, 1959, Gamito, 1998) is quite common,

$$\frac{dW}{dt} = kW^{1-b} \qquad (6)$$

where k is a constant related with the metabolic loss of an individual unit weight and the achievement of assimilated food for growth and b is a constant ($0<b<1$). The model given by (6) assumes that the allometric growth rate decreases with time due to the decrease that occurs in metabolic rate with increasing fish size and that W increases without limit (Gamito, 1998).

From (3), (4) and (6), we obtain the associated seasonal model

$$\frac{dW}{dST} = k \cdot W^{1-b}, \qquad (7)$$

i.e.

$$\frac{dW}{dt} = k \cdot T(t) \cdot W^{1-b} \qquad (8)$$

In what follows, we assume that the time, t, is given in days (d), the units of the constant rate k (>0) are $g^{b} \cdot (^{o}C \cdot d)^{-1}$, $T(t)$ is the function that provides the water temperature at each time (°C), and the allometric exponent b ($0<b<1$) is dimensionless.

If we suppose initially that $t=t_0$, $ST(t_{0,} t_0)=0$ and $W=W_0$, the solution of (7) is

$$W^{b}(t) = W_0^{b} + k \cdot b \cdot ST(t_0, t). \qquad (9)$$

An immediate discrete version of (9) can be obtained by considering for each day, i, $i=1,2,\ldots,n$, the mean of the daily temperature, T_i. so we have the model

$$W_n^{b} = W_0^{b} + k \cdot b \cdot \sum_{i=1}^{n} T_i, \quad n = 1,2,\ldots \quad (10)$$

If $b=1/3$ in (10), and denote $k=TGC/b$, we obtain the classic TGC-model (Cho, 1992)

$$W_n^{\frac{1}{3}} = W_0^{\frac{1}{3}} + TGC \cdot \sum_{i=1}^{n} T_i \qquad (11)$$

which was developed from empirical results without any mathematical or dynamical previous consideration (Dumas *et al.*, 2007).

Equations (7) or (8) allow modelling the indeterminate seasonal growth. The function $T(t)$ can take different expressions depending on environmental conditions (Akamine, 1993).

The integral solution of equation (8) is given by the expression

$$W^b(t) = W_0^b + k \cdot b \cdot \int_{t_0}^{t} T(t) \cdot dt \qquad (12)$$

i.e.

$$W(t) = \left(W_0^b + k \cdot b \cdot \int_{0}^{t} T(t)\, dt \right)^{\frac{1}{b}} \qquad (13)$$

As mentioned above, the temperature function *T*(*t*) depends on the context. In the case of marine farms in fixed locations, fish live in an environmental where the water temperature evolves according to regular annual cycles. A simple one-year periodic expression, which allows us to include the seasonal influence of temperature on growth in the model, is based on the sinusoidal function (14) used in different studies

$$T(t) = T_m + T_D \cdot \sin\left(\frac{2\pi}{365} \cdot (t - \alpha) \right) \qquad (14)$$

where $t \geq 0$, and T_m is the average annual temperature, T_D is the amplitude and α is a tuning parameter· From (14), we obtain a compact expression for the cumulative temperature function in the time interval $[t_0, t]$,

$$ST(t_0, t) = \int_{t_0}^{t} T(t)\, dt = T_m\, (t - t_0) - T_D\, \frac{365}{2\pi} \left(\cos\left(\frac{2\pi\, (t - \alpha)}{365} \right) - \cos\left(\frac{2\pi\, (t_0 - \alpha)}{365} \right) \right) \qquad (15)$$

In the case of gilthead sea bream, it is more appropriate to use the effective temperature, *T*(*t*)-12, instead of *T*(*t*) (Mayer *et al.* 2008), which only involves replacing T_m by T_m -12 in (15).

$$ST(t_0, t) = \int_{t_0}^{t} (T(t) - 12)\, dt = (T_m - 12)(t - t_0) - T_D\, \frac{365}{2\pi} \left(\cos\left(\frac{2\pi\, (t - \alpha)}{365} \right) - \cos\left(\frac{2\pi\, (t_0 - \alpha)}{365} \right) \right) \qquad (16)$$

By substituting (16) in (13), and solving the integral, we obtain an expression for the weight in the instant *t* (Dumas and France, 2008)

$$W(t) = \left(W_0^b + k \cdot b \cdot \left((T_m - 12) \cdot (t - t_0) - T_D\, \frac{365}{2\pi} \left(\cos\left(\frac{2\pi\, (t - \alpha)}{365} \right) - \cos\left(\frac{2\pi\, (t_0 - \alpha)}{365} \right) \right) \right) \right)^{\frac{1}{b}} \qquad (17)$$

i.e.

$$W(t) = \left(W_0^b + TGC_b \cdot \left((T_m - 12)\cdot(t - t_0) - T_D \frac{365}{2\pi} \left(\cos\left(\frac{2\pi\,(t-\alpha)}{365} \right) - \cos\left(\frac{2\pi\,(t_0-\alpha)}{365} \right) \right) \right) \right)^{\frac{1}{b}} \qquad (18)$$

where $TGC_b = k\,b$.

Dumas and France, (2008) obtained good results for describing the growth of different species of ectotherms, using models analogous to that given by equation (18), assuming different values of b for different species and contexts, but fixing different values for different time periods under study.

From equation (18), three models were developed in order to simulate the seasonal indeterminate growth of gilthead sea bream. Two of them were obtained by fitting the data to equation (11), assuming the values b = 1/3 and b = 2/3, based on actual values of accumulated temperature. The third model is built by aggregation of the two models mentioned above, establishing two stages of growth.

2.2. Data description

Models have been developed considering data on weight and accumulated temperature from 20 batches of farmed gilthead sea bream in real conditions of growth (Mayer *et al.* 2008).

To validate the models the weight data from 6 batches of gilthead sea bream (Table 1) were used. The production conditions of these 6 batches were similar to those described in Mayer *et al*. (2008) and corresponded to an initial production period between April and October (Table 1).

TABLE 1

2.3 Statistical analysis and design of the models

A preliminary exploratory analysis of the data from the 20 batches was performed, considering the discrete model

$$W_f = \left(W_0^b + TGC_b \cdot ST \right)^{\frac{1}{b}} \qquad (19)$$

where parameters b and TGC_b were estimated from available actual data of accumulated effective temperatures, by the Levenberg-Marquard iterative method available in

Statgraphics© plus version 5.1. The exploratory analysis studying the model (19) with both b=1/3 and b=2/3 was continued using a least squares fit after linearisation, obtaining the values for the *TGC*, named $TGC_{1/3}$ and $TGC_{2/3}$, respectively.

For integrating two models it was necessary to establish the transition point of change in the dynamics of growth we consider the expression (7) with b=1/3 and b=2/3, and solve the equation

$$k_{1/3} \cdot W^{2/3} = k_{2/3} \cdot W^{1/3} \qquad (20)$$

Note that in (7) we must distinguish two values of k which are different for the two values of b, so $k_b = TGC_b/b$ for b=1/3, b=2/3, respectively. The non-zero solution for W in (20), W_c= 1/8 $(TGC_{2/3} / TGC_{1/3})^3$ is a theoretical critical value of the weight for which the instantaneous rate of change in terms of weight depending on accumulated temperature is the same for both models (see Figure 1). We assumed the hypothesis that the critical weight obtained indicates a smooth transition from the dynamics described by the model given by equation (19) with b=1/3 to the dynamics described by the model with b=2/3.

To estimate the final weight of gilthead sea bream, two simulation models were developed from equation (18) with b=1/3 ($TGC_b=TGC_{1/3}$) and b=2/3 ($TGC_b=TGC_{2/3}$), respectively, and from the temperature function, $T(t)$, given in (14). These models were designated the seasonal 1/3-*TGC* model and seasonal 2/3-*TGC* model, respectively. The parameters T_m, T_d and α, of the temperature function $T(t)$, (14) were adjusted for the environmental conditions where the studied batches were located. This was done using a large sample of daily temperatures of sea water for a period of three years (March 1998-March 2001) and the Levenberg-Marquardt algorithm available in MATLAB© v. 5.3 was used.

From the seasonal models1/3-*TGC* and 2/3-*TGC*, taking into account the transition value of the weight obtained previously ($W_c \approx$117 g), a new simulation model was designed which is a combination of both of the previous versions, named seasonal Mixed-*TGC* model. To analyse and validate the seasonal models, 1/3-*TGC*, 2/3-*TGC* and Mixed-*TGC*, various techniques were applied, using the statistical package Statgraphics© plus 5.1.

The three models were tested using the 6 batches described in Table 1, which were not used in model development. We have considered jointly, the actual average weight data, from different samples taken in the 6 batches obtaining a single large sample. Samples in each batch were taken at different times of the production cycle. The estimated weights for the three models, from the initial weight and for each batch, for the same times in which samples were taken, were computed. Finally the actual and estimated values were compared. On the one hand, we contrasted the equality of the means of the absolute errors (absolute value of the difference between real values and estimated values of weight) for the three models by means of an ANOVA, using the *t*-test. On the other hand, the differences between actual weights and estimated weights were also studied considering contrasts for paired values (using the *t*-test). It was thus verified whether each model estimated suitably, overestimated or underestimated the final actual weight.

Finally, by contrasting hypotheses about the equality of standard deviations of the absolute errors, it was determined which model estimates more accurately the actual weight.

3.-Results

Considering the data from the 20 batches and equation (19), the parameters b and TGC_b were estimated from real data of actual accumulated temperatures. A value for b=0.6478 very close to 2/3 was obtained, with the 95%-asymptotic confidence interval being for b, (0.5576, 0.7180), and a value for TGC_b=0.014437, with the 95%-asymptotic confidence interval being for TGC_b (0.007744, 0.021129) and R^2=97.8%. Asymptotic confidence intervals showed that the parameters were significant and the coefficient of determination indicated a strong model fit to the data. These results led us to propose the viability of the *TGC* model with $b = 2/3$.

The results for the value TGC_b obtained by least squares, after linearisation, for models with b=1/3 and b=2/3, respectively, are shown in Table 2. Obviously, *TGC* values are different in the two models, $TGC_{1/3}$ = 0.00164 and $TGC_{2/3}$ = 0.01609, but remain highly significant.

TABLE 2

Figure 1 shows graphs corresponding to the instantaneous rates of growth, *dW/dST*, depending on the weight, *W*, given in (6), for the cases b=1/3 (1/3-model) and b=2/3 (2/3-model), considering the values $k=k_{1/3}$ and $k=k_{2/3}$, showed in Table 2, respectively. Both curves allow us to compare the dynamics of the evolution of weight for both models. Instantaneous growth rates based on the cumulative effective temperature (*dW*/*dST*, g $^{o}C^{-1}$) are equal for the non-zero intercept point corresponding to the value of weight $W \approx 117$ g (transition value of weight). From W=0 to W=117 g, instantaneous growth rate of weight with respect to the cumulative effective temperature is higher and grows faster for the 2/3-model. After W=117g, the instantaneous growth rate is higher for 1/3-model. These results clearly suggest a pattern of gilthead sea bream growth in two stages.

FIGURE 1

The fitted values for the parameters of the temperature function $T(t)$, described in (14), are T_m=18.8525, T_D=-6.6997 and α=-312.4609. Figure 2 shows the temperature function $T(t)$ and actual temperature data over a period of time established by the available actual data (available time interval started at day 69, March 10). Note that by periodicity, the first day of January would be day $1+365 \cdot j$, where j is any integer value.

FIGURE 2

So, two seasonal models were established based on equation (18), in order to describe the growth of gilthead sea bream; the seasonal 1/3-*TGC* model (b=1/3, $TGC_{1/3}$=0.001646) and the seasonal 2/3-*TGC* model (b=2/3, $TGC_{2/3}$=0.016095). From the former models, 1/3-*TGC* and 2/3-*TGC*, we constructed the seasonal Mixed-*TGC* model, which is defined in (21) and (22).

$$W_f(t) = \left(W_0^{\frac{1}{3}} + TGC_{1/3} \cdot ST(t_0, t) \right)^3, \text{ if } W_f(t) < 117 \qquad (21)$$

$$W_f(t) = \left(W_0^{\frac{2}{3}} + TGC_{2/3} \cdot ST(t_0, t) \right)^{\frac{3}{2}}, \text{ if } W_0(t) \geq 117 \qquad (22)$$

To estimate final weights greater than 117 g from initial weight less than 117 g, first we calculated the value t_1 for reaching 117 g using the 1/3-*TGC* model and the expression

(23), and then we estimated the final weight using the 2/3-*TGC* model and expression (24).

$$W_f(t_1) = \left(W_0^{\frac{1}{3}} + TGC_{1/3} \cdot ST(t_0, t_1) \right)^3 = 117 \qquad (23)$$

$$W_f(t) = \left(117^{\frac{1}{3}} + TGC_{2/3} \cdot ST(t_1, t) \right)^{\frac{3}{2}} \qquad (24)$$

Therefore, until a final weight less than 117, the Mixed-*TGC* model coincides with the 1/3-*TGC* model. In the case of an initial weight greater than or equal to 117g, the Mixed-*TGC* model coincides with the 2/3-*TGC* model. The Mixed-*TGC* model leads to a continuous curve for representing the final weight of the gilthead sea bream. Moreover, the curve is also differentiable at all time because the Mixed-*TGC* model is constructed so that when the weight is exactly 117 g, the derivatives of the functions that define the models 1/3-*TGC* and 2/3-*TGC* coincide. Thus, the transition from the 1/3-*TGC* model to the 2/3-*TGC* model occurs smoothly, without sharp points.

Figure 3 shows actual weight points together with estimated weight curves obtained from the three models, 1/3-*TGC*, 2/3-*TGC*3 and Mixed-*TGC*, for the six new batches reserved for validating the theoretical models.

FIGURE 3

Table 3 shows the results for the averages of absolute errors of estimation for the complete cycle (long-term using data from all monthly samples), for the periods before the critical weight (W_f<117) and after the critical weight (W_0>117) and for final weight at the end of the cycle. The estimated absolute error (absolute value of the difference between real and estimated value), is a measure of the adjustment of the values estimated by models to the real data. The results show a lower value of the average of the absolute errors for the 1/3-*TGC* model than 2/3-*TGC* when W<117 g, and for the 2/3-*TGC* model than 1/3-*TGC* when W≥117 g, but if the complete production cycle is considered and the Mixed *TGC* model is compared with the 1/3-*TGC* model and the 2/3-*TGC* model, differences were not statistically significant. When final weight was estimated from initial weight with three models, differences were not significant.

TABLE 3

Finally, Table 4 shows the outcomes of the hypothesis tests considering the resulting variable by subtracting the actual weight minus the estimated weight, $D=W_{real}-W_{est}$. When considering the sign of the difference between real and estimated weight, we can determine if a model overestimates or underestimates real weight. Analysis distinguishes the case in which the real final weight is less than 117 g (first stage) from that where the real initial weight is greater than or equal to 117 g (second stage). The Mixed-*TGC* model does not appear in the analysis because for final weights less than 117g the Mixed-*TGC* model coincides with 1/3-*TGC* model, and if initial weight is greater than or equal to 117, then Mixed-*TGC* model coincides with 2/3-*TGC* model.

TABLE 4

For a significance level $\alpha = 0.05$, on the one hand the results indicate that there are no statistically significant differences between the real weight and the weight estimated by the 1/3-*TGC* model for the first stage, and that the 1/3-*TGC* model tends to overestimate the final weight in the second stage of growth. On the other hand, the 2/3-*TGC* model overestimated the final weight in the first stage of growth while there were no statistically significant differences between the real weight and weight estimated by the 2/3-*TGC* model in the second stage of growth.

4.-Discussion

Final weight of gilthead sea bream in real conditions of production, seems to be better explained using the *TGC* model with *b*=2/3 than with *b*=1/3, because estimated value of exponent was *b*=0.648, very close to 2/3. Lupatsch and Kissil (1998) developed a growth model for gilthead sea bream and obtained a coefficient for weight similar to 2/3 (*b*= 0.613), although in a new model (Lupatsch *et al.*, 2003) the coefficient was lower (*b*=0.514).

When the two models, 1/3-*TGC* and 2/3-*TGC* were assayed, a change in the pattern of growth for gilthead sea bream under commercial production conditions was noted, as the presence of a transition weight value from around 117 grams was detected, which indicates a turning point for the dynamics of growth in weight of fish. If we start with an initial weight of 10 grams, this value can be matched with a value of the sum of effective temperatures *ST* =1670 ºC. We cannot explain the hypothetical physiological process of change that occurs at 117 grams. The results indicate the need to address a more detailed study of allometric growth of gilthead sea bream under production conditions. Nevertheless, the reasons for the change in the pattern of growth should be related with aspects such as the compensatory growth, genetic potential, allometric growth, nutrients or physiology of reproduction. Dumas *et al.* (2007) showed that to describe the growth of rainbow trout over a full cycle of production, there are three stanzas with different values for *b*. Growth changes associated with these stages are explained by morphological changes due to muscle growth dynamics, nutrient utilisation and reproduction investment. It seems clear that parameter *b* should not be considered a priori as a constant for a *TGC* model intended to explain the growth of gilthead sea bream in a full production cycle. Specifically, in the case of gilthead sea bream, when considering a complete production cycle, the *TGC*-1/3 model tends to overestimate the final weight (Mayer *et al.* 2008).

The 1/3-*TGC* model gives statistically significant better results for the estimated weight of fish in early stages, to lower final weights of 117 g, while 2/3-*TGC* model gives better results in estimating the final weight of fish with initial weights higher than 117 g. The result is consistent with the fact that the1/3-*TGC* model is based on the model proposed by Iwama and Tautz (1981) for fingerling growth in hatcheries. If we assume that the temperature varies continuously over time, therefore the model assumes that the growth rate is allometrically related to the weight, *W*, and the allometric constant of proportionality is directly related to temperature that varies during the rearing period.

When we compare the real weight with the estimated weights along the complete growth cycle, we cannot establish statistically significant differences, because the large dispersion of the absolute errors corresponding to the 1/3-*TGC* model and 2/3-*TGC* model (Figure 4). The three models seems to provide acceptable results for estimating the long term weight, as evidenced by the analysis of absolute errors of estimation (Table 3). If we consider only the weights at the end of the cycle, the absolute error analysis does not allow statistically significant differences between the three models, but the final error clearly seems to be lower with Mixed or 2/3-*TGC* models than 1/3-*TGC*-model. In view of the graphs in Figure 3, it seems clear that both 2/3-*TGC* model and further the 1/3-*TGC* model, tend to overestimate the weights at the final of the cycle of production. Notably, the absolute errors for models 1/3-*TGC* and 2/3-*TGC* show a wide dispersion, which prevents us establishing significant discrepancies in absolute errors considering the complete cycle in the three models. In Figures 4 and 5, the value for the standard error reflects the variation within each sample, and we can observe that the mean of absolute errors for the Mixed-*TGC* model is the lowest.

FIGURE 4 – FIGURE 5

It appears that the errors for the Mixed-*TGC* model have statistically significant lower dispersion when estimating the average absolute errors. Indeed, to test if the differences between standard deviations of the errors are statistically significant when considering the whole cycle, hypothesis tests were performed comparing the standard deviations of the long-term absolute errors of estimation for 1/3-*TGC*, 2/3-*TGC* and Mixed-*TGC* models. First, we tested the null hypothesis that the standard deviation from the absolute error for 1/3-*TGC* model is equal to that corresponding to the Mixed-*TGC* model (H_0: $\sigma_{1/3} = \sigma_{Mix}$), against the alternative hypothesis that the standard deviation of the absolute error for the 1/3-*TGC* model is greater (H_1: $\sigma_{1/3} > \sigma_{Mix}$), obtaining the *p*-value=0.0073 which leads to rejection of H_0. When the null hypothesis that the standard deviations of the absolute errors for 2/3-*TGC* model and Mixed-*TG*C model are equal (H_0: $\sigma_{2/3} = \sigma_{Mix}$) was tested against the alternative hypothesis that the standard deviation is greater for the model 2/3-*TGC* (H_1: $\sigma_{2/3} > \sigma_{Mix}$), it yielded a *p*-value=0.0009, which also led to rejection of H_0. Statistically significant differences between the standard deviations of the absolute errors cannot be set for the 1/3-*TGC* and 2/3-*TGC* models. The results of these contrasts showed lower uncertainty in estimates from the Mixed-*TGC* model and confirmed what Figure 4 seemed to show. Moreover, the 1/3-*TGC* model and the 2/3-*TGC* model tended to overestimate the weight in the second stage of growth and in the first stage of growth, respectively.

From the above considerations, the Mixed-*TGC* model clearly seems to be the most appropriate for describing the growth over the complete production cycle.

5.-Conclusions

The family of *TGC* seasonal models obtained by considering different values for the *b* parameter in the metabolic equation provides a framework for studying and explaining indeterminate growth patterns. In the case of gilthead sea bream, the use of 1/3-*TGC* model is useful for estimating the weight in the initial period of growth (in this case the 1/3-*TGC* model matches with the Mixed-*TGC* model). In the case where the initial weight exceeds 117 grams, it is advisable to use the 2/3-*TGC* model to estimate the weight (which in this case also coincides with the Mixed-*TGC* model). The study of *TGC* models has revealed a change in the growth pattern that occurs when the fish reaches a weight around 117g.

A continuous growth curve including temperature function and integrating the two models was developed to establish a practical tool for fish farmers.

The results indicate that the Mixed-*TGC* model is the most appropriate for long-term and final weight estimations along the complete cycle of growth.

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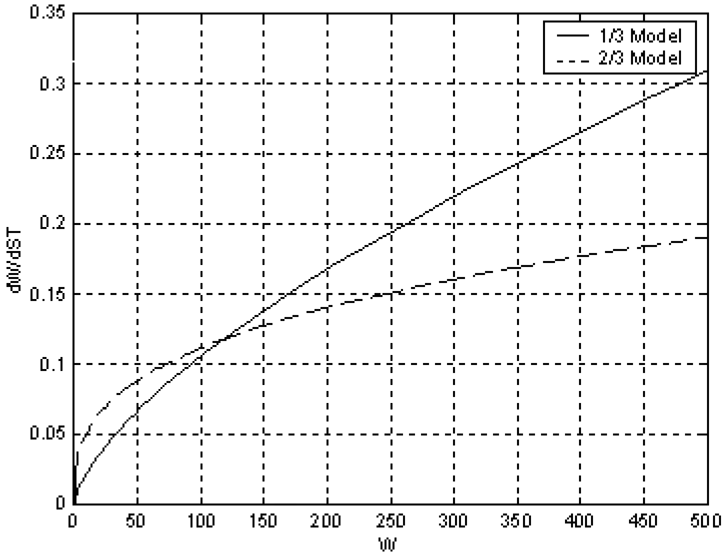

Figure 1. Curves representing instantaneous growth rate *dW/dST* for two models (1/3-*TGC and* 2/3-*TGC*)

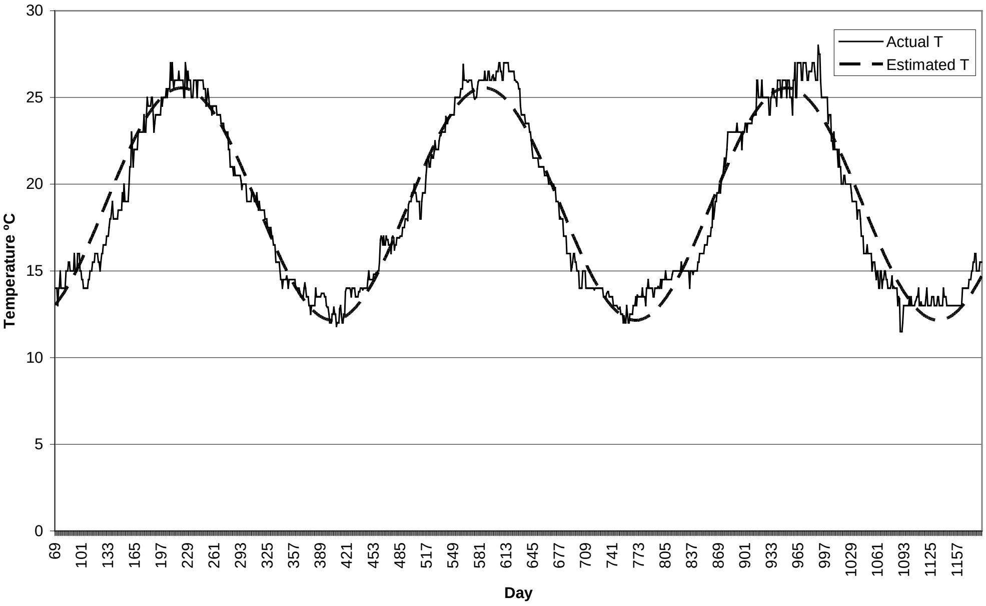

Figure 2. Temperature curve obtained and available actual data from Mediterranean Sea in Spanish southwest coast

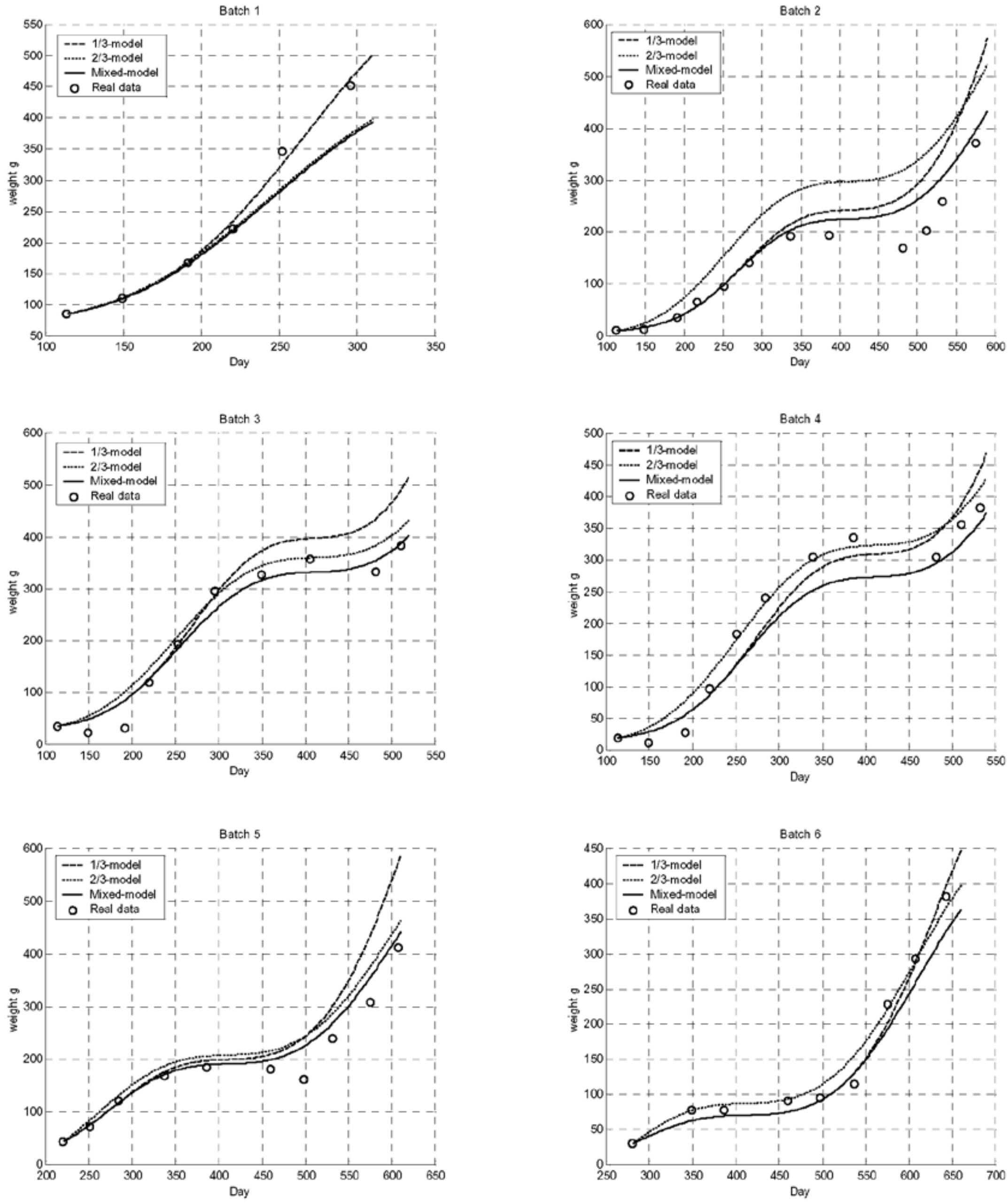

Figure 3. Growth curves generated with three models (*1/3-TGC* and *2/3-TGC* and Mixed) and real data from six new batches. (The abscissa axis shows the value of the time variable *t* day within a year. So, *t*=1 corresponds to the first day of the year, January 1 and a time value *t* >365 indicates a transition from a year to the next)

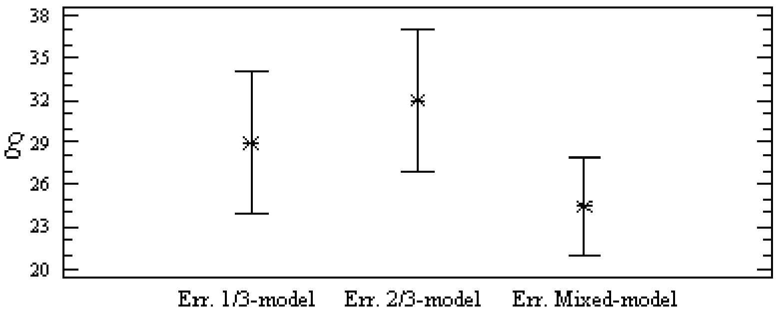

Figure 4. Mean and standard errors for absolute error of long term estimation using three models and real values of new six batches.

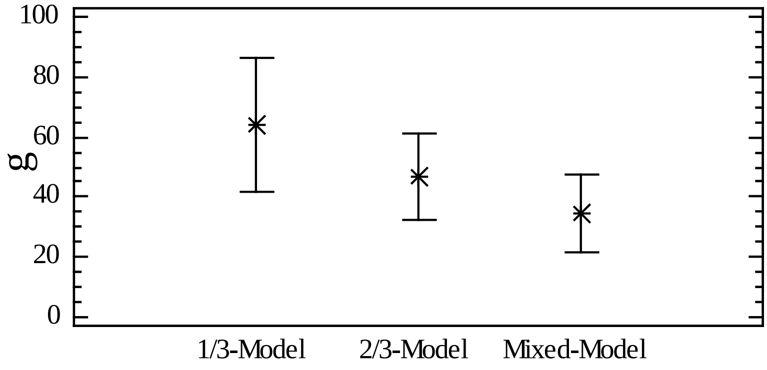

Figure 5. Mean and standard errors for absolute error of final weight estimation using three models and real values of new six batches.

Table 1. Description data from six new batches used for validation of models

Batch	Samples along the period	Initial weight, W_0 (g)	Final weight, W_f (g)	*Days*	*T* (ºC)	*ST* (ºC)	*TGC* ($g^{1/3}$ ºC^{-1})	*SGR* (% d^{-1})
1	5	85.1	452.1	183	23.7	2141.1	0.001530	0.91
2	10	10.0	371.4	463	19.8	3611.4	0.001394	0.78
3	7	35.0	384.4	399	19.0	2793	0.001432	0.60
4	9	19.3	382.8	420	19.2	3024	0.001514	0.71
5	8	43.8	411.6	388	19.7	2987.6	0.001310	0.57
6	8	30.0	381.4	364	20.6	3130.4	0.001324	0.69

T: average temperature of the period, *ST*: cumulative effective temperature (effective temperature is temperature in degrees Celsius minus 12), *TGC*: Thermal Growth Coefficient $\left(\frac{W_f^{1/3} - W_0^{1/3}}{ST}\right)$, *SGR*: Specific Growth Rate $\left(\frac{\ln(W_f) - \ln(W_0)}{Days}\right)$)

Table 2. Thermal Growth Coefficients obtained using the two models (*b*=1/3 and *b*=2/3) considering growth data from 20 batches

Model	TGC_b	95% TGC_b Confidence interval	K_b	R^2
$\left(b = \frac{1}{3}\right)$	$TGC_{1/3}$=0.00164561	0.00156 - 0.00174	0.0049368	97.3%
$\left(b = \frac{2}{3}\right)$	$TGC_{2/3}$=0.0160949	0.0153 - 0.0169	0,02414235	98.1%.

Table 3. ANOVA results for the averages of absolute errors of estimation (g), and the three models, for the complete cycle (long-term using data from all monthly samples), for the periods before the critical weight ($W_f<117$) and after the critical weight ($W_0>117$) and for final weight at the end of the cycle, considering data from six new batches.

	1/3-*TGC* Model	2/3-*TGC* Model	Mixed-*TGC* model
Long-term	28.9	31.9	24.4
$W_f<117$ (1)	9.5 ^{a}	28.8 ^{b}	-
$W_0 \geq 117$ (2)	48.7 ^{a}	29.0 ^{b}	-
Final Weight	64.1	46.6	34.6

The results must be interpreted by row. (1) *P*-Value = 0.0328 (2) *P*-Value= 0.0349

Table 4. **Hypothesis tests for paired variables distinguishing two stages of growth: first W_f < 117 g and second W_0 ≥117 g**

Model	W_f < 117 g	W_0 ≥117 g
1/3-*TGC*	H_0: D=0 H_1: D≠0 P-value=0.890 Not Reject H_0	H_0: D=0 H_1: D<0 P-value=0.0005 Reject H_0
2/3-*TGC*	H_0: D=0 H_1: D<0 P-value=0.0021 Reject H_0	H_0: D=0 H_1: D≠0 P-value=0.60 Not Reject H_0