Recurrence in Linear dynamics

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Resumen en inglés

A bounded and linear operator is said to be *hypercyclic* if there exists a vector such that its orbit under the action of the operator is dense. The first example of a hypercyclic operator on a Banach space was given in 1969 by Rolewicz who showed that if B is the unweighted unilateral backward shift on l^2, then λB is hypercyclic if and only if $|\lambda| > 1$. Among its features, we can mention for example that finite-dimensional spaces cannot support hypercyclic operators, proved by Kitai. On the other hand, several people have shown in different contexts, in the Hilbert space frame, that the set of hypercyclic vectors for a hypercyclic operator is a G_δ dense set.

This thesis is divided into four chapters. In the first one, we give some preliminaries by mentioning some definitions and known results that will be of great help later.

In chapter 2, we introduce a refinement of the notion of hypercyclicity, relative to the set $N(U, V) = \{n \in \mathbb{N} : T^{-n}U \cap V \neq \emptyset\}$ when belonging to a certain collection $\mathscr{F}$ of subsets of $\mathbb{N}$, namely a bounded and linear operator T is called *$\mathscr{F}$-operator* if $N(U, V) \in \mathscr{F}$, for any pair of non-empty open sets U, V in X. First, we do an analysis of the hierarchy established between $\mathscr{F}$-operators, whenever $\mathscr{F}$ covers those families mostly studied in Ramsey theory. Second, we investigate which kind of properties of density can have the sets $N(x, U) = \{n \in \mathbb{N} : T^n x \in U\}$ and $N(U, V)$ for a given hypercyclic operator, and classify the hypercyclic operators accordingly to these properties.

In chapter three, we introduce the following notion: an operator T on X satisfies property $\mathcal{P}_{\mathscr{F}}$ if for any U non-empty open set in X, there exists

$x \in X$ such that $N(x,U) \in \mathscr{F}$. Let $\overline{\mathcal{BD}}$ the collection of sets in $\mathbb{N}$ with positive upper Banach density. We generalize the main result of a paper due to Costakis and Parissis using a strong result of Bergelson and Mccutcheon in the vein of Szemerédi's theorem, leading us to a characterization of those operators satisfying property $\mathcal{P}_{\overline{\mathcal{BD}}}$. It turns out that operators having property $\mathcal{P}_{\overline{\mathcal{BD}}}$ satisfy a kind of recurrence described in terms of essential idempotents of $\beta\mathbb{N}$ (the Stone-Čech compactification of $\mathbb{N}$). We will discuss the case of weighted backward shifts satisfying property $\mathcal{P}_{\overline{\mathcal{BD}}}$. On the other hand, as a consequence we obtain a characterization of reiteratively hypercyclic operators, i.e. operators for which there exists $x \in X$ such that for any U non-empty open set in X, the set $N(x,U) \in \overline{\mathcal{BD}}$.

The fourth chapter focuses on a refinement of the notion of disjoint hypercyclicity. We extend a result of Bès, Martin, Peris and Shkarin by stating: B_w is $\mathscr{F}$-weighted backward shift if and only if $(B_w, \dots, B_w^r)$ is d-$\mathscr{F}$, for any $r \in \mathbb{N}$, where $\mathscr{F}$ runs along some filters containing strictly the family of cofinite sets, which are frequently used in Ramsey theory. On the other hand, we point out that this phenomenon does not occur beyond the weighted shift frame by showing a mixing linear operator T on a Hilbert space such that the tuple (T, T^2) is not d-syndetic. We also, investigate the relationship between reiteratively hypercyclic operators and d-$\mathscr{F}$ tuples, for filters $\mathscr{F}$ contained in the family of syndetic sets. Finally, we examine conditions to impose in order to get reiterative hypercyclicity from syndeticity in the weighted shift frame.