Effective flexural stiffness of slender reinforced concrete columns under axial forces and biaxial bending

J.L. Bonet [a], M.L. Romero[a,*], and P.F. Miguel [a]

[a] Instituto de Ciencia y Tecnología del Hormigón (ICITECH). Universidad Politécnica de Valencia, Spain

ABSTRACT

Most of the design codes (ACI-318-2008 and Euro Code–2-2004) propose the moment magnifier method in order to take into account the second order effect to design slender reinforced concrete columns. The accuracy of this method depends on the effective flexural stiffness of the column. This paper proposes a new equation to obtain the effective stiffness EI of slender reinforced concrete columns. The expression is valid for any shape of the cross sections, subjected to combined axial loads and biaxial bending, both for short-time and sustained loads, normal and high strength concretes, but it is only suitable for columns with equal effective buckling lengths in the two principal bending planes. The new equation extends the proposed EI equation in the "Biaxial bending moment magnifier method" by Bonet et al [6], which is valid only for rectangular sections. The method was compared with 613 experimental tests from the literature and a good degree of accuracy was obtained. It was also compared with the design codes ACI-318 (08) and EC-2 (2004) improving the precision. The method is capable to verify and design with sufficient accuracy slender reinforced concrete columns in practical engineering design applications.

Key words: reinforced concrete, columns, biaxial bending, strength, design.

Corresponding author: Tel: +34-963877007(ext:76742) Fax: +34-963879679,
E-mail address: mromero@mes.upv.es

1 INTRODUCTION

The codes ACI-318 [1] and Euro Code–2[2] propose the moment magnifier method in order to take into account the second order effect to design slender reinforced concrete columns. The accuracy of this method depends on the effective flexural stiffness EI of the column. Such parameter depends on cracking, creep and non-linear material behaviour.

Over the last three decades, many authors and national codes have proposed different methods to determine the column stiffness for short-time and sustained loads.

Thus, the ACI-318 (08) code [1] proposes an equation that is independent from the loads applied to the column. However, the EC-2 code [2] and most authors, such as Mavichak and Furlong [3], Mirza[4], Westerberg [5], Bonet et al[6], Tikka and Mirza[7],[8] and so on, claim that flexural stiffness EI depends on the loads applied by means of the relative eccentricity or else through the axial load. Table 1 compares the different EI equations from the literature and the design codes. As it is shown, there is no homogeneity between the different proposals regarding the variables analyzed and the functions used.

Most of the proposed EI equations by these authors are only applicable for rectangular cross sections. Only, Ehsani et al [9] and Sigmon et al [10] propose an equation of EI for circular sections and instantaneous loads. Such authors agree that the EI formula proposed by the ACI-318 (08) [1] is very conservative for this type of columns. Furthermore, most of the EI equations were obtained for normal strength concretes. Since, the mechanical behavior of high strength concrete cannot be extrapolated from the normal strength one, it is necessary to update the applicability of such expressions to any range of strength.

In this paper a new equation to calculate the stiffness EI in reinforced concrete columns with any cross section shape subjected to axial load and uniaxial bending is proposed. It attempts to fill the gap in the equations presented in the bibliography because they are only valid for rectangular and normal strength concrete, and in practice there are sections with different shapes: rectangular, circular, ovoid, cross shape, hexagonal or thin-walled box.

Moreover, many reinforced concrete sections are subjected to biaxial bending and axial loads as a result of their position in the structure, the shape of the cross-section or the source of the external loads. For those cases, the ACI-318 (08) code [1] amplifies the first order bending moments in each flexure plane independently. The design of the cross-section of the columns is based on these magnified forces. Although the EC-2 code [2] also magnifies the bending moment separately in each direction, the design is performed using the "load contour method" by Bresler [11]:

$$\left(\frac{M_{tx}}{M_{ux}}\right)^{\gamma} + \left(\frac{M_{ty}}{M_{uy}}\right)^{\gamma} \leq 1 \qquad (1)$$

where M_{ux}, M_{uy} are the nominal bending moment strength around the "x" and "y" axes, respectively.

M_{tx}, M_{ty} are the nominal bending moments that are applied in the critical cross-section of the column considering the second order effects.

γ axial load contour exponent. It depends on the shape of the cross-section.

For biaxial bending those methods can produce unsafe situations of design for axial load levels close to the the ultimate axial load of the column if the most important bending force corresponds to the direction of the lower slenderness (bending with respect to the strong axis). Such effect was confirmed experimentally by Pallares et al [12]. Such methods do not take into account the interaction that both curvatures have in the structural behavior of the member. Hence, Bonet et al [6] proposed the “Biaxial bending moment magnifier method”, where an equation of the effective flexural stiffness was introduced for biaxial bending and rectangular sections. It included the interaction between both axes of bending.

This paper extends the proposed EI equation by Bonet et al [6] which was valid only for rectangular sections to any shape of the cross section. The novelty again is focussed in the addition of the interaction between both axes of curvature, in distinction from what the methods from the ACI-318 code [1] and the EC-2 code [2] do.

A new equation of EI for biaxial bending is proposed, because it does not exist in the literature for a general cross-section shape. The method will be limited to the case where the effective buckling length of the column is equal in the two principal bending planes. It will be applicable if there are one or two axes of symmetry (rectangular, thin-walled box, ovoid, or C-shape cross sections), but also if there is not any symmetry (“L” cross section p.e.).

The “biaxial bending moment magnifier” method was based on the magnification of the first order bending moment applied in the critical section of the column:

$$M_t = \delta_{ns} \cdot M_d \tag{2}$$

where M_t is the total vector modulus for design

$$M_t = \sqrt{M_{tx}^2 + M_{ty}^2} \tag{3}$$

M_d vector modulus of the first order bending moment

$$M_d = \sqrt{M_{dx}^2 + M_{dy}^2} \tag{4}$$

δ_{ns} magnification factor

$$\delta_{ns} = \frac{1}{1 - N_d / N_{cr}} > 1 \tag{5}$$

N_d design axial load

N_{cr} critical buckling load, which is a function of the flexural stiffness of the column EI and of the effective length (l_p)

$$N_{cr} = \frac{\pi^2 EI}{l_p^2} \tag{6}$$

The effective flexural stiffness EI of the column represents the equivalent stiffness of a fictitious column with constant stiffness, whose effective buckling length (l_p) and critical axial load (N_{cr}) agree with those of the real column. Such column flexural stiffness EI represents the global behaviour of the total element and not of just one particular section.

The flexural stiffness EI equation was inferred from the results obtained with the numerical simulation described in the next section. The adjustment of the proposed equation was compared with 613 experimental tests from the literature.

2. NUMERICAL SIMULATION

The flexural stiffness EI of the column was obtained from the utilisation of a general method of structural analysis for reinforced concrete using finite elements. This numerical method includes the following main issues:

- 1-D finite element with non-constant curvature: the finite element has 13 degrees of freedom (d.o.f's), Marí [13]. This element has three nodes, with 6 d.o.f.'s in the initial and final nodes (three rotations and three displacements), while the mid-span node has only one degree of freedom in the axial direction to capture the variable curvature of the element, Figure 1.a.

- The numerical integration of the cross section is performed using the Green's theorem, Bonet et al. [14], Figure 1.b.
- Non-linear concrete behaviour (Model Code-90[15], CEB-FIP [16])
- Non-linear steel behaviour: bilinear diagram. (ModelCode-90[15])
- Geometric non-linearity: The geometric stiffness matrix and the update of the displacements are included in the definition of the model.
- Time-dependent effects: creep and shrinkage (CEB[17],[18])

The numerical model was verified with 613 tests from the bibliography ([3], [19-41]). The experiments correspond to reinforced concrete columns pinned-pinned subjected to axial load and both to uniaxial and biaxial bending. In those tests, the magnitude and the direction of the eccentricity are fixed, evaluating the maximum axial load of the column. The shape of the cross-sections are rectangular, square, box with one or two cells, ovoid, "C"-section or "L"-section. The length of the columns and the size of the cross-sections are the same than the experiments. A even number of finite elements were used because the applied load was symmetric. Moreover, it was verified that with a length of the finite element equal to the height of the section the results obtained had reasonable accuracy.

Table 2 shows the variation of parameters studied in the experiments. The accuracy of the numerical model is evaluated through the ratio between the axial load of the test N_{test} and the axial load from the numerical simulation N_{NS}.

Table 3 presents the accuracy of the numerical model for both the type of load (short-term and instantaneous) and the type of the cross-section (rectangular or non-rectangular). Table 4 shows the accuracy for the type of curvature (uniaxial or biaxial bending) and for the type of load. It can be seen that that an average ratio of 1.06 (safe side) and a variation coefficient of 0.13 was obtained when all the cases are analyzed.

The scatter of the results is the typical for this type of laboratory experiments. It was verified that the degree of accuracy is similar for all the parameters considered in this study.

The previous calibrated numerical model was used here to perform the analysis of the main variables that exert an influence on the stiffness *EI*. Table 5 shows the analysed parameters and their variation coefficients, which when combined produced 7360 numerical tests.

3. PROPOSAL OF A FLEXURAL STIFFNESS "EI"

a) Flexural Stiffness of a column for axial loads and uniaxial bending under short-term loads.

The estimation of the stiffness *EI* of the column subjected to short-term loads is obtained through the well-known equation:

$$EI = \alpha \cdot E_c \cdot I_c + E_s \cdot I_s \qquad (7)$$

where E_c is the short-term secant elastic modulus of the concrete and equal to $22.000 \cdot (f_{cm}/10)^{0.3}$ (in MPa), where f_{cm} is the mean compressive strength of concrete (in MPa); E_s is Young's modulus of reinforcement and equal to 200 000 MPa; I_c, I_s are the moments of inertia of the gross section of concrete and of the longitudinal reinforcement with respect to the centre of gravity of the gross section and, in this research, α is termed **"effective stiffness factor"**. This coefficient needs to be adjusted against numerical results. The concrete and steel elastic modulus were obtained from the Euro Code-2 [2]. A mean strength f_{cm} equal to the strength of the concrete from the numerical test "f_c" was chosen to perform the fitting of the coefficient α.

If a numerical simulation (N.S.) is performed for a slender column ($\lambda_m \neq 0$) subjected to axial loads and uniaxial bending, the ultimate first order bending moment $(M_1)_{NS}$ can be obtained for a particular axial load N_i. Likewise, it is also possible to compute the ultimate bending moment $(M_u)_{NS}$ of the cross-section of the column ($\lambda_m = 0$) for the same axial force, Figure 2.

From both values the effective stiffness factor "α" can be calculated by performing the following steps in sequence:

a) First, the magnification factor is obtained:

$$(\delta_{ns})_{NS} = (M_t)_{NS} / (M_d)_{NS} \tag{8}$$

b) This value allows the critical buckling load of the column to be computed by reordering equation 5:

$$(\mathrm{N_{cr}})_{\mathrm{NS}} = \frac{N_i}{1 - 1/(\delta_{ns})_{NS}} \tag{9}$$

c) The flexural stiffness of the column can be computed from equation 6:

$$(EI)_{NS} = \frac{(N_{cr})_{NS} \cdot l_p^2}{\pi^2} \tag{10}$$

d) Finally, the effective stiffness factor "α"can be obtained from equation 7:

$$(\alpha)_{SN} = \frac{(EI)_{NS} - E_s I_s}{E_c I_c} \tag{11}$$

Figure 3 presents (as an example) the "α" coefficient graphically in terms of the first order relative eccentricity η and of the mechanical slenderness λ_m, for the particular case of a circular section with 12 reinforcing bars, mechanical reinforcement ratio (ω) equal to 0.5, and for a concrete strength of f_c=30 MPa.

The first order relative eccentricity can be computed as:

$$\eta = \frac{(M_1)_{NS} / N_i}{4 \cdot i_c} = \frac{e_0}{4 \cdot i_c} \tag{12}$$

where i_c is the radius of gyration of the concrete section with respect to the axis of bending and e_0 is the first order eccentricity.

The effective stiffness factor α presents a non-linear behaviour in terms of the relative eccentricity η and the slenderness λ_m. In fact, the effective stiffness factor α is independent of the slenderness λ_m if the relative eccentricity η is equal to 0.2, as can be deduced from Figure 3. It can also be inferred from this figure that the performance of α is different if the relative eccentricity η is lower or higher than 0.2. Thus, if "η" is higher than 0.2 α decreases and is appreciably independent of the slenderness and can be approximated by only one straight line. Otherwise, α depends strongly on the slenderness and has to be approximated by straight lines whose slope is non-constant in terms of the mechanical slenderness λ_m.

For high values of the relative eccentricity ($\eta > 0.2$), the failure is produced by the ultimate strength of the section. Consequently, α is not influenced by the slenderness. In this case, when η is increased, the cross-section of the column reaches higher deformations and it produces a decrease in the stiffness of the column.

However, for small values of η and high slenderness, the failure is produced by the instability of the column. Therefore, α depends on the slenderness. For this case, when the slenderness is increased (maintaining constant the eccentricity) the possibility to reach an unstable position is higher and, in consequence, the cross-section is less deformed and the stiffness increases.

Finally, for small values of the relative eccentricity and slenderness, the factor α decreases in terms of the relative eccentricity. In this case the column is very compressed and the failure is due to the ultimate strength of the section. Thus, when the relative eccentricity η decreases, the column has higher compression and the difference between the real elastic modulus of the materials and the tangent elastic modulus adopted in equation 7 is higher. The parameter α corrects this difference.

The least square adjustment of the lines α- η from the numerical simulation enables the following equations to be proposed for the effective stiffness factor (α):

$$\begin{aligned} \alpha &= (1.95 - 0.035 \cdot \lambda_m) \cdot (\eta - 0.2) + \left(\frac{f_c}{225} + 0.11\right) \not< 0.1 \quad \text{for} \quad \eta < 0.2 \\ \alpha &= \left(\frac{f_c}{110} + 0.45\right) \cdot (0.2 - \eta) + \left(\frac{f_c}{225} + 0.11\right) \not< 0.1 \quad \text{for} \quad \eta \geq 0.2 \end{aligned} \tag{13}$$

Figure **4** shows a comparison between the method proposed in this paper and the codes ACI-318[1], Euro Code 2 [2], and also with the method proposed by Westerberg[5] and Tikka and Mirza [8] for the same cross-section (used in Figure 3). In order to apply the equation from Tikka and Mirza [8] to a circular section an equivalent height of the section (h_{eq}) was used, equal to $\sqrt{12}$ times the radius of gyration of the concrete circular cross section (i_c).

It can be noticed that both design codes propose an effective stiffness factor α independent of the relative eccentricity η and the mechanical slenderness λ_m. However, the other authors include the dependence of α in terms of the eccentricity or the slenderness. In general, these authors propose an equation of α that decreases with η. Only the formula from Westerberg [5] shows that for small values of η, the parameter α increases with this parameter. The proposals from these authors confirm the non-linear behavior of the parameter α.

b) Flexural Stiffness of a column for axial loads and uniaxial bending subjected to sustained loads.

The stiffness EI equation for sustained loads is achieved in a similar manner to the equation from the previous section:

$$EI = \alpha \cdot \frac{E_c}{1+\varphi} \cdot I_c + \frac{E_s}{1+\xi_\varphi(\lambda_m,\varphi)} \cdot I_s \tag{14}$$

where φ is the creep coefficient and ξ_φ is a reduction function of the tangent steel elastic modulus for sustained loads.

In the equation 14, the elastic modulus of concrete (E_c) is reduced through the expression $E_c/(1+\varphi)$. Moreover, the design value of the modulus of elasticity of the reinforcement (E_s) is reduced with the factor ξ_φ. It is a reduction function that according to the numerical simulation depends on the mechanical slenderness (λ_m) and on the creep coefficient (φ), in such a way that if the creep coefficient is increased and the slenderness is decreased, the steel deformation is reduced. This function is obtained by least squares from the results of the numerical simulation:

$$\xi_\varphi = 1.9 \cdot \varphi \cdot \exp(-\lambda_m/25) \tag{15}$$

In the end, taking into account the effect of the creep for small eccentricities (η<0.2) gave rise to appreciable modifications in equation 13:

$$\begin{aligned} \alpha &= (1.95 - 0.035 \cdot \lambda_m - \mathbf{0}.25 \cdot \varphi) \cdot (\eta - 0.2) + \left(\frac{f_c}{225} + 0.11\right) \nless 0.1 \quad && if \quad \eta < 0.2 \\ \alpha &= \left(\frac{f_c}{110} + 0.45\right) \cdot (0.2 - \eta) + \left(\frac{f_c}{225} + 0.11\right) \nless 0.1 && if \quad \eta \geq 0.2 \end{aligned} \tag{16}$$

For the case where the permanent load applied to the column is different to the total load, the creep coefficient (φ) from equations 14, 15 and 16 will be replaced by the effective creep ratio (φ_{eff}). According to the clause 5.8.4 from the Euro Code–2[2], this coefficient is the creep coefficient times the ratio between the first order bending moment in quasi-permanent load combination, SLS (M_{0Eqp}) and the first order bending moment in design load combination, ULS (M_{0Ed}).

c) Flexural Stiffness of a column subjected to axial load and biaxial bending.

It is important to notice that if the column is subjected to axial loads and biaxial bending, the magnification of the bending moment is performed in accordance with the bending plane (Figure 5, equation 2). The equation of the column stiffness EI for axial loads and uniaxial bending was expanded for the biaxial case:

$$EI = \alpha \cdot \frac{E_c}{1+\varphi_{eff}} \cdot I_{ce} + \frac{E_s}{1+\xi_\varphi} \cdot I_{se} \tag{17}$$

where α is the effective stiffness factor:

$$\begin{aligned} \alpha &= (1.95 - 0.035 \cdot \lambda_m - \mathbf{0.25} \cdot \varphi_{eff}) \cdot (\eta - 0.2) + \left(\frac{f_c}{225} + 0.11\right) \not< 0.1 \quad & if \quad \eta < 0.2 \\ \alpha &= \left(\frac{f_c}{110} + 0.45\right) \cdot (0.2 - \eta) + \left(\frac{f_c}{225} + 0.11\right) \not< 0.1 \quad & if \quad \eta \geq 0.2 \end{aligned} \tag{18}$$

η is the first order relative eccentricity:

$$\eta = \frac{e_0}{4 \cdot i_c} = \frac{M_d}{N_d \cdot 4 \cdot i_c} \tag{19}$$

where e_0 is the first order eccentricity

$$e_0 = M_d / N_d \tag{20}$$

M_d is the vector modulus of the first order bending moment (Figure 5.b)

$$M_d = \sqrt{M_{dx}^2 + M_{dy}^2} \tag{21}$$

M_{dx}, M_{dy} first order bending moments with respect to the axes of coordinates “x” and “y” of the section, respectively

N_d design axial load

i_c critical radius of gyration of the cross-section (Figure 5.a). The minimum radius of gyration of the gross section with respect of the principal axes of inertia is selected (i_{cu}, i_{cv})

$$i_c = \min(i_{cu}, i_{cv}) \qquad (22)$$

λ_m mechanical slenderness of the column

$$\lambda_m = l_p / i_c \qquad (23)$$

I_{ce} equivalent moment of inertia of the gross section

I_{se} equivalent moment of inertia of the reinforcing bars

ξ_φ reduction factor of tangent steel modulus E_s for sustained loads

$$\xi_\varphi = 1.9 \cdot \varphi_{eff} \cdot \exp(-\lambda_m/25) \qquad (24)$$

The equivalent moments of inertia of the gross section (I_{ce}) and of the reinforcement (I_{se}) are obtained by interpolating the moments of inertia of the section:

$$I_e = I_u \cdot \delta + I_{v\cdot} \cdot (1-\delta) \qquad (25)$$

where I_u, I_v are the moments of inertia with respect to the principal strong and weak axis respectively (Figure 5.a) and δ is an interpolating function.

In order to compute the direction which corresponds to the principal strong axis of inertia (Figure 5.a) with respect to the "x" axis, the following equation must to be solved:

$$(I_{c1} - I_{c2}) \cdot sin(2 \cdot \theta_p) - 2 \cdot cos(2 \cdot \theta_p) \cdot I_{cxy} = 0 \quad \Rightarrow \quad \theta_p \qquad (26)$$

where: I_{c1}, I_{c2} are the maximum and minimum moments of inertia of the section with respect to the "x" and "y" axes of the concrete section, respectively.

$$I_{c1} = \max(I_{cx}, I_{cy}); \quad I_{c2} = \min(I_{cx}, I_{cy}) \tag{27}$$

I_{cx}, I_{cy} moments of inertia of the section with respect to the "x" and "y" axes of the concrete section

I_{cxy} product of inertia of the section with respect to the "x" and "y" axes of the concrete section

If the moments of inertia of the section with respect to the "x" and "y" axes are equal, then from equation 26 it is obtained that $\theta_p = \pi / 4$.

Otherwise:

$$\theta_p = \frac{1}{2} atan\left(\frac{2 \cdot I_{cxy}}{I_{c1} - I_{c2}} \right) \tag{28}$$

The angle (θ_p) is positive for counter-clockwise (Figure 5.a) and the mechanical properties are calculated with respect to the centre of gravity of the concrete section.

In the equation 26, it was considered that the principal axis with higher inertia agrees with the "x" axis ($I_{c1} = I_{cx}$). Otherwise, the angle θ_p will be increased in $\pi/2$.

The principal moments of inertia of the section with respect to the strong axis (I_{cu}) and weak axis (I_{cv}) can be obtained with the following equations:

$$\begin{aligned} I_{cu} &= 0.5 \cdot \left(I_{cx} + I_{cy} \right) + \sqrt{\left(\frac{I_{cx} - I_{cy}}{2} \right)^2 + I_{cxy}^2} \\ I_{cv} &= 0.5 \cdot \left(I_{cx} + I_{cy} \right) - \sqrt{\left(\frac{I_{cx} - I_{cy}}{2} \right)^2 + I_{cxy}^2} \end{aligned} \tag{29}$$

The radii of gyration of the concrete section with respect to the principal axes of inertia are calculated with:

$$i_{cv} = \sqrt{I_{cu} / A_c}; \quad i_{cu} = \sqrt{I_{cv} / A_c} \tag{30}$$

where A_c is the area of the concrete section

The moments of inertia of the reinforcements (I_{su}, I_{sv}) with respect to the principal axes of inertia "u" and "v" are obtained with:

$$\begin{aligned} I_{su} &= 0.5\cdot(I_{sx}+I_{sy})+0.5\cdot(I_{sx}-I_{sy})\cdot\cos(2\cdot\theta_p)-\sin(2\cdot\theta_p)\cdot I_{sxy} \\ I_{sv} &= 0.5\cdot(I_{sx}+I_{sy})+0.5\cdot(I_{sx}-I_{sy})\cdot\cos(2\cdot(\theta_p+\pi/2))-\sin(2\cdot(\theta_p+\pi/2))\cdot I_{sxy} \end{aligned} \quad (31)$$

where: I_{sx}, I_{sy} are the moments of inertia of the reinforcements with respect to the axes "x" and "y" of the concrete section.

I_{sxy} is the product of inertia of the reinforcements with respect to the axes "x" and "y" of the concrete section.

The calculation of the principal axes of inertia of the section and its centre of gravity are performed with respect to the concrete section alone for simplicity without considering the contribution of the reinforcement bars. A more rigorous calculation could be done with respect to the homogenized section.

Equation 25 takes into account the interaction between both axes of curvature. Thus, as it was observed from the numerical simulation, if the column is not braced and the relative eccentricity (η) tends to zero, the critical axial load of the column (N_{cr}) is about the weak axis, and consequently the flexural stiffness of the member EI corresponds to the weak axis. Besides, if the column is subjected to biaxial bending with zero axial load, the relative eccentricity (η) is infinite, and in this case, the flexural stiffness corresponds to an intermediate value between the strong axis and the weak axis. This stiffness will be equal to the weak axis if the column bends with respects to this axis, and equal to the strong axis if the member bends with respect to the strong axis. For other loading conditions, the stiffness of the column will correspond with an intermediate value between both axes of curvature.

Consequently, the interpolation function δ depends on the relative eccentricity (η) and on the relative biaxial bending angle (β_d).

The interpolation function δ was obtained from least squares fit of the numerically simulated data:

$$\delta = \cos^2 \beta_d \cdot \frac{\eta}{\eta + 2} \tag{32}$$

where β_d is the relative biaxial bending angle. It is positive in counter-clockwise sense.

$$\beta_d = \text{atan}\left(\frac{M_{dv} \cdot i_{cv}}{M_{du} \cdot i_{cu}}\right) \tag{33}$$

M_{du}, M_{dv} are the first order bending moments with respect to the principal axes of the section "u" and "v" respectively (Figure 5.b):

$$\begin{aligned} M_{du} &= M_{dx} \cdot cos\,\theta_p + M_{dy} \cdot sin\,\theta_p \\ M_{dv} &= -M_{dx} \cdot sin\,\theta_p + M_{dy} \cdot cos\,\theta_p \end{aligned} \tag{34}$$

Equation 25 represents the behaviour of an unbraced column subjected to an axial load and both single and double curvature. Such a function takes into account the interaction between both flexural axes.

On the other hand, if the column is braced and is subjected to single curvature bending with an axial load, the equivalent moment of inertia (I_e) corresponding to its flexure axis (I_u or I_v) will be selected.

4. VERIFICATION OF THE PROPOSED METHOD

Because of the simplifications that were adopted, it becomes necessary to analyse the accuracy obtained using the proposed equation of the stiffness EI with respect both to the numerical simulation and to experimental results from the literature.

4.1. Verification with the numerical results.

The accuracy of the proposed equation EI in this paper can be evaluated using the ratio of the first order bending moments obtained with numerical simulation $(M_1)_{NS}$ and the proposed method $(M_1)_{method}$ (Figure 2). However, this procedure is not appropriate for cases subjected to the critical axial load, where this ratio tends to infinite. To overcome these inadequacies, the ratio ξ_{NS} is selected as reference to evaluate the accuracy.

$$\xi_{NS} = \frac{R_{NS}}{R_{method}} \tag{35}$$

where: R_{NS} $= \sqrt{\left(N_i/(N_{uc})_{NS}\right)^2 + \left((M_1)_{NS}/(M_{u,máx})_{NS}\right)^2}$

R_{method} $= \sqrt{\left(N_i/(N_{uc})_{NS}\right)^2 + \left((M_1)_{method}/(M_{u,máx})_{NS}\right)^2}$

$(N_{uc})_{NS}$ critical axial load of the section in simple compression

$(M_{u,max})_{NS}$ bending maximum capacity of the cross section obtained from the numerical simulation (Figure 2)

Table 6 and Table 7 show the accuracy with respect to the numerical simulation in terms of the type of load, cross-section and curvature. The same cases used to infer the proposed EI equation (Table 5) were used for the verification. It can be seen that the average for all the experiments is 1.09 with a variation coefficient of 0.14. The same accuracy is observed for the different types of load, cross-section and curvature. It can be noticed that the proposed method adjusts accurately to the numerical results.

4.2. Verification with experimental results.

To evaluate the accuracy (ξ) with respect to the experimental results, the following strength ratio was adopted:

$$\xi = \frac{N_{test}}{N_{method}} \tag{36}$$

where: N_{test} maximum experimental axial load

N_{method} maximum axial load using the proposed method

The proposed equation (equation 17) was compared with the same 613 experimental tests from the literature ([3], [20]-[42]) that were used to validate the numerical model (section 2). If ξ (equation 36) has a value greater than one, the proposed method is on the safe side. Table 2 shows the range of variation of the parameters studied in the experimental results.

To calculate the ultimate bending moments of the cross-section, the parabola-rectangle diagram for concrete under compression defined in the EC-2 (2004) code [2] was applied (clause 3.1.7 from Euro Code 2 (2004) [2]). The concrete strength (f_c) in each experimental test was taken as the value of the mean compressive strength in order to calculate the elastic concrete modulus E_c (equation 7).

Table 8 lists the authors that performed the experimental tests, as well as the accuracy degree ξ of the proposed method both for short-term and sustained loads (average ratio, variation coefficient, percentile 5% and 95%). The evaluation of the method independently of the type of load and type of cross-section has also been included in this table. It can be seen that an average ratio for short-term loads of 1.10 with a variation coefficient of 0.14 was obtained. For sustained loads, an average ratio of 1.11 with a variation coefficient of 0.12 was obtained. Finally, for all the experiments, an average ratio of 1.10 with a corresponding coefficient of variation of 0.15 was obtained. Table 8 shows that the accuracy is slightly better for rectangular sections than for non-rectangular sections.

Figure 6 shows the ratio distribution ξ and its trend line in terms of the most important parameters. The accuracy degree is analyzed with the same reference variables that the selected for the comparison with the numerical results.

For all the graphs, the trend line is placed in a position of ξ that is slightly higher than one, the results lying on the safe side. Generally, the trend line seems to be decreasing, apart from the yielding stress of the steel (f_y) and the relative first order eccentricity (η), where the trend line seems to be increasing. Consequently, the proposed method detects the variation of such variables properly.

Finally, a comparison between the results from proposed EI equation and the methods proposed by the ACI-318(08) [1] and Euro Code-2) [2] was carried out in connection with the experimental results from the literature. Table 1 shows the E.I equations used in both design codes

The method from the ACI-318(08) code [1] suggests the use of the magnifier method for the design of unbraced columns. In order to take into account the second order effects, the following magnification factor is proposed:

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{N_d}{\phi \cdot N_{cr}}} \tag{37}$$

where:

- C_m is the coefficient for calculating the equivalent uniform bending moment. It is equal to one for the case of columns subjected to an equal bending moment at both ends causing symmetric single curvature bending.
- ϕ is the strength reduction factor. It is set to a value of one to perform this comparative analysis.
- $N_{cr} = \pi^2 \cdot EI/l_p^2$ where EI is the flexural stiffness of the column. The EI is calculated with equation 10.14 from the ACI-318 (08) [1]. The following expression is used to calculate the Young modulus of concrete: $E_c = 4700 \cdot \sqrt{f_c}$ ("f_c" in MPa).

The first order bending moment is magnified in each direction independently for the case of biaxial bending.

The method that was proposed from the EC-2 code [2] suggests also the magnification factor in order to take into account the second order effects (Section 5.8.7.3. EC-2 (2004) code [2]):

The effective elastic modulus of the concrete section is obtained from:

$$E_{cd,eff} = E_{cm} / \{\gamma_{cE} \cdot (1 + \varphi_{eff})\} \tag{38}$$

γ_{cE} partial safety factor (equal to 1.2). For this comparative study, it has a fixed value of 1.

E_{cm} concrete secant elastic modulus:

$$E_c = 22000 \cdot (f_{cm}/10)^{0.3} \quad (f_{cm} \text{ en MPa}) \tag{39}$$

f_{cm} mean value of concrete cylinder compressive strength. In this analysis, it is equal to the strength of concrete (f_c) for each experiment.

φ_{eff} equivalent creep coefficient:

$$\varphi_{eff} = \kappa \cdot \varphi \tag{40}$$

φ creep coefficient

κ ratio between the quasi-permanent and the total load

If the column is subjected to axial loads and biaxial bending, then equation 1 is applied.

To compute the ultimate bending moment of the section in the ACI-318(08) code [1] the equivalent rectangular concrete stress distribution was used. While the parabola-rectangle diagram for concrete under compression was used for the code EC-2 (2004) code [2].

Table 9 and Table 10 show a comparison between the results from the proposed method and the methods from the ACI-318 (08) code [1]and the EC-2 code [2], with respect to the experimental results. In general, the proposed method achieves an average ratio that is closer to one, the lowest variation coefficient, and it presents an essential improvement for sustained loads, mainly with regard to biaxial bending. It is important to observe that the method proposed by the ACI-318(08) code [1] appears to be more conservative.

Regarding the results obtained for non-rectangular section, the proposed method presents a better accuracy degree than the design codes, that is, lower variation coefficient, higher 5% percentile, lower 95% percentile and an average value close to 1.10.

If the 5 % percentile of the proposed method is compared with the design codes, it can be observed that it is higher for short-term loads and lower for sustained loads. The accuracy degree of the codes is different for short-term loads from for sustained loads, being more conservative for the last sustained loads (mainly in biaxial bending). However, with the proposed method the value of the 5% percentile is 0.9 for almost all types of curvatures.

5. **EXAMPLE**

In order to illustrate the practical application of the proposed method, the longitudinal reinforcement of an unbraced column is calculated. The column has a buckling length of 5 meters and it is subjected to constant forces along the length of the element corresponding to the ultimate limit state for the permanent or variable state.

These are N_d = 1000 kN, M_{dx} = 24 kN.m and M_{dy} = 40 kN.m with respect to the centre of gravity of the gross section. The cross-section is presented in Figure 7. The mechanical properties of the materials are f_{ck} = 30 MPa and f_{yk} = 500 MPa. The creep coefficient (φ) is equal to 2 and the ratio between the quasi-permanent and the total axial load *(N_{sg}/N_{tot})* is equal to 0.6.

The size of the reinforcement is obtained by following the steps explained in sections 1 and 3 using the basic hypothesis from the EC-2 (2004) code [2] to compute the ultimate bending moments.

Initially, the following parameters are computed:

$$f_{cd} = f_{ck} / \gamma_c = 30 / 1.5 = 20 \; MPa$$
$$f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 434.78 \; MPa$$
$$\varphi_{eff} = \left(N_{sg} / N_{tot} \right) \varphi = 0.6 \cdot 2 = 1.2$$

The flexural stiffness of the column *EI* is obtained using equation 17, for which the following computations must be performed:

-Moment of inertia (I_{cx}, I_{cy}) and product of inertia (I_{cxy}) of the gross section in m^4 with respect of its centre of gravity:

$$I_{cx} = I_{cy} = 0.001466$$
$$I_{cxy} = -5.33333 \cdot 10^{-4}$$

- Angle of the principal strong axis of inertia (θ_p) with respect to the x-axis. It is positive in the counter-clockwise sense (eq. 26).

$$I_{cx} = I_{cy} \quad \Rightarrow \quad \theta_p = \pi/4 \quad rad.$$

- Principal moments of inertia of the gross section (I_{cu}, I_{cv}) in m^4 with respect of its centre of gravity: (eq. 29):

$$I_{cu} = 0.002; \quad I_{cv} = 9.\widehat{3} \cdot 10^{-4}$$

- Radius of gyration of the concrete section with respect to the principal axes of inertia in m (eq. 30):

$$i_{cu} = 0.0881917; \quad i_{cv} = 0.129099$$

- Moment of inertia of the reinforcement bars (I_{sx}, I_{sy}) in m^4 with respect to the centre of gravity of the gross section. For the steel distribution presented in *Figure 7*, such a moment of inertia can be expressed in terms of the total area of reinforcement (A_s) in cm^2:

$$I_{sx} = I_{sy} = 0.01138 \cdot 10^{-4} A_s$$

- Moment of inertia of the reinforcement bars (I_{su}, I_{sv}) in m^4 with respect to the principal axes of inertia of the concrete section (u,v) in terms of the total area of reinforcement (A_s) in cm^2(eq. 31):

$$I_{su} = 0.015833 \cdot 10^{-4} \cdot A_s; \quad I_{cv} = 0.006944 \cdot 10^{-4} \cdot A_s$$

- Design bending moments with respect to the principal axes of inertia of the concrete section (eq. 34):

$$M_{du} = 45.25\ kN.m; \quad M_{dv} = 11.31\ kN.m$$

- Critical radius of gyration of the concrete section (eq. 22):

$$M_{dv} \neq 0 \quad \Rightarrow \quad i_c = i_{cu} = 0.0881917\ m$$

- Mechanical slenderness of the column (eq. 23):

$$\lambda_m = l_p / i_c = 56.69$$

- First order relative eccentricity (η) (eq. 19):

$$\eta = \frac{e_0}{4 \cdot i_c} = \frac{M_d}{N_d \cdot 4 \cdot i_c} = 0.1322$$

$$M_d = \sqrt{M_{dx}^2 + M_{dy}^2} = 46.65\ kN.m$$

- Relative biaxial bending moment (β_d) (eq. 33):

$$\beta_d = \tan^{-1}\left(\frac{M_{dv}\cdot i_{cv}}{M_{du}\cdot i_{cu}}\right) = 0.3508\ rad.$$

- Effective stiffness factor α (eq. 18) for $\eta < 0.2$:

$$\begin{aligned}\alpha &= (1.95 - 0.035\cdot\lambda_m - 0.25\cdot\varphi_{eff})\cdot(\eta - 0.2) + \left(f_{cd}/225 + 0.11\right) \nless 0.1 \\ &= (1.95 - 0.035\cdot 56.69 - 0.25\cdot 1.2)\cdot(0.1322 - 0.2) + \left(20/225 + 0.11\right) = 0.2215 \nless 0.1\end{aligned}$$

- Reduction factor of the stiffness of the reinforcement ξ_φ (eq. 24):

$$\xi_\varphi = 1.9\cdot\varphi_{eff}\cdot\exp(-\lambda_m/25) = 1.9\cdot 1.2\cdot\exp(-50.69/25) = 0.236$$

- Interpolation coefficient (eq.32):

$$\delta = \cos^2\beta_d\cdot\frac{\eta}{\eta+2} = 0.05469$$

- Equivalent moments of inertia inm^4 (ec.25):

$$\begin{aligned}&-\text{Gross Section} \quad I_{ce} = I_{cx}\cdot\delta + I_{cy}\cdot(1-\delta) = 9.9167\cdot 10^{-4} \\ &-\text{Reinforcement} \quad I_{se} = I_{sx}\cdot\delta + I_{sy}\cdot\cdot(1-\delta) = 0.007430\cdot 10^{-4}\ A_s\end{aligned}$$

- Secant concrete elastic modulus (E_{cd}) for design

The EC-2 (2004) code [2] adopts a value of 1.2 for the safety factor of the concrete elastic modulus (γ_{cE}). Moreover, if the real value of the mean concrete compressive strength is unknown (f_{cm}), it is computed using the following equation: $f_{cm}=f_{ck}+8$ (in MPa).

$$E_{cd} = E_c/\gamma_{cE} = 22000\cdot\left(f_{cm}/10\right)^{0.3}/1.2 = 27363.81\ MPa$$

- Elastic modulus of the longitudinal reinforcement:

$$E_s = 200000\ MPa$$

- Flexural stiffness of the column EI in $kN\cdot m^2$ (eq.17):

$$EI = \alpha\cdot\frac{E_{cd}\cdot I_{ce}}{1+\varphi_{eff}} + \frac{E_s\cdot I_{se}}{1+\xi_\varphi} = 2732.63 + 120.23\cdot A_s$$

The critical axial load (N_{cr}) in kN is equal to (eq.6):

$$N_{cr} = \frac{\pi^2 EI}{l_p^2} = 1078.80 + 47.46 \cdot A_s$$

The magnification factor is equal to (eq.5):

$$\delta_{ns} = \frac{1}{1 - \left(N_d / N_{cr}\right)} = \frac{1078.80 + 47.46 \cdot A_s}{78.80 + 47.46 \cdot A_s} \geq 1.0$$

The magnified bending moment in kN.m is equal to (eq.2):

$$M_d^* = \delta_{ns} \cdot M_d = \frac{50323.5 + 2214.10 \cdot A_s}{78.80 + 47.46 \cdot A_s} \nless M_d = 46.65\ kN.m$$

To determine the required longitudinal reinforcement, the design forces (N_d, M_d*) and the ultimate forces of the section (N_u, M_u) are matched and a non-linear system of two equations and two unknowns (A_s, x) is thus obtained. This system of equations can be solved by using the well-known "Regula Falsi" method. Figure 8 shows the variation of M_d^* and M_u in terms of A_s for the given axial load N_d. The intersection between both curves determines the required area of reinforcement A_s to be equal to 20.13 cm^2, which is equal to 12 rebars with a diameter ϕ=16 mm (24.12 cm^2).

$$\begin{aligned} N_d &= N_u(A_s, x) \\ M_d^*(A_s) &= M_u(A_s, x) \end{aligned}$$

6. CONCLUSIONS

This paper proposes a new equation to obtain the effective stiffness EI of slender reinforced concrete columns both for verification and design subjected to combined axial loads and biaxial bending that is valid for short-time and sustained loads, and for both normal and high strength concretes. The method is only valid for columns with equal effective buckling lengths in the two principal bending planes.

The new equation extends the proposed EI equation in the “Biaxial bending moment magnifier method” by Bonet et al [6], which was valid only for rectangular sections to sections with any shape of the cross-section.

Furthermore, a new EI equation under uniaxial bending and axial load valid for any type of cross-section is proposed.

The proposed formulation for biaxial bending is an extension of the general flexural stiffness equation EI for uniaxial bending obtained by calculating the equivalent moment of inertia of the gross section and the reinforcing bars.

Such formulation includes the existing interaction between both flexural axes and the case of the axial load and single curvature. The effect of braced structures is taken into account in the behaviour of the column subjected to an axial load and uniaxial bending with respect to the strong axis.

The method was compared with 613 experimental tests and it proved to be reasonably accurate for practical engineering design application.

A noticeable improvement in the prediction accuracy of column strength was achieved using the new flexural equation of EI when compared with the current equations of the ACI-318 (08) code [1] and the EC-2 (2004) code [2]. It is important to highlight that this improvement is more relevant for sustained loads and biaxial bending. For the case of single bending curvature and sustained loads, the average and variation coefficient are 15% and 75% lower than the Euro Code 2 [2] respectively. For biaxial bending and sustained loads the average obtained with the proposed method is 20% lower than the Euro Code 2 [2] being still conservative, while the variation coefficients are similar. Otherwise, the ACI-318 [1] is more conservative and has higher scattering tan the Euro Code 2 and the proposed method.

The equations proposed in this paper are more complex than the proposed by other authors or design codes; however, from the practical point of view its application is very easy with spreadsheets or small computer programs. A more economical design is obtained with a higher accuracy degree than with the actual design codes.

The method is useful for structures in buildings since it presents a high degree of accuracy for application in professional practice, such as checking reinforced concrete sections or in the design phase.

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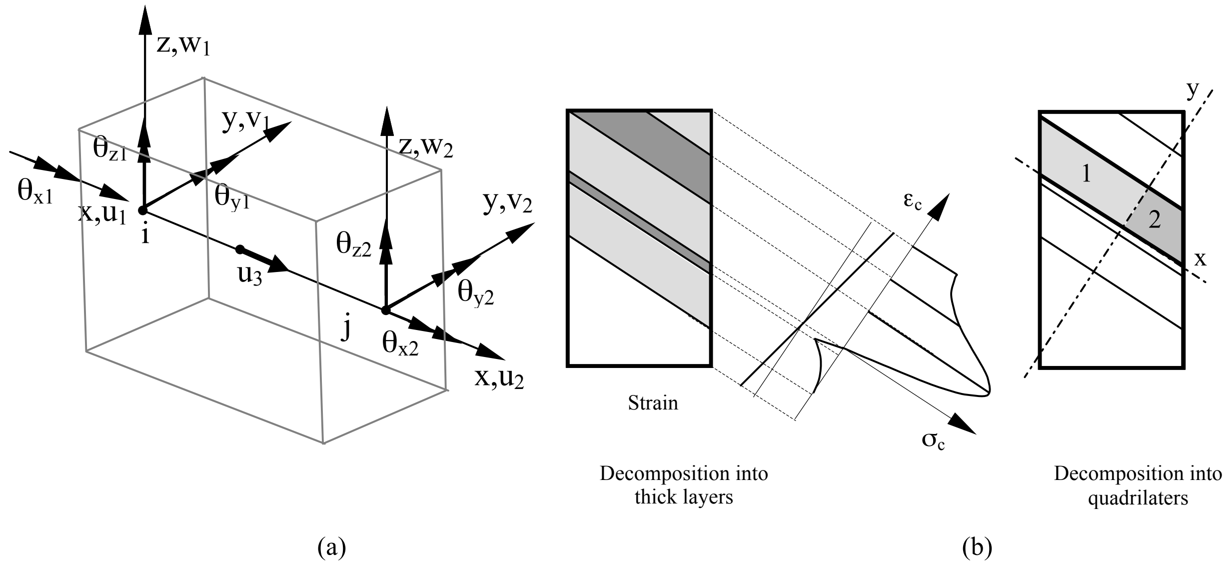


Figure 1. Finite element model: a) general arrangement, b) Cross section integration, Bonet et al. [14].

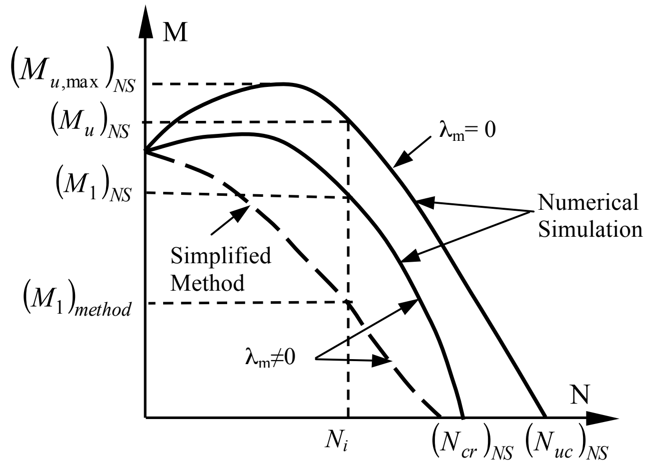


Figure 2. Magnifier bending moment method

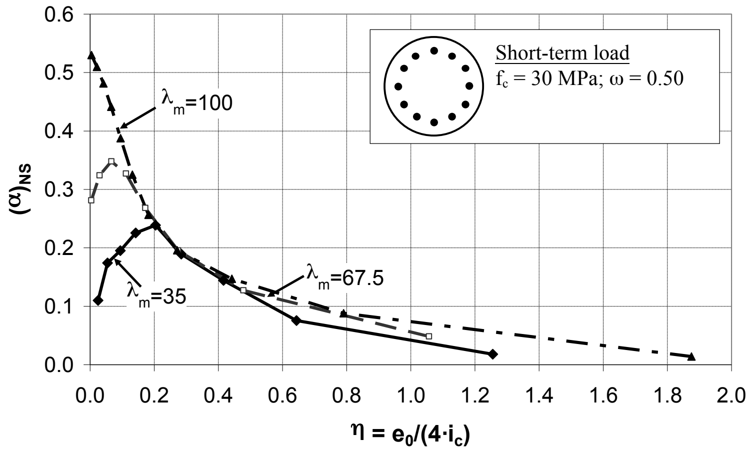


Figure 3. Effective stiffness factor α

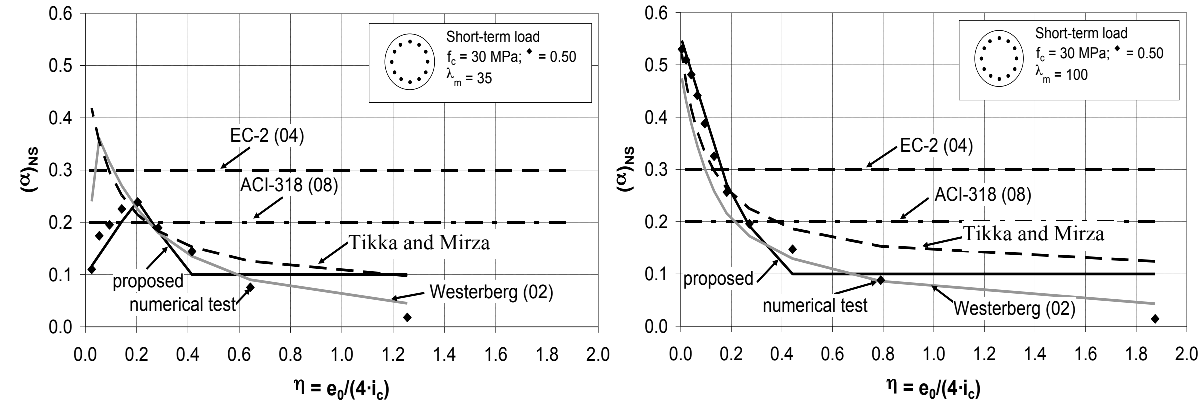
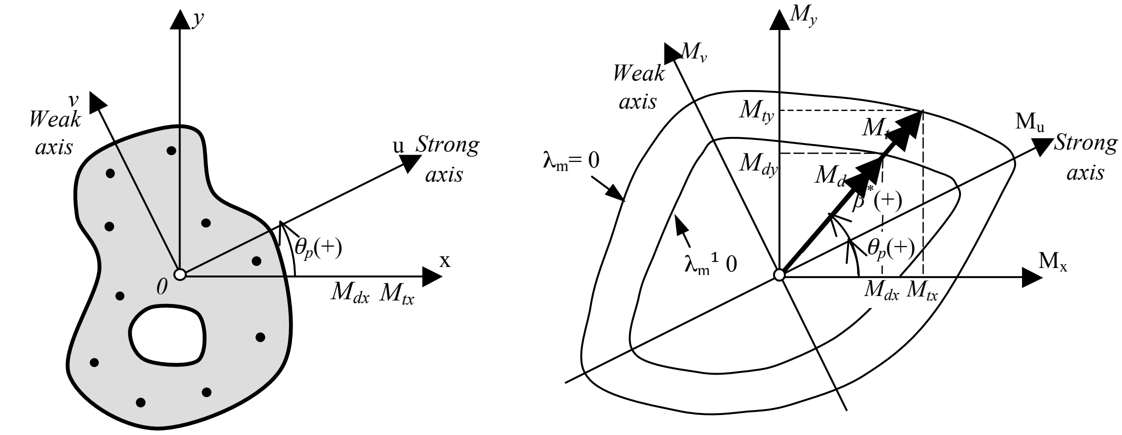


Figure 4. Comparison of "α"with different authors and design codes.



(a) Cross-section of the column *(b) Interaction diagram (M_x,M_y) for design axial load of N_d*

0 ≡ centre of gravity of the concrete gross section
x, y ≡ coordinate axes of the section
u, v ≡ principal axes of inertia of the concrete gross section

Figure 5. Proposed simplified method

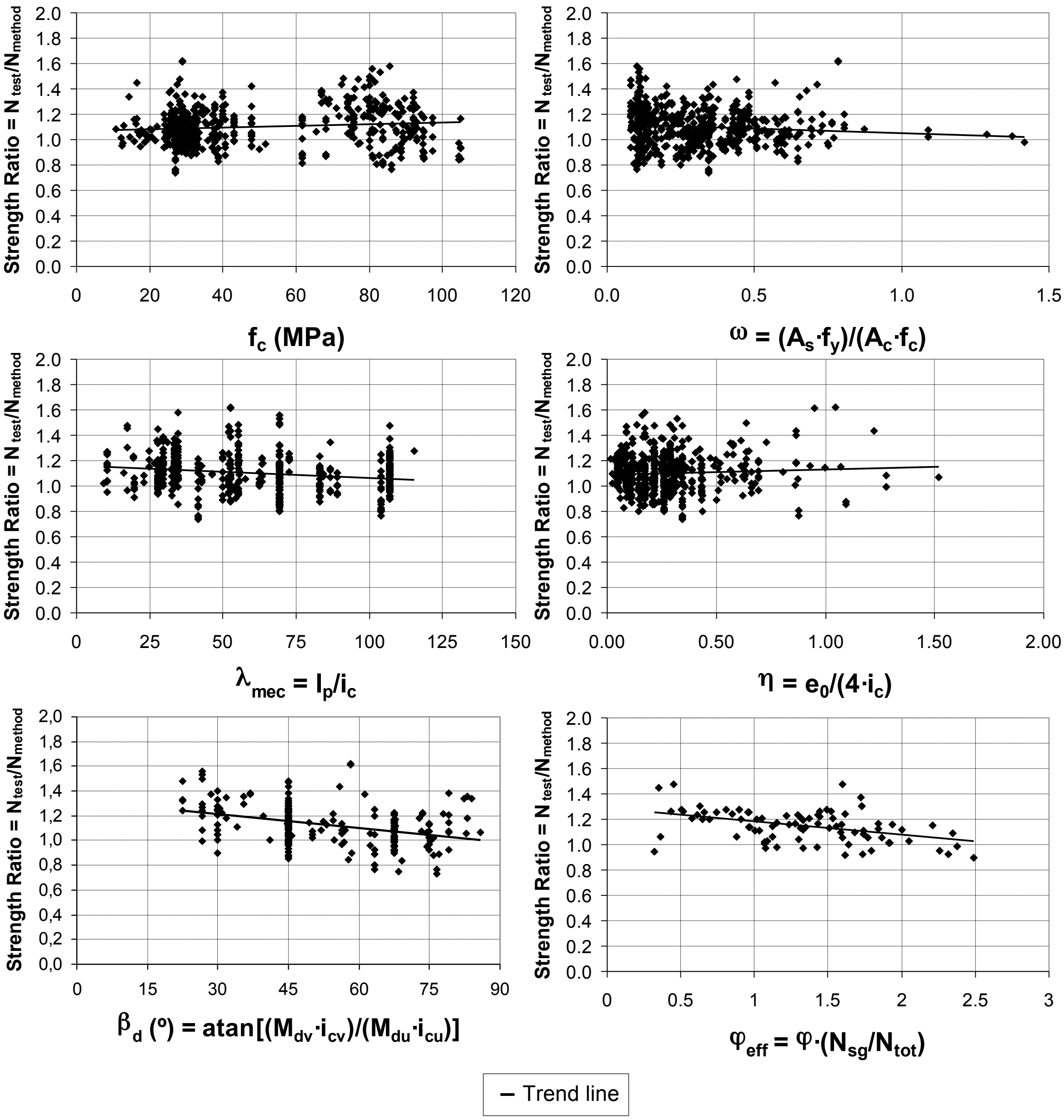


Figure 6. Comparison of the proposed method with the experimental results

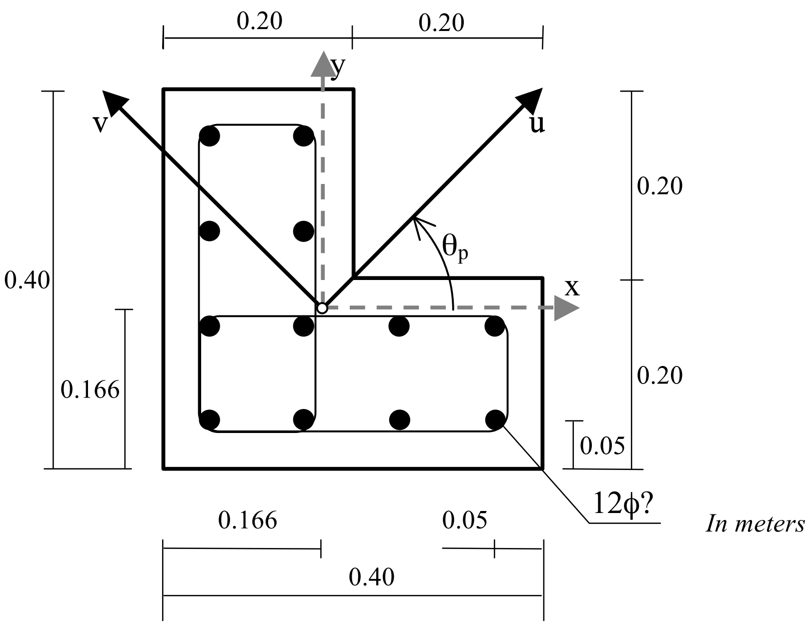


Figure 7. Example. Cross-section of the column

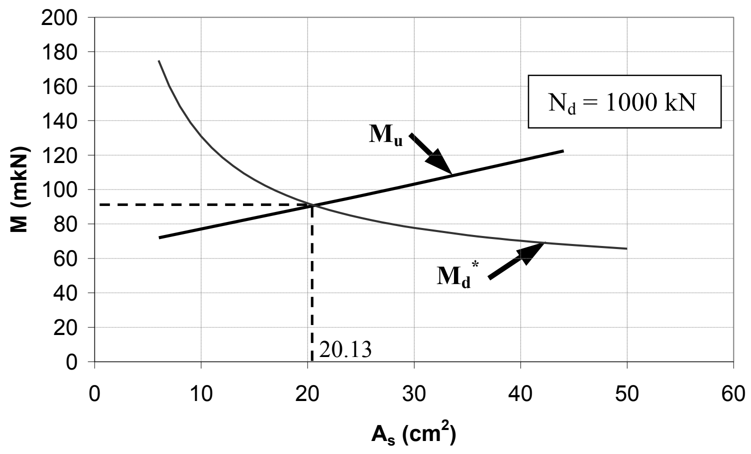


Figure 8. Example. Reinforcement ratio calculation

Table 1. Comparison of different E.I. equations.

Author	Parameter	Effective flexural stiffness *EI*
Mavichak and Furlong [3]	N/N_{uc}	$EI = \frac{0.2 \cdot E_c \cdot I_c + E_s \cdot I_s}{1.6 \cdot (1 - 2 \cdot N/N_{uc})}$
Mirza[4]	e/h; l/h; β_d	$EI = \frac{\alpha \cdot E_c \cdot I_c + E_s \cdot I_s}{1 + \beta_d}$ $\alpha = (0.27 + 0.003 \cdot (l/h) - 0.3 \cdot (e/h) \geq 0$ *or* $\alpha = (0.3 - 0.3 \cdot (e/h) \geq 0$
Westerberg [5]	ν; λ_m; φ_{eff}; f_c; ω	$EI = \alpha_\varphi \cdot \alpha_e \cdot E_c \cdot I_c + E_s \cdot I_s$ $\alpha_e = 0.08 \cdot \nu \cdot f_c^{0.6} \cdot e^{\frac{\lambda_m}{100} - 2 \cdot \omega} \leq (1 + \omega) - \nu$; $\alpha_\varphi = 1 - 0.8 \cdot \varphi_{eff} \cdot \left(1 - \frac{\lambda_m}{200}\right) \cdot \omega^{0.25}$
Bonet et al [6]	e/h; l/h; φ_{eff}; f_c	$EI = \alpha \cdot \frac{E_c}{1 + \varphi_{eff}} \cdot I_c + \frac{E_s}{1 + \eta} \cdot I_s$ *for* $e/h < 0.2$ $\alpha = (-0.14 \cdot \frac{l}{h} + 2.5 - 0.35 \cdot \varphi_{eff}) \cdot (e/h - 0.2) + \alpha_p \nless 0.1$ *for* $e/h \geq 0.2$ $\alpha = \alpha_p \cdot (1.2 - e/h) \nless 0.1$ $\alpha_p = \left(\frac{f_c}{200} + 0.12\right)$; $\eta = 1.9 \cdot \varphi_{eff} \cdot \exp\left(-0.1 \cdot \frac{l}{h}\right)$
Tikka and Mirza[7]	e/h; ρ_l; β_d	$EI = \frac{\alpha \cdot E_c \cdot I_c}{1 + \beta_d} + 0.8 \cdot E_s \cdot I_s$ $\alpha = \left(0.47 - 3.5 \cdot (e/h) \cdot \left(\frac{1}{1 + \beta \cdot (e/h)}\right)\right) \geq 0$ $\beta = 7$ *for* $\rho_l \leq 2\%$; $\beta = 8$ *for* $\rho_l > 2\%$
Tikka and Mirza[8]	e/h; l/h; ρ_l; β_d	$EI = \frac{\alpha \cdot E_c \cdot (I_c - I_s - I_{ss})}{1 + \beta_d} + 0.85 \cdot E_s \cdot (I_s + I_{ss})$ $\alpha = \left(0.48 - 3.5 \cdot (e/h) \cdot \left(\frac{1}{1 + \beta \cdot (e/h)}\right)\right) + 0.002 \cdot \frac{l}{h} \geq 0$ $\beta = 7$ *for* $\rho_l \leq 2\%$; $\beta = 8$ *for* $\rho_l > 2\%$ *and* $\beta = 9$ *for composite columns.*
ACI-318 (08) [1]	β_d	$EI = \frac{0.2 \cdot E_c \cdot I_c + E_s \cdot I_s}{1 + \beta_d}$ *or* $EI = \frac{0.4 \cdot E_c \cdot I_c}{1 + \beta_d}$
EC-2 code [2]	ν; λ_m; φ_{eff}; f_{ck}; ρ_l	$EI = K_c \cdot E_c \cdot I_c + K_s \cdot E_s \cdot I_s$ $\rho_l \geq 0.2\%$ $K_s = 1$ $K_c = k_1 \cdot k_2 / (1 + \varphi_{ef})$; $\rho_l \geq 1\%$ $K_s = 0$ $K_c = 0.3/(1 + 0.5 \cdot \varphi_{ef})$ $k_1 = \sqrt{\frac{f_{ck}}{20}}$; $k_2 = \nu \cdot \frac{\lambda_m}{170} \leq 0.2$

EI = flexural stiffness of compression member; E_c = modulus of elasticity of concrete; E_s = modulus of elasticity of reinforcement; I_c = moment of inertia of gross concrete section; I_s = moment of inertia of reinforcement; N = axial load; N_{uc} = maximum load capacity; e/h = eccentricity ratio; l/h = geometrical slenderness ratio; β_d = ratio of the maximum factored axial sustained load to the maximum factored axial load associated with the same load combination; ω = mechanical reinforcement ratio; ν = relative normal force; λ_m = mechanical slenderness ratio; φ_{eff} effective creep ratio; f_c = concrete strength; ρ_l = geometrical reinforcement ratio; f_{ck} = characteristic compressive cylinder strength of concrete at 28 days.

Table 2 . *Parameter variation in the experimental tests*

Parameter	**Range**
Compressive concrete strength [$f_{c.}$ (MPa)]	10.76 MPa – 104.84 MPa
Steel strength [f_y (MPa)]	298.55 MPa – 684 MPa
Mechanical reinforcement ratio [ω]	0.07 – 1.42
Geometrical reinforcement ratio [ρ_g]	0.01 – 0.05
Type of section	Rectangular / Square / "L"-shape / Box/ "C"-shape / Ovoid
Mechanical Slenderness [λ_m]	9.13 – 115.47
Ratio between the principal radii of gyration [i_{cv}/i_{cu}]	1 – 3
Relative eccentricity [$\eta = e_0/4/i_c$]	0.016 – 1.52
Relative axial load [ν]	0.04 – 1.25
Relative biaxial angle [β_d]	0 º – 90 º
Creep coefficient [φ]	0.32 – 2.90
Equivalent creep coefficient [φ_{eff}]	0.32 – 2.49
Ratio between the axial load from the permanent force and the axial load from the total [N_{sg}/N_{tot}]	0.42 – 1.00

Table 3 . Calibration of the numerical model. Analysis in terms of the shape of the section and the type of load.

	Section	N^{er}	ξ_m	V.C	P_5	P_{95}
Short-term Loads	R	468	1.06	0.13	0.87	1.31
	NR	49	1.09	0.10	0.89	1.24
	All	517	1.06	0.13	0.87	1.31
Sustained Loads	R	96	1.02	0.10	0.88	1.21
	NR	-	-	-	-	-
	All	96	1.02	0.10	0.88	1.21
Total	R	564	1.05	0.13	0.87	1.28
	NR	49	1.09	0.10	0.89	1.24
	All	**613**	**1.06**	**0.13**	**0.87**	**1.28**

R =Rectangular or Square; NR= Non- Rectangular

ξ_m: Average ratio; V.C.: variation coefficient; P_5: Percentile 5%; P_{95}:Percentile 95%;

Table 4 . Calibration of the numerical model. Analysis in terms of the type of load and type of curvature.

Type of curvature	**Type of load**	**Number of tests**	ξ_m	**V.C.**	P_5	P_{95}
Uniaxial	Short-term	313	1.06	0.14	0.86	1.36
	Sustained	60	0.99	0.08	0.88	1.14
Biaxial	Short-term	202	1.06	0.10	0.89	1.24
	Sustained	38	1.06	0.10	0.91	1.23

ξ_m: Average ratio; V.C.: variation coefficient;
P_5: Percentile 5%; P_{95}:Percentile 95%;

Table 5 . *Parameter variation.*

Parameters	Values
Column mechanical slenderness (λ_m)	• λ_m = 35, 52.5, 70, 87.5 and 100
Cross-section shape	• Rectangular /Circular /Hexagonal / Cross/ • Box / Ovoid / "L"-shape
Ratio between the principal radius of gyration [i_{cv}/i_{cu}]	• i_{cv}/i_{cu} = 1 a 2.5
Biaxial bending angle (β_d) with respect to the strong axis	– Circular β_d = 0º – "L" -shape with one axis of symmetry β_d = -60º, -30º, 0º, 30º, 60º, 90º – "L"-shape without any axis od symmetry – β_d = 0º, 45º, 90º, 225º, 270º, 315º – Other types of sections: – *βd = 0º, 22.5º, 45º, 67.5º, 90º*
Reinforcement distribution	– Circular: Uniformly distributed (with 6 and 12 bars) – Cross, hexagonal, "L", Box In the corners – Ovoid Uniformly distributed – Rectangular Doubly symmetric at four corners. Doubly symmetric and uniformly distributed at four faces Symmetric at opposite faces
Structural typology	• Isolated element with pinned ends.
Axial load	• 10 values for equivalent steps, starting from a zero axial load to the ultimate capacity for pure compression.
Compressive concrete strength (f_c)	• f_c = 30 MPa, 80 MPa and 100 MPa
Steel strength (f_y)	• f_y = 400 MPa and 500 MPa
Mechanical reinforcement ratio (ω)	• ω = 0.06, 0.25, 0.50, 0.75
Creep coefficient (φ)	• φ = 1, 2, 3

Table 6 . Verification of the proposed EI equation with respect to the numerical simulation. Analysis in terms of the cross-section type and the type of load.

	Short-term loads					**Sustained loads**					**Total**				
Section	**Number**	**$\xi_{NS,m}$**	**V.C**	**P_5**	**P_{95}**	**Number**	**$\xi_{NS,m}$**	**V.C**	**P_5**	**P_{95}**	**Number**	**$\xi_{NS,m}$**	**V.C**	**P_5**	**P_{95}**
Rectangular	834	1.10	0.14	0.97	1.37	1583	1.11	0.20	0.90	1.44	2417	1.10	0.19	0.93	1.41
Circular	294	1.02	0.07	0.92	1.15	261	1.04	0.09	0.90	1.18	555	1.03	0.16	0.88	1.26
Cross-shape	242	1.00	0.07	0.88	1.14	218	1.01	0.12	0.82	1.22	460	1.01	0.10	0.84	1.17
Hexagonal	200	1.09	0.11	0.99	1.30	216	1.03	0.07	0.92	1.18	416	1.06	0.10	0.93	1.26
Hollow Rectangular	749	1.09	0.17	0.92	1.33	707	1.14	0.15	0.95	1.43	1456	1.10	0.19	0.91	1.39
Ovoidal	798	1.09	0.12	0.94	1.37	397	1.13	0.15	0.92	1.46	1195	1.10	0.18	0.91	1.40
L-section	522	1.09	0.13	0.95	1.35	346	1.10	0.16	0.91	1.43	868	1.09	0.17	0.92	1.38
All	**3639**	**1.08**	**0.13**	**0.94**	**1.33**	**3728**	**1.10**	**0.14**	**0.91**	**1.40**	**7367**	**1.09**	**0.14**	**0.92**	**1.36**

$\xi_{NS,m}$:: Average ratio; V.C.: variation coefficient; P_5: Percentile 5%; P_{95}:Percentile 95%;

Table 7 . Verification of the proposed EI equation with respect to the numerical simulation. Analysis in terms of the type of load and type of curvature.

Type of curvature	Type of load	Number	$\xi_{NS,m}$	V.C	P_5	P_{95}
Uniaxial	Short-term	2105	1.05	0.12	0.92	1.25
	Sustained	2347	1.08	0.16	0.90	1.36
Biaxial	Short-term	1534	1.11	0.12	0.98	1.37
	Sustained	1381	1.13	0.15	0.90	1.45
All		7367	1.09	0.14	0.92	1.36

$\xi_{NS,m}$:: *Average ratio; V.C.: variation coefficient;* P_5: *Percentile 5%;* P_{95}:*Percentile 95%;*

Table 8 . Authors and accuracy of the experimental tests.

		Short-term loads					Sustained load					Total				
Author	**Section**	**N^{er}**	**ξ_m**	**V.C**	**P_5**	**P_{95}**	**N^{er}**	**ξ_m**	**V.C**	**P_5**	**P_{95}**	**N^{er}**	**ξ_m**	**V.C**	**P_5**	**P_{95}**
Galano et al (2008) [20]	S	60	1.00	0.13	0.85	1.21	-	-	-	-	-	60	1.00	0.13	0.85	1.21
Pallarés et al (2006) [21]	R	56	1.16	0.19	0.82	1.52	-	-	-	-	-	56	1.16	0.19	0.82	1.52
Germain (2005) [22]	S	12	1.15	0.06	1.07	1.24	-	-	-	-	-	12	1.15	0.06	1.07	1.24
Sarker et al (2000) [23]	S	12	1.23	0.11	1.04	1.38	-	-	-	-	-	12	1.23	0.11	1.04	1.38
Kim et al (2000) [24]	S	30	1.03	0.14	0.81	1.27	-	-	-	-	-	30	1.03	0.14	0.81	1.27
Claeson et al (2000) [25]	S	4	1.02	0.07	0.93	1.06	2	1.1	0	(*)	(*)	6	1.06	0.08	0.94	1.14
Claeson et al (1998) [26]	S	12	1.03	0.12	0.85	1.25	-	-	-	-	-	12	1.03	0.12	0.85	1.25
Foster et al (1997) [27]	S	54	1.18	0.09	1.03	1.34	-	-	-	-	-	54	1.18	0.09	1.03	1.34
Lloyd et al (1996) [28]	S / R	36	1.15	0.12	0.95	1.43	-	-	-	-	-	36	1.15	0.12	0.95	1.43
Taylor et al (1995) [29]	Box	30	1.03	0.14	0.81	1.27	-	-	-	-	-	30	1.03	0.14	0.81	1.27
Kim et al (1995) [30]	S	14	1.31	0.07	1.18	1.45	-	-	-	-	-	14	1.31	0.07	1.18	1.45
Hsu et al (1995) [31]	S	3	1.02	0.08	(*)	(*)	-	-	-	-	-	3	1.02	0.08	(*)	(*)
Tsao et al (1994) [32]	S	6	1.04	0.11	0.91	1.18	-	-	-	-	-	6	1.04	0.11	0.91	1.18
Tsao et al (1994) [32]	L-section	7	1.14	0.07	1.05	1.22	-	-	-	-	-	7	1.14	0.07	1.05	1.22
Wang et al (1992) [33]	S	8	1.12	0.10	1.04	1.25	-	-	-	-	-	8	1.12	0.10	1.04	1.25
Hsu (1987) [34]	C-section	11	1.05	0.07	0.94	1.15	-	-	-	-	-	11	1.05	0.07	0.94	1.15
Iwai et al (1986) [35]	S / R	36	1.03	0.09	0.92	1.16	-	-	-	-	-	36	1.03	0.09	0.92	1.16
Hsu (1985) [36]	L-section	9	1.28	0.23	1.00	1.62	-	-	-	-	-	9	1.28	0.23	1.00	1.62
Poston et al (1985) [37]	Box	4	1.30	0.06	1.20	1.36	-	-	-	-	-	4	1.30	0.06	1.20	1.36
Wu et al (1977) [38]	S	11	1.02	0.06	0.93	1.09	17	1.3	0.1	1.2	1.3	28	1.16	0.12	0.97	1.30
Mavichak et al (1976) [3]	R	9	1.07	0.13	0.90	1.25	-	-	-	-	-	9	1.07	0.13	0.90	1.25
Mavichak et al (1976) [3]	0	15	1.07	0.13	0.85	1.23	-	-	-	-	-	15	1.07	0.13	0.85	1.23
Drysdale et al (1971)[39]	S	27	1.09	0.06	0.98	1.17	30	1.07	0.11	0.92	1.23	57	1.08	0.09	0.92	1.22
Goyal et al (1971) [40]	S	26	0.94	0.04	0.89	1.00	20	1.1	0.1	1	1.2	46	1.05	0.10	0.91	1.19
Breen et al (1969) [41]	R	10	1.12	0.10	1.04	1.25	-	-	-	-	-	10	1.12	0.10	1.04	1.25
Viest et al (1956) [42]	S	15	1.1	0.1	0.9	1.2	27	1.1	0.1	0.9	1.2	42	1.07	0.11	0.9	1.21

Total	R / S	415	1.10	0.14	0.85	1.36	96	1.11	0.12	0.94	1.29	434	1.10	0.15	0.87	1.35
	NR	102	1.13	0.15	0.90	1.42	-	-	-	-	-	179	1.13	0.15	0.90	1.42
	All	**517**	**1.10**	**0.14**	**0.86**	**1.37**	**96**	**1.11**	**0.12**	**0.94**	**1.29**	**613**	**1.10**	**0.15**	**0.87**	**1.37**

Type of section: S = square; R =Rectangular; O = Ovoid; NR= Non-rectangular
ξ_m: Average ratio; V.C.: variation coefficient; P_5: Percentile 5%; P_{95}:Percentile 95%;

Table 9 . Comparative study between the proposed method and the methods suggested by the ACI-318 (08) code [1] and the EC-2 (2004) code [2], with respect to the experimental tests. Analysis in terms of the cross-section type and the type of load.

	Section	Short- term loads					Sustained loads					Total				
		N^{er}	ξ_m	V.C	P_5	P_{95}	N^{er}	ξ_m	V.C	P_5	P_{95}	N^{er}	ξ_m	V.C	P_5	P_{95}
Proposed EI	R	468	1.10	0.14	0.85	1.36	96	1.11	0.12	0.94	1.29	564	1.10	0.15	0.87	1.35
	NR	49	1.13	0.15	0.90	1.42	-	-	-	-	-	49	1.13	0.15	0.90	1.42
	All	517	1.10	0.14	0.86	1.37	96	1.11	0.12	0.94	1.29	**613**	**1.10**	**0.15**	**0.87**	**1.37**
EC2 (2004) [2]	R	468	1.17	0.25	0.74	1.63	96	1.26	0.21	0.97	1.57	564	1.19	0.28	0.79	1.59
	NR	49	1.22	0.24	0.83	1.78	-	-	-	-	-	49	1.22	0.24	0.83	1.78
	All	517	1.17	0.25	0.76	1.66	96	1.26	0.21	0.97	1.57	**613**	**1.19**	**0.28**	**0.80**	**1.64**
ACI-318 (08) [1]	R	468	1.21	0.29	0.78	2.00	96	1.32	0.25	0.98	1.69	564	1.24	0.34	0.83	1.93
	NR	49	1.08	0.23	0.73	1.64	-	-	-	-	-	49	1.08	0.23	0.73	1.64
	All	517	1.20	0.29	0.77	1.97	96	1.32	0.25	0.98	1.69	**613**	**1.22**	**0.34**	**0.79**	**1.87**

R =Rectangular or Square; NR= Non- Rectangular

ξ_m*: Average ratio; V.C.: variation coefficient;* P_5*: Percentile 5%;* P_{95}*:Percentile 95%;*

Table 10 . Comparative study between the proposed method and the methods suggested by the ACI-318 (08) code [1] and the EC-2 (2004) code [2], with respect to the experimental tests. Analysis in terms of the type of load and type of curvature.

	Type of Curvature	**Type of load**	**N^{er}**	**ξ_m**	**V.C**	**P_5**	**P_{95}**
Proposed EI	Uniaxial	Short-term	313	1.09	0.14	0.86	1.33
		Sustained	60	1.13	0.10	0.98	1.30
	Biaxial	Short-term	202	1.11	0.15	0.85	1.40
		Sustained	38	1.17	0.12	0.92	1.30
EC2 (2004) [2]	Uniaxial	Short-term	313	1.18	0.28	0.73	1.79
		Sustained	60	1.26	0.15	1.00	1.56
	Biaxial	Short-term	202	1.17	0.17	0.89	1.48
		Sustained	38	1.40	0.12	1.08	1.59
ACI-318 (08) [1]	Uniaxial	Short-term	313	1.12	0.25	0.73	1.64
		Sustained	60	1.27	0.17	1.00	1.61
	Biaxial	Short-term	202	1.34	0.30	0.92	2.20
		Sustained	38	1.49	0.09	1.26	1.69

ξ_m: Average ratio; V.C.: variation coefficient; P_5: Percentile 5%; P_{95}:Percentile 95%;